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EXAMINING “MATHEMATICS FOR TEACHING” THROUGH AN ANALYSIS OF TEACHERS’ PERCEPTIONS OF STUDENT “LEARNING PATHS”¹

Donna Kotsopoulos
Susan Lavigne

ABSTRACT. How teachers think about student thinking informs the ways in which teachers teach. By examining teachers’ anticipation of student thinking we can begin to unpack the assumptions teachers make about teaching and learning. Using a “mathematics for teaching” framework, this research examines and compares the sorts of assumptions teachers make in relation to “student content knowledge” versus actual “learning paths” taken by students. Groups of teachers, who have advanced degrees in mathematics, education, and mathematics education, and tenth grade students engaged in a common mathematical task. Teachers were asked to model, in their completion of the task, possible learning paths students might take. Our findings suggest that teachers, in general, had difficulty anticipating student learning paths. Furthermore, this difficulty might be attributed to their significant “specialized content knowledge” of mathematics. We propose, through this work, that examining student learning paths may be a fruitful locus of inquiry for developing both pre-service and in-service teachers’ knowledge about mathematics for teaching.

KEYWORDS. Content Knowledge, Learning Paths, Mathematics for Teaching, Pedagogical Content Knowledge, Students, Teachers.

INTRODUCTION

Presently in mathematics education there is a growing interest concerning the kinds of knowledge mathematics teachers ought to know to teach mathematics effectively; it is known as “mathematics for teaching” (Adler & Davis, 2006; Deborah L Ball, Bass, Sleep, & Thames, 2005; Davis & Simmt, 2006). Adler and Davis (2006) explain that the theoretical foundation of mathematics for teaching is based upon the “epistemological assumption that . . . there is a specificity to the mathematics that teachers need to know and how to

¹ A synopsis of this research is to be presented at the Symposium on the Occasion of the 100th Anniversary of the International Commission on Mathematical Instruction (ICMI), Rome, 2008.

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use” (p. 271). Scholarship in the area of mathematics for teaching is a fairly recent research effort. Theorization about mathematics for teaching is an important endeavor in order to make sense of the many complexities involved in effective mathematics instruction. The resulting outcomes of such theorization can lead to important insights that would ultimately benefit students.

Ball, Bass, Sleep and Thames (2005) propose a framework that describes the knowledge associated with mathematics for teaching. The framework consists of four “distinct domains” (Deborah L Ball et al., 2005, p. 3): (1) common content knowledge (CCK) — the mathematical knowledge of the school curriculum, (2) specialized content knowledge (SCK) — the mathematical knowledge that teachers use in teaching that goes beyond the mathematics of the curriculum itself, (3) knowledge of students and content (KSC) - the intersection of knowledge about students and knowledge about mathematics, and (4) knowledge of teaching and content (KTC) - intersection of knowledge about teaching and knowledge about mathematics (p. 4).

This research specifically examines the relationship between Ball et al.’s (2005) second domain, specialized content knowledge (SCK), and the third domain, knowledge of students and content (KSC). The locus of this research rests on the third domain, KSC, as the central domain of analysis. The reason being, this particular domain is largely defined by students and student learning, which is the foremost interest of mathematics education. The authors explain that “KSC includes knowledge about common student conceptions and misconceptions, about what mathematics students find interesting or challenging, and about what students are likely to do with specific mathematics tasks” (Deborah L Ball et al., 2005, p. 3).

The third domain has another interesting feature. The other domains, although described as “distinct” by Ball et al. (2005), intersect, if not overlap, most closely within this particular domain. For example, within this domain, knowledge of content intersects with CCK, while the knowledge about students intersects with KTC. Also, domains CCK and KTC intersect within KSC, as does SCK in that teachers may or may not have advanced education in mathematics, which may impact all the other domains, and most particularly the third.

The intersections of the domains, particularly with KSC, are not surprising and are anticipated since the collective of the domains can be viewed as what mathematics teachers do when teaching mathematics. Increased knowledge in any of the domains can generally be
viewed as beneficial to the teaching and learning of mathematics, which is one of the central
goals of the mathematics for teaching movement. This having been said, in earlier research
with pre-service mathematics teachers, Ball (1989) found that teachers with advanced degrees
in mathematics (or a related field), or to use Ball et al.’s (2005) domains, high SCK, were not
necessarily any better at teaching mathematics. This finding resonated with us and we
wondered whether there were instances were SCK might actually interfere with KSC and the
development of the other domains.

As Ball (1989) herself demonstrated, teachers without sufficient SCK (or other
domains) are able to learn both pedagogy and content and become effective teachers of
mathematics, hence supporting the mathematics for teaching movement. Our research focus is
important in that it deviates from the general concerns over insufficient SCK amongst
teachers. Therefore, the research questions guiding this work are: (1) what are the
assumptions teachers with sufficient or high SCK make about student thinking (i.e., KSC) or
the “learning paths” that students take? And, (2) in analyzing such assumptions, what
conceptual and pedagogical insights might be mined to support knowledge development in
the other domains defined by Ball et al.? To explore these questions, pairs of students and
pairs of teachers were given a common mathematical task. Teachers were asked to model the
learning paths students might take.

LITERATURE REVIEW

Three bodies of literature inform this research. The foremost contributor shaping this
research is the growing body of scholarship known as “mathematics for teaching,” which was
introduced earlier, and will briefly elaborate upon. This scholarship suggests that there is a
complex, interrelated, and multi-faceted core knowledge required for teaching mathematics
that ought to inform how mathematics teacher education is conceived of and how ongoing
professional development amongst teachers occurs.

Adler and Davis (2006) suggest that the mathematics for teaching movement is a
relatively new way of thinking about mathematics education. However, the term
“mathematics for teaching” has been appropriated within scholarship by many and is
currently in vogue in terms of research imperatives within this field. However, the underlying
epistemology to the various appropriations of mathematics for teaching is fairly consistent
and generally agreed upon. Perhaps the most compelling and cohesive aspect of the
underlying epistemology, amongst those that theorize about mathematics for teaching, is the
view that teachers must know mathematics in such a way to be able to know how to use
mathematics to develop student understanding (Adler & Davis, 2006; D. Ball & Bass, 2001; Deborah Loewenberg Ball, 1989; Deborah L Ball et al., 2005; Ball Loewenberg, 2000; Davis & Simmt, 2006).

Although there is arguably a cohesive underlying epistemology to mathematics for teaching, some researchers, in their conceptualization, frame the movement in alternative ways. Davis and Simmt (2006) frame their conceptualization of mathematics for teaching through the “complexity science” lens. This lens views the relationship between teaching and learning as inherently nested, with learning as a collective endeavor. The authors propose alternative domains to those defined earlier in the theoretical framework proposed by Ball et al. (2005). For example, Davis and Simmt describe domains as “nested,” and offer different conceptualizations of the domains as follows: (1) mathematical objects, (2) curriculum structures, (3) classroom activity, and (4) subjective understanding. The different emphases between theorists may be attributed to differing loci of attention, or as a related function to the types of questions that particular researchers are exploring. For the purpose of this research, we follow the domains defined by Ball et al. (2005).

Another point of consensus is the underpinning of the movement to the theorizations of Lee S. Shulman (1986; 1987) – the second body of literature informing this research. Shulman was among the first to begin making distinctions between the types of knowledge needed for teaching in his conceptualization of Pedagogical Content Knowledge (PCK). Informed by earlier theorizations by Dewey (1969) and Bruner (1966) around teacher’s subject knowledge, according to Shulman PCK “goes beyond knowledge of subject matter . . . to the dimension of subject matter knowledge for teaching [author’s emphasis]” (Shulman, 1986, p. 9). He says emphatically that teachers must have “ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9). Yet, as some have argued, knowledge of subject content alone will not necessarily enable an individual to teach that knowledge to another (Deborah Loewenberg Ball, 1989; Sfard, 1997).

Ball et al. (2005), in their conceptualization of mathematics for teaching, attribute their proposed domains of KSC and KTC to Shulman (1986; 1987). Ball et al. state that KSC and KTC “are closest to what is often meant by ‘pedagogical content knowledge’ — the unique blend of knowledge of mathematics and its pedagogy” (p. 4). The importance of PCK for the education of mathematics teachers has been well document (e.g., Ball Loewenberg, 2000; Langrall, Thornton, Jones, & Malone, 1996). Likewise, the importance of enacting PCK for ongoing teacher practice has also been well documented (e.g., Lampert, 1990; Marks, 1990).

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3 For a more detailed description of “complexity science” see Davis and Simmt (2003).
The final body of literature informing this research considers teachers' beliefs in relation PCK, or in other words teachers' anticipation of student thinking (Feiman-Nemser & Parker, 1990; Murata & Fuson, 2006; Nesbitt Vacc & Bright, 1999). Beliefs and “anticipation,” for the purpose of this research are seen as mutually dependent occurrences given that the former shapes the later. Teachers’ beliefs often form the very basis of the decisions teachers make.

Some research has shown that largely through professional development, pre-service teacher education, alternative experiences, and so forth, teachers are able to shift early beliefs about teaching, learning, and mathematics that may be negative or situated in a more teacher-directed model (Nesbitt Vacc & Bright, 1999). However, more compelling are the claims that teachers, while seemingly open and motivated to shifting beliefs about teaching and mathematics, continue to be remarkably unchanged in terms of belief systems formed from early experiences (Cohen & Ball, 1990; Gadanidis & Namukasa, 2005; McDiarmid, 1990; Norton, McRobbie, & Cooper, 2000; Stipek, Givvin, Salmon, & MacGyvers, 2001). Despite motivation and openness to change in order to improve teaching and, most importantly, student learning, teachers often resort to what is familiar in terms of how and what to teach. This is despite, and often because of, curricular, standardized achievement, and pedagogical pressures.

This current research elaborates upon research that examines teacher belief systems in relation to PCK, with a particular focus on beliefs related to student thinking. The importance of teachers being able to anticipate student content knowledge based upon the students’ perspectives rather than teachers’ own content knowledge and appraisals of their own classroom teaching has been identified as a key yet under explored issue in the PCK literature (Feiman-Nemser & Parker, 1990; Murata & Fuson, 2006; Nesbitt Vacc & Bright, 1999).

Successful anticipation of student thinking might suggest that classroom practices have adequately met the needs of students and the goals of learning. Alternatively, unsuccessful anticipation of student thinking might reveal to teachers’ need to rethink the ways in which learning occurred in the classroom. Equally important is a teacher’s ability of to further accommodate student learning when there is a mismatch between the teacher’s and the students’ perspectives. From a research perspective, examining instances of mismatch may shed significant insight on those areas of mathematics for teaching that likely need increased attention, whether in pre-service teacher education or through professional development for in-service teachers.

Murata and Fuson (2006) propose that successful anticipation of student thinking need not be overly complex. In their research examining the thinking of Japanese first
graders, they argue that in relation to understanding students’ mathematical thinking “there
are not 20 to 35 different learning paths [authors’ emphasis] or strategies for teachers to
understand and assist” (p. 424). Rather, student thinking and learning can be isolated to a few
specific and predictable trajectories, or learning paths. The authors propose that:

for many mathematics topics, there are a few typical errors that stem from partial but
incomplete understandings and some other more random errors from momentary
lapses of attention or effort. Likewise, there are usually several solution methods, but
these are limited in number and vary in their sophistication, generalizability, and ease
of understanding. (p. 424)

Murata and Fuson make clear that these few predictable trajectories are not a closed
set, and that other trajectories are possible; hence, teachers must be open to these other
trajectories.

Notwithstanding Murata and Fuson’s (2006) clarification of learning paths as
possibly open sets, teachers routinely in their pedagogy routinely anticipate the predictable
learning paths of students (e.g., in lesson planning, assessment, etc.). We contend an
examination of the concurrences and contradictions between teachers’ anticipation of student
learning paths and actual learning paths of students is an important area of research in the
mathematics for teaching movement. Accordingly, this research contributes to the existing
research.

Examined in this research are teachers’ anticipation of students’ “learning paths” by
interrogating the intersection between Ball et al.’s (2005) conceptualization of SCK and KSC.
Through the context of a common mathematical task given to pairs of teachers with high SCK
and pairs of students, we examine the assumptions teachers make about student learning paths
when teachers are asked to complete a task as they would anticipate a student might complete
the task. More simply put, we asked teachers to develop a solution for the task akin to a
“solution set” used to evaluate student work. We examine the assumptions teachers make, and
consider the tensions between what teachers are thinking students ought to be thinking versus
what actually occurred in response to the common task. We discuss the various assumptions
teachers demonstrated, and the implications this evidence might have for the education of
mathematics teachers and mathematics for teaching.

METHODOLOGY

Data for this research was collected over the 2005-2006 school years at multiple sites.
Students, their parents, and teachers who participated in this research were provided with
information about the study, then consents to participate were secured, in accordance with the research policy requirements for the Board of Education in which the research took place.

Participants

In order to examine teachers’ anticipation of student learning paths, a common task, as the primary instrument of data analysis was administered to groups of tenth-grade students and a group of mathematics teachers. The students (n\text{total} = 51) were from two classes (n_1 = 25, n_2 = 25), each taught by one of the authors of this paper. The composition of students varied according to gender, ethnicity, and socio-economic status. Although, the school where the research was conducted, located in a large urban setting, was known to be of higher socio-economic standing largely based upon location and community demographics.

The course, in which the students were enrolled, was an “advanced” mathematics course, geared toward students who were anticipating post-secondary education. This having being said, the range of abilities within the classes varied. However, the majority of students achieved at least a Level 3- (72%) in this course.

We were the regular mathematics classroom teachers for these students. Aside from the collaboration on select tasks throughout the duration of the course, the actual methods used in each of our classrooms were independently decided upon. The course was provincially set with common curricular goals, assessment guidelines, and so forth. Approved textbooks, as determined by our Board of Education for this course, were largely narrowed to two – with one in particular being the most commonly used textbook in the province.

The mathematics teachers (n = 27) who participated in this research were all mathematic department heads from various secondary schools (n = 30), some urban and some rural, of one particular board of education. The data was gathered during a monthly organizational meeting where the department heads, in addition to discussing administrative and curricular issues, engaged in a professional development component. We had contacted the system-wide coordinator and requested permission to attend this session and to invite the teachers to participate in our research. In addition to the teachers’ “headship” responsibilities (i.e., course assignments, staffing, etc.), the teachers all taught mathematics.

All of the teachers except for one had greater than 10 years of teaching experience, with 19 of the teaches having more than 15 years of teaching experience. All of the teachers, except for one, had advanced teacher training and qualifications in mathematics education.\footnote{Each of the teachers, in addition to their university degrees and teaching degrees, also had additional qualifications in mathematics instruction, except for one teacher whose additional qualifications were in biology and chemistry.}
The post-secondary educational background for the teachers varied. Most of the teachers had Bachelors of Science degrees (n = 12). Many of the teachers had Bachelors of Mathematics (n = 7). One teacher held a degree in engineering. The remainder of the teachers had undefined Bachelors degrees (n = 7). These undefined degrees likely had a significant mathematical component given that the majority of the teachers also had advanced qualifications in mathematics education that required a minimum amount of post-secondary mathematics education. Consequently, we describe this group of teachers as having high SCK. This sample of teachers is a purposive effort to neutralize concerns over low SCK in relation to mathematics for teaching and to elaborate on contemporary research about teaching mathematics that largely focuses on teachers with low SCK.

The mathematical task that formed the primary artifact of analysis, describe in the next section, involved quadratic relations, which all of the teachers would have taught on multiple occasions. Although the mathematical task was explored in this study with tenth-grade students, quadratics appears in each of the subsequent grades of mathematics instruction in our province. Therefore, the teachers, in this research, likely taught this material on multiple occasions, in multiple courses, during the current school year, as well as during their extensive careers.

The task: What is water pressure?

The teachers and the students in this research completed a common mathematical task – What is water pressure? We developed the task as a final assessment for a quadratics unit. The unit itself spanned approximately four weeks.

The mathematical task involved modeling the water flow from the drinking taps in a school, where the projection of the water from the spout forms a parabolic arch. Pairs of teachers and pairs of students (n_teachers = 13, n_students = 23) were asked to determine various forms of the quadratic relations (e.g., standard form, factored form, and vertex form) that modeled the current flow of the particular tap they were investigating. Additionally, the pairs were asked to determine the quadratic relation representing an arbitrarily set ‘ideal’ water flow of 3 cm above the faucet guard at the fountain. The underlining impetus for determining the ideal water flow was based on notions of water conservation; that is, a reduced water flow is potentially more cost-effective in terms of overall water consumption.

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5 In the province of Ontario, most universities that grant additional qualifications in mathematics instruction require nine full undergraduate courses in mathematics or six full courses plus five full courses in another subject area.

6 See AUTHOR 1 and AUTHOR 2 (2007) for the complete task and assessment tool that was provided to the students and the teachers.
The task itself was open-ended given that each fountain had a differing flow-rate. The task for the students was a final summative assessment for a unit on quadratic relations. Teachers and students had 70 minutes (one class period) to complete the mathematical task. Teachers and students were randomly paired and assigned a fountain in either their school or the school in which the department heads’ meeting was taking place. Each pair was given a chart paper to record their full solution. The group as a whole read through the mathematical task. The assessment rubric was provided with the mathematical task and reviewed prior to beginning the inquiry.

Instructions to the teachers were modified. Teachers were informed of the nature of our inquiry (i.e., anticipation of student thinking). As well as reviewing the task and the assessment rubric, as was done with the students, teachers were given the additional instructions to complete the task as a “level 3” response that might be anticipated from a tenth-grade student. We also relayed to the teachers that students were not permitted to use graphing calculators on the task, but were permitted to use their own scientific calculators. Following the completion of the task, the teachers reconvened for a brief, focus group session to discuss the task, possible teaching and learning dilemmas, and their assumptions.

**Data collection and analysis**

The primary artefact of analysis was the actual solutions submitted by the teachers and students. However, researcher notes were also made of the focus group discussion. Researcher field notes were taken during the completion of the task for each group of participants as well, documenting other non-mathematical aspects of completing the task (i.e., time at the taps, returning to the taps after initially coming back to the classroom, etc.).

A content analysis, involving the coding and counting of features within the artifacts, was completed for all the solutions (Berg, 2004). A common coding structure was established by initially each independently completing a possible “answer key” for the task. From our individual solutions, we discussed features that might be important or noteworthy if missed or omitted. We recognized that the student and teacher solutions may evoke the development of additional codes or alternative learning paths, as suggested by Murata and Fuson (2006), thus we remained open to this during our coding. One particular pair of teachers’ solution did just this, which is outline in our results. Following the establishment of the common coding scheme, each author independently coded all the solutions from all the pairs of teachers and students. We then compared our coding with each other and contemplated differences and

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7 Level 3 represents the acceptable Ministry of Education standards of achievement (i.e., approximately 75% average) in mathematics.
omissions with a goal of reaching consensus. Upon agreement of the codes applied to the artifacts, the collective set of student data was compared against the teacher data by a calculation of means. Finally, we engaged in an analysis of the overall results – including means and artifacts.

We acknowledge that, as mathematics teachers and as teachers of the students in the research, our own beliefs also influenced the codes established in relation to our own anticipations of student learning paths. In our examination of the complete data set, we examine the teachers’ anticipation of student thinking, and unavoidably our own anticipation of student thinking as well. The codes used, therefore, emerged from general hypotheses we made about potential learning paths of students. In the results section, we detail each of these hypotheses as we report the findings from the coding in an effort to be explicit about the assumptions we made – both about student learning paths, and how teachers might anticipate learning paths in relation to these codes.

In total, there were seven codes used for the analysis. Evidence of an error, either in the solutions or the graphical representations (e.g., incorrect $x$-intercepts on the graphical representation), was coded as *conceptual error*. Solutions that did not include a diagram, make use of the available physical model, were highly abstract in reasoning (i.e., factoring or using the quadratic formula to find the zeros of the quadratic relation), and/or achieved via graphing calculator (i.e., quadratic regression) were coded as *theoretical reasoning*. The theoretical reasoning code could be viewed as an overly sophisticated response, one of the variances of learning paths proposed by Murata and Fuson (2006). Solutions that were incomplete in one or more of the requirements of the task were coded as *incomplete*, regardless of the extent that the solution was incomplete.

Both researchers, as mathematics teachers emphasized the use of fractions in our courses, as this was consistent with current ministry curricular expectations. Therefore, we thought it was important to see who opted to use decimals over fractions, suggesting, from our classroom experience, perhaps an underdeveloped understanding of operations with fractions. Consequently, solutions that were completed using fractions as opposed to decimals were coded as *fractions*. We had falsely predicted that this code would be used predominantly for the student work.

We anticipated that some solutions might reflect the physical model, as seen, while other solutions might show a transformation of the fountain’s water flow to the first quadrant to facilitate more straightforward calculations. Transformation of graphical representations of water fountains that appear to flow into the second quadrant of the Cartesian plane, to the first quadrant were coded as *transformed model*. We also coded instances in which solutions
showed the intentional use of friendly numbers, indicating that students or teachers understood the numbers used were somewhat arbitrary and could be manipulated slightly to facilitate easier calculations.

Finally, we coded, using the Achievement Chart from the current curriculum documents in mathematics (Ontario Ministry of Education/OME, 2005), the overall level of communication of mathematical findings of the solutions as either a L1, L2, L3, or L4 – with L3 representing current acceptable ministry standards, L4 exceeding standards, and L1 significantly below ministry standards. This code for the overall level of communication is consistent with what Murata and Fuson (2006) describe as “ease of understanding” (p. 424) as a possible variance in student learning paths.

Our results are grouped according to our coding categories, drawing relevant examples from both the teacher and the student solution sets, where necessary to effectively illustrate the result. Overall results from the coding as percentages, for the teachers and students, are also reported in a table format (Table 1). We begin each section of the results stating our assumptions, as hypotheses, of the learning paths that we anticipated might have emerged in relation to each of the codes. A concluding comparison of both sets of solutions is in the subsequent section.

**RESULTS**

To recall, the task involved determining the quadratic relations that would model the water flow from a school drinking fountain. The task also involved determining the ideal water flow, at an arbitrary height, from the faucet guard of the water fountain. The bowl or the basin of the water fountain, potentially formed the x-axis with the y-axis either being set as the side of the faucet handle or transformed, if the handle was on, say, the right side of a fountain, making the flow of water into the second quadrant of the Cartesian plane. Depending on the flow of each individual fountain, the ideal height, set at 3 cm above the faucet guard, would have resulted in a transformation of the x-intercepts if the fountain were either higher or lower than the ideal. The one intercept from the spout of the fountain would remain constant.

Many of the teachers, much like the students in our respective classes, did not initially anticipate the amount of time that would be required at the fountains in order to gather data. Like some of our students, some teachers returned quickly to the classroom where we had gathered for this meeting. However, they realized rapidly that they did not have the necessary measurements to adequately complete the task, thus returned once again to their fountains.
At the fountains, the teachers appeared to be engaged with the task in much the same ways we had observed with our students. In both groups, we saw much discussion and debate at the fountains while various measurements were taken, as well as later in the classroom where their solutions were being written. During the focus group session, the teachers described the activity as “very interesting” and “highly engaging.” There was consensus that this task would be welcome in their classrooms, in their departments, and useful for other grade levels investigating quadratics. We make these points now to suggest that the results we are about to report ought not to be attributed to lack of engagement or interest from either the teachers or the students.

CONCEPTUAL ERRORS

Hypothesis: Approximately 25% of students would perform conceptual errors in their solutions. However, teachers would not make conceptual errors in their anticipation of student learning paths.

Conceptual errors included calculation errors, as well as errors in the graphical representations (e.g., incorrect $x$-intercepts on the graphical representation) (see Figures 1 and 2). Students, as predicted, did make conceptual errors (<26%), albeit slightly higher than predicted. However, more surprising was the extent to which conceptual errors were made by the teachers (69%). It was remarkable to see the number of conceptual errors throughout the teacher solutions. Although an argument could be made that the students may have been more focused on the task given that it was an assessment, our observations, of the teachers engaged in the task, was that there was significant interest in the task and in performing well for each other as colleagues.

The most common and significant error for teachers occurred in relation to the zeros, $x$-intercepts, or zeros of the graphical representation. One pair of teachers raised this question in their solutions: “Would kids worry about the zeros changing with the maximum?” Unlike the majority of the students, who showed in their graphical representations a transformation of the zeros (i.e., either widening or narrowing) as the flow of the fountain was manually regulated with the handle (i.e., change in the maximum height), many of the teachers did not. This suggests the teachers did not feel that students would “worry” about the transformation of the zeros, despite the changes that occurred in the physical model.

Consequently, all the conceptual errors made by the teachers included inaccurate graphical representations showing that, despite increases or decreases in water flow, the points at which the water hit the basin of the water remained constant. Provided that the
graphical representation was used for the remainder of the task, the incorrect intercepts made the rest of the solution also incorrect. In all cases, for the pairs of teachers and the pairs of students, the error in the intercepts was not the only error. Other errors in basic calculations, distributive properties, factorization, and so forth, plagued the teachers’ responses and select student responses.

We suggest here that the significant amount of conceptual errors by the teachers was not intentional. That is, teachers did not construct solutions to the task infused with conceptual errors because that is what they would anticipated student learning paths to have evidenced, even though this was the explicit request of the task. The high number of conceptual errors made by teachers with high SCK is noteworthy, nevertheless.

THEORETICAL REASONING

Hypothesis 2: Few students might use theoretical reasoning, largely in the form of the quadratic formula, in their solutions given the presence of a physical model. Teachers would use the physical model fully given the parameters of the task and therefore would not show evidence of theoretical reasoning.

Theoretical reasoning is overly sophisticated responses (Murata & Fuson, 2006). Solutions that were highly theoretical in their reasoning included: (1) solutions that did not include a diagram, (2) solutions that did not make use of the available physical model, (3) solutions that were highly abstract in reasoning (i.e., factoring or using the quadratic formula to find the zeros of the quadratic relation), or (4) solutions achieved via graphing calculator (i.e., quadratic regression). In the case of the first, second, and fourth points only teacher responses showed evidence of these. The third point was evidenced in both teacher and student solutions. In comparison to the students’ responses of 13% reflecting theoretical reasoning, 31% of the teachers’ responses showed evidence of theoretical reasoning.

In two instances, teachers used the quadratic formula rather than the physical model to determine the various forms (e.g., standard form, vertex form, and factored form) of the quadratic relation. Even though students did have the quadratic formula at their disposal, they did not resort to the formula given the accessibility to the physical model. Evidence of theoretical reasoning is closely linked to conceptual errors. Teachers in their theoretical (rather than applied) reasoning did not go to the physical model, and therefore missed the transformation of the zeros in their graphical representation and subsequent calculations.

Two pairs of teachers used graphing calculators for their solutions, in spite of having been told explicitly that students did not have graphing calculators available to them during
their completion of the task. The use of graphing calculators was evidenced in their solutions. For example, one pair of teachers said that the vertex form of their solution was obtained by “using the curve of best fit.” This pair of teachers also reported the $r^2$ value as an indication of fit in relation to the points on the curve.

The same pair of teachers, who used graphing calculators and reported the $r^2$ value, also had unique method obtaining measurements at the fountain. These teachers indicated on their solution that the curve representing the water flow was “obtained by holding paper behind water and inserting pen through water to mark paper.” Unlike all the other pairs of teachers and students, who used a ruler aligned either horizontally with the basin of the fountain (i.e., $x$-axis) or vertically with the spout of the fountain (i.e., $y$-axis), these two teachers put the graphing paper into the water fountain so that the actual water flow hit the paper and made a watermark from which the measurements were then taken.

The use of a graphing calculator, the efforts to achieve maximum accuracy in the measurement of the flow from the fountain, the indication of the $r^2$ value as a measure of fit, and having put the paper into the water fountain, suggested that this last pair of teachers’ reasoning was highly theoretical and precision was ultimately an important factor in the successful completion of the task. Overall, teachers were deeply attached to theoretical reasoning approaches and were less practical than the students. Given the goals of the task, this result was unexpected.

**INCOMPLETE**

Hypothesis 3: Some students might have incomplete solutions. However, few, if any, teachers would have incomplete solutions – despite their anticipation that some students might not complete the task.

Solutions that were incomplete in one or more of the requirements were coded as incomplete, regardless of the extent to which the solution was incomplete. The main error by both teachers and students was determining the ideal flow. More students (44%) than teachers (31%) did not complete the task.

We did not anticipate that some pairs of teachers would not be able to complete the task fully. We conclude here that incompletion, on behalf of some of the teachers, may have been as a result of over analysis of the problem. In other words, the teachers that did not complete the task may have been overly distracted with producing a sophisticated result, hence missed some key components of the task (e.g., determining the equation for the ideal flow). We did not perceive the incompletions by the teachers to be intentional.
From our observational notes, teachers seemed highly engaged in the task – in fact, teachers wanted to succeed at the task. During the focus group session, teachers agreed that the task was useful and highly engaging. Furthermore, teachers reported “wanting to do well” given that the results would be under scrutiny by their peers (i.e., the authors of this paper, as colleagues in teaching). Teachers were invested in the task; nevertheless, the results show almost one-third of the teachers had incomplete solutions.

**FRACTIONS**

_Hypothesis 4: Students that struggle with fractions will convert fractions to decimals, in spite of being encouraged throughout their mathematics course to work with fractions rather than convert to decimals. Teachers, however, would predominantly work through the solutions using fractions._

Students who are less comfortable with operations on fractions will convert fractions into decimals in order to facilitate, most particularly, the use of a calculator. In our own teaching of mathematics, we emphasize the importance of being able to work with fractions and discourage the practice of converting fractions to decimals. However, we frequently see those students less comfortable with fractions convert these to decimals. During the focus group session with the teachers following the task, we queried the teachers on their perspectives about fractions – which by consensus were aligned with our own anticipation of student learning paths. Yet, more than half of the teacher pairs (54%) and almost half of the student pairs (48%) used fractions for their calculations during the task. Consequently, it was unanticipated that more than half of the teachers converted their fractions into decimals.

**TRANSFORMED MODEL**

_Hypothesis 5: Students would construct their graphical representations of the physical models as they saw it (i.e., in the second quadrant if the spout was on the right side of the fountain). Teachers would also do the same._

In our initial discussions, about possible learning paths that might be demonstrated, we had agreed that students would likely create graphical models that closely resembled the actual physical model. Surprisingly, we saw a significant amount of students (74%) transform the physical model from the second quadrant to the first quadrant when constructing the graphical representation. This learning path was also unanticipated by the teachers, who
predominantly constructed the model as they saw it (38%). The transformation of the model by the students evidenced more sophisticated, problem solving strategies than expected.

A novel finding emerged during the coding of the transformations. Our initial concern was with transformations between quadrants (e.g., flips across the $y$-axis); however, the student responses had one additional transformation that was completely unanticipated by us, and by the teachers. In more than half of the student solutions, students also transformed the $x$-axis from the basin of the fountain to the point at which the water left the spout (Figure 1). This enabled much more straightforward calculations. We did not see any evidence of this vertical slide in the teachers’ responses, suggesting that the teachers also did not anticipate this as a possible student learning path. As Murata and Fuson (2006) state, alternative learning paths are possible; however, given that more than half of the students did this transformation, we do not view this as an alternative learning path, but rather one that ought to be anticipated by teachers.

**FRIENDLY NUMBERS**

_Hypothesis 6: Teachers would anticipate that students would select “friendly numbers” (i.e., rounding up or rounding down a measurement to the nearest whole number) when determining measurements for their models of the water fountain, to ease calculations._

Both students and teachers predominantly used friendly numbers. In most instances, each group of pairs rounded to the nearest whole number. For instances in which students or teachers used decimals, the decimals were self-restricted to two decimal points. Therefore, students and teachers rounded were necessary, showing recognition of the somewhat arbitrary aspect of the measurements taken at the fountain.

**OVERALL LEVEL OF COMMUNICATION OF MATHEMATICAL FINDINGS**

_Hypothesis 7: Learning paths of students, with respect to overall level of communication, would be predominantly level 3. The teachers’ solutions, given the high SCK, would be predominantly higher._

Using the Achievement Chart for the Province of Ontario (Ontario Ministry of Education/OME, 2005) as our guide, we coded the overall level of communication of mathematical findings for all the samples submitted by students and teachers, with level 3 representing current, acceptable ministry standards of achievement. This code did not assess the overall correctness of the solution (i.e., graphical representation, calculations, etc.), but
rather took into account whether there was evidence of a systematic approach, using
disciplinary conventions and discursive practices of mathematics, in the solving of the task.
Therefore, a solution may have achieved a level 3 or 4 in our coding, but as a result of the
incompleteness and/or conceptual and other errors throughout the solution, it may have
achieved a lower level when taking these other factors into consideration.

All of the students achieved level 3 or better. Surprisingly, the same was not true of
the teacher responses. Although more than 70% of teachers achieved a level 3 or 4, on their
communication of mathematical findings, 30% did not. We reiterate here that the teachers
appeared to be engaged and motivated to produce good work on the actual task given that
their solutions were being examined by peers and within a research context. We also restate
that these teachers had both significant academic training in mathematics and mathematics
education, plus extensive classroom experience. Despite these important contextual
circumstances of the teachers, almost one-third of the teachers were unable to anticipate
student learning paths with respect to communication of mathematical findings. Again, we are
not suggesting that this was intentional by the teachers. Nevertheless, this result is striking.

**IMPLICATIONS FOR MATHEMATICS FOR TEACHING**

Our purpose in this research was to examine teachers’ anticipation of student learning
paths across a common task, and to consider the implications of the results in relations to the
mathematics for teaching framework proposed by Ball et al. (2005). This research centered on
KSC and SCK, as the central domains of analysis. Earlier research has shown that teachers
with advanced degrees in mathematics (or a related field) were not necessarily better at
teaching mathematics (Deborah Loewenberg Ball, 1989). We wondered then whether there
were instances when SCK may then interfere with KSC, and what insight this may shed on
the development of the other domains.

To recall then, the research questions guiding our work were: Therefore, the research
questions guiding this work are: (1) what are the assumptions teachers with sufficient or high
SCK make about student thinking (i.e., KSC) or the “learning paths” that students take? And,
(2) in analyzing such assumptions, what conceptual and pedagogical insights might be mined
to support knowledge development in the other domains defined by Ball et al. (Deborah L Ball
et al., 2005)?

As our results show, the teachers in this study with high SCK and CCK, and
significant experience with KCT, were not able to easily anticipate the learning paths of
students. In other words, they had difficulty anticipating the KSC. We suggest that this is
demonstrated by learning paths evidenced in the teachers’ solutions demonstrates just this and can be attributed largely to high levels of SCK.

The conceptual errors and the theoretical reasoning of the teachers, in this research, are intertwined and directly related to SCK. Although many teachers neglected to transform the $x$-intercepts of their graphical representations, we felt that this error was not in relation to the actual mathematics, but rather to the perception that the model was perhaps unnecessary in order for them to complete the task. Consequently, the physical model was not fully incorporated into their solution. The teachers showed more dependency on their theoretical understanding of the quadratic relation (or high SCK). Their results were less precise because of this, although we believe that the teachers’ intentions were increased precision. We contend, however, that for both teachers and students, complex calculations do not necessarily imply complex and/or task-related thinking.

Students adapted to the task somewhat more concretely and did better overall because of their use of the physical model. Students’ concrete thinking was further evidenced by the creative ways in which their graphical representations were transformed in order to make the calculations more straightforward. This was surprisingly not anticipated by the teachers or us - the researchers. This leads us to question whether student learning paths might potentially be restricted or develop at all, if higher SCK informs the decisions teachers make in relation to KTC.

Also compelling is the evidence that many teachers, with significant SCK, were unable to complete the task. Again, here we assert that over-theorization was the contributing factor. In spite of this assertion, we wonder how teachers come to zoom in on the particulars of CCK and KTC in order to develop an understanding of KSC, given the interference of SCK.

CONCLUDING THOUGHTS

Our research demonstrates that more research is needed to examine, not only those with limited SCK, but also those with significant SCK. Indeed, a different type of mathematics for teaching education may be required for those with high SCK. These teachers, in our opinion, would have been the most likely to anticipate student learning paths, given their personal histories. Yet, the learning paths of these students were not well anticipated, which we contend was not intentional.

However, as concluded by the second author of this paper, who brings more than 30 years of classroom teaching experience to this analysis, despite a teacher’s experience and
high levels of SCK, student thinking can still be surprising and reveal alternative ways of thinking about mathematics. We surmise, as well, that having high SCK might permit teachers to look at alternative learning paths and see the validity and merit in those learning paths. The ability of those with limited SCK may not have the same sort of elasticity within the domains defined by Ball et al. (2005).

Our findings demonstrate that indeed the loci of mathematics for teaching, within Ball et al.’s (2005) domains, does rest in KSC. Teachers’ knowledge of the other domains can be examined, by extension, through an analysis of KSC. Furthermore, we propose that our research suggests that perhaps a fruitful starting point for mathematics teacher education may be through an examination of student learning paths or KSC.

In our research, we did not return to the teachers and present the student results in relation to their own projection of student learning paths. This remains an important area of further inquiry and, as mentioned earlier, could be a productive way of educating both current and in-service teachers about the various domains of mathematics for teaching. Furthermore, we do not explore the factors that do contribute to student learning. Further analysis is needed to see what other factors contribute to student understanding, particularly when the learning paths of students and differ from those projected by teachers.

Our hypotheses of student learning paths, on the whole, coincided with the student samples. We recognize that our role as these students’ regular classroom teachers may have influenced this. However, aside from the common task, the individual choices each of us made within our classrooms, as teachers, were not common or shared. We each made independent decisions about our classroom practices. However, like our colleagues, who engaged in this research, our choices are restricted by the current policies in mathematics education (i.e., curriculum, assessment, and authorized texts) so there is a commonality in pedagogy to an extent. This commonality between our fellow teachers and us should have resulted in a more cohesive set of learning paths between the students and the teachers. However, this was not the case.

How teachers think about student thinking potentially correlates to the ways in which teachers teach. By examining teachers’ anticipation of student thinking, we can then begin to unpack the assumptions teachers make. Furthermore, we can begin to understand the kinds of additional knowledge that teachers might need to be more effective at teaching mathematics.
Table 1: Overall results from the coding, as percentages, for the teachers and students.

<table>
<thead>
<tr>
<th>Code</th>
<th>Teachers</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual error</td>
<td>69%</td>
<td>26%</td>
</tr>
<tr>
<td>Theoretical reasoning</td>
<td>31%</td>
<td>13%</td>
</tr>
<tr>
<td>Incomplete</td>
<td>31%</td>
<td>44%</td>
</tr>
<tr>
<td>Fractions</td>
<td>54%</td>
<td>48%</td>
</tr>
<tr>
<td>Transformed model</td>
<td>38%</td>
<td>74%</td>
</tr>
<tr>
<td>Friendly numbers</td>
<td>92%</td>
<td>96%</td>
</tr>
<tr>
<td>Overall level of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>communication of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematical findings</td>
<td>L4: 62%</td>
<td>L4: 74%</td>
</tr>
<tr>
<td></td>
<td>L3: 8%</td>
<td>L3: 26%</td>
</tr>
<tr>
<td></td>
<td>L2: 22%</td>
<td>L2: 0</td>
</tr>
<tr>
<td></td>
<td>L1: 8%</td>
<td>L1: 0</td>
</tr>
</tbody>
</table>

Figure 1: Student sample – x- and y-intercepts transformed.

Figure 2: Teacher sample – x-intercepts not transformed.
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REVISITING THE INFLUENCE OF NUMERICAL LANGUAGE CHARACTERISTICS ON MATHEMATICS ACHIEVEMENT: COMPARISON AMONG CHINA, ROMANIA, AND U.S.

Jian Wang
Emily Lin
Madalina Tanase
Midena Sas

ABSTRACT. Eastern Asian students repeatedly outperform U.S. students in mathematics. Some suggest that number-naming languages consistent with the base-10 number system found in many Eastern Asian countries presumably contribute to their students’ better understanding of the base-10 system and consequential performance. Such language features do not exist in English or other Western languages. The current study tests this assumption by comparing base-10 knowledge of students in kindergarten and first-grade from China, Romania, and U.S. who have developed number-naming language abilities but received relatively little formal school instruction. It is expected that since Chinese number-naming is linguistically more transparent and consistent with the base-10 system while English number-naming language is least consistent. However, the analysis of this study revealed that although Chinese children outperformed both Romanian and U.S. counterparts in accomplishing base-10 tasks, there were no significant differences between Romanian and U.S. children. This finding suggests that the extent to which number-naming language is linguistically transparent and consistent with the base-10 system may not necessarily align with the level of children’s understanding of the base-10 system and relevant mathematics performances.

KEYWORDS. Asian, American, Romanian, Language, Cognitive Representation.

INTRODUCTION

As shown in many international comparisons, Eastern Asian students are typically the top international mathematics performers across various grade levels (Mullis et al., 1997; Mullis et
al., 2004; Mullis et al., 2000; Programme for International Student Assessment, 2004; Stigler et al., 1990). However, how they attain such performances is less well understood and this has resulted in a multitude of interpretations and lines of research (Wang & Lin, 2005).

Much of the mainstream research points to differences in schooling factors between Eastern Asian countries and U.S. as an interpretation. This line of research suggests that in comparison with Eastern Asian countries, U.S. school mathematics curriculum materials are less focused and more repetitive when examining content coverage, instructional requirements, and structures (Mayer et al., 1995; Schmidt et al., 1999; Schmidt et al., 1996b). U.S. curriculum policy is also less authoritative, specific, and consistent (Cohen & Spillane, 1992; Eckstein, 1993; Wang, 2001). In addition, teachers in Eastern Asian countries are able to develop a deeper understanding about mathematics and its representations (Ma, 1999) and provide clearer explanations, make more efficient use of their class time, develop smoother pedagogical flow, and engage more students in inquiry using whole class instruction (Perry, 2000; Schmidt, et al., 1996a; Stigler & Hiebert, 1999; Stigler et al., 1987). Furthermore, Eastern Asian teachers are able to plan, observe, and reflect on each other’s instruction to improve their teaching practices (Lewis, 2000; Stigler & Hiebert, 1999; Wang & Paine, 2003). These findings have exerted important influences on U.S. mathematics education reform featuring the establishment of mathematics curriculum standards and engagement of teachers to examine and reflect on one another’s instruction (Romberg, 1997; 1999).

Other researchers focus on non-schooling factors to explain the performance differences between Eastern Asia and U.S. (Wang & Lin, 2005) since solely focusing on schooling factors cannot explain why Asian students in the U.S., who have little or no exposure to the kinds of teaching and curriculum in Eastern Asia, are still better mathematics performers than other American racial groups (Chen & Stevenson, 1995; Kaufman et al., 1998; Sanchez et al., 2000; Stevenson et al., 1990). These non-schooling factors include student intelligence (Flynn, 1991; Lynn, 1991); self-esteem and self-efficacy (Leung, 2002; Stevenson et al., 1993; Wilkins, 2004); academic expectations and effort (Chen & Stevenson, 1995; Chiu, 1987; Tuss et al., 1995); family education values, expectations, and support (Crystal & Stevenson, 1991; Hess & Others, 1987; Huntsinger et al., 2000; Patterson et al., 2003); and language clarity, word structure, and patterns (Geary et al., 1993; Han & Ginsburg, 2001; Li & Nuttall, 2001; Miller et al, 2000; Miura et al., 1988; Rasmussen et al, 2006).

Along the line of research on language pattern influences, some scholars explain Eastern Asians’ higher mathematics performance over U.S. students to be the result of Eastern Asian
number-naming languages, such as Chinese, Japanese, and Korean, that are more linguistically transparent (Rasmussen et al, 2006) in number names and consistent with the base-10 structure (Bell, 1990; Fuson & Kwon, 1991; Geary, 1996; Miller & Stigler, 1987). Such base-10 knowledge is assumed to be the basis for students to learn and perform better in other relevant mathematics content areas (Ho & Cheng, 1997).

This assumption was supported by the comparisons of base-10 knowledge between pre-school and first grade students from Eastern Asian and those from U.S. and other European countries where number-naming languages are inconsistent with a base-10 system (Ho & Fuson, 1998; Miller & Stigler, 1987; Miller et al., 1995; 2000; Miura et al., 1988; Rasmussen et al., 2006). These comparisons together imply that the nature and extent of number-naming language’s transparency and consistency with the base-10 system is associated with students’ performance levels in base-10 knowledge and other relevant mathematics areas. However, existing studies predominantly examine students whose languages are either consistent or inconsistent with a base-10 system with little attention to languages that are partially consistent with the base-10 system, like that found in the Romanian language. That is, Chinese oral number-naming is linguistically more transparent and consistent while Romanian is somewhat consistent and English is least consistent with the base-10 system. If the transparency and base-10 consistency of the Chinese number-naming system determines mathematical performance, Romanian children would be expected to be intermediate between Chinese-speaking and English-speaking children in their understanding of base-10 system and representation of relevant mathematics concepts (Bloom, 2000; Siegler, 1998).

However, the findings from some large scale comparative studies contradicts the above expectation which showed that despite the somewhat transparent and consistent nature of the Romanian number-naming system with the base-10 system, Romanian students still performed below their U.S. peers in mathematics at various grade levels (Beaton et al., 1996; Mullis et al., 1998; Programme for International Student Assessment, 2004). Since these comparisons only measure Romanian students at upper elementary to high school levels, they are unable to delineate schooling and non-schooling factors, such as language, on their performances. This study is designed to test whether there is a relationship between students’ performance in base-10 knowledge tasks and the transparency of number-naming language consistent with the base-10 system by examining Chinese, Romanian, and U.S. pre-school and early first grade students who have relatively little influence from formal schooling.
NUMBER-NAMING LANGUAGE AND MATHEMATICS PERFORMANCE

Central to the research on the influences of number-naming language consistent with the base-10 system on student mathematics performance is the controversial Sapir-Whorf hypothesis about the relationship between language structure and thinking. It posits that the structures of a language strongly influences or even determines the way in which its native speakers perceive the world in a highly habituated manner (Sapir, 1949; Whorf, 1956). In the field of linguistics, such an assumption spawned substantial research to examine if different language structures, such as color terms, kinship terms, ethno-biological taxonomies, and obligatory morphosyntactic categories, like noun, numeral classifiers, and tense, can produce distinctive thinking (Gumperz & Levinson, 1996). The findings from this line of research challenge the extreme form of Sapir-Whorf hypothesis and propose that language and thinking are not significantly related at the grammar, individual speaker, and habitual levels (Lucy, 1996; Michael, 2002; Towse & Saxton, 1997).

The research on the influences of number-naming language (Miura et al., 1988) assume that numbers are mentally represented and stored through language and in Eastern Asian languages such as Chinese, Japanese, and Korean, numerical names are organized in a way that is congruent with the traditional base-10 numeration system. However, in other languages, the level of this transparency and consistency can be different. Such differences in linguistic transparency and consistency in different languages can be illustrated by comparing the number names for teen quantities between Chinese, English and Romanian languages.
As shown in Table 1, in Chinese number-naming, the teen words are represented as ten plus some ones: 11 is simply "ten one," 12 is "ten two" and 13 is "ten three." This pattern continues into the decade numbers where 20 is "two ten," 30 is "three ten" and 45 is "four ten five." Such a number-naming language makes it relatively transparent that the number system is base-10.

Romanian number-naming words are somewhat similar to Chinese number words in terms of a base-10 system except for the following variations. First, in Romanian, the word representing 11 to 19 are reversed, with ones coming before the ten. For example, in the Romanian number naming language: 11 is “one over ten”, 12 is “two over ten”, 20 is “two times ten”, 30 is “three times ten”, and 45 is “four times ten and five.” This structure is much more consistent with base-10 system when compared with the English language. Second, there are three irregularities in the Romanian “teen” number words that might interfere with some children’s understanding of the pattern: in “unsprezece” (11), “un” is used to represent “1”
instead of “unu” (1); in “paisprezece” (14), “pais” is used instead of “patru” (4); and in “saisprezece” (16), “sais” is used instead of “sase.”

However, these variations are comparatively small and more transparent when contrasted with the English number-naming language. For instance, English speakers learn the numeral 11 as “eleven”, 12 as “twelve”, 14 as “fourteen”, 20 as “twenty”, 30 as “thirty” and 45 as “forty-five” where these numbers and words that represent them have different forms in pronunciation and structure connections. For instance, most English speakers are not able to see connections from ten to eleven and twelve from a base-10 perspective.

Thus, it can be argued that Chinese oral number naming is the most transparent and consistent with the base-10 system. Romanian number-naming language to a great extent is transparent and consistent with base-10 system while the English number-naming language is least consistent with base-10 pronunciation and structures. Therefore, if the transparency and consistency of base-10 number-naming system determines mathematics performance on tasks related to base-10 knowledge, Romanian children would be expected to be intermediate between Chinese-speaking and English-speaking children in their mathematics performance. The comparison of performance in representing symbolic numbers among Chinese, Romanian, and U.S. children, who have relatively little influences from formal schooling, would help explore the influences of number-naming language on students’ understanding of place values and base-10 systems.

Since the late 1980’s, a series of studies were developed to test the influence of number-naming language on children’s understanding of base-10 systems and thus, relevant mathematics performance. These studies shared two similarities in their research design. First, they all choose pre-school or early elementary students as their participants so that the influence of number-naming language can be presumably isolated from formal schooling and teaching factors. Second, they compared students’ base-10 understanding from high mathematics performing countries where number-naming languages are consistent with base-10 system with students from the lower mathematics performing countries where number-naming language is incongruent with the base-10 system. In this way, the hypothetical relationship between children’s number-naming language, their base-10 understanding, and their mathematics performance can be presumably verified or rejected.

Three types of studies have been conducted following the above research design. The first focuses on single country comparisons. By comparing pre-school children in Taiwan with those in U.S. on abstract counting and object counting, Miller and Stigler (1987) suggested that
the Chinese number-naming language, consistent with base-10, favored Chinese children in their object counting while English number-naming language, inconsistent with base-10, did not favor U.S. students in accomplishing abstract counting tasks. Later, Miller et al. (1995) confirmed this findings by comparing children from Mainland China and U.S. Other researchers (Miura & Okamoto, 1989) tested first grade students from Japan and U.S. on their number representation using tens and ones blocks and found that Japanese students performed substantially better than their U.S. counterparts, which is assumed to be contributed by Japanese number-naming language congruent with the base-10 system.

The second type of study includes comparisons between a single country and multiple countries in an attempt to test the assumption across national lines. By comparing U.S. first graders with Chinese, Japanese, and Korean first graders as well as Korean kindergartners on their number representation using tens and ones blocks, Miura and her colleagues (1988) found that U.S. children tended to use ones unit blocks while Chinese, Japanese, and Korean children used correct combinations of tens and ones unit blocks to represent symbolic numbers. The study claimed that Chinese, Japanese, and Korean number-naming languages consistent with base-10 systems contribute positively to their children’s performances. Ho and Fuson (1998) tested Chinese preschoolers with different IQ’s on their counting sequence, assessed Chinese and U.S. peers on their differences in counting sequence with embedded-ten cardinal understanding, and in the end, compared children from U.S. and England on their embedded-ten cardinal understanding. They found that Chinese children surpassed their English and U.S. peers in rote counting, place value, and embedded-ten cardinal counting and were able to apply this understanding to solve simple addition problems. The researchers proposed that Chinese children’s number-naming system influenced these differences in performance rather than their IQ’s.

The third type of study includes the comparisons between groups of multiple countries. Miura and her colleagues (1993) repeated their earlier studies by comparing Japanese and Korean first graders with French, Swedish, and U.S. students. Their study concluded that Eastern Asian students with number-naming language consistent with base-10 tended to use more correct combinations of tens and ones blocks to represent symbolic numbers than their Western counterparts whose number-naming languages were inconsistent with the base-10 system. Later, they confirmed this finding by including Chinese first grade students in their comparison and tested all the students in their native language in two consecutive trials (Miura et al., 1994).

All these studies suggest that pre-school or first grade children whose number-naming languages consistent with the base-10 system tend to develop better understanding about place
values and base-10 functions than those whose number-naming languages were inconsistent with the base-10 system (Fuson & Kwon, 1991). This implies that the consequence of these place values and base-10 ideas developed among Eastern Asian students may contribute greatly to their learning and performance in the areas of mathematics that rely on a deeper understanding and flexible utilization of place values and a base-10 system (Bloom, 2000; Siegler, 1998).

However, these research findings can be challenged conceptually and methodologically in three ways. First, the use of pre-school and first grade students, although able to control for the influence of formal schooling factors, does not isolate the influence of family and other social factors, such as formal mathematics instruction at home and in preschool environments, from number-naming languages. A longitudinal study (Huntsinger et al., 2000) that followed Chinese-American and Caucasian American children from kindergarten to fourth grade using survey, observation, and interview data, suggested that Chinese-American parents tended to use formal instruction to teach their children mathematics even before they entered schools. An observational and interview study (Yang & Cobb, 1995) also suggested that Chinese first grade students’ advantage in composite multiunit numerical concepts can be contributed, in part, to arithmetic learning activities at home and, in part, to their classroom instruction. A reasonable question arising from these studies would be whether the performance difference in base-10 understanding between children from Eastern Asian and U.S. and other Western countries is due to their family and preschool influences or to the additive influences of both family and preschool and number-naming language. To examine this question, comparison of base-10 understanding may be made between children with English as their first and only language and their peers from different racial groups that are influenced by varying culture and family environments, such as that found among Caucasian, Hispanic, and African American children in the U.S. (Ogbu, 1983; Ogbu & Simons, 1994).

Second, the factors that impact preschool and first grade students’ understanding of place value and base-10 system may differ substantially since the former presumably have little school influence while the later, although relatively short, may be influenced by teaching that focus on place value and base-10 in early formal schooling. Considering little distinction was made between pre-school children and first grade students in the existing studies, it is reasonable to question whether children’s understanding of place values and base-10 systems in Eastern Asian countries is the result of their number-naming language or explicit family and school influences. To address this schooling influence, the question of whether there exists performance differences between preschool and first grade students from countries included in previous comparative
studies need to be explored. For instance, were there any differences between preschool and first grade students in China?

Third, the existing studies tend to focus only on the comparison of students in countries who use transparent number-naming languages consistent with base-10 and have higher mathematics performance with those children in the countries where their number-naming language is inconsistent with a base-10 system and possess lower mathematics performances in international comparisons. Little attention is given to the children whose number-naming language are somewhat transparent and consistent with a base-10 system but exhibit lower mathematics performances than those whose number naming language is inconsistent with the base-10 system. Romanian students represent this population since they use a number-naming language that can be classified as being transparent and consistent with the base-10 system except for a few minor irregularities. Nevertheless, Romanian students still scored lower in mathematics performance than both their Asian and U.S. counterparts in international comparisons (Beaton et al., 1996; Mullis et al., 1998; Programme for International Student Assessment, 2004). Countries like Romania limit the interpretative power of the above mentioned language influence research since it is not able to explain why their number-naming language failed to help them perform better in the international comparisons, even though they use a number-naming language that is somewhat consistent with the base-10 system. Alternatively, it may be that the direct relationship between number-naming language consistent with the base-10 system and mathematics performance is weak or does not exist.

To resolve the above three issues, the following question must be answered: Do differences exist in base-10 knowledge among Romanian, East Asian, and U.S. children? This study is designed to examine the aforementioned questions by (1) comparing Romanian, Chinese and U.S. students; (2) examining the differences between kindergarten and first grade students in China, and (3) exploring the differences in base-10 knowledge among Caucasian, African American, and Hispanic students in the U.S.

**METHODOLOGY**

**Subjects**

To examine and reduce the interactive effects of school factors and number-naming language, both pre-school and first graders early in their formal schooling from China were included. Additionally, first grade children from Romania and U.S. as well as U.S. Caucasians,
Hispanic, and African American children were included in order to explore possible differences among different racial groups possessing a similar number-naming language. The U.S. subjects were recruited from two urban elementary schools in a large southwest city. Romania subjects were from an urban area in a large coastal city, and the Chinese subjects were from a suburban area in a large southern city.

There were 20 Chinese subjects (10 preschoolers and 10 first graders; 12 boys and 8 girls) with a mean age of 6.43 years old (range = 5 to 7.5 years old). There were 18 first grade Romanian subjects (14 boys and 12 girls) with a mean age of 6.87 years old (range = 6.5 to 7.5 years old). Twenty-six first grade American subjects including 8 Caucasians, 10 African Americans, and 8 Hispanics (14 boys and 12 girls) were selected from the U.S. with a mean age of 6.81 years old (range = 6.5 to 7 years old) and whose first language was English.

**Data collection**

Tests were conducted with all the subjects during the beginning or the first half of their academic year to ensure that school influences on base-10 understanding were minimized. Subjects were individually interviewed in their native language using an identical set of base-10 blocks. These included short blocks representing units of ones and long blocks representing units of tens. The base-10 blocks had not been used during any formal instructional processes with any of the subjects prior to the testing in each country.

Replicating Miura et al’s (1994) study, two trials were conducted in this study. During the first trial, researchers met the subjects individually, introduced them to the set of base-10 blocks, and explained that the blocks could be used for counting and constructing numbers. The subjects were then instructed on how to represent three numbers (2, 7, and 15) using correct combinations of base-10 blocks. Each subject was then asked to construct representations with the right combination of ones and tens blocks using the number: 2, 7, and 15 as in the first trial. In the end, the subjects were asked to construct the five numbers (11, 13, 28, 30, and 42) in random order.
As in Miura et al’s study (1994), each child’s responses were scored if they correctly represented the numbers with the right combination of blocks. Correct responses were categorized separately for Trials 1 and 2 using the following categories (Ross, 1986): (1) Unit collection: the representation used only ones blocks. For example, for the number 42, subjects would use 42 ones blocks. (2) Canonical base-10 representation: subject’s representation included the correct number of tens blocks and ones blocks with no more than 9 ones blocks in the ones position. For instance, for the number 42, subjects would use 4 tens blocks and 2 ones blocks. (3) Non-canonical base-10 representation: subjects would use the correct number of ten and unit blocks with more than 9 units in the ones place, such as 3 tens blocks and 12 ones blocks for number 42. Each subject’s incorrect answers were also calculated and listed along with the different categories of correct answers.

ANALYSIS AND RESULTS

Cross-national comparison of number representation

*Overall comparison on both trials*

To compare cross-national number representations for each category for both trials, we calculated the mean of each child’s responses in each category for both trials. Then, we calculated the percentage of correct representations and incorrect presentations by dividing the possible correct and incorrect representations with actual correct representations in each category and incorrect representations for each national group. The results of this analysis are shown in Table 2, which suggested the following findings:

<table>
<thead>
<tr>
<th>Trial 1+2</th>
<th>N</th>
<th>Unit</th>
<th>Canonical</th>
<th>Non-canonical</th>
<th>Incorrect</th>
<th>Total Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>20</td>
<td>23%</td>
<td>65%</td>
<td>12%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Romania</td>
<td>18</td>
<td>44%</td>
<td>34%</td>
<td>15%</td>
<td>7%</td>
<td>93%</td>
</tr>
<tr>
<td>U.S.</td>
<td>26</td>
<td>40%</td>
<td>38%</td>
<td>11%</td>
<td>11%</td>
<td>89%</td>
</tr>
</tbody>
</table>

First, Chinese subjects constructed 65% correct representations for canonical base-10 category while Romanian subjects had 34% and the U.S. subjects had 38% in both Trials. These scores indicated that Chinese subjects were more likely to use canonical base-10 to represent numbers than their Romanian and U.S. peers while Romanian and U.S. subjects showed no substantial differences with only a slightly higher percentage for U.S. subjects.
Second, for the unit representation category, Romanian and U.S. subjects showed comparable correct representations (44% and 40%, respectively) which were higher than their Chinese counterparts 23% in both trials. This showed that Chinese subjects were less likely to use unit representations to represent a number than their Romanian and U.S. peers while Romanian and U.S. subjects again showed no differences in this category.

Third, in the non-canonical category, there were no substantial differences across the three national groups with 12%, 15%, and 11% for Chinese, Romanian, and U.S. subjects, respectively. However, U.S. students tended to make more incorrect representations (11%) than Romanian subjects (7%) who had, in turn, more incorrect answers than their Chinese peers (0%).

One-way ANOVA of the three national group means in both trials were conducted for each categorical representation. The means and standard deviations for each national group in using each category of number representations for each trial are shown in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>China (n=20)</th>
<th>Romania (n=18)</th>
<th>United States (n=26)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trial 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit representations</td>
<td>1.40</td>
<td>2.61</td>
<td>2.23</td>
</tr>
<tr>
<td></td>
<td>1.88</td>
<td>2.30</td>
<td>1.90</td>
</tr>
<tr>
<td>Canonical Base 10</td>
<td>2.90</td>
<td>1.78</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>3.25</td>
<td>1.96</td>
<td>1.95</td>
</tr>
<tr>
<td>Non-canonical Base 10</td>
<td>0.70</td>
<td>0.56</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>1.04</td>
<td>0.85</td>
</tr>
<tr>
<td>Incorrect</td>
<td>0</td>
<td>0.06</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.24</td>
<td>0.89</td>
</tr>
<tr>
<td><strong>Trial 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit representations</td>
<td>0.90</td>
<td>1.78</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>1.41</td>
<td>1.56</td>
<td>1.86</td>
</tr>
<tr>
<td>Canonical Base 10</td>
<td>3.60</td>
<td>1.67</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>1.54</td>
<td>1.64</td>
<td>1.70</td>
</tr>
<tr>
<td>Non-canonical Base 10</td>
<td>0.50</td>
<td>0.94</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>1.11</td>
<td>1.04</td>
</tr>
<tr>
<td>Incorrect</td>
<td>0</td>
<td>0.61</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.46</td>
<td>1.64</td>
</tr>
</tbody>
</table>

*Note: The maximum number possible in each category was 5.*

ANOVA results revealed that there were significant differences for two categories across the three countries: the unit and canonical representations. F-test results revealed F = 6.39 and F = 10.93 for respective unit and canonical representation category, both of which are larger than the F 0.01 (2, 61 df) = 4.98. For the non-canonical and incorrect representations categories, there was no significant difference across the three national groups.
Subsequent to the ANOVA, a post hoc test using the Tukey's Honestly Significant Difference (HSD) was conducted to check the significance of every pairwise difference for the three countries in both the unit and canonical representations. The findings revealed:

First, Chinese subjects performed better in both trials than Romanian and U.S. subjects and at significant level with HSD = 5.86 and HSD = 5.64 respectively, each of which is greater than HSD 0.01 = 3.76. Second, Romanian and U.S. subjects scored higher in using unit representations in both trials than their Chinese peers at significant levels with HSD = 4.63 and HSD = 4.10 respectively, each of which is higher than HSD 0.01 = 3.76. Third, there were no significant differences between Romanian and U.S. groups in both trials in either the canonical or unit representation categories with HSD = 0.91 and HSD = 0.73 respectively, each of which is smaller than HSD 0.05 = 2.83.

These cumulative results suggest that Chinese subjects tended to use more canonical representations and made fewer incorrect representations while Romanian and U.S. subjects preferred to use unit representations and made more errors to represent numbers, even though there was no significant difference among the three national groups in using non-canonical representations.

**Comparisons on each trial**

To compare cross-national number representations in each trial for each category, percentages of correct representation in each category and incorrect representation were determined by dividing the possible correct and incorrect representations with actual correct and incorrect representations in each category for each national group for the first trial and then, the second trial. These results are shown in Table 4 which suggest following:

<table>
<thead>
<tr>
<th>Trial 1</th>
<th>N</th>
<th>Unit</th>
<th>Canonical</th>
<th>Non-canonical</th>
<th>Incorrect</th>
<th>Total Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>20</td>
<td>28%</td>
<td>58%</td>
<td>14%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Romania</td>
<td>18</td>
<td>52%</td>
<td>36%</td>
<td>11%</td>
<td>1%</td>
<td>99%</td>
</tr>
<tr>
<td>U.S.</td>
<td>26</td>
<td>45%</td>
<td>41%</td>
<td>8%</td>
<td>6%</td>
<td>94%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trial 2</th>
<th>N</th>
<th>Unit</th>
<th>Canonical</th>
<th>Non-canonical</th>
<th>Incorrect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>20</td>
<td>18%</td>
<td>72%</td>
<td>10%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Romania</td>
<td>18</td>
<td>36%</td>
<td>33%</td>
<td>19%</td>
<td>12%</td>
<td>88%</td>
</tr>
<tr>
<td>U.S.</td>
<td>26</td>
<td>35%</td>
<td>35%</td>
<td>15%</td>
<td>15%</td>
<td>85%</td>
</tr>
</tbody>
</table>
First, Chinese subjects exhibited increasingly more canonical representations (from 58% to 72%) than Romanian and U.S. subjects who showed gradually fewer canonical representations (36 to 33% and 41% to 35%, respectively) from the first trial to the second trial. Second, all three groups reduced their use of unit representations from the first to the second trial with 28% to 18% for Chinese subjects, 52% to 36% for Romanian subjects, and 45% to 35% for U.S. subjects. Third, Chinese subjects used slightly fewer non-canonical representations (14% to 10%) from the first to the second trial while Romanian and U.S. subjects tended to use slightly more non-canonical representations from 11% to 19% and from 8% to 15%, respectively, from the first to the second trial. Fourth, while Chinese subjects did not make any incorrect representations in either trial, Romanian and U.S. subjects tended to make increasingly more incorrect representations from the first trial to second trial (1% to 6% and 12% to 15% correspondently).

F-test results for each representation category in each trial across the three national groups showed a significant difference for only the canonical representation category across the three countries in the second trial. F-test results for the second canonical representation trial revealed $F=11.30$, which is greater than $F_{0.01} (2, 61 \text{ df}) = 4.98$. Tukey’s Honestly Significant Difference analysis revealed that Chinese subjects scored statistically significant better than Romanian and U.S. subjects in the second trial for canonical presentation with HSD =5.75 and HSD =5.95 respectively, each of which is greater than HSD $0.01=3.76$. There was no significant difference between Romanian and U.S. subjects in using canonical representations in the second trial with HSD = 0.32, which is smaller than HSD $0.05 =2.83$.

Together, these findings suggest that although all three groups learned to use fewer unit representations by the second trial, only the Chinese students increased the use of more canonical representations without mistakes, which differs from their Romanian and U.S. peers at significant level. In contrast, Romanian and U.S. subjects had a tendency to use more non-canonical representations and make more incorrect representations even after repeated demonstration by the researchers.

**Within group comparisons in number representation**

*Comparison between two Chinese groups.* The comparison of each categorical number representation between Chinese first grade and preschool subjects in each trial are shown as percentages in Table 5. This table suggests:
Table 5: Percentage of Representations in Each Category for Two Chinese Groups

<table>
<thead>
<tr>
<th>Trial 1</th>
<th></th>
<th>Canonical</th>
<th>Non-canonical</th>
<th>Incorrect</th>
<th>Total Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Grade</td>
<td>10</td>
<td>24%</td>
<td>70%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>Pre-School</td>
<td>10</td>
<td>32%</td>
<td>46%</td>
<td>22%</td>
<td>0%</td>
</tr>
<tr>
<td>Trial 2</td>
<td></td>
<td>Canonical</td>
<td>Non-canonical</td>
<td>Incorrect</td>
<td>Total</td>
</tr>
<tr>
<td>First Grade</td>
<td>10</td>
<td>18%</td>
<td>78%</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td>Pre-School</td>
<td>10</td>
<td>18%</td>
<td>66%</td>
<td>16%</td>
<td>0%</td>
</tr>
</tbody>
</table>

First, Chinese first graders constructed 70% canonical representations in the first trial while Chinese preschoolers had only 46%. By the second trial, Chinese first graders’ canonical representations increased to 78% while Chinese preschoolers had 66%.

Second, for the unit representation category, Chinese first graders and preschoolers had very small differences in the initial trial (24% and 32%, correspondently). By the second trial, both groups reduced their unit representations to 18%.

Third, Chinese preschoolers tended to use more non-canonical representations than their first grade peers in the first trial (22% and 6%, respectively). By second trial, both group reduced their non-canonical representations to 16% and 4% respectively.

F-test results showed no significant difference between the two Chinese groups in any of the categorical representation across the two trials. F-test results ranged from F = 0.003 to F = 0.642 correspondently, each of which is smaller than F.05 (1,18 df) = 4.41.

These findings suggested that although both Chinese groups preferred more canonical representations compared to the other two categories, unit and non-canonical representations, Chinese first graders were more likely to use canonical representations than their preschool peers during the first trial. However, after the demonstration, Chinese preschoolers increased their use of canonical representations substantially as exhibited in their second trial performance.

Comparison among three U.S. groups. Reminiscent of the three national group analyses, comparisons of each categorical number representation among Caucasian, African American, and Hispanic American groups in the U.S. for each trial were also conducted using percentages. These results shown in Table 6 suggest the following:

Table 6: Percentage of Representations in Each Category for Three U.S. Groups

<table>
<thead>
<tr>
<th>Trial 1</th>
<th></th>
<th>Canonical</th>
<th>Non-canonical</th>
<th>Incorrect</th>
<th>Total Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caucasian Americans</td>
<td>8</td>
<td>43%</td>
<td>35%</td>
<td>0%</td>
<td>23%</td>
</tr>
<tr>
<td>African Americans</td>
<td>10</td>
<td>50%</td>
<td>34%</td>
<td>16%</td>
<td>0%</td>
</tr>
<tr>
<td>Hispanic Americans</td>
<td>8</td>
<td>40%</td>
<td>55%</td>
<td>5%</td>
<td>0%</td>
</tr>
<tr>
<td>Trial 2</td>
<td></td>
<td>Canonical</td>
<td>Non-canonical</td>
<td>Incorrect</td>
<td>Total</td>
</tr>
<tr>
<td>Caucasian Americans</td>
<td>8</td>
<td>25%</td>
<td>33%</td>
<td>10%</td>
<td>33%</td>
</tr>
<tr>
<td>African Americans</td>
<td>10</td>
<td>36%</td>
<td>42%</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>Hispanic Americans</td>
<td>8</td>
<td>45%</td>
<td>30%</td>
<td>25%</td>
<td>0%</td>
</tr>
</tbody>
</table>
First, Caucasian and African American subjects preferred to use unit representations rather than canonical or non-canonical alternatives (43% and 50%, respectively). In contrast, Hispanic subjects preferred to use canonical representation (55%) rather than the other two representations during the initial trial.

Second, by the second trial, Caucasian and African American subjects tended to use canonical more than the other representational options (33% and 42%, respectively), while Hispanic American subjects preferred to use unit representation more (45%).

Third, all three U.S. groups tended to use fewer non-canonical representations than the other two representational options as shown by 0% for Caucasians, 16% for African Americans, and 5% for Hispanic Americans in the first trial. However, by the second trial, Caucasian and Hispanic American subjects increased their use of non-canonical representation (10% and 25%, respectively) while African American subjects reduced their use of this representation (10%).

Fourth, while Hispanic subjects made no incorrect representations in both trials which are similar to their Chinese counterparts, Caucasian and African American subjects increased their incorrect representations from the first to the second trials (23% to 33% and 0% to 12%, respectively). In addition, the Caucasian group made the most incorrect representations among all three groups in both trials.

F-test results for each categorical representation showed that there were significant differences among the three U.S. groups in using non-canonical representation and making incorrect representations in the initial trial. F-test results for the first non-canonical representation trial revealed $F = 3.96$, which is larger than $F.05 (2, 23 \, df) = 3.42$. For the first incorrect representation trial, $F = 6.26$, which is greater than $F.01 (2, 23 \, df) = 5.66$. Tukey’s Honestly Significant Difference analysis revealed that African American subjects were more likely than their Caucasian peers to use non-canonical representations at statistically significant levels during the initial trial with $HSD=3.11$, which is larger than $HSD 0.05= 2.92$. Caucasian subjects were more likely than African and Hispanic American subjects to make errors during the first trial at statistically significant level with $HSD=4.48$ and $HSD= 4.25$ respectively, each of which is greater than $HSD 0.01= 3.96$. In addition, these findings also suggested that the repeated demonstration by the researcher had different effects on their use of canonical representations.
DISCUSSION AND CONCLUSION

This study was conducted with two foci in mind. The first was to determine whether children in China, Romania, and U.S. had different levels of base-10 knowledge. The second was to examine whether there were differences between Chinese pre-school and first grade children and among different U.S. racial groups in light of base-10 knowledge. Our exploration is to help further scrutinize the assumptions made about the influences of structural characteristics of particular numerical languages on children’s number representations, which may, in turn, explain the variations seen in mathematics understanding and performances in international comparisons.

Findings from the comparisons of the three national groups suggested that, on the one hand, Chinese children outperformed both Romanian and U.S. children in using base-10 systems and Chinese children’s performance in this area increased substantially as they progressed from the first to the second trial. On the other hand, Romanian and U.S. children were not different from each other in using the base-10 system. Instead, they both preferred to use unit representations and even after repeated instruction, they both failed to yield any substantial gains in their base-10 task performance. These findings not only support the claim that the intuitive influence of language structure on specific thinking process is weak in the field of linguistics (Gumperz & Levinson, 1996; Rasmussen, 2006), but also challenge the assumed relationship between number-naming language and student understanding of base-10 knowledge and consequential mathematics performance in several ways.

First, the Romanian number-naming language is somewhat transparent and consistent with the base-10 system. It is intermediate between Chinese and English number-naming languages. Although such language features helped to explain why Romanian children did not perform as well as Chinese peers, it failed to explain why Romanian children did not perform better in place values and base-10 tasks than their U.S. peers who were presumably disadvantaged with their number-naming language.

Second, Chinese pre-school children tended to use substantially fewer canonical representations than their first grade peers in first trial but with repeated instruction in the second trial, they significantly increased their performance. This finding suggests that even a short period of schooling may possibly affect children performance on base-10 tasks, which leads to further questions about the possible differences that exist between preschoolers and first grade children in the Eastern Asian countries and which deserve future research exploration. The finding seems to suggest that instead of exerting influence on children’s performance in base-10 tasks in a habitual
manner, the influence of number-naming language may need to be activated by activities with a clear intention.

Third, while helping Chinese children produce more canonical representations, the instruction given in the second trial did not seem to help produce more canonical representations for the Romanian children over their U.S. counterparts. These findings are surprising considering that both Chinese preschoolers and Romanian children share some similarities in number-naming language advantages than U.S. participants. Thus, this raises the question of whether number-naming language advantages can be activated to help children perform better in using base-10 knowledge without considering the impact of other influential factors, such as family and other social influences outside of schools.

The exploration of the differences within each group also revealed some interesting findings. These findings came from the comparison among the three U.S. groups during their first trial. First, African American and Caucasian students tended to have similar levels of canonical 10 performances. Second, African American students were more likely than their Caucasian peers to use non-canonical representations. Third, Hispanic American students tended to use more canonical representations than both their African American and Caucasian peers while Caucasian students tended to make the most incorrect representations. These differences among the three is reminiscent of the findings in the 1980’s that showed that the failure of African American children to perform well in mathematics was not necessarily due to their lack of informal mathematics knowledge developed in their home or cultural environments. Rather, it may be that their informal mathematics knowledge may not be enhanced in school learning (Ginsburg & Allardice, 1984). Thus, further comparisons among the different racial groups within the U.S. context are valuable not only for verifying the language influences on mathematics learning but also for a deeper understanding about the various levels and ways of understanding base-10 knowledge among different groups.

As implied previously, one of the major limitations in our study is the use of small sample sizes. Because of this, generalization of our findings is limited. However, our study extends the same line of inquiry and provides a check on the reliability of previous studies. As well, the limitation in sample sizes is also prevalent in all of the previously conducted studies in this area that often use fewer than 100 children in each groups in the comparison. Future research should focus on greater sample sizes and more refined comparisons between and within national groups. Another limitation of these studies, also found in our present study, is the experimental treatment itself. The type of treatment used may prevent researchers from directly exploring the
relationship between number-naming language structure and mathematics. This is shown in two recent studies which reported that a simple addition of twenties blocks to the tens and ones blocks used in the experimental process (Towse & Saxton, 1997) or a change in the numbers used in the demonstration part of the treatment (Alsawaie, 2004) may cause substantial differences in the use of canonical representation for children whose number-naming languages are not consistent with the base-10 system. Thus, instead of settling the debate about the relationship between language structures and mathematics learning, this study can serve as an inspiration for future studies to use more creative research designs for more discriminating comparisons in this area.

REFERENCES


THE EFFECTS OF GRADE LEVEL, GENDER, AND ETHNICITY ON ATTITUDE AND LEARNING ENVIRONMENT IN MATHEMATICS IN HIGH SCHOOL

Thienhuong N. Hoang

ABSTRACT. The purpose of this study was to investigate different factors (grade level, gender, and ethnicity) that might affect the attitudes and learning environment perceptions of high school mathematics students in Los Angeles County, California, USA. The study involved the administration of the What Is Happening In This Class? (WIHIC) questionnaire and an attitude questionnaire based on the Test of Mathematics-Related Attitude (TOMRA) to 600 Grades 9 and 10 mathematics students in 30 classes in one high school. Quantitative research method was used in collecting information from the sample. The quantitative data were statistically analyzed using ANOVA and MANOVA. The results showed that male consistently reported slightly more positive perceptions of classroom environment and attitudes than did females. Anglo students’ scores consistently are a little higher than Hispanic students’ scores. There is strong evidence of associations between students’ attitudes and the learning environment.

KEYWORDS. Mathematics, Gender, Ethnicity, Grade Level, Attitude, Learning Environment.

INTRODUCTION

The objective of this study was to investigate factors (grade-level, gender, and ethnicity) that might affect the attitudes and learning environment perceptions of high school mathematics students in Los Angeles County, California, USA. The study involved the administration of the What Is Happening In this class? (WIHIC) questionnaire and an attitude questionnaire based partly on the Test of Mathematics-Related Attitudes (TOMRA) to 600 Grade 9 and 10 mathematics students in 30 classes in one high school. Quantitative research method was used in collecting information from the sample.

The quantitative data consisted of the administration of the questionnaire to the sample (N=600 students). The data gathered were statistically analyzed. Effect sizes, ANOVA, and MANOVA were used to determine grade-level, gender, and ethnic differences in students’
attitudes toward mathematics and their perceptions of the learning environment. Finally, simple correlation and multiple regression analysis were conducted to determine the relationship between the nature of the classroom environment as assessed with the WIHIC and students’ attitudes.

Objectives

The purpose of this study was to investigate different factors that affect the attitudes and learning environment perceptions of high school mathematics students. The specific research questions are listed below:

1. Are there differences in the learning environment perceptions and attitudes of high school mathematics students according to: a) grade level, b) gender, and c) ethnicity?
2. Is there a relationship between the nature of the classroom environment as assessed with the WIHIC and students’ attitudes?

Background

Today, minority students form a large and growing portion of the school-aged population in the United States. However, they are underrepresented at every level from elementary to graduate school. Furthermore, they are the ones who are most left out of science and mathematics. Currently, we face the potential of a serious shortfall in the number of minorities entering the fields of science and mathematics. Ultimately, the lack of preparation in science and mathematics among underrepresented minority groups in the early elementary grades undermines success rates in secondary-level school programs and in college and career choices later in life. Adequate preparation in science and mathematics enables students to develop socially and intellectually, and to participate fully in a technological society as informed citizens (Clark, 2006). However, minority students less frequently study science and mathematics.

A basic understanding of these subjects is essential for all students, not only those pursuing careers in scientific and technical fields. Minority students are depriving themselves of many career choices, including skilled technical and computer-oriented occupations as well as access to high-salaried occupations dominated by white males.

According to the National Science Foundation (2004), in 2000, racial and ethnic minorities in the USA constituted 22% of the civilian labor force, but only 14% of the science and engineering labor force. Underrepresented minorities (Blacks, Hispanics, and American Indians)
represented 19% of the total labor force and 8% of the science and engineering labor force. There are several factors that contribute to unequal participation of minorities in science and mathematics education which include the understaffed and under-equipped schools that usually are found in minority communities, tracking, judgments about ability, the number and quality of science and mathematics courses offered, access to qualified teachers, access to resources, and curricular emphasis (National Science Foundation, 2006).

As more women have decided to enter the workforce instead of playing the role of homemaker, it has been noted that approximately 85% of new entrants to the workforce in the United States have been females. However, the number of women represented in the scientific and technology professions is not proportional to the whole population of females. For instance, statistics reveal that women are underrepresented in science-related and mathematics-related careers. According to the National Science Foundation (2004), women made up a staggering 46% of the labor force in all occupations, but only 22% of the science and engineering labor force.

In considering the statistics, one wonders why these ethnic and gender differences exist in the area of mathematics and science. Could these differences be a product of schooling in America? Do educators provide all students, regardless of ethnicity and gender, with a positive learning environment that instills positive attitudes toward mathematics and science? These questions were the starting point of our study, which aimed to investigate ethnic and gender differences in learning environment and attitudes in mathematics. By investigating gender and ethnic differences, this study hopefully will clarify factors that influence students’ attitudes towards mathematics and perceptions of the classroom environment.

**Theoretical Framework**

*Learning Environments*

In the past, most research in mathematics education focused on student academic achievement. Very little attention was devoted to studying the learning environment as a determinant of learning outcomes. However, over the last 30 years, remarkable progress has been made in conceptualizing, assessing, and investigating the learning environment (Taylor, Fraser, & Fisher, 2007).

The evolution of the field of learning environments in the past three decades has seen the development of a variety of instruments that can be used to assess the classroom environment. The evolution of classroom environment instruments has facilitated learning environments
research at the secondary and post-secondary level, such as in the studies conducted by Kim, Fisher, and Fraser (2005). The literature suggests that learning environments research is needed at all grade levels. Therefore, this study assessed high school mathematics students’ perceptions of the classroom learning environment using the WIHIC and sought to investigate if grade level, gender, and ethnic differences in learning environment perceptions existed.

**Student Attitudes toward Mathematics**

The low number of women in scientific professions has become a national concern. It has been proposed that the attitude of girls toward mathematics is one factor that influences their lack of participation in science-related careers. This concern has resulted in a variety of studies designed to identify gender differences that could affect the number of girls in the scientific pipeline (Oaks, 2000). Particularly in the United States, boys hold more positive attitudes toward mathematics than do girls (Kahle, 2003; Kurth, 2007). These gender differences seem to predominate as students move from the elementary to the high school level (Kanai & Norman, 2007). For instance, research studies indicate that gender differences in attitudes toward mathematics do not exist in the elementary grades. In the middle school grades, gender differences begin to appear in attitudes toward mathematics and boys are more likely than girls to find mathematics interesting (American Association of University Women, 2002; Lockheed, Thorpe, Brooks-Gunn, Casserly, & McAloon, 2005; Oakes, 2000). By high school, few young women consider mathematics and science-related careers as desirable options. Experts attribute this phenomenon to the fact that, during the middle school years, adolescents formulate their gender identities and career aspirations (American Association of University Women, 2002; Oakes, 2000; Sadker, Sadker, & Klein, 2001). Therefore, this study aimed to investigate if gender and grade-level differences in attitudes toward mathematics exist among high school students.

**METHODS**

**Instruments**

Versions of the What Is Happening In this Class? (WIHIC) questionnaire and an attitude questionnaire based on the Test of Mathematics-Related Attitudes (TOMRA) were used in this study. The WIHIC is an instrument developed for assessing perceptions of classroom learning environments in terms of seven dimensions (Student Cohesiveness, Teacher Support, Involvement, Investigation, Task Orientation, Cooperation, and Equity). The WIHIC
questionnaire was originally developed by Fraser, McRobbie, and Fisher (1996) by combining the scales from past learning environment instruments that have been proven to be useful and significant reactors of learning outcomes with scales that are salient in modern-day classrooms such as cooperative learning and equity. The WIHIC was highly valid and reliable when used with high school students (Aldridge & Huang, 1999; Zandvliet, 2004).

I added a third scale, Student Self-Efficacy, to the attitude questionnaire to measure students’ self-concept as it relates to their mathematics ability. A sample item from the Self-Efficacy scale reads “I am good at mathematics.” This Self-Efficacy scale was based on Aldridge and Huang’s (2003) adaptation of a scale developed by Jinks and Morgan (2000).

**Research Approach**

The research approach taken in this study made use of quantitative research method. The quantitative data were collected through the administration of the two questionnaires to investigate students’ perceptions of the learning environment and their attitudes toward mathematics.

**Sample**

The data were gathered in an urban high public school located in a middle-class to upper-middle-class neighborhood in the Los Angeles County area of South California. The total number of participants in the study consisted of 600 students in Grades 9 and 10 in 30 integrated mathematics classrooms.

**Data Analysis Method**

Average item mean, average standard deviation, effect sizes (magnitude of differences), and F values from MANOVA were calculated for each of the scales of the WIHIC and attitude questionnaire utilizing the data gathered from the 600 high school mathematics students. These analyses were used to investigate differences in attitudes and learning environment perceptions of high school mathematics students according to grade level, gender, and ethnicity. To investigate the relationship between students’ perceptions of the mathematics learning environment and their attitudes toward mathematics, simple correlation and multiple regression analyses at two units of analysis were conducted (individual and class mean).
RESULTS

In order to investigate grade level, gender, and ethnic differences in learning environment and attitude scores, a three-way MANOVA was conducted. The set of dependent variables consisted of seven learning environment scales assessed by the WIHIC and the three attitude scales. The independent variables were the two-level grade-level variable (Grades 9 and 10), the two-level gender variables (female and male), and a two-level ethnicity variable (Hispanic and Anglo).

Of the 600 students in the sample, 510 students could be reliably classified as either Hispanic or Anglo. The other 90 students consisted of relatively small groups with a variety of other ethnic backgrounds (e.g. Asian, Native American, and Multiracial). However, the smallness of the number of students in these other ethnic groups made it impossible to conduct meaningful data analyses, so these 90 students were excluded from the MANOVA. Because of the MANOVA yielded significant results for the set of 10 dependant variables as a whole (using Wilks’ lamda criterion), the univariate ANOVA results were interpreted for each dependent variable. Table 1 shows ANOVA results obtained for each main effect and each criterion effect for each of the 10 classroom environment and attitude scales. Table 1 indicates that, overall, the MANOVA and 10 individual ANOVAs yielded a total of 10 cases in which the F ratio was statistically significant (p<0.05). Except for one significant ethnicity effect for Equity and a significant grade x ethnicity interaction, all of the other statically significant effects occurred for the grade effect. The results are discussed in subsections below, which are devoted to (1) grade-level differences, (2) gender differences, and (3) ethnic differences.

Table 1: MANOVA Results for Grade Level, Gender, and Ethnic Differences in Mathematics Students’ Scores on the Seven Scales of the WIHIC and Three Attitude Scales.

| Scale                  | Grade | Gender | Ethnicity | \( F \) | Grade \(
\times
\) Gender | Grade \(
\times
\) Ethnicity | Gender \(
\times
\) Ethnicity | Grade \(
\times
\) Gender \(
\times
\) Ethnicity |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom Environment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Grade</td>
<td>Gender</td>
<td>Ethnicity</td>
<td>Grade Ethnicity</td>
</tr>
<tr>
<td>Student Cohesiveness</td>
<td>10.53**</td>
<td>0.10</td>
<td>0.45</td>
<td>0.13</td>
<td>0.44</td>
<td>3.47</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Teacher Support</td>
<td>19.03**</td>
<td>0.35</td>
<td>1.18</td>
<td>0.12</td>
<td>3.94*</td>
<td>0.12</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Involvement</td>
<td>1.08</td>
<td>0.83</td>
<td>1.43</td>
<td>2.19</td>
<td>0.21</td>
<td>1.21</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Task Orientation</td>
<td>44.80**</td>
<td>0.11</td>
<td>3.04</td>
<td>0.06</td>
<td>3.70</td>
<td>0.78</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>3.44</td>
<td>0.65</td>
<td>0.03</td>
<td>0.27</td>
<td>0.10</td>
<td>0.14</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>4.37*</td>
<td>3.26</td>
<td>5.73*</td>
<td>0.40</td>
<td>2.86</td>
<td>1.33</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Investigation</td>
<td>1.26</td>
<td>2.09</td>
<td>0.70</td>
<td>1.24</td>
<td>0.11</td>
<td>2.08</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Attitude</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Grade</td>
<td>Gender</td>
<td>Ethnicity</td>
<td>Grade Ethnicity</td>
</tr>
<tr>
<td>Attitude to Inquiry</td>
<td>8.92**</td>
<td>3.28</td>
<td>2.35</td>
<td>0.23</td>
<td>1.73</td>
<td>0.35</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Enjoyment of Mathematics</td>
<td>0.20</td>
<td>1.26</td>
<td>0.71</td>
<td>0.00</td>
<td>1.41</td>
<td>2.75</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>Lessons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Grade</td>
<td>Gender</td>
<td>Ethnicity</td>
<td>Grade Ethnicity</td>
</tr>
<tr>
<td>Student Self-Efficacy</td>
<td>7.05**</td>
<td>0.36</td>
<td>2.21</td>
<td>0.28</td>
<td>0.95</td>
<td>0.22</td>
<td>1.14</td>
<td></td>
</tr>
</tbody>
</table>

*p<0.05 **p<0.01
N=298 Hispanics, 212 Anglos, 234 males, 276 females, 245 Grade 9 students, and 265 Grade 10 students.
Grade-Level Differences

Regarding grade-level differences, the MANOVA results in Table 1 show that statistically significant result ($p<0.05$) occurred for the four learning environment scales of Student Cohesiveness, Teacher Support, Task Orientation and Equity and for the two attitude scales of Attitude to Inquiry and Student Self-Efficacy.

Table 2: Average Item Mean, Average Item Standard Deviation, and Effect Size for Grade-Level Differences for Seven WIHIC Scales and Three Attitude Scales.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Grade 9</th>
<th>Grade 10</th>
<th>Grade 9 Standard Deviation</th>
<th>Grade 10 Standard Deviation</th>
<th>Difference Between Grades</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Classroom Environment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Cohesiveness</td>
<td>3.15</td>
<td>3.42</td>
<td>1.20</td>
<td>0.77</td>
<td>0.27**</td>
<td></td>
</tr>
<tr>
<td>Teacher Support</td>
<td>4.30</td>
<td>3.90</td>
<td>0.84</td>
<td>1.00</td>
<td>-0.43**</td>
<td></td>
</tr>
<tr>
<td>Involvement</td>
<td>4.05</td>
<td>3.95</td>
<td>0.76</td>
<td>0.91</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>Task Orientation</td>
<td>4.25</td>
<td>3.68</td>
<td>0.82</td>
<td>0.93</td>
<td>-0.65**</td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>4.09</td>
<td>3.93</td>
<td>0.78</td>
<td>0.92</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>3.88</td>
<td>4.02</td>
<td>0.86</td>
<td>1.01</td>
<td>0.15*</td>
<td></td>
</tr>
<tr>
<td>Investigation</td>
<td>4.03</td>
<td>3.93</td>
<td>0.78</td>
<td>0.92</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td><strong>Attitudes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude to Inquiry</td>
<td>3.95</td>
<td>4.16</td>
<td>0.86</td>
<td>0.94</td>
<td>0.23**</td>
<td></td>
</tr>
<tr>
<td>Enjoyment of Math Lessons</td>
<td>4.16</td>
<td>4.17</td>
<td>0.91</td>
<td>1.00</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Student Self-Efficacy</td>
<td>4.11</td>
<td>3.87</td>
<td>0.91</td>
<td>0.88</td>
<td>-0.27**</td>
<td></td>
</tr>
</tbody>
</table>

N=245 Grade 9 and N=265 Grade 10 mathematics students.

a Average item mean = Scale mean divided by the number of items in that scale.

* $p<0.05$, ** $p<0.01$

b Significance levels are taken from the MANOVA results in Table 1.

The interpretation of the results for grade-level differences is illustrated in Table 2, which provides the average item mean and average item standard deviation for each scale in Grade 9 and Grade 10. (The average item mean is simply the scale mean divided by the number of items in that scale, and it facilitates meaningful comparisons between scales containing differing number of items.) The last column in Table 2 reports the magnitudes of grade-level differences (as distinct from their statistical significance) in terms of effect sizes. The effect size for a scale is the difference between the Grade 9 and Grade 10 mean divided by the pooled standard deviation. It expresses a difference between grades in standard deviation units.

Finally, the statistically significance results from MANOVA in Table 1 have been included in the last column of Table 2 as significance levels. Table 2 shows that the effect sizes for the six statistically significant grade-level differences range from 0.15 to 0.65 standard deviations. These magnitudes suggest that there are some educationally noteworthy grade-level differences. Whereas there was an increase in Student Cohesiveness, Attitude to Inquiry, and Equity scores between Grades 9 and 10, there was a decline between Grades 9 and 10 on each of the other three scales (Teacher Support, Task Orientation, and Student Self-Efficacy).
Gender Differences

Regarding gender differences, Table 1 shows that MANOVA revealed no statistically significant gender differences for any of the 10 learning environment and attitude scales. Table 3, which reports average item means and effect sizes, confirms this pattern in that the magnitudes of the gender differences on the 10 scales are quite small (ranging from only 0.00 to 0.16 standard deviations). Nevertheless, although the magnitude of the gender difference is quite small for each scale, the direction of the difference is in a consistent direction. Females consistently reported slightly more positive perceptions of classroom environment and attitudes than did males.

Table 3: Average Item Mean, Average Item Standard Deviation, and Effect Size for Gender Difference for Seven WIHIC Scales and Three Attitude Scales.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Average Item Meana</th>
<th>Average Item Standard Deviation</th>
<th>Difference Between Gradesb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom Environment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Cohesiveness</td>
<td>3.29</td>
<td>3.29</td>
<td>0.99</td>
</tr>
<tr>
<td>Teacher Support</td>
<td>4.07</td>
<td>4.12</td>
<td>0.87</td>
</tr>
<tr>
<td>Involvement</td>
<td>4.03</td>
<td>3.97</td>
<td>0.82</td>
</tr>
<tr>
<td>Task Orientation</td>
<td>3.97</td>
<td>3.94</td>
<td>0.88</td>
</tr>
<tr>
<td>Cooperation</td>
<td>4.04</td>
<td>3.98</td>
<td>0.91</td>
</tr>
<tr>
<td>Equity</td>
<td>4.03</td>
<td>3.89</td>
<td>0.91</td>
</tr>
<tr>
<td>Investigation</td>
<td>4.04</td>
<td>3.93</td>
<td>0.83</td>
</tr>
<tr>
<td>Attitudes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude to Inquiry</td>
<td>4.13</td>
<td>3.99</td>
<td>0.80</td>
</tr>
<tr>
<td>Enjoyment of Math Lessons</td>
<td>4.21</td>
<td>4.13</td>
<td>0.90</td>
</tr>
<tr>
<td>Student Self-Efficacy</td>
<td>4.02</td>
<td>3.96</td>
<td>0.88</td>
</tr>
</tbody>
</table>

N=245 Grade 9 and N=265 Grade 10 mathematics students in Los Angeles County, California.

a Average item mean=Scale mean divided by the number of items in that scale.
b Significance levels are taken from the MANOVA results in Table 1.

Ethnic Differences

Tables 1 and 4 show that ethnic differences on the 10 environment and attitude scales are small (with effect sizes ranging from 0.02 to 0.22 standard deviations) and are statistically significant only for the Equity scale. However, an interesting pattern is that the direction of the small ethnic difference is consistent for all 10 scales. For each environment and attitude scale, Anglo students’ scores are a little higher than Hispanic students’ scores.
Table 4: Average Item Mean, Average Item Standard Deviation, and Effect Size for Ethnic Difference for Seven WIHIC Scales and Three Attitude Scales.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Hispanic Mean</th>
<th>Anglo Mean</th>
<th>Hispanic SD</th>
<th>Anglo SD</th>
<th>Difference Between Grades</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Classroom Environment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Cohesiveness</td>
<td>3.27</td>
<td>3.31</td>
<td>1.01</td>
<td>1.01</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Teacher Support</td>
<td>4.05</td>
<td>4.16</td>
<td>1.01</td>
<td>0.85</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Involvement</td>
<td>3.96</td>
<td>4.05</td>
<td>0.86</td>
<td>0.80</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Task Orientation</td>
<td>3.89</td>
<td>4.05</td>
<td>0.98</td>
<td>0.83</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>Cooperation</td>
<td>4.00</td>
<td>4.02</td>
<td>0.86</td>
<td>0.85</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>3.87</td>
<td>4.07</td>
<td>0.98</td>
<td>0.88</td>
<td>0.22*</td>
<td></td>
</tr>
<tr>
<td><strong>Investigation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitudes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude to Inquiry</td>
<td>4.00</td>
<td>4.13</td>
<td>0.94</td>
<td>0.86</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Enjoyment of Math Lessons</td>
<td>4.14</td>
<td>4.21</td>
<td>1.01</td>
<td>0.88</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Student Self-Efficacy</td>
<td>3.93</td>
<td>4.06</td>
<td>0.92</td>
<td>0.86</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

N=245 Grade 9 and N=265 Grade 10 mathematics students in Los Angeles County, California.

*p < 0.05

Average item mean=Scale mean divided by the number of items in that scale.

Significance levels are taken from the MANOVA results in Table 1.

Associations between each of the three attitude scales (Attitude to Inquiry, Enjoyment of Mathematics Lessons, and Student Self-Efficacy) and the seven learning environment scales were explored using simple correlation and multiple regression analyses. The simple correlation analyses provide information about the bivariate association between each learning environment scale and each attitude scale. The multiple regression analyses reduces the Type I error rate and provided a more parsimonious picture of the joint influence of correlated learning environment scales on attitudes. All analyses were conducted twice, once with the individual student as the unit of analysis and once with the class mean as the unit of analysis. Table 5 shows that the simple correlation between an environment scale and an attitude scale is statistically significant in all 21 cases with the student as the unit of analysis, and in 17 cases with the class as the unit of analysis (with the exception being the Attitude to Inquiry with Teacher Support and Task Orientation and for Student Self-Efficacy with Student Cohesiveness and Equity). In every case, statistically significant correlations are positive, confirming a positive relationship between attitude and learning environment scales.

The bottom of table 5 shows that the multiple correlation between each attitude scale and the set of seven learning environment scales is statistically significant with either the individual or the class as the unit of analysis. In order to interpret which individual learning environment scales were responsible for explaining the significant multiple correlations, the standardized regression weights in Table 5 were examined, with the following findings:
Table 5: Simple Correlation and Multiple Regression Analyses for Relationship between Learning Environment and Attitude Scales for Two Units of Analysis

<table>
<thead>
<tr>
<th>Scale</th>
<th>Unit of Analysis</th>
<th>Attitude to Inquiry</th>
<th>Enjoyment of Mathematics Lessons</th>
<th>Student Self-Efficacy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r$</td>
<td>$\beta$</td>
<td>$r$</td>
</tr>
<tr>
<td>Student Cohesiveness</td>
<td>Individual</td>
<td>0.38**</td>
<td>0.70**</td>
<td>0.35**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.55**</td>
<td>0.36</td>
<td>0.56**</td>
</tr>
<tr>
<td>Teacher Support</td>
<td>Individual</td>
<td>0.58**</td>
<td>-0.06</td>
<td>0.61**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.34</td>
<td>-0.43</td>
<td>0.59**</td>
</tr>
<tr>
<td>Involvement</td>
<td>Individual</td>
<td>0.73**</td>
<td>0.33**</td>
<td>0.70**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.61**</td>
<td>0.30</td>
<td>0.73</td>
</tr>
<tr>
<td>Task Orientation</td>
<td>Individual</td>
<td>0.58**</td>
<td>0.07</td>
<td>0.06**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.30</td>
<td>0.18</td>
<td>0.54**</td>
</tr>
<tr>
<td>Cooperation</td>
<td>Individual</td>
<td>0.71**</td>
<td>0.25**</td>
<td>0.70**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.63**</td>
<td>0.32</td>
<td>0.72**</td>
</tr>
<tr>
<td>Equity</td>
<td>Individual</td>
<td>0.61**</td>
<td>0.16**</td>
<td>0.56**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.60**</td>
<td>0.35*</td>
<td>0.44*</td>
</tr>
<tr>
<td>Investigation</td>
<td>Individual</td>
<td>6.67**</td>
<td>0.10*</td>
<td>0.68**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.58**</td>
<td>-0.06</td>
<td>0.82**</td>
</tr>
<tr>
<td>Multiple Correlation (R)</td>
<td>Individual</td>
<td>0.79**</td>
<td>0.77**</td>
<td>0.75**</td>
</tr>
<tr>
<td></td>
<td>Class</td>
<td>0.82**</td>
<td>0.88**</td>
<td>0.91**</td>
</tr>
</tbody>
</table>

* $p<0.05$  ** $p<0.01$

N=600 students from 30 classes in one high school in Los Angeles County, California.

- For Attitude to Inquiry and with the student as the unit of analysis, Student Cohesiveness, Involvement, Cooperation, Equity and Investigation were all statistically significant independent predictors of attitudes when the other environment scales were mutually controlled.

- For Attitude to Inquiry and with the class of the unit of analysis, only Equity was a statistically significant independent predictor of attitudes.

- For Enjoyment of Mathematics Lessons and with the student as the unit of analysis, Teacher Support, Involvement, Task Orientation, Cooperation and Investigation were all statistically significant independent predictors of attitudes.

- For Enjoyment of Mathematics Lessons and with the class mean as the limit of analysis, Student Cohesiveness and Investigation were statistically significant independent predictors of attitudes.

- For Student Self-Efficacy and with the student as the unit of analysis, Involvement, Task Orientation, Cooperation and Investigation were all statistically significant independent predictors of attitudes.

- For Student Self-Efficacy and with the class as the unit of analysis, only Task Orientation was a statistically significant independent predictor of student attitudes.
It is noteworthy that every statistically significant regression weight in Table 5 is positive. Overall, the results in Table 5 provide strong evidence of associations between students’ attitudes and their perceptions of classroom environment as assessed by the WIHIC.

**CONCLUSION**

The data gathered were analyzed to investigate grade level, gender, and ethnic differences in learning environments perceptions and attitudes. Additionally, the relationships between the nature of the classroom learning environment as assessed with the WIHIC and students’ attitudes toward mathematics were explored. The key findings are (1) Some educationally noteworthy grade-level differences were found. For instance, an increase in Student Cohesiveness, Attitude to Inquiry, and Equity scores between Grades 9 and 10 were found. Also, a decline between Grades 9 and 10 on Teacher Support, Task Orientation, and Student Self-Efficacy were found; (2) Small and statistically nonsignificant gender differences for each learning environment and attitude scale were found. However, males consistently reported slightly more positive perceptions of classroom environment and attitudes that did females; (3) Small ethnic differences on the 10 environment and attitude scales were found, with a statistically significant difference only on the Equity scale. However, for each environment and attitude scale, Anglo students’ scores consistently are a little higher the Hispanic students’ scores; and (4) Strong evidence of associations between students’ attitudes and the learning environment was found.

This study is of significance to the field of learning environments. First, the present research study provides further results about ethnic, gender, and grade-level differences in high school mathematics students’ perceptions of the learning environment and their attitudes toward mathematics. This is noteworthy because very few learning environment studies have looked at these three factors when studying the learning environment and/or student outcomes. Additionally, results of this research add another study to the strong and common line of past classroom environments research, namely, the investigation of outcomes-environment associations.

Most importantly, this study is significant because it provides information about how students from different grade levels, ethnicities, and gender perceive their mathematics classroom environment and what attitudes they have toward mathematics. The results are likely to be especially helpful for educators, educational policy-makers, parents, and/or community members who will be able to read the results and determine if the findings can assist them in advocating for
funding at local, state and federal venues to increase educational reform that will help narrow the
gap between grade levels, gender, and ethnicities in classroom environment perceptions and
attitudes toward mathematics.

With the increasing need for more workers in science-related and mathematics-related
careers, ensuring that more high school students, regardless of grade level, ethnicity, and/or
gender status, perceive a more positive mathematics classroom environment and have better
attitudes toward mathematics is a worthwhile goal to pursue.

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ABSTRACT. This study examined the effect of behavioral objective-based (BOBIS) and study question-based (SQBIS) instructional strategies on students' attitude towards Senior Secondary Mathematics. The three hypotheses for the study were tested at 0.05 level of significance. The issue of attitudinal changes of student in mathematics classroom is an evergreen topic which cannot be wished away. It is therefore important to search for more and simple methods/ways by which teachers could continually inspire positive attitude in mathematics classroom. The research adopted a pre-test, post-test, control group quasi experimental design. There were three treatment groups which are - two experimental groups (behavioral objective-based (group1, N=117) and study question-based (group II, N=95) instructional strategies) and a control group (group III, N=100). A total of 312 students were involved in the study. The classrooms were randomly selected in each school and all the students in the selected classroom constitute the sample (intact class). Students’ Attitude Questionnaire (SAQ) has a reliability coefficient of $r = 0.81$. Findings revealed a significant effect of treatments (BOBIS & SQBIS) on students’ attitude towards Mathematics. The result was $(F (2,311) = 72.95, P < 0.05)$. There was a significant difference in attitude between behavioural objective-based instructional strategy group and the control group with the BOBIS group having far better attitude to mathematics than the control group. Similarly, significant difference was found between the attitude of SQBIS group and the control group but no significant difference in attitude was found between BOBIS group and SQBIS group. Behavioral objective-based and Study-question-based groups were found to have similar attitude towards. In other words, there was significant differences between the attitudes of subjects exposed to behavioural objectives and control group and between those exposed to study question and the control group and no significant difference in attitude between the behavioural objective and study question groups. Both experimental groups (BOBIS and SQBIS) proved to be superior to the control group. Based on the findings, behavioral objective-based and study question-based instructional strategies were found to be viable instructional strategies that could promote positive attitude towards mathematics. The implication of the result is that teachers’ method of instruction in classroom is important in changing students’ attitude and habits towards mathematics.

KEYWORDS. Instructional Methods, Behavioural Objectives, Study Questions, Attitudes towards Mathematics.
INTRODUCTION

It is generally believed that students’ attitude towards a subject determines their success in that subject. In other words, favorable attitude result to good achievement in a subject. A student’s constant failure in a school subject and mathematics in particular can make him to believe that he can never do well on the subject thus accepting defeat. On the other hand, his successful experience can make him to develop a positive attitude towards learning the subject. This suggests that student’s attitude towards mathematics could be enhanced through effective teaching strategies. It has in fact been confirmed that effective teaching strategies can create positive attitude on the students towards school subjects Bekee (1987), Balogun and Olarewaju (1992), Akinsola (1994), Akale (1997), Olowojaiye (1999), (2000).

Attitudes are psychological constructs theorized to be composed of emotional, cognitive, and behavioral components. Attitudes serve as functions including social expressions, value expressive, utilitarian, and defensive functions, for the people who hold them (Newbill, 2005). To change attitudes, the new attitudes must serve the same function as the old one. Instructional design can create instructional environments to effect attitude change. In the greater realm of social psychology, attitudes are typical classified with affective domain, and are part of the larger concept of motivation (Greenwald, 1989d). Attitudes are connected to Bandura’s (1977) social cognitive learning theory as one of the personal factors that affect learning (Newbill, 2005).

The definition of attitude depends on the purpose of the definition. Most attitudes researchers include the concept of evaluation as the basis for the definition (e.g. Boliner & Wanke, 2002, Eagly & Chaiken, 1993). To Petty and Caciopppo (1986) attitude are general evaluations of people hold in regard for themselves, other people, object, and issues. To Greenwald (1989b), attitudes are pervasive, predict behaviors, are a force in perception and memory, and they serve various psychological functions. Though there is an ongoing debate about the structure of attitudes (Newbill, 2005), however instructional designers have long assumed that attitudes is made up of three components; a cognitive component, an emotional component, and a behavioral component (e.g., Bednar & Levie, 1993, Kamradt & Kamradt, 1991). The debate of the existence of the component structure of attitude may never be completely resolved because attitudes are constructs and are therefore not directly observable (Newbill, 2005). The measurement of attitudes is inextricably tangled with theoretical debate on the nature of attitudes.

Social psychologists has notice that people respond to objects (ideas) with different degrees of positive to negative evaluations. Responses could be affective (e.g., frown or smiling);
cognitive (e.g., stating rational thoughts) or behavioral (clapping or running away). Social psychologists conceived of a driving force behind these responses, and name it – attitude. They proceeded to measure attitude by measuring what they conceived to be the effects of it. It is important to note that all responses are technically behaviors (Ajzen, 1989).

Definitions of attitude towards mathematics are numerous as researchers’ and thinkers’ conceptions, ideas and perspectives vary. According to a point of view, the attitude towards mathematics is just a positive or negative emotional disposition towards mathematics (Zan & Martino, 2007). Hart (1989), considering attitudes towards mathematics from a multidimensional point define an individual’s attitude towards mathematics as a more complex way by the emotions that he/she associates with mathematics, his/her beliefs towards mathematics, which could be either positive or negative and how he/she behaves towards mathematics. Research on attitude in mathematics education has been motivated by the belief that ‘something’ called “attitude” plays a crucial role in learning mathematics but the goal of highlighting a connection between positive attitude and mathematics achievement has not been reached conclusively (Zan & Martino, 2007). It is therefore imperative to continue to search for linkages between instructional methods that could facilitate the development of more positive attitude towards the learning of mathematics. Hence this research.

Several studies in the area of mathematics have shown that instruction, especially at the secondary school level remains overwhelmingly teacher-centered, with greater emphasis being placed on lecturing and textbook than on helping students to think critical across subject area and applying their knowledge to real-world situation (Butty, 2001). There is a need to adopt some of the recent reform-based instructional strategies, along with some traditional practices that have been overlooked and underutilized in secondary mathematics (National Council of Teachers’ of Mathematics, 2000). Such practices include individual exploration, peer interaction, and small group work each of which emphasizes the use of multiple approaches to problem solving, active student inquiry, and the importance of linking mathematics to students’ daily life (Butty, 2001). A key component in reform is the movement from traditional to reform instructional practices in mathematics is the importance of examining the effects and relationship among types of instructional practices that student receives and their resulting achieving and attitudes towards mathematics. Studies related to instructional practices and academic achievement have suggested that the quality of teachers’ instructional messages affects children’s task involvement and subsequent learning in mathematics (Cornel, 1999, Butty, 2001). The National Council of Teacher of Mathematics (NCTM, 2000) has advocated for the development of inquiry-based
mathematics tradition. According to Fennema, Carpenter, and Peterson (1989), students who experience this reform tradition are encouraged to explore, develop conjectures, prove, and solve problem. The assumption is that student learns best by resolving problematic situations that challenge them through conceptual understanding. In the study by Stein, Grover, & Henninssen (1996), investigated the use of enhanced instructions as a means of building student capacity for mathematics thinking and reasoning concluded that students must first be provided with opportunities, encouragement, and assistance before they can engage in thinking, reasoning, and sense making in mathematics classroom. Consistent engagement in such thinking practices, they maintained, should lead students to a deeper understanding of mathematics as well as increased ability to demonstrate complex problem solving, reasoning, and communication skill upon assessment of learning outcomes. They concluded that the tasks used in mathematics classroom highly influence the kinds of thinking processes students employ, which in turn influence learning outcomes. Perhaps this is the reason why the mode of questioning in mathematics classroom becomes relevant.

It is therefore imperative for teachers to appreciate and inculcate in students positive attitude towards mathematics by using improved and appropriate instructional strategy. It is believed that the lack of specific directives has one way or the other hindered learning achievement among students.

However, behavioral objective when properly formulated and communicated to students could function to remedy the problem of effective teaching and learning of Mathematics. Since behavioral objective or related study question projects specific learning outcome, the knowledge of behavioral objective or a study question related to it can be useful in indicating to the learner what is actually required of them instead of wondering over the learning materials and as a result relevant learning achievement and attitude are promoted. Mager (1962) popularized the use of behavioral objectives in his classic on preparing instructional objectives. According to him if a learner is provided with a copy of behavioral objectives the teacher does less work. Melton (1978) had supported the use of behavioral objective by pointing out that behavioral objectives clearly indicate to students what is required of them and as a result relevant learning is enhanced. He argued that behavioral objectives and inserted questions are very much similar in that both show students what they should be able to do as a result of learning process.

Nzewi (1994) noted that teachers should no longer be satisfied with only having a statement of behavioral objectives in their lesson notes. They should also make it a point to let their students know these objectives, and if possible, the students should be given these objectives
in a written form. He also noted that teacher should refer to the objectives in the course of teaching. This seemed to be in line with Duchastel and Merril (1973) who opined that objectives would certainly make no difference if the student pays no attention to them in the learning situations. Presenting students therefore with behavioral objectives of a lesson topic or the study questions related to these objectives at the beginning of instruction can alert their sensitivity to the learning situation. Referring students to these objectives or related questions at every stage of information presentation can serve as an evaluating role for teachers teaching as well as students learning, thus, helping to promote learning and positive attitude.

In 1912, Stevens stated that approximately eighty percent of a teacher's school day was spent asking questions to students. More contemporary research on teacher questioning behaviors and patterns indicate that this has not changed. Teachers today ask between 300-400 questions each day (Leven and Long, 1981).

Teachers ask questions for several reasons (from Morgan and Saxton, 1991):
1. the act of asking questions helps teachers keep students actively involved in lessons;
2. while answering questions, students have the opportunity to openly express their ideas and thoughts;
3. questioning students enables other students to hear different explanations of the material by their peers;
4. asking questions helps teachers to pace their lessons and moderate student behavior;
and 5 questioning students helps teachers to evaluate student learning and revise their lessons as necessary.

Classroom questioning is an extensively researched topic. The high incidence of questioning as a teaching strategy, and its consequent potential for influencing student learning, have led many investigators to examine relationships between questioning methods and student achievement and behavior (Cotton, 2001)

Cotton (2001) suggested a variety of purposes for classroom questioning that include:

- To develop interest and motivate students to become actively involved in lessons
- To evaluate students' preparation and check on homework or seatwork completion
- To develop critical thinking skills and inquiring attitudes
- To review and summarize previous lessons
- To nurture insights by exposing new relationships
• To assess achievement of instructional goals and objectives
• To stimulate students to pursue knowledge on their own

As one may deduce, questioning is one of the most popular modes of teaching. For thousands of years, teachers have known that it is possible to transfer factual knowledge and conceptual understanding through the process of asking questions. Unfortunately, although the act of asking questions has the potential to greatly facilitate the learning process; it also has the capacity to turn a child off to learning if done incorrectly. (Brualdi, 1998).

Statement of the problem

Teachers often state behavioral objectives in their lesson notes when preparing to teach and give students questions to practice after teaching. They however, fail to realize that behavioral objective and study question could better be utilized to stimulate the learners for possible better learning outcomes. The study therefore, investigated the effect of behavioral objective-based and study question-based instructional strategies on students’ attitude towards mathematics.

Hypothesis:

The hypotheses below were tested at 0.05 level of significance.

H1: There will be no significant difference in attitude scores on the behavioural objective-based group posttest between students who have been given knowledge of behavioural objectives prior to instruction and students who do not have prior knowledge of such objectives.

H2: There will be no significant difference in attitude scores on the study questions-based group posttest between students who have been given knowledge of the study questions prior to instruction and students who do not have prior knowledge of such study questions.

H3: There will be no significant difference in attitude scores on the behavioural objective-based group posttest between students who have been given knowledge of behavioural objectives prior to instruction and students who are given study question prior to instruction.

METHOD

Research Design: A pre-test post-test control group quasi experimental design was employed. Two experimental groups I (behavioural objective-based group, n =) and II (study
question-based group, \( n = \) and a control group III (conventional method, \( n = \)) were used. Students in group I were exposed to behavioral objective treatment only, group II were exposed to study question treatment only while the control group students were exposed to the conventional teaching method.

**Subjects:** The subjects constituted a total of 312 (184 male & 128 female) senior secondary school two students from six co-education schools selected by using stratified random sampling technique from three Local Education District (LED) of Lagos State, that is, two schools from each LED.

The selected schools in each LED were assigned randomly to a treatment group so as to avoid interaction that may occur among the groups if two or more treatment groups are located in the same school. To avoid disrupting the school program or arrangement, intact classes (that is, students as find in the class) were used and the selection of the classes used was done in each school through simple random sampling technique (that is, a arm of the class is selected by random sampling in each school).

**Instrument:** Basically, the instrument used for the study was Students’ Attitude Questionnaire (SAQ).

A stimulus instrument (instructional guide) for the teachers was also used. The SAQ is made up of two sections, that is, section A which has to do with questions that seek for the background information about students like name of school, class, sex and age, and section B which consists of 22 items covering the students’ cognitive, affective and behavioral attitude components. Students method of response to the items was the closed response mode of 4 points scale of strongly agree, agree, disagree and strongly disagree. Scoring was therefore from 4 to 1 mark, that is, 4 marks for strongly agree, 3 marks for agree, 2 marks for disagree and 1 mark for strongly disagree of the item if positively warded. Where the item is negatively warded, scoring was in reverse order. The reliability coefficient of the instrument was established using Cronbach coefficient alpha reliability method and was found to be 0.81.

**Procedure:** The Students’ Attitude Questionnaire was administered as pre-test on students in the six schools. The senior secondary school two Mathematics teacher from each of the selected schools received training in the use of the strategy appropriate for his group for two weeks. Materials were then given to the teachers. Only the teachers for experimental group I (BOBIS) were provided with the list of behavioral objectives of the lesson topics while only the teachers for experimental group II (SQBIS) were provided with the list of study questions relating
to the behavioral objectives. The teachers for the control group were not provided with either behavioral objectives or study questions lists. Having administered the pre-test, training teachers and providing them with the necessary materials, teaching commenced and lasted for 8 weeks. For experimental group I, the teacher started lessons by presenting the list of behavioral objectives of the lesson topic to the students. While teaching, he makes use of the behavioral objectives by drawing the attention of the students to the relevant objectives where necessary.

For experimental group II (SQBIS), study questions were presented to students at the beginning of instruction and were used exactly the same way that behavioral objectives were used for group I (BOBIS). The control group III (CON) neither has the benefits of objectives nor study questions; instruction was purely the conventional type. At the end of instruction, the pre-test instrument, that is, SAQ, was used as post-test to all groups to measure the attitude that has taken place, thus marking the end of the experiment.

**Data Analysis:** The SAQ scores formed the basis of data analysis. The research hypothesis was tested by employing Analysis of Covariance (ANCOVA) with pre-test score as covariates. Multiple Classification Analysis (MCA) technique was used to detect the magnitude and direction of the difference among the groups. The Scheffe post hoc analysis procedure was also employed to determine the relationship between means of different pairs of groups and the direction of significant difference observed on the ANCOVA.

**RESULTS AND DISCUSSION**

**Hypothesis:** There is no significant effect of treatment on students’ attitude towards mathematics.

**Table 1:** ANCOVA Summary Table for Post-test Attitude Scores by Treatment with Pre-test as Covariates

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Sign of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>15126.880</td>
<td>1</td>
<td>15126.880</td>
<td>606.110</td>
<td>.000</td>
</tr>
<tr>
<td>PREATT</td>
<td>15126.880</td>
<td>1</td>
<td>15126.880</td>
<td>606.110</td>
<td>.000</td>
</tr>
<tr>
<td>Main Effects</td>
<td>3622.192</td>
<td>2</td>
<td>1811.096</td>
<td>72.568</td>
<td>.000 *</td>
</tr>
<tr>
<td>TRT</td>
<td>3622.192</td>
<td>2</td>
<td>1811.096</td>
<td>72.568</td>
<td>.000 *</td>
</tr>
<tr>
<td>Explained</td>
<td>18749.072</td>
<td>3</td>
<td>6249.691</td>
<td>250.415</td>
<td>.000 *</td>
</tr>
<tr>
<td>Residual</td>
<td>7686.848</td>
<td>308</td>
<td>24.957</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26435.920</td>
<td>311</td>
<td>85.003</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = significant at P < 0.05
Table I present’s the analysis of covariance of students’ attitude toward mathematics by treatment. The table reveals a significant effect of treatment on students’ attitude towards Mathematics (F (2, 311 = 72.568, P < 0.05). Thus the null hypothesis is rejected.

Table 2: Multiple Classification Analysis (MCA) of Post-test Attitude Scores by Treatment with Pre-test as Covariates

<table>
<thead>
<tr>
<th>Variable Category</th>
<th>N</th>
<th>Unadjusted Dev’n</th>
<th>Eta</th>
<th>Adjusted for Independent Covariates Dev’n</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group I (BOBIS)</td>
<td>117</td>
<td>1.95</td>
<td>1.14</td>
<td></td>
<td>.35</td>
</tr>
<tr>
<td>Group II (SQBIS)</td>
<td>95</td>
<td>2.48</td>
<td>3.59</td>
<td></td>
<td>.37</td>
</tr>
<tr>
<td>Group III (CON)</td>
<td>100</td>
<td>-4.64</td>
<td>-4.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple R Square</td>
<td></td>
<td></td>
<td>.709</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple R</td>
<td></td>
<td></td>
<td>.942</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Group I: Behavioral Objective-based Instructional Strategy (BOBIS)
Group II: Study Question-based Instructional Strategy (SQBIS)
Group III: Conventional Method (CON)

The related Multiple Classification Analysis (MCA) in table 2 shows that the Study Question-Based Instructional Strategy (SQBIS) group scored the highest adjusted mean score of 71.96, Behavioral Objective-Based Instructional Strategy (BOBIS) group came second with an adjusted mean score of 71.43 while the control (CON) group came last with an adjusted mean score of 64.84. The table also shows that treatment accounted for 13.64 (0.37)^2 of variation in students’ attitude towards mathematics. Since significant effect was observed, the Scheffe post-hoc analysis procedure was further carried out on the data in order to find out where the significant difference lies.

Table 3: Scheffe Post-Hoc Analysis on Post-test

<table>
<thead>
<tr>
<th>Attitude Mean Score by Treatment Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>64.8400</td>
</tr>
<tr>
<td>71.9684</td>
</tr>
<tr>
<td>71.4359</td>
</tr>
</tbody>
</table>

* indicate significant difference between pair of groups at 0.05 level of significance.

Table 3 shows that there is no significant difference between BOBIS and SQBIS groups. However, the BOBIS and SQBIS groups are significantly different from CON group. That is BOBIS > SQBIS > CON for attitude measure.
Studies investigating the relationship between instructional practices and students’ attitude toward mathematics report that classroom organization and instructional variables correlates more strongly with students achievement, while measures of teachers’ personal qualities correlate higher with students’ attitudes towards mathematics (Butty, 2001).

The result of this study indicates that the attitude of the experimental groups, that is, BOBIS and SQBIS groups are akin and significantly better than that of the control group. This implies that the two strategies have functioned to develop in students’ positive attitude towards Mathematics. The result shows that instructional method employed in the mathematics classroom play a central role in developing students positive attitude towards mathematics learning. The result gives an unequivocal support to Bekee (1987), Guat & Tel (1987), Balogun and Olarewaju (1992), Akinsola (1994), Akale (1997), Olowojaiye (1999), (2000). It however contradicts those of Ibegbulam (1980), Nzewi (1994).

By utilizing behavioral objective-based and study question-based instructional strategies on students learning outcome, the teacher has established a structural framework which helps students to organize their learning in a systematic way for more efficient study thus, reducing the time spent on irrelevances.

In this way, students were not bored with the lesson; there was that eagerness to study more. No wonder, the improvement in attitude. The knowledge of behavioral objectives or study questions may have helped the students to perceive learning as relevant and meaningful thus, fostering a positive attitude in them towards mathematics.

Since attitudes refers to those actions that results from and are influenced by emotion, consequently, the affective domain relates to emotion, attitudes, appreciations, and values. In the mathematics classroom the affective domain is thus concerned with students’ perceptions of mathematics, their feelings towards solving problems, and their attitudes about school and education in general. Pleasant experience through innovative and clearly understood instructional methods employed by the teacher will surely facilitates positive attitude toward mathematics. Personal development, self-management and ability to focus on important aspect of classroom learning are key areas which instructional delivery pattern could be used to enhance, promote and facilitate mathematics learning. Attitude cannot be easily separated from learning because they are acquired through the process of learning. Learning is a process of acquiring and retaining attitudes, knowledge, understanding, skills and capabilities (Farrant, 1994). Since learners are not
born with attitudes but instead they acquire them when they got in contact with the new world thus attitude can be learn and teachers should strive hard to develop the right attitudes in their students through various means especially instruction strategy. If learners are not assisted or encouraged to perceive positively most of the things they learning in mathematics classes, their performance will be affected. It depends entirely on the teacher to help learners develop positive attitudes towards the learning of mathematics.

IMPLICATION AND RECOMMENDATION

Evidence abounds that the conventional teaching method which is the traditional method commonly used in schools, is inadequate for improved students attitude towards Mathematics. This suggested the need to shift from the conventional method of teaching and embrace some other instructional strategies that have been found to have facilitative effect in promoting students’ positive attitude towards Mathematics. The results of this study reveals that BOBIS and SQBIS are potent to bring about the desirable attitude towards the subject, both strategies influence attitude in a similar manner and exhibited superiority over the conventional method. It is therefore suggested that the teacher can use either strategy or a combination of both to increase positive attitude towards mathematics especially as the study questions are questions related to the behavioral objectives of the lesson topics.
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WHAT'S WORTH FIGHTING FOR IN EDUCATION?
by Andy Hargreaves and Michael Fullan,  
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To meet the standards and expectations of today’s growing and rapidly changing world, it became vital to bring about significant improvements in teaching and learning within schools. However, for such educational changes to take place, it is essential that educators and the public move towards each other. What’s Worth Fighting for in Education presented by Andy Hargreaves and Michael Fullan is a provoking book in this aspect. It underlines the importance of establishing a school culture of collaboration, and building strong, open and interactive connections with communities beyond school in order to make considerable and long lasting changes in education.

The book consists of four chapters. Chapter 1 begins with explaining the need for change in schools through collaboration. It states that schools should not close their doors, and become indifferent to the kinds of living and working awaits their students in the outside world. Instead, the boundaries between school and its environment should be permeable and transparent so that children can be prepared for a wider livelihood that is satisfying and worthwhile. To do this, it is affirmed that educators must ‘go wider’ and ‘go deeper’ in their thinking and practice.
In chapters 2 and 3, the book gives details about why and how to go deeper and wider in thinking and practice. By going wider, it is meant that reframing and developing new relationships with parents, employers, universities, technology, and the broader profession with the purpose, passion and power of collaboration. By going deeper, it is meant that reaching into the heart of practice by discovering the passion and moral purpose that makes teaching and learning exciting and effective.

The last chapter of the book contains concrete strategies for teachers, governments, and parents for implementation. For instance, the book suggests teachers to make students their prime partners, to respond to parents’ need and desire as if they were their own, and to develop their profession all the time. For governments, the book advises to invest in the long term and put capacity building without considering the ideologies of left and right. Lastly, the book recommends parents not to demand for the kind of education they remember having themselves, however to call for significant improvements in education and to press government for the implementation of such kind of changes.

Overall, What’s Worth Fighting for in Education offers vigorous and reasoned arguments about the implementation of educational change, and gives messages to all concerned entities such as teachers, parents, community, and government to be proactive in this process. The message of this book is that if you are looking for positive educational change in your country that will benefit all pupils and motivate the teaching profession, do not expect the reform movements to provide it. Instead, turn to yourself, and look for what you can do for promoting educational change. In this aspect, the book tries to capture public imagination, focus on relationships, and suggest practical strategies.

The strategies offered in What’s Worth Fighting for in Education can be effective at every grade level and in any school demographic. Therefore, the book should appeal to teachers, parents, and policy makers who would like to come together as a collaborative community with the common goal of accomplishing a healthy education system.

REFERENCES

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ICMI STUDY 19: PROOF AND PROVING IN MATHEMATICS EDUCATION

Announcement and Call for Contributions

The International Commission on Mathematical Instruction (ICMI) announces its next ICMI Study: Proof and Proving in Mathematics Education.

The Study Conference will be held in Taipei, Taiwan, from May 10 to May 15, 2009.

Participation in the Conference is by invitation to the authors of accepted contributions following a refereeing process. The printed proceedings, available at the conference, will contain the accepted refereed submissions of all participants and will form the basis of the study’s scientific work. The Conference will be a working one; every participant will be expected to be active. We therefore hope that the participants will represent a diversity of backgrounds, expertise, experience and nationalities.

Call for contributions

The International Program Committee (IPC) invites individuals or groups to submit original contributions. A submission should represent a significant contribution to knowledge about learning and teaching proof. It may address questions from one or more of the study themes, or further issues relating to these, but it should identify its primary focus. The Study themes are set out in the Discussion Document which is available on the ICMI Study 19 website (still under construction but functional) http://jps.library.utoronto.ca/ocs/index.php?cf=8 (or via Google: ‘ICMI 19’).

Submissions will be a maximum of 6 pages, including references and figures, written in English, the language of the conference. Further technical details about the format of submissions will be available on the Study website.
**Important dates:**

- **By 30 June 2008:** Potential authors upload their papers to the conference website.
- **By 15 November 2008:** Potential authors receive the result of the refereeing process. Invitations to participate in the conference are sent to authors whose papers are accepted.

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