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ANNEX TO

A PRACTICAL APPROACH TO PROBABILITY IN THE CONTEXT OF A SCIENCE FAIR

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To article

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SCIENCE FAIR OF THE SCIENCE MUSEUM UNIVERSUM

To article
Top

In this paper we are dealing particularly with the content of a project developed for the science fair "Feria de las Ciencias del Museo de las Ciencias Universum" (Science Fair of the Science Museum UNIVERSUM), which is an already established yearly event for high school students in Mexico. In this project, an existing game dealing with concepts of conditional probability and Bayes' theorem was used, modifying the game's rules to fit into a sequence.

In our experience, working with students with a project for a science fair starts by encouraging them to participate in such an event. Sometimes that could not be easy, since the science fair is an extra-curricular activity, which means that they would not get a bonus for their grade, and students tend to work only if extra points could be gained, as it has been reported frequently. So, the teacher has to offer some ideas that may be challenging and motivating for the pupils in such a way that they get "trapped" and deeply involved with the project.

Once the students have committed on working and/or researching with some topic, the teacher must provide certain guidelines to follow, based on the objectives of the science fair; this means, following the guidelines of the fair itself. In the case of the science fair in which our students participated, the guidelines, disseminated with the projects' call describe the following conditions for the paper's structure:

- A front page, provided on registration
- Title
- Abstract
- Theoretical frame
- Problem
- Objectives
- Hypothesis (if any)
- Development
- Results
- Analysis and results interpretation
- Conclusions
- References.

Under this structure, the teacher has to guide the students through the process of fulfilling the document, and, at the same time, to control the experimental process, including collecting data, constructing devices for stating some knowledge or applying some knowledge, etc.

The intention of the science fair is to encourage participating students to become familiar with the type of work involved in scientific research by developing an investigative project and presenting their results to the public at the fair. The construction of suitable instruments for problem solving is also encouraged by the organizing board. In earlier years, projects at the science fair included research mainly from the disciplines of physics, chemistry, biology, health sciences, and related areas; more recently, however, this event has expanded to mathematics as well.

The participation process is organized in two stages: In the first phase the students have to present to the organization committee their research project plan, which should follow some pre-fixed guidelines. In the call for the science fair, it was established that any student interested should register a Project, and on a deadline, to submit the paper to a committee that evaluates whether the document can participate at the science fair. If the paper is accepted, the students would continue their investigations and participate at the science fair, presenting their research, findings, and devices they may have used or developed in their work to the audience and the to the evaluation committee.

The guidelines that appeared on the call poster of the science fair on November 2006, for those with a paper accepted to the fair, comprised the following rules (http://www.dgdc.unam.mx/casita/assets/files/conv_xv_feria.pdf):

- The best projects of each area, classification and category will pass the final stage of the contest. In the modality of documental research they will be selected only by category.
- To the authors and tutors of the accepted projects, which did not pass to the final stage, a certificate of participation will be handed over. In case that in some project a lack of guidance is detected, the tutors will not get any certificate.
- The expositors will set their exhibition at the pre-assigned place, on May the 4th (2007) from 8:00 to 10:00 o'clock. Surveillance of materials, instruments, and equipment presented will be the liability of the expositors.
- The Fair's opening will be on May the 4th at 10:00 o'clock. After that, during the day, the authors of the finalist works will have to expose them before an evaluation committee.
- The committee will be composed by teachers of the three high-school level sub-systems, teachers of college level, and prestigious researchers; their decisions will be incontrovertible.

- During the days of the exposition to the audience (Saturday and Sunday), at least one of the authors of the research has to be present at the stand in order to explain the work. Those who do not fulfil this requirement will be disqualified.
- The authors and tutors of the exposed works will get a certificate of participation in the final of the Science Fair. The three winners will get a diploma identifying also their rank. Depending on the quality of the works, the committee may give an honorific mention to one of them, to declare ties, or to declare empty places if they consider pertinent.
- The awards ceremony will be held on May the 6th at 18:00 o'clock, during the closing ceremony.

The project must be supported by a teacher. If the work is selected, then the students will present it at the science fair. The second stage involves the development of an exposition and the defence of their work before a board of selected researchers specialized in different areas of science followed by presenting the project to the general public in an open session. At the end, the best projects in each discipline receive an award.

This whole procedure involves the students in the real process of scientific research at a level, which is adequate to their age and knowledge. And if the teacher oversees the project development adequately, the student has a rich educational experience, enabling him or her to handle college-level work (where greater responsibility is placed on one's own learning) with better skills for acquiring knowledge. And this process also allows fulfilling the idea that learning occurs through hands-on interaction rather than through direct instruction in which the student passively receives knowledge (Cobb 1994, Mills 2003).

The students participating at the fair were from the same group of Statistics and Probability subject, one female and one male, María and Cuauhtémoc. They both were high school seniors, seventeen years old. None of them appeared along the course as gifted students, but both, and mainly he, were responsible and committed with the school work. They started their research early November 2006, while in the first semester of the course, so they had to study some themes either out of the scholar program for the two semesters or as part of the second semester's program.

Their grades at the first semester were 90% for Cuauhtémoc and 80% for María, while in the second semester, in which the fair was carried out, they both got 100% grades. Furthermore, since they started the second semester in Statistics and Probability with certain knowledge of their research, they were able to understand better and faster the topics taught in class that second

semester, they showed a higher sophistication in their probability reasoning than the rest of the group while answering the class' tasks.

And, since Bayes' theorem – a topic on which they had to work within their project – is not part of the educational program in statistics and probability at their school in neither one of the two semesters, they had gained a bonus of extra knowledge. The latest information about them is that Cuauhtémoc is studying a major in Psychology and María is going to major in Biology. A copy of the work they've developed for the fair is available on the Internet at http://www.cch.unam.mx/ssaa/naturales/pdf/bayesiana.pdf

This is of particular interest if we consider that the whole group of their course (of senior year of high school) had encountered the subject for the first time, with the exception of some very basic concepts acquired earlier.

For their participation in the science fair, the teacher first suggested some activities and lines of research to the students; they then decided to work with these suggestions in order to develop their project.

The importance of students' own reconstruction of the concepts in the learning process (cf. von Glasersfeld 1987) seems undisputable. However, this does not mean that following such ideas about an active role for learners guarantees complete and immediate easy-to-reach learning results; the students faced a lot of difficulties, the main of them being the calculations for the conditional probability tables, or the idea of the use of prior information, and the development of Bayes' theorem.

Such situations of learning difficulties establish a most delicate issue for the involved teacher; the students have the possibility of asking the teacher, but the development of the projects and their finding must be done by them, so the teacher just may help by giving some ideas, like "Why not trying this?" or "Have you checked those values in your table? Look carefully for the chances on the dice". The teacher's role diverts to a counsellor, he no longer is the prime and sole source for knowledge.

To article
Top

PROJECT DEVELOPMENT

To article
Top

For the students, their work started by choosing one out of three different projects proposed by the teacher. Earlier, the whole group was informed about the science fair, and just the two students which finally participated in the project were the only interested. At the beginning, they were asked to play and to study the device involved in the project they had chosen; the device being the game "Shut the box" with some twists in the rules.

It is worth mentioning here that a game was involved in every one of the three projects offered to the students, since a game is more attractive for the young people, and a game could be attractive for the audience at the fair as well; and it was not just a strategy for catching attention (what proved to be effective since the game is not known in Mexico, and the dice different from the six-faced dice are still something rare), but it was also a way of providing a concrete context for probability calculations.

After that first stage of observation, they were asked to find how probability could fit into the game. They explored some basic ideas, calculated some probabilities and at the end they noticed that working just with single events would be of some limitation, so, some steps further were taken, under the advice of the teacher. After some discussions, it became clear that working on the concept of conditional probability would be of major benefit for the project, since the probability of single events could be complemented in that way, so the students had to learn about the topic, with a proper guidance about what to research for, presenting their references to the teacher and relating the subject directly to the game.

Part of the teacher's guidance was given in a way of certain suggestions, like "Okay, you have noticed now that the possibilities with single events and their probabilities are narrowly limited. Why don't you search for some information about conditional probability?" Later, after the students researched about that topic and discussed some ideas and how these can be useful, a new guidance like "Now try to find something on Bayes' theorem or related issues. Let's see what you find and then figure out if it is actually useful and how it might be used". In such way, they constructed some tables and calculations in order to compare different strategies by their success probabilities.

They also noticed that the whole work done just for showing some behaviour on the first die would be nothing compared to trying to analyze questions related to the second tossing of the

die and further, so they decided to stop just with the analysis for the first toss. At that point, some open questions were discussed with the teacher; first of all, that there are more types of conditioning events than those already analysed by the students; second, that the first toss's result will be also a conditioning event.

For example, if the player gets a result at the first tossing that leads to one decision, and then gets the very same result at the second toss, then it could be that only one option remains for the player or it could even signify the end of the game; and third, that the decision made by the player at the first toss would also be a conditioning event, since there will be different results usable to close further boards at the second toss after the first decision. And then, their findings about Bayes' theorem would be applied more directly for their calculations.

After they finished their research, they started to report their findings according to the structure guidance for the particular science fair in which they were to participate.

During the science fair, the students stayed for two days showing their findings and confronting the audience with some of their ideas. Basically, the audience came to their stand asking about the project they have done, so the students presented some ideas involving decisions under randomness, presented their problem and the game, invited the audience to play, and then presented the ideas on conditional probability involved and how they had dealt with them during the development of the project.

The fair was held in the outer premises of a science museum that organized the fair, since there were more than one hundred projects, of different areas, participating. During the fair, the students had to stay at their stand all the time, at least one of them. They offered three games with the set of dice on a table, and a poster explaining the basic facts of their investigation, which should also serve to attract attention from the visitors.

The audience was composed by participant students' parents, the students participating with other projects, some teachers, and visitors to the museum. The dynamic at the fair was that the audience was free to come and go along the stands with the projects, and to stop at any stand for questioning the projects; it usually happened that the audience got to the stand asking for the students to demonstrate their devices and findings. In this way, the students started presenting the game and rules to the audience, letting them to choose the pair of dice to play with, and letting them play for a while.

After that, they started to talk about the game being an opportunity of using probabilities to make decisions, presenting some ideas about making decisions based on previous information

and presenting the developing of their research on probability, showing some calculations and tables, concluding with the importance they find in using previous information on making decisions under randomness and speaking about the tables as a way of deciding which pair of dice would be the most favourable to win the game and a certain strategy on closing the boards for the first toss; at this point they also spoke of their own experience on actually playing with the box.

Particularly, the explanations about conditional probability were accompanied by some examples, like the how "logic" it is that the probability of a car accident to happen increases if it starts to rain. Cross-tables were used as a mean of explaining the reasoning for stating the probability formula for A given B, just mentioning that this formula lead them to the understand Bayes' theorem. In some cases they avoided to explain to the audience the whole process of proving the theorem; in some other cases, when the audience was more involved, the students used their to demonstrated how conditional probabilities are used in a proof of the theorem.

After all the explanations and conclusions, which were of about ten to fifteen minutes, the students gave a copy of their paper to the audience member, pointing out that they were the only group working on such questions at the whole fair.

Most of the time, the audience member listened to the explanations without questioning them. There were some enquiries about the idea of dependence of events, or about the game (i.e. if it were actually possible to win?), or if and when the students actually use probability calculations to make their decisions.

To article
Top

QUESTIONS USED BY THE STUDENTS AT THE SCIENCE FAIR

To article
Top

At the Science fair, the students communicated the following problems to the audience:

First Stage: Prior Knowledge

The students discussed with the audience how subjective knowledge about everyday situations might influence the private judgement of the probability of specific events (accidents, being late etc) and the decisions as a consequence of the changed situation (make a detour, starting earlier etc).

> Every morning we leave home at a certain time to get to school or to our job on time; we know all well that, often, something unusual may happen and cause a delay on our way. We can ask the following:
>
> "How can randomness influence our lives? What decision would we make if we knew that there was a construction work on our way during our trip to school or job? Or, if we hear on the radio before leaving home that a car accident has just occurred precisely at some point of our usual route?"

> "In our example, the fact that a car accident happens on our route will make the probability of traffic jams more likely, or will reduce our probability of getting on time to our destination".

> "Is it possible to check out how specific events modify the probability of other events? That is, how can we assess whether two events can be named dependent or independent of each other?"

Second Stage: Conditional Probability

To discuss the concepts of dependent and independent events, and the concept of conditional probability, two-way tables like Table 5 were presented, in which events A and B were considered along with their complements, denoted as $\overline{A}$ and $\overline{B}$:

Table 5. Cardinality of events A and B from the same sampling space.

	A	$\overline{A}$	sum
B	$n(A \cap B)$	$n(\overline{A} \cap B)$	$n(B)$
$\overline{B}$	$n(A \cap \overline{B})$	$n(\overline{A} \cap \overline{B})$	$n(\overline{B})$
sum	$n(A)$	$n(\overline{A})$	N

"Based solely on this information, is it possible to calculate the probability of event A to happen, for instance? In which cases could you consider events A and B as dependent or independent?"

"If we consider now not only two events, but three or more, what would happen with the formula for conditional probabilities?"

Third Stage: Bayes' formula

The students tried to explain the formal procedure to revise probabilities in the light of new events or information. They imbedded this discussion in playing the board game "Shut the box" and used the results of the previous stages to discuss the relative merits of various strategies of the game. The audience could learn from playing the game that strategies based on probabilistic concepts are better than their intuitive approach.

Figure 1. The game Shut the Box – used as "Bayesian Box".

The reader is referred to the article for the details how the students coped with this issue.

See the article

POSTER USED BY THE STUDENTS AT THE SCIENCE FAIR

To article
Top

CAJA BAYESIANA

Objetivo

Revisar si la probabilidad condicional es aplicable dentro de la estrategia de un juego, a través del análisis bayesiano, con el fin de establecer la viabilidad de este enfoque en la toma de decisiones.

Desarrollo

En el presente trabajo se establece el uso del juego "Shut the box" o "Cierra la caja" como una vía para promover el razonamiento probabilístico. Se ha incluido una variante al juego, que es ofrecer al jugador en potencia la posibilidad de jugar con los dos dados de seis caras, con un dado de cuatro caras y uno de ocho, con un dado de diez caras y uno de dos, o con un solo dado de doce caras.

A partir de estas reglas, hemos aplicado la Definición Clásica de Probabilidad y el Teorema de Bayes para tratar de encontrar la mejor estrategia en el juego. Dada la complejidad en el desarrollo del juego basado en decisiones previas, solo realizamos el análisis del comportamiento del juego para el primer lanzamiento de los dados.

Resultados

El cálculo de las probabilidades de puntos por sumas o por caras resultó ser muy limitado, de manera que se requirió trabajar también con probabilidades condicionales. Todos los resultados debieron vaciarse en tablas y verificar su validez a partir de las propiedades de la probabilidad enunciadas por Laplace y sistematizadas por Kolmogorov. Igualmente recurrimos a la experimentación como una vía de validación, dado la complejidad que este estudio adquirió a medida que avanzamos en la búsqueda de resultados.

Hipótesis

Consideramos que en el juego con el que pretendemos simular la toma de decisiones es aplicable el cálculo de probabilidades condicionales y no solo el cálculo simple de probabilidades no condicionadas.

Thomas Bayes (1702–1761) fue un clérigo inglés, y estableció su teoría de probabilidad en 1764. Sus conclusiones fueron aceptadas por Laplace en 1781, redescubiertas por Condorcet, y que permanecieron sin ser cuestionadas hasta que Boole lo hizo. Desde entonces, las técnicas de Bayes han sido objeto de controversia.

Pierre-Simon Laplace (1749-1827) Matemático francés que, entre otros logros notables, dio un gran impulso a la teoría de la Probabilidad Matemática.

Conclusiones

El cálculo de probabilidades efectivamente es de ayuda en la toma de

XV Concurso Universitario Feria de las Ciencias

Mayo 4, 5 y 6 de 2007. Museo de las Ciencias Universum, Ciudad Universitaria, México.

Andrey Nikolaevich Kolmogorov (1903–1987). Matemático ruso y uno de los impulsores de la Teoría Moderna de la Probabilidad. Estableció los tres axiomas de la Probabilidad, llamados Axiomas de Kolmogorov.

decisiones, pero que no siempre es factible aplicarlo de manera directa, ya que la ocurrencia de algún determinado evento puede modificar la idea que teníamos en un principio respecto a la viabilidad de ocurrencia de otro evento y llevarnos eso a tomar una estrategia equivocada.

Teorema de Bayes

Sea $\{A_1, A_2, ..., A_k\}$ una partición del espacio muestral S, tales que $P(A_i) > 0$, $i = 1, 2, ..., k$. Entonces

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i)P(A_i)}$$

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To article
Top

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