International Electronic Journal of Mathematics Education
2025, 20(4), em0859
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article

OPEN ACCESS

What do 'good' geometry lessons look like? A comparative analysis of prospective primary school mathematics teachers' images

Ana Kuzle [1]*

[1] University of Potsdam, GERMANY
***Corresponding Author:** kuzle@uni-potsdam.de

Citation: Kuzle, A. (2025). What do 'good' geometry lessons look like? A comparative analysis of prospective primary school mathematics teachers' images. *International Electronic Journal of Mathematics Education, 20*(4), em0859. https://doi.org/10.29333/iejme/17289

ARTICLE INFO

Received: 26 May 2025

Accepted: 17 Sep 2025

ABSTRACT

Research has shown that teachers' perceptions and beliefs about good teaching are closely related to their teaching style and, consequently, to effective classroom learning and student achievement. Although much research has been conducted on good mathematics teaching, its sub-discipline, geometry, is often neglected. Therefore, this longitudinal study focuses on geometry and uses participant-produced drawings to examine the images prospective primary school mathematics teachers have of good geometry lessons in terms of modes of instruction (teacher-directed, student-directed, balanced), both before and after attending a two-semester course, 'Geometry and Teaching Geometry'. The study participants were trainee primary school teachers in their second year of a mathematics teacher education program at one German university department. Trends that emerged from the study sample (N = 101) showed that most of the prospective primary school mathematics teachers' perceptions of what makes a good geometry lesson were aligned with teacher-directed mode of instruction, with the two-semester geometry course prompting a move towards a student-directed mode of instruction. Thus, the findings indicate that carefully designed university teacher training courses may stimulate changes in the belief systems of prospective primary school mathematics teachers, contributing to a more modern understanding of geometry teaching and learning.

Keywords: geometry lessons, images, mathematics, mode of instruction, prospective primary school teachers

INTRODUCTION

The widely cited meta-analysis study by Hattie (2009) shows that teachers make an important contribution to students' learning outcomes and have a decisive role in designing good lessons. Research (e.g., Corbett & Wilson, 2002; Thomas, 1998; Weinstein, 1989) has shown that good teachers possess important intrapersonal qualities such as patience, empathy, and kindness. Profound content knowledge, the ability to explain content understandably, professional background and teaching experience have also been shown to be crucial (Strickland et al., 1986; Voss et al., 2015).

In a (mathematics) classroom, the teacher guides students' acquisition of knowledge through the purposeful planning and design of lessons, which can be characterized by the prevailing teaching pedagogies (modes of instruction,[1] learning environment, teaching and learning philosophies). The construct of *modes of instruction* refers to the way teaching and learning take place concerning the teacher's role in the teaching-learning process (Gulek, 1999; Hatisaru, 2019; Rott et al., 2023; Sinclair et al., 2013; Utley et al., 2020). In a classroom, the mode of instruction can range from strictly teacher-directed to primarily student-directed. This is dependent on whether it is the teacher or the student who plays a primary role in the teaching-learning process (Gulek, 1999; Segall & Wilson, 1998; Sinclair et al., 2013; Utley et al., 2020), respectively, or can fall somewhere in between these two polarities (Sinclair et al., 2013). Regardless of the mode of instruction, the teacher's role is to support students as a learning guide or facilitator, to keep them motivated, and to support them both emotionally and professionally in their mathematics learning (Kuzle, 2021). Thus, the institution of school has a significant impact on children's cognitive and emotional development (Evans et al., 2009; Kuzle, 2021), highlighting the importance of the mode of instruction.

Although aspects of student-directedness are taught in most teacher education programs, this may not be the way prospective teachers have been trained as students (Sinclair et al., 2013). According to Calderhead and Robson (1991), a respected teacher reinforces what a prospective teacher thinks is a good teacher and good teaching. Thus, a prospective teacher's image of a 'good' or 'bad' teacher and classroom is shaped by years of personal classroom experiences as a student. Therefore, it is not surprising that "one of the primary goals of a teacher preparation program is to teach individuals how to become good teachers" (Murphy et

[1] In this paper the construct 'mode of instruction' is used throughout.

al., 2004, p. 69). The present study aims to contribute to research on the effect of a teacher training program on prospective primary school mathematics teachers' perceptions and already-established belief systems about good (mathematics) teaching. According to Brunner et al. (2006), the training of good teachers requires not only practical experience but also a dynamic interaction between different types of professional knowledge, beliefs, motivational orientations, and self-regulatory skills.

Despite the increased research in the field of primary school geometry in the past decade (Kuzle, 2023; Sitter, 2019; Wiese, 2016), there are still many research gaps, especially concerning good instructional practices in school geometry. Therefore, the focus here is on the understanding of good teaching and learning in primary school geometry from the perspective of prospective primary school mathematics teachers who were enrolled in their second year of a mathematics teacher education program. The objectives of the study are twofold:

(1) to examine prospective primary school mathematics teachers' perceptions of the mode of instruction in good geometry lessons prior to attending a two-semester course titled 'Geometry and Teaching Geometry' and how this change after attending the course, and

(2) to evaluate the extent to which the two-semester geometry course stimulated changes in the belief systems of prospective primary school mathematics teachers.

THEORETICAL BACKGROUND

The following section first examines the construct of modes of instruction, followed by modes of instruction in good mathematics lessons, then research-based criteria for good geometry lessons are discussed. The section ends with the two research questions that guided the study.

Modes of Instruction

The current state of research shows that teaching quality is measured above all by whether student learning processes are initiated and to what extent these are sustainable (Helmke, 2009). Hence, 'good' teaching is equated with effective teaching practices. The school institution, and thus also teaching, is subject to constant change. This is due to the development of scientific and pedagogical theories, which consequently influence the teaching practices in the classroom (Domović, 2003; Rathgeb-Schnierer et al., 2023). The current development is characterized by a move away from traditional teaching (teacher-directed lessons) to a modern constructivist type of teaching (student-directed lessons) (Domović, 2003; Kuzle et al., 2023; Rathgeb-Schnierer et al., 2023). Teacher-directed lessons are reduced to the reproduction of knowledge and its acquisition through drill, whereas student-directed lessons emphasize orientation towards the students through multilateral communication, and active forms of learning by employing diverse classroom resources (Sinclair et al., 2013). This shift consequently influences teachers' actions and activities in supporting students in their learning process (Organisation for Economic Co-operation and Development [OECD], 2010, 2016). The role of the teacher resulting from changes in the mode of instruction serves as the primary definition of modes of instruction and their categorization into three main categories (Hatisaru, 2019; Sinclair et al., 2013).

A teacher-directed classroom

A teacher-directed classroom takes a traditional approach to teaching and learning (Sinclair et al., 2013). It is based on the systematic preparation of subject-specific content (Reiss & Hammer, 2013), which is conveyed by the teacher, who is at the forefront of all teaching and learning processes (Dole et al., 2016; Sinclair et al., 2013). The teacher plays an active part in the teaching process and takes over the students' work, whereas the learners find themselves in a passive role as recipients of the lesson content (Dole et al., 2016). The main task of the teacher is to impart knowledge to the students and to discipline them (Sinclair et al., 2013), resulting in a time-efficient and error-avoiding classroom environment (Reiss & Hammer, 2013). As the overarching goal is focused on increasing the subject-specific knowledge (Sinclair et al., 2013), independent thinking and social learning processes are barely addressed or are neglected entirely (Fromm, 2017; Trigwell et al., 1999). A teacher-directed classroom is characterized by explanations and demonstrations by the teacher and by questions that are mostly closed (Fromm, 2017; Hatisaru, 2019). The use of examples in contrast to counterexamples is a prototypical feature of this teaching principle (Fromm, 2017; Hatisaru, 2019), as is the primary use of blackboards and textbooks as teaching resources (Sinclair et al., 2013). The teacher's questions are a component of teacher-directedness, whereas questions from the students are considered undesirable (Sinclair et al., 2013). Furthermore, the students' thoughts have no influence on what happens in the classroom, and the teacher exerts motivational control over the students through reward and punishment systems (Crain, 2000; Sinclair et al., 2013). The teacher is most often positioned at the blackboard or the teacher's desk, and the students sit in rows with little, if any, interaction between the students and teachers (Gulek, 1999). Furthermore, Gulek (1999), and Fromm (2017) emphasize that learners primarily work individually and receive very little support in the learning process.

A student-directed classroom

A student-directed classroom takes a constructivist approach to teaching and learning. It is characterized by a stronger emphasis on the active role of learners and their learning processes (Reiss & Hammer, 2013; Sinclair et al., 2013), and thus a counterpart to the teacher-directed approach (Aktas, 2010; Hatisaru, 2019; Reiss & Hammer, 2013). The learning processes are designed and initiated by the students (Sinclair et al., 2013) with teachers taking on a moderating role as facilitators of students' learning and activities (Aktas, 2010; Reiss & Hammer, 2013; Sinclair et al., 2013). Typical for student-directed classrooms are social and playful forms of learning in which students work and learn together in groups, student presentations (Hatisaru, 2019), as well as open-ended, problem-solving, and project-oriented work (Dole et al., 2016). Furthermore, students are encouraged to question

facts, make conjectures, and discover learning objects on their own, which helps them become proficient in mathematics (National Council of Teachers of Mathematics, 2014) as they are given an active part in class discussions (Gulek, 1999). Unlike in a teacher-directed classroom, the students' desks are clustered (e.g., group tables), and the lessons are characterized by ongoing student-student and student-teacher interaction (Gulek, 1999). In the classroom, the teacher positions himself or herself within reach of the students so that they can experience continuous support during active and collaborative learning (Gulek, 1999).

In their pure form, both approaches have strengths and weaknesses. Educators (Meyer, 2019; Reinmann & Mandl, 2006; Reiss & Hammer, 2013; Sinclair et al., 2013), therefore, recommend a combination of instructional practices that emphasize active and social learning which is supported with targeted instruction from the teacher to achieve optimal student learning success in the classroom. Meyer (2019), for instance, stated that "mixed forest [...] is better than monoculture" (p. 9). Balanced learning is recognized as a teaching and learning approach of modern, thoughtful, and responsible teaching (Sinclair et al., 2013). It is understood as a link between the traditional teacher-directed approach and the constructivist student-directed approach. This is reflected in lesson planning, which takes into account the current state of research and allows students to benefit from new knowledge as well as new procedures and technologies in their learning process (Sinclair et al., 2013). As a combination of the best practices of these two modes of instruction, balanced learning meets both Vygotsky's theory of the zone of proximal development, which is the basis for teacher-directedness, and Dewey's theory of collaborative and self-regulated learning, which is the basis for student-directedness (Sinclair et al., 2013).

Mode of Instruction in Good Mathematics Lessons

A comparison of the studies focusing on students' and teachers' perspectives on good (mathematics) teaching revealed a discrepancy between the actual and the preferred mode of instruction. Picker and Berry (2000) pointed out that students often experience a teacher-directed mathematics classroom. Hatisaru (2019, 2020) came to a similar conclusion, where Turkish lower secondary students' drawings predominantly illustrated a teacher-directed mode of instruction in mathematics lessons. Rott et al. (2023) conducted a similar study with 104 Grade 4 German students. The criterion-based analysis of the drawings revealed a recessivity of the characteristics related to a student-directed classroom. Additionally, the criteria relating to teacher-directedness were represented in almost all points with exception of the representation of learners in all drawings (Rott et al., 2023). Picker and Berry (2000), Hatisaru (2019, 2020), and Rott et al. (2023) pointed out that, from the students' perspective, desirable student-directed teaching rarely took place. Instead, teacher-directed practices seemed to primarily exist, manifesting in various factors (Hatisaru, 2019, 2020, 2022). However, it is not only educational researchers who call for teaching in the sense of the (social) constructivist understanding of teaching and learning; students themselves also described good teaching as student-directed (Murphy et al., 2004).

In addition to investigating Grade 2 students' perspectives, the study by Murphy et al. (2004) used complementary forms of data (i.e., survey data, drawings, and open-ended questions) to examine pre-service and in-service teachers beliefs about the characteristics of good teaching and the developmental aspects of such beliefs. The results showed that both the students and the (prospective) teachers preferred active learning situations in which students worked independently, and the teacher supported them. Sinclair et al. (2013) examined the type of perceptions undergraduates, interns, alternatively certified, and graduate-level students in teacher education courses in the US held about classroom teachers and teaching. The analysis of 150 drawings showed a dominance of balanced learning with 50%, followed by teacher-directedness with 39% (Sinclair et al., 2013). Only 11% of the drawings illustrating graduate students' mental models of teachers and teaching suggested student-directed teaching (Sinclair et al., 2013). A similar picture emerged in the study by Utley et al. (2020), with nearly two-thirds of prospective elementary teachers exhibiting beliefs aligned with teacher-directed approaches to teaching mathematics at the end of their teacher education program.

Even though in studies by Murphy et al. (2004) and Sinclair et al. (2013), the question of the extent to which such student-directed types of instruction were used in the classroom remained unanswered, they suggested prevailing subjective attitudes toward good teaching, which also influence teachers' actions in the classroom. In other words, the described stability of teacher beliefs has proved to be problematic concerning the existing beliefs of in-service mathematics teachers, who often have an unfavorable belief system that is difficult to change (Swars Auslander, 2007). According to Kelkel and Peschel (2023), many prospective mathematics teachers have a traditional understanding of teaching, which has usually been shaped by personal, often instructional, teaching experiences during their own school years.

Despite the difficulties in changing already established belief systems, there are indications that university teacher training programs can be important starting points for potential change (Debreli, 2012; Kelkel & Peschel, 2023; Murphy et al., 2004). Approaches that encourage students to actively engage with their own beliefs appear particularly promising (Debreli, 2012; Kelkel & Peschel, 2023; Rumpf & Schöps, 2013; Safrudiannur et al., 2021; Steinmann & Oser, 2012; Swars Auslander, 2007). Rumpf and Schöps (2013) explain that the "profound and memorable experiences gained in university teaching facilitate the transfer to one's future school practice"[2] (p. 34).

In a long-term study, Swars Auslander (2007) examined the pedagogical beliefs of 103 prospective elementary mathematics teachers and their development during a two-year training program. The training program consisted of several mathematics courses covering the mathematical content areas of arithmetic, geometry and statistics, mathematical methods, and methodology courses. The latter emphasized a constructivist approach to teaching and learning in which students were able to transfer newly learned methods to practical teaching and learning situations. The results showed a significant change in the prospective mathematics teachers' pedagogical beliefs toward a social constructivist understanding of teaching and learning

[2] Translated from German to English by the author.

according to Wygotski (1987) by the end of their training program. They viewed the role of the teacher as a learning facilitator who supports students in constructing knowledge rather than simply presenting the content, as well as the notion that children can construct their mathematical knowledge instead of just being recipients of knowledge (Swars Auslander, 2007). Likewise, they sought that mathematics skills should not be taught in isolation but in relation to understanding and problem solving. Here, method courses immersed prospective teachers in a reform-oriented approach to teaching and learning, and their field placements provided opportunities to implement these practices (Swars Auslander, 2007). Additionally, Swars Auslander (2007) also saw the role model function of the mathematics teacher educators' student-oriented forms of teaching and learning as a contributing factor to the change process.

Criteria for Good Geometry Lessons

In a subject-specific view of mathematics teaching, where there is growing interest in research in geometry (Kuzle, 2023; Sitter, 2019; Wiese, 2016), high-quality teaching is achieved when it enables learners to grasp and understand everyday life situations mathematically, and to solve real-world problems (Vollrath & Roth, 2012). Three central pillars that deal with problem solving, mathematical communication, and argumentation form the competence-related core of geometry lessons (Franke & Reinhold, 2016). Consequently, continuous interaction between the students and the teacher is essential to acquire the principles of geometry teaching through the linguistic training of the learners by way of explanations, but also through justifications and discussions (Radatz & Rickmeyer, 1991).

Köhler (1998) summarizes the characteristics of good geometry lessons in elementary education in eight central quality criteria. In a good geometry lesson, the students are involved in activities through which they develop enactive representations for the learning objects using different materials and manipulatives (Köhler, 1998). Kuzle (2021) emphasized the relevance of action orientation and discovery learning in geometry lessons as an opportunity for children to learn across disciplines as well as to be active participants in the teaching-learning environment. Similarly, Radatz and Rickemeyer (1991) emphasized action orientation in geometry lessons as this teaching principle encourages students to be active in the teaching-learning process. Furthermore, they emphasized the adoption of an exploratory attitude toward geometry, enabling students to construct their geometric knowledge, which is reflected in discovery-based learning as an underlying concept of school geometry. In addition to the use of measuring and drawing tools in the drawing/construction tasks, playful forms of teaching and learning are emphasized for the initiation of targeted geometric actions (Köhler, 1998).

While the design of the lesson plays a central role, the choice and linking of lesson content according to the fundamental ideas of geometry should not be neglected, as these emphasize the coherence and diversity of the content (Köhler, 1998; Kuzle et al., 2023). The understanding of visually conveyed information that supports the learning process can be promoted by employing drawings, blackboard illustrations, or topic-specific posters, which can simultaneously provide linguistic support and promote the construction of geometry concepts (Köhler, 1998). To do justice to all learners with their individual needs and levels of knowledge, learning in a student-directed classroom with partial and targeted support from the teacher appears to be the most effective (Alfieri et al., 2011; Reinmann & Mandl, 2006; Reiss & Hammer, 2013; Streit & Royar, 2014). In summary, good geometry lessons comprise various characteristics that are made up of multiple facets and in which the focus is always on the learners, and therefore, the teaching principle of student-directedness is the most effective (Radatz & Rickmeyer, 1991).

Research Questions

This study examined prospective primary school mathematics teachers' perspectives on good geometry lessons using participant-produced drawings. As discussed previously, there is little research on good teaching that focuses on school geometry. Most research relates to mathematics in general, although even here there is a lack of consistency, as shown in the results of studies by Murphy et al. (2004), Sinclair et al. (2013), Swars Auslander (2007), and Utley et al. (2020). Furthermore, in-service teachers' belief systems are shaped by years of personal classroom experience as students. However, teacher training programs can influence prospective teachers' perceptions and already-established belief systems (Kelkel & Peschel, 2023; Swars Auslander, 2007). With these goals in mind, the following two research questions guided the study:

RQ1 What images do the prospective primary school mathematics teachers have of good geometry lessons at the beginning and at the end of the two-semester 'Geometry and Teaching Geometry' course, as illustrated in their participant-produced drawings?

RQ2 How do the prospective primary school mathematics teachers' images of good geometry lessons change after attending the two-semester 'Geometry and Teaching Geometry' course?

METHOD

Research Design and Subjects

For this study, an explorative longitudinal mixed-methods research design (Patton, 2002) using participant-produced drawings (Kearney & Hyle, 2004) was chosen. The study participants were prospective primary school mathematics teachers (Grades 1 to 6) who were in their second year of a teacher education program at a large German university. This group of prospective teachers was optimal for the purposes of the study. They were chosen because they took two geometry courses in the first semester ('Geometry and Teaching Geometry in Early Childhood Education' and 'Geometry and Teaching Geometry 1'), and a further geometry course in the second semester ('Geometry and Teaching Geometry 2') of their second year. Of 117 prospective primary school mathematics teachers who took the course, 101 were included in the study. In terms of gender, there were 90

female (89.1%) and 11 (10.9%) male students, which reflected the usual gender ratio in the mathematics primary teacher education program.

Data Collection Instruments and Procedures

Drawing assignment

Drawings as a visual research method are one of the crucial data collection tools, gaining on their importance over the last two decades due to richness, and the quality of the data providing a holistic insight into participants' everyday lives (Einarsdóttir, 2007). In mathematics education, drawings have been successfully used in various research areas, such as to explore participants' beliefs about mathematics and mathematicians (Lin, 2018; Murphy et al., 2004; Picker & Berry, 2000), perceptions of the classroom climate (Kearney & Hyle, 2004; Kuzle, 2021; Kuzle et al., 2023), and perceptions of mathematics lessons (Aktas, 2010; Hatisaru, 2019, 2020). Despite the many advantages of using drawings as a data source, limitations have also been identified in literature. For instance, participants may find it difficult to draw, dislike drawing, or draw objects that are easy to illustrate (Kuzle et al., 2023). Furthermore, the analysis of drawings is challenging as the researcher must interpret the meanings the drawer gave to the depicted situations and objects (Kearney & Hyle, 2004). In order to avoid the researcher's own interpretation, data triangulation is strongly advised (Kearney & Hyle, 2004).

The study data were collected at two different measuring points, namely at the beginning of and after completing the two-semester course 'Geometry and Teaching Geometry' using participant-produced drawings (Kearney & Hyle, 2004). At each measuring point, the participant-produced drawings consisted of a drawing (1), and a short descriptive text of the drawing (2). Regarding (1), the participants were given the following instructions:

> What image do you have of a good geometry lesson? On a blank sheet of A4 paper draw a classroom with a teacher and students who are working on something, as you imagine what a good geometry lesson should look like. Colored pens are permitted.

After the drawing assignment, the participants were asked to describe their illustrations in writing (2) by answering the following three questions:

1) In what ways does your drawing represent a good geometry lesson?
2) Why did you include these elements in your drawing? and
3) Is there anything you did not draw, but still want to say about a good geometry lesson?

The instructions for the drawing assignment were based on those in similar studies where the focus was also on individuals' perceptions of teachers and/or teaching (Gulek, 1999; Hatisaru, 2019, 2020, 2021, 2022; Kuzle, 2019, 2021; Kuzle et al., 2023; Murphy et al., 2004; Sinclair et al., 2013). While the participant-produced drawings' methodology (Kearney & Hyle, 2004) uses drawings as a catalyst for a semi-structured interview, this study instead used descriptive texts. Their purpose, as with a semi-structured interview, was to increase the validity of the results (Einarsdóttir, 2007; Utley et al., 2020).

Context of 'Geometry and Teaching Geometry' course

As outlined earlier, the participants were enrolled in two geometry courses during the study, each consisting of a lecture and an exercise. The lectures and exercises in the 'Geometry and Teaching Geometry in Early Childhood Education' course were given every two weeks for one semester. The ones in 'Geometry and Teaching Geometry' were given weekly for 15 weeks per semester. In both courses, subject-specific and pedagogical content on school geometry was almost equally conveyed. The lectures focused on subject-specific and pedagogical aspects as well as practical exercises (e.g., analysis of schoolbooks and student work) and thought experiments. Both were taught in close relation to each other. The exercises deepened the content and pedagogical content knowledge presented in the lectures, in particular by using various analog and digital materials and by working with relevant learning manipulatives, giving the participants the opportunity to experience them from the learner's perspective.

The underlying concept of both courses was for the prospective teachers to experience a constructivist approach to teaching and learning geometry. This was achieved especially in terms of being able to transfer newly learned methods to practical teaching and learning situations. Furthermore, they were given opportunities to model student-oriented forms of teaching and learning in both the lectures and exercises. Through the practical, application- and action-oriented design of the exercises, which were mostly structured as group work, the students were additionally given the opportunity to develop the action skills needed for their future careers.

The data collection was conducted as part of the weekly homework assignments in the two-semester course. In the winter semester, the students received a preparatory homework assignment via a learning platform one week before the start of the 'Geometry and Teaching Geometry 1' course, which was to be submitted prior to the first lecture. At the end of the summer semester, the students received a similar worksheet with the same tasks for their final homework assignment after completing the second part of the course. Due to the presentation of very personal ideas and concepts, the students' work was anonymized. For this second submission, the code was supplemented depending on the completion of the 'Geometry and Teaching Geometry in Early Childhood Education' course in order to determine which geometry courses the students had already attended. Students who had completed the two-semester 'Geometry and Teaching Geometry' course as well as 'Geometry and Teaching Geometry in Early Childhood Education', and submitted both homework assignments on time, were matched in pairs using their anonymized code, resulting in a total of 101 students.

Data Analysis

Operationalizing indicators of modes of instruction

The drawings were analyzed after all the data had been collected using an analytical tool, which was based on the works of Gulek (1999), Hatisaru (2019, 2020, 2022), Lin (2018), Sinclair et al. (2013), and Rott et al. (2023). It comprised the characteristics of the manifestations of the three modes of instruction (i.e., teacher-directed, student-directed, balanced), forming the basis for category analysis of the drawings on the continuum from high teacher-directedness, moderate teacher-directedness, balanced, moderate student-directedness to high student-directedness. It is important to note that the two outer gradations of high teacher- and student-directedness do not represent the poles of the continuum in the sense of exclusive teacher- or student-directedness (Bohl & Kucharz, 2010). A strong teacher- or student-directed approach may also contain individual characteristics of the other mode of instruction. The teacher-directed approach is characterized by the focus on the teacher as the person designing the lesson and the rather passive role of the learners (Reiss & Hammer, 2013). As the two teacher-directed gradations do not exclusively contain teacher-directed characteristics, student-directed aspects (e.g., active learning) can also be noted the closer one moves along the continuum towards the student-directed side. Furthermore, the two gradations of student-directedness may also contain individual characteristics of teacher-directedness. For example, the teacher acts as a learning guide compared to purely student-directed instruction without learning support. The continuum of the modes of instruction is illustrated in **Figure 1**.

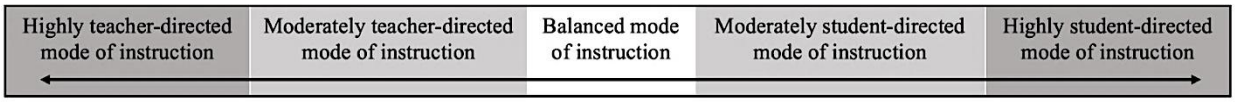

Figure 1. Modes of instruction on a continuum (Source: Author's own elaboration)

The continuum of the modes of instruction was used as the basis for the development of the analytical tool by considering the following aspects in each drawing: teacher activity, student activity, teacher-student interaction, student-student interaction, position of the teacher, and classroom and environment physical appearance (see **Table 1**). For all six aspects, characteristic features of the two categories, namely teacher- and student-directedness, were formulated based on previous work (Gulek, 1999; Hatisaru, 2019, 2020, 2022; Lin, 2018; Rott et al., 2023; Sinclair et al., 2013). The third category, which captured balanced learning (i.e., a mixed form of teacher- and student-directedness), as in the study of Lin (2018), was not added to the coding guide. The mixed form arose when aspects of both teacher- and student-directedness were present in the drawings and, therefore, did not require a separate column in the analytical tool.

Concerning indicator (1) *teacher activity*, the role of the teacher as a lesson designer or learning facilitator (Lin, 2018), was expanded by the specific activities of the teacher (e.g., presenting lesson content) according to Hatisaru (2019), and Reiss and Hammer (2013).

Indicator (2) *student activity* described the active learning of students on the part of student-directedness (Lin, 2018), and is also supplemented by concrete examples (e.g., handling materials) (Franke & Reinhold, 2016). On the part of teacher-directedness, the role of the information recipient (Lin, 2018) was replaced by passive learning as the counterpart to active learning, according to Winter (1991) and Gulek (1999). Passive learning was evident, for example, in the lack of materials shown or the students just listening (Gulek, 1999; Winter, 1991).

The indicator (3) *teacher-student interaction* was characterized differently from Lin (2018), due to contradictory study findings. Lin (2018) assumed that there is no teacher-student interaction in teacher-directed lessons unlike in student-directed lessons. However, the study by Utley et al. (2020) showed that teacher-student interaction was present in both teacher-directed and student-directed lessons. Reiss and Hammer (2013) and Swars Auslander (2007) outlined differences in the initiation of teacher-student interaction and in the proportion of speaking. While in teacher-directed lessons the teacher-student interaction was predominantly initiated by the teacher, for example, in the classic question-answer pattern, and the teacher's speaking time increased by the presentation of the learning content, the teacher-student interaction in student-directed lessons was primarily initiated by the students (Hammer, 2013; Swars Auslander, 2007). The proportion of speaking time is more balanced or higher on the part of the students, for example, when they report on findings.

The findings of Reiss and Hammer (2013) and Nührenbörger and Verboom (2005) served as a basis for the additional indicator (4) *student-student interaction*. Compared to teacher-directed lessons, in student-directed lessons, there are opportunities for students to interact with each other, for example, in the form of partner and/or group work.

Indicator (5) *position of the teacher* was characterized in a similar way to the coding guide of Lin (2018). With regard to teacher-directedness, the description was supplemented by concrete examples of the teacher's position (e.g., at the blackboard) (Gulek, 1999; Hatisaru, 2019).

Indicator (6) *classroom and environment physical appearance* was also largely adopted from Lin (2018). It was expanded with regard to the category of student-directedness so that not only group tables, but also other open seating arrangements (e.g., sitting circle and chair circle) could be recorded (Hatisaru, 2021).

Table 1 provides a detailed description of the attributes relevant to identifying the modes of instruction illustrated in the drawings, pertaining to all six instructional indicators.

Table 1. Analytical tool for analyzing the modes of instruction

Indicators	Teacher-directedness (Result: -1)	Student-directedness (Result: 1)
Teacher activity	The teacher acts as a lesson organizer and presents the lesson content.	The teacher acts as a learning facilitator and supports the students.
Student activity	Passive learning is evident (e.g., through the lack of learning materials, students sit at desks working on their assignments). Traditional learning tools are used, e.g., paper/worksheet, pencil, ruler.	Active learning is evident (e.g., through the handling of learning materials).
Teacher-student interaction	The teacher-student interaction is predominantly initiated by the teacher (e.g., by asking questions, directing, or disciplining the students). The teacher's speaking part predominates. No interaction with students.	The teacher-student interaction is predominantly initiated by the students (e.g., through questions, insights). The proportion of speaking is balanced or higher on the part of the students.
Student-student interaction	There is no social exchange between the students (e.g., individual work).	Social exchange between the students is apparent (e.g., partner or group work).
Position of the teacher	The teacher stands away from the students (e.g., near the blackboard or the teacher's desk or in front of the class, or in the center of the classroom).	The teacher is close to the students or among the students.

Scoring procedure

The data analysis followed a similar procedure as in the study of Lin (2018). Each drawing was assigned a score of -1 for each teacher-directedness indicator or 1 for each student-directedness indicator. If characteristics of both teacher- and student-directedness were illustrated, the indicator was assigned a 0. Similar to the results of Hatisaru (2019), the students' short descriptive texts were used to confirm or refute the decision to include them in the selected category and thus generate more reliable results. If an indicator could not be determined based on the drawing, the corresponding short descriptive text was consulted. If no further information was obtainable from the text, the indicator was assigned a dash and was therefore not included in the analysis (n = 14 pre-drawings, n = 11 post-drawings). This was most often the case with the indicator 'teacher-student interaction' (n = 22). In a few cases, the indicators 'teacher activity' (n = 9) and 'position of the teacher' (n = 7) could not be determined. The assigned scores were then added up to a total score, which ranged from -6 to 6. The gradations, according to Gulek (1999), were assigned to the corresponding scores based on Lin (2018). In the range from -6 to 6, high teacher-directedness comprised the score -6, -5, and -4, moderate teacher-directedness -3, -2, and -1, balanced learning 0, moderate student-directedness 1, 2, and 3, and high student-directedness 4, 5, and 6.

Using the analytical tool, the author of the paper and another researcher analyzed half of the drawings to determine the mode of instruction that was depicted in each drawing. Results were compared, and interrater agreement was high (89% agreement). The inventory was discussed to obtain full agreement, which led to adding and/or refining descriptors for each aspect. The final interrater reliability was high (100% agreement). Thus, analyst triangulation contributed to the verification and validation of qualitative analysis (Creswell & Miller, 2000; Patton, 2002) concerning measuring the appearance of the modes of instruction in good geometry lessons on the continuum from a teacher-directed to a student-directed classroom. Subsequently the descriptive statistics were calculated to answer both research questions.

Figure 2 and **Figure 3** exemplary illustrate the coding of the depicted mode of instruction. The drawings do not represent prototypical drawings but rather have been selected based on data richness, versatility, and coding difficulties. Specifically, **Figure 2** was selected because the drawn elements can clearly be assigned to all six indicators, making it particularly suitable for explaining the coding. Concerning aspect: (1) teacher activity, the teacher in the drawing is shown as a learning facilitator because she is turned toward the students. This is also stated explicitly in the descriptive text. Regarding aspect (2) student activity, active learning is evident. The students interact with each other during group work and engaging with the geometry manipulatives (e.g., body models). The teacher-student interaction (3) is initiated by the students; in the drawing, a student is seen reporting, and, according to the short description, the teacher takes on an observing and supporting role. Concerning student-student interaction (4), it is clear from both the drawing and the description that the students are interacting with each other. For example, the students at table 3 are facing each other. The teacher is depicted close to the pupils, which shows the position of the teacher (5). Lastly, the classroom and environment physical appearance (6) shows tables that are arranged in groups. Thus, indicators for each aspect reflect a student-directed classroom. According to the continuum of modes of instruction, the figure was given a score of 6, which indicates a highly student-directed mode of instruction.

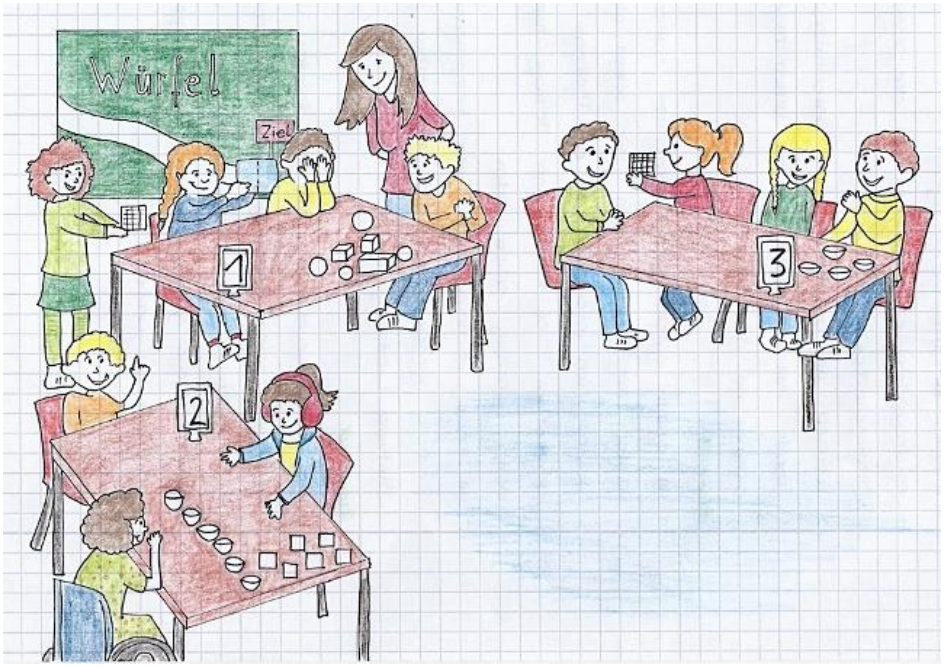

Figure 2. Exemplary coding of a highly student-directed mode of instruction (Score 6) (Source: Field study)
Note. Score 6: teacher activity +1, student activity +1, teacher-student interaction +1, student-student interaction +1, position of the teacher +1, classroom and environment physical appearance +1.

Figure 3 illustrates difficulties in coding, given that not all aspects could be identified, and how these difficulties were resolved. Due to the absence of a teacher in the drawing, no assignment could be made for three indicators: (1) teacher activity, (3) teacher-student interaction, and (5) teacher position. Concerning indicator (2), student activity, the drawing shows a variety of geometric teaching materials that are actively used by the students. This was reaffirmed in the short descriptive text, which contained a detailed description of the individual stations depicted in the drawing. Concerning student-student interaction (4), the drawing shows both individual work (e.g., a student at station 2 folds a geometric body), and partner work (e.g., two students at the bottom of the drawing are working together and communicating with each other). Thus, no clear assignment to the teacher-directed or student-directed category could be made. Indicator (6), classroom and environment physical appearance, was assigned to the category of student-directedness. Although no group tables are depicted, the seating arrangement shows an open way of working as materials are shown on the tables, and the students move freely around the room. Thus, the indicators for each coded aspect reflect a student-directed classroom. In terms of the continuum of modes of instruction, the figure was given a score of 2, which indicates a moderately student-directed mode of instruction.

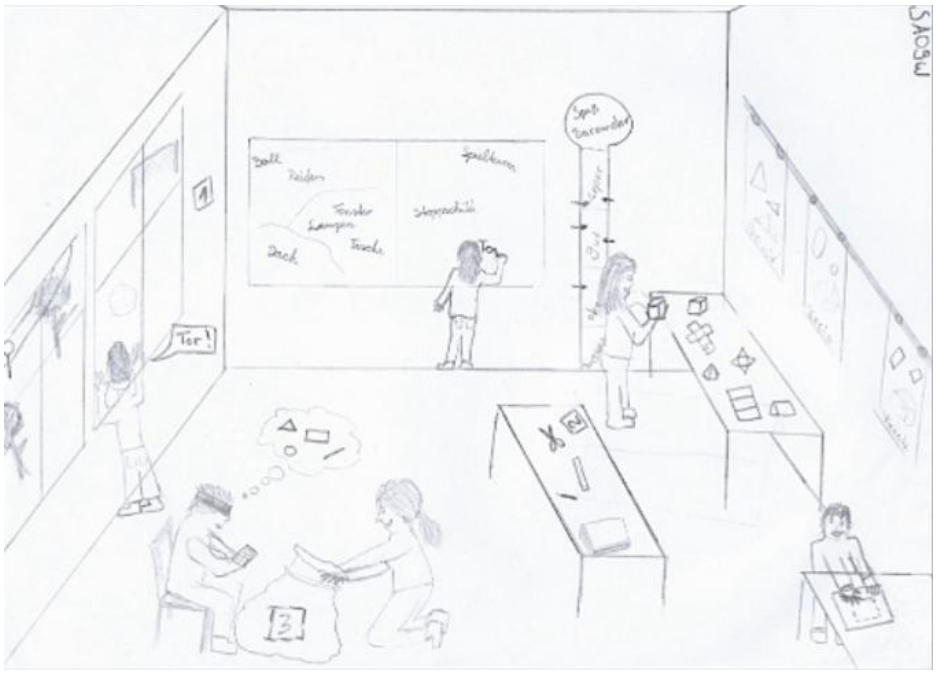

Figure 3. Exemplary coding of a moderately student-directed mode of instruction (Score 2) (Source: Field study)
Note. Score 2: teacher activity -, student activity +1, teacher-student interaction -, student-student interaction 0, position of the teacher -, classroom and environment physical appearance +1.

RESULTS

In this section, results are presented pertaining to the images that prospective primary school mathematics teachers have of good geometry lessons before and after attending a two-semester course titled 'Geometry and Teaching Geometry'. It also discusses how these images changed following the two-semester course.

Prospective Primary School Mathematics Teachers' Images of Good Geometry Lessons before and after Attending a Two-Semester 'Geometry and Teaching Geometry' Course

To understand the evolution of prospective primary school mathematics teachers' pedagogical perspectives, the six indicators are first examined independently. **Table 2** summarizes the results concerning the modes of instruction without taking the gradation of each mode of instruction into consideration. The majority of pre-drawings (39.6%) illustrated good geometry lessons as teacher-directed, with 11 pre-drawings (10.9%) and 29 pre-drawings (28.7%) depicting highly teacher-directed geometry lessons, and moderately teacher-directed geometry lessons, respectively. Pre-drawings illustrating balanced (29.7%), and student-directed classrooms (30.7%) as examples of good geometry lessons were almost equally distributed with approximately one-third of drawings each. However, out of 31 student-directed drawings, 25 (24.8%) illustrated a moderately student-directed classroom, and six (5.9%) a highly student-directed classroom. A completely different picture emerged from the post-drawings. Here, three-fourths of the drawings (n = 76) depicted good geometry lessons as student-directed, with almost two-thirds of the drawings (n = 49) portraying a highly student-directed classroom. Only 25 post-drawings illustrated teacher-directed (n = 15) or balanced (n = 10) classrooms as examples of good geometry lessons. Furthermore, no post-drawing illustrated a highly teacher-directed classroom.

Table 2. Prospective primary school mathematics teachers' drawing scores regarding the modes of instruction

Modes of instruction	Highly teacher-directed	Moderately teacher-directed	Balanced	Moderately student-directed	Highly student-directed
Pre-drawings (N = 101)	11 (10.9%)	29 (28.7%)	30 (29.7%)	25 (24.8%)	6 (5.9%)
Post-drawings (N = 101)	0 (0%)	15 (14.9%)	10 (9.9%)	27 (26.7%)	49 (48.5%)

When comparing pre- and post-drawings a trend can be observed. On the one hand, the percentage of teacher-directed drawings, both highly teacher-directed and moderately teacher-directed, decreased from 39.6% to 14.9%. A similar trend applies to balanced learning with a decrease of 19.8%. On the other hand, the percentage of student-directed drawings increased from 30.7% to 75.2%, and a noticeable increase in highly student-directed drawings is seen, from 5.9% to 48.5%. The number of drawings pertaining to moderately student-directed drawing remained stable.

To provide a more holistic view of pedagogical orientation shifts, the drawings were also analyzed based on their total scores on the instructional continuum (see **Figure 1**). **Table 3** summarizes the results concerning the modes of instruction by taking the gradation of each mode of instruction into consideration. With respect to a teacher-directed mode of instruction all degrees but -6 were present in the pre-drawings. However, in the post-drawings, the range included only degrees -3, -2, and -1. Furthermore, a trend regarding teacher-directedness can be observed. Whereas there is a decrease in drawings with gradation from -5 to -2, there is an increase in drawings with -1. With respect to the student-directed mode of instruction, all degrees were present in both the pre- and post-drawings. Gradations 1 and 3 show a slight increase in pre- and post-drawings, whereas a noticeable increase in drawings with gradations 4 to 6 can be observed. Furthermore, one-fifth of post-drawings (n = 21) had a 5-gradation degree, which was followed by gradation degree 6 in 16 drawings, and gradation degree 3 in 13 drawings.

Table 3. Prospective primary school mathematics teachers' drawing scores regarding the gradation of the modes of instruction

Modes of instruction	Highly teacher-directed			Moderately teacher-directed			Balanced	Moderately student-directed			Highly student-directed		
Gradation	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Pre-drawings (N = 101)	0	3	8	13	13	3	30	7	9	9	3	2	1
Post-drawings (N = 101)	0	0	0	2	6	7	10	8	6	13	12	21	16

To exemplify drawings in which good geometry lessons are represented as student-directed classrooms, **Figure 2**, **Figure 3** and **Figure 4** are provided. In the drawings assigned to either moderately or highly student-directed classrooms, students were always shown working with geometry materials (e.g., plane figures, shapes, models, manipulatives) in small groups at various learning stations and were communicating with each other. The tables were arranged in groups or in a circle. The position of the teacher varied, from being at the front of the class to being amongst students observing their work. The short descriptive texts reflected the importance of clear structure provided by the teacher at the beginning of the lesson, the teacher explaining the goal of the lesson before students engage in group work, and the teacher supporting the students when needed by helping them with the tasks or by answering their questions.

The drawer of a 'good' geometry lesson illustrated in **Figure 4** exemplified these ideas:

> In my picture, the teacher is standing in the background while the task, which has been written on the board, is being carried out. The children have formed groups and have been asked to match the objects in front of them to their corresponding shapes. While the teacher remains in the background, the children are free to discuss the task and exchange ideas.

Furthermore, she wrote:

> Children can work and learn effectively without a frontal teaching approach. In fact, it is often beneficial for them to communicate with each other independently.

She also stressed that:

> Good geometry lessons should always use examples from children's everyday lives. This makes lessons more engaging and encourages active participation based on personal experience.

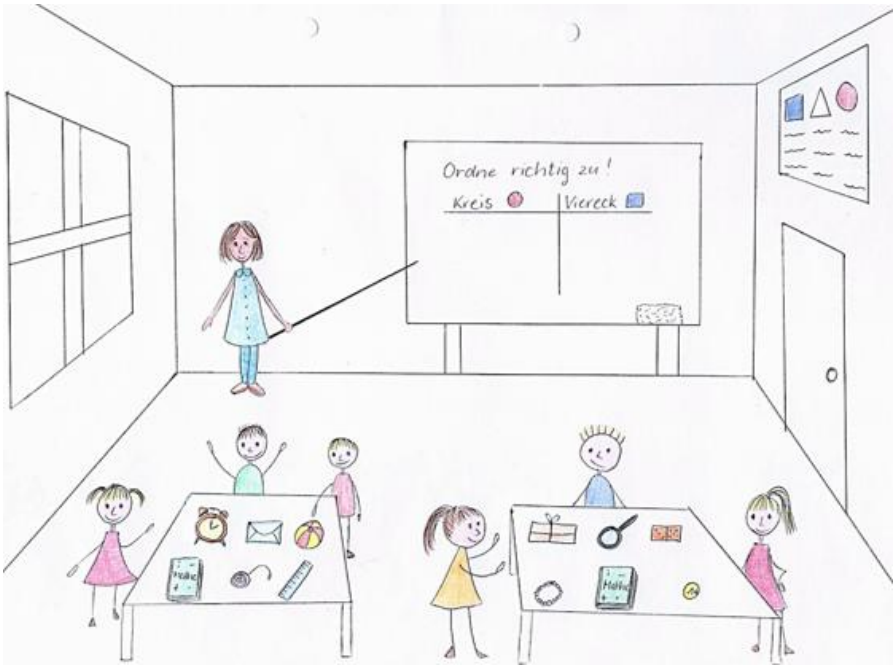

Figure 4. Drawing of a highly student-directed mode of instruction in a good geometry lesson (Score 5) (Source: Field study)
Note. Score 5: teacher activity +1, student activity +1, teacher-student interaction +1, student-student interaction +1, position of the teacher 0, classroom and environment physical appearance +1.

To exemplify a typical drawing portraying a highly teacher-directed geometry classroom, **Figures 5** is provided. A teacher stands in front of the blackboard and transmits the content, with one student in front and one sitting. Two tables are illustrated in one row. No communication between the students is shown. However, manipulatives are illustrated which reflects active learning. The accompanying short descriptive text confirmed the illustration of a good geometry lesson:

> My drawing represents a good geometry lesson because the teacher is standing in front of the blackboard and drawing the tasks for the children. The students are all sitting facing the blackboard and can follow the lesson well. My primary school teacher for Grades 1-3 is a role model for me in many ways.

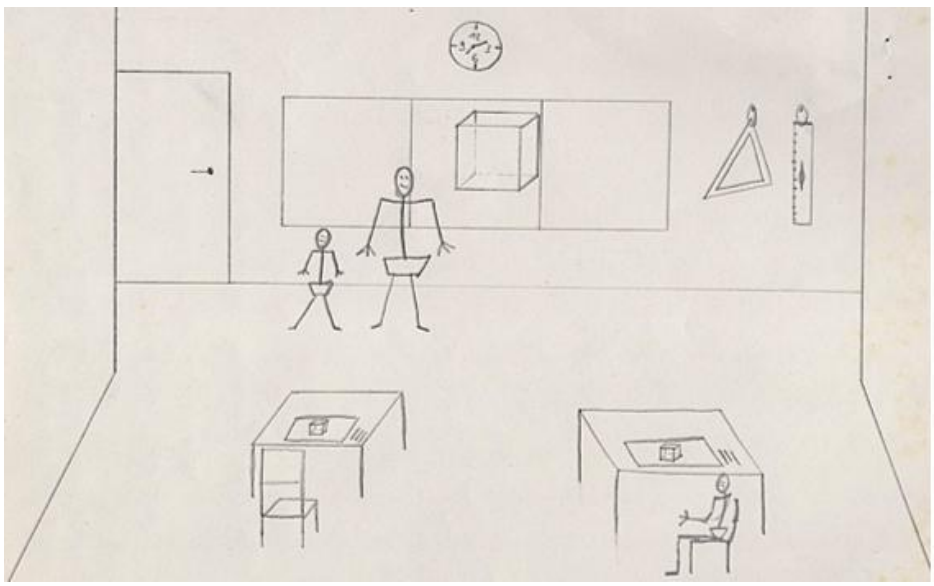

Figure 5. Drawing of a highly teacher-directed mode of instruction in a good geometry lesson (Score -4) (Source: Field study)
Note. Score -4: teacher activity -1, student activity +1, teacher-student interaction -1, student-student interaction -1, position of the teacher -1, classroom and environment physical appearance -1.

The last statement makes especially clear the influence of the learning experiences during the student's own school years.

Lastly, the drawings placed in the middle of the continuum (balanced classroom) indicated characteristics of teacher-directed geometry lessons (e.g., teacher in front of the classroom/at the blackboard, teacher lecturing, individual student work) as well as of a student-directed classroom (e.g., non-traditional table grouping such as group tables, station learning with manipulatives). The latter is exemplary illustrated in **Figure 6**. The drawer's short descriptive text emphasized station learning in good geometry lessons as it:

> Allows children to explore the topic on their own and try different learning manipulatives.

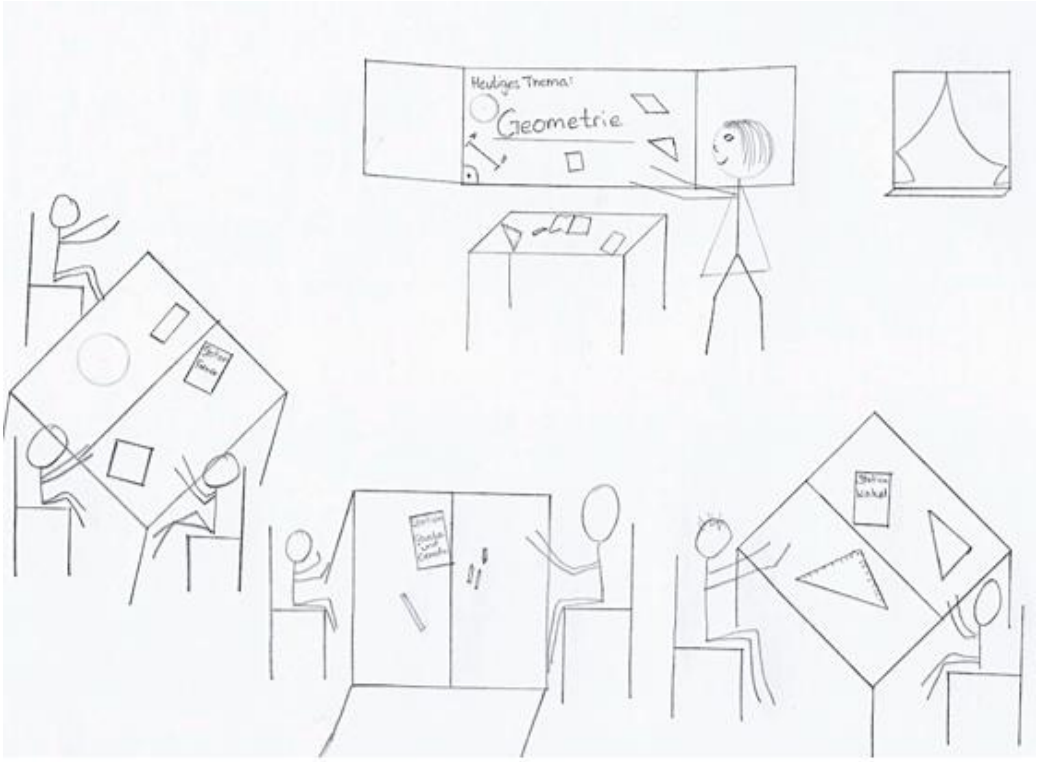

Figure 6. Drawing of a balanced mode of instruction in a good geometry lesson (Score 0) (Source: Field study)
Note. Score 0: teacher activity 0, student activity +1, teacher-student interaction -, student-student interaction -1, position of the teacher -1, classroom and environment physical appearance +1.

Changes in Prospective Primary School Mathematics Teachers' Images of Good Geometry Lessons after Attending the Two-Semester 'Geometry and Teaching Geometry' Course

To determine the extent to which the prospective primary school mathematics teachers' individual images of a good geometry lesson changed, the individual pairs of drawings (i.e., each participant's pre- and post-drawings) were analyzed with regard to changes in the mode of instruction on the continuum in the drawings (see **Table 4**). In terms of the continuum of the modes of instruction (see **Figure 1**), a more student-directed change describes a shift toward the pole of pure student-directedness, and a more teacher-directed change describes a shift toward the pole of pure teacher-directedness. If the mode of instruction in the post-drawing did not change from the mode of instruction of the pre-drawing, the drawing was assigned to the same category. It is important to note that **Table 4** records the changes in the gradations of the modes of instruction according to Gulek (1999) and not the changes in the individual degrees of expression according to Lin (2018). Here, a subsequent drawing may have a lower or higher degree of proficiency but was only assigned to a more teacher- or student-directed category if the gradation of the mode of instruction also changed. The orientation toward the mode of instruction takes into account potential deviations around the determined gradation and thus promotes the generation of reliable results.

Table 4. Comparison of individual pre- and post-drawing Pairs (N = 101)

Shift in the mode of instruction	Absolute and relative frequencies of drawing pairs
Toward teacher-directed classroom	2 (2%)
Toward balanced classroom	6 (5.9%)
Toward student-directed classroom	45 (44.6%)
Unchanged	48 (47.5%)

That said, almost half of the students (47.5%) drew the same mode of instruction (i.e., teacher-directed, balanced, student-directed) in their pre- and post-drawings. Similarly, almost half of the students (44.6%) chose a more student-directed mode of instruction in the post-drawings than in their pre-drawings. These changes applied to pre-drawings portraying either teacher-directed or balanced classrooms as examples of good geometry lessons. In two drawing pairs (2%) a shift toward a teacher-directed classroom from a balanced classroom was illustrated. Six drawing pairs (5.9%), which showed teacher-directed

classrooms as examples of good geometry lessons in the pre-drawings, demonstrated a shift toward a balanced learning environment.

The most noticeable changes in the post-drawings were connected to the teacher's activity and the classroom and physical appearance of the environment. In all but two drawings, the teacher acted as both an organizer and learning facilitator, or purely as a learning facilitator supporting students in their learning process. Only four post-drawings illustrated a traditional seating arrangement. All other drawings illustrated group table arrangements and indicated active learning using typical geometry teaching materials and tools (e.g., models of 2D shapes, models of 3D-solids). This was also reported in the short descriptive texts where the students emphasized the importance of enactive hands-on teaching and learning practices in geometry lessons such as discovery learning and explorations. The picture of teacher-student and student-student interaction was somewhat ambiguous as both were illustrated and/or reported to be an individual or group activity. However, social relationships within the classroom were more evident in student-teacher communication than in student-student communication.

The degree of change was also taken into consideration, regardless of whether the direction was toward teacher or student-directedness (see **Table 5**). The degree of change indicates the extent to which the mode of instruction in the post-drawing differs from the pre-drawing by one, two, three, or four gradations. The change can be toward the pole of pure teacher-directedness as well as student-directedness. After deducting the 48 pairs of drawings that showed no change, the 53 pairs that showed a shift in the mode of instruction were considered. The majority of these pairs of drawings (37.7%) showed that the mode of instruction in the post-drawing had shifted by two gradations toward a balanced classroom, and the pole of pure student-directedness compared to the pre-drawing. This was followed by a simple shift of one gradation toward the pole of a pure teacher-directed classroom, balanced classroom and a pure student-directed classroom compared to the pre-drawing, which was observed in almost one-third of the pairs of drawings (30.2%). A change in the mode of instruction by three and four gradations was observed in 22.6% (n = 12) and 9.4% (n = 5) of the pairs, respectively.

Table 5. Gradation of change in individual pairs of drawings (n = 53)

Change in the mode of instruction	Absolute and relative frequencies of pairs of drawings
By one gradation	16 (30.2%)
By two gradations	20 (37.7%)
By three gradations	12 (22.6%)
By four gradations	5 (9.4%)

Two of the pairs of drawings (see **Figure 7** and **Figure 8**) show particularly clearly the differences in the pre- and post-drawings where the mode of instruction shifted by four gradations toward the pole of pure student-directedness. In both cases, the pre-drawing illustrated a highly teacher-directed geometry classroom. The change was especially evident in the reversed role and position of the teacher, and the seating arrangement. The short descriptive text to **Figure 7** reflects the teacher's role in organizing the lesson and presenting its content:

> I chose this drawing because it demonstrates active participation in the lesson, with all the children finding the correct solution to task 1. This suggests that the teacher had explained and introduced the various geometric shapes well, as well as explained the task clearly.

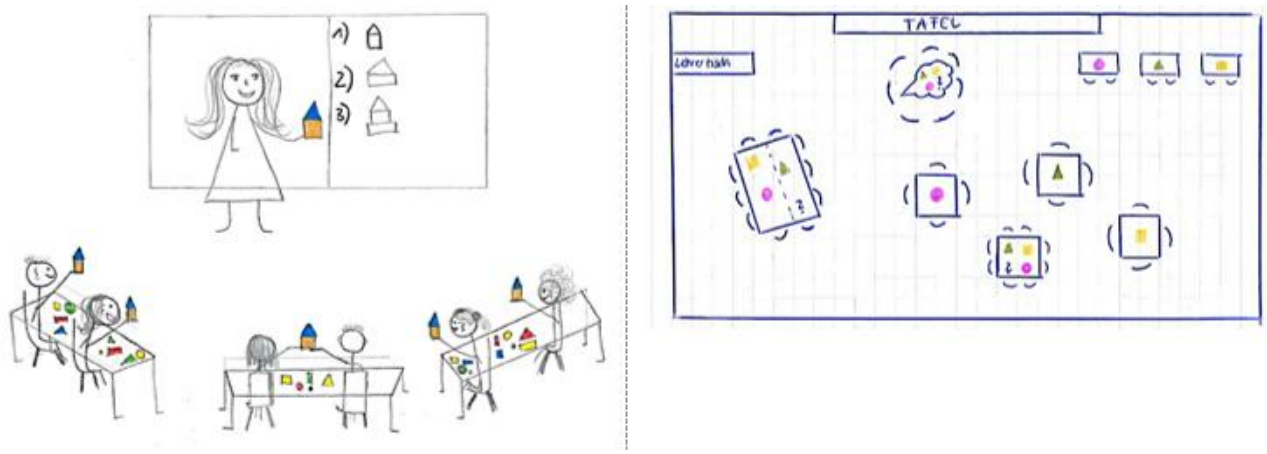

Figure 7. Exemplary four gradation mode of instruction change in the drawing pair CO08W (-6) and CO08WJ (5) (Source: Field study)

Note. Score (-6): teacher activity -1, student activity -1, teacher-student interaction -1, student-student interaction -1, position of the teacher -1, classroom and environment physical appearance -1 (left). Score (5): teacher activity 0, student activity +1, teacher-student interaction +1, student-student interaction +1, position of the teacher +1, classroom and environment physical appearance +1 (right).

Whereas in the pre-drawing the teacher acts as the lesson organizer transmitting the content, in the post-drawing the teacher takes the role of organizer (at the beginning and the end of the lesson) but also acts as a learning facilitator during group work. The group work was characterized by student individual responsibility as follows:

> All students work on the same assignment. However, the students decide on their own how: sequence of tasks, task difficulty (differentiation). Exchange with other students and the teacher is possible.

This was also reflected in the arrangement of the desks, which were positioned much more openly in the form of group desks in both of the post-drawings. The short descriptive text to **Figure 8** emphasized the benefits of such classroom appearance:

> In this drawing, I would like to show the different approaches that can be used in geometry lessons to convey the content in a playful way. The approaches can be structured as a station work and used to support students in their learning and encourage them to gain their own insights in a playful way, e.g., through experiments. Above all, however, working in small or large groups is far more effective than pure frontal teaching.

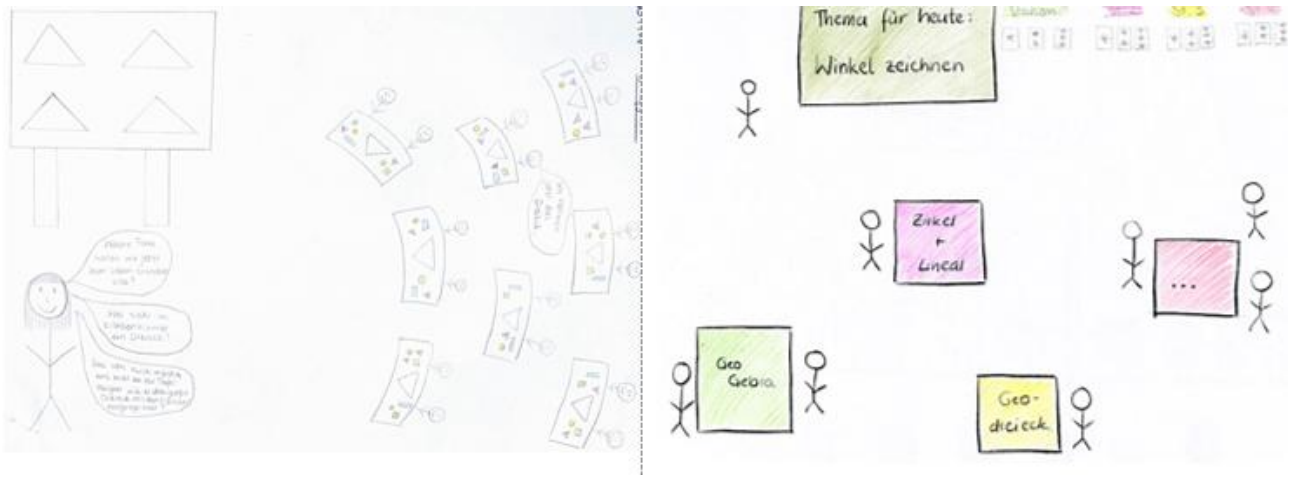

Figure 8. Exemplary four gradation mode of instruction change in the drawing pair DO11W (-6) and DO11W (5) (Source: Field study)
Note. Score (-6): teacher activity -1, student activity -1, teacher-student interaction -1, student-student interaction -1, position of the teacher -1, classroom and environment physical appearance -1 (left). Score (5): teacher activity 0, student activity +1, teacher-student interaction +1, student-student interaction +1, position of the teacher +1, classroom and environment physical appearance +1 (right).

Furthermore, the teacher-student interaction in the pre-drawings was limited to the teacher posing questions and students showing individual solutions to the questions posed. The communication is not illustrated in the post-drawings, however, the short descriptive texts report that teacher-student, as well as student-student communication, was initiated by the students.

To summarize, at the beginning of the two-semester course 'Geometry and Teaching Geometry' most of the prospective primary school mathematics teachers' ideas about what constitutes a good geometry lesson were aligned with a teacher-directed mode of instruction. By the end of the two-semester course, however, this image had, changed with most of the prospective primary school mathematics teachers now associating good geometry lessons with a student-directed mode of instruction. Thus, the two-semester geometry course prompted a shift towards a student-directed mode of instruction (75.2%), reflecting a greater emphasis on discovery learning and explorations in good geometry lessons. The most noticeable changes in the post-drawings were connected to teacher's activity and classroom and physical appearance of the environment, but also to social relationships within the classroom, which became more evident.

DISCUSSION

The final section discusses first the findings pertaining to the first research question, namely prospective primary school mathematics teachers' images of good geometry lessons in terms of modes of instruction before and after attending a two-semester course 'Geometry and Teaching Geometry'. This is then followed by a discussion of the findings related to the second research question, namely change in prospective primary school mathematics teachers' images of good geometry lessons after attending the two-semester course. Lastly, the limitations of the study are considered, along with some possible future research directions and educational implications.

Prospective Primary School Mathematics Teachers' Images of Good Geometry Lessons across a Two-Semester Course 'Geometry and Teaching Geometry'

The research findings of Sinclair et al. (2013), Swars Auslander (2007) and Utley et al. (2020), indicated that most prospective mathematics teachers hold a traditional understanding of teaching mathematics. The present study, however, showed that before the two-semester geometry course 30.7% of the prospective primary school mathematics teachers held a desirable understanding of teaching according to the social constructivist teaching-learning perspective and that this increased to 75.2% after the two-semester geometry course. This was reflected in the depiction of active student learning in the form of group, partner or station work. Thus, as in the study by Murphy et al. (2004) the prospective primary school mathematics teachers associated good geometry lessons with student-directed instruction.

In line with good geometry practices advocated by mathematics educators (Franke & Reinhold, 2016; Nührenbörger & Verboom, 2005), the prospective primary school mathematics teachers emphasized the importance of social exchange for the student's learning process as well as the role of the teacher as a learning guide in active learning settings:

> [T]he teacher supports the learning process. He/she moves from table to table and helps with questions and insights.
>
> (GE05W)

Although in some drawings the teacher's role appeared rather passive and reserved, the short descriptive texts pointed out that a thematic introduction in the sense of an instructionally guided introductory phase and additional explanations of the content by the teacher are relevant, as emphasized by Reusser (2016):

> Of course, a good geometry lesson not only involves the children working independently but also the teacher first familiarizing them with the various topics/task and explaining basic things.
>
> (BR08W)

Nevertheless, it was found that the role of the teacher as a learning guide was not subject to a standardized understanding of learning support. The uncertainties described in the study of Streit and Royar (2014) regarding the correct learning support were also reflected in this study:

> [The teacher] is happy that the children are working quietly at the stations [...].
>
> (BR08W)

and

> I [the teacher] go around and make sure that they draw neatly and use the materials correctly.
>
> (TA08W)

In both quoted descriptions of the teacher's role, the focus is more on disciplinary and organizational instructions than on supportive impulses and thought-provoking suggestions.

In addition to the student-directed approach to teaching geometry, 29.7% of pre-drawings and 9.9% of post-drawings revealed an understanding of teaching and learning that favored a balanced combination of teacher- and student-directed approaches in good geometry lessons as was also the case in the study by Sinclair et al. (2013), and also recommended by education educators (e.g., Meyer, 2019; Reinmann & Mandl, 2006). In the drawings, this was particularly evident in discovery-based learning, in student individual work, and in the teacher's alternating role as facilitator and lesson designer. Both aspects are aligned with good geometry practices (Alfieri et al., 2021; Köhler, 1998; Kuzle, 2021; Radatz & Rickemeyer, 1991). Although active learning was emphasized, this was not always the case with student-student communication:

> My drawing depicts geometry lessons as station work. This has the advantage that each pupil can work on different tasks individually and in a differentiated way.
>
> (JA12M)

Other than in the studies by Sinclair et al. (2013), Swars Auslander (2007) and Utley et al. (2020) in which about two-thirds of prospective teachers held a pronounced traditional understanding of teaching and learning, in this study, only 39.6% of the prospective mathematics teachers' participant-produced drawings showed a traditional understanding of teaching in good geometry lessons before attending the two-semester geometry course, and 14.9% after attending the course. These drawings reflected a reversed understanding of the teacher's role in a geometry classroom, in which the teachers were the active lesson organizers and presented the knowledge and the students took on a more passive role:

> My picture represents good geometry lessons because the teacher stands at the board and draws the tasks for the children.
>
> (IS04W)

Furthermore, especially in the drawings with a strong teacher-directed approach, active and social components were not evident at all.

One possible explanation for the somewhat contradictory findings may be due to differences in the subjects studied as well as in the individual sub-disciplines of mathematics. Kuhl et al. (2013) and Pajares (1992) already pointed out that the beliefs about good teaching and learning are subject-specific. As a result, the study findings of Murphy et al. (2004) and Swars Auslander (2007) are difficult to compare with those of the present study. In contrast to the present study, the study by Murphy et al. (2004) surveyed prospective teachers regardless of their teaching subject. Although Swars Auslander (2007) also investigated mathematics teachers, there are major differences in the sub-disciplines of mathematics. When comparing arithmetic with geometry, the geometric content offers more opportunities for practical activities (e.g., building, folding and laying) (Franke & Reinhold, 2016; Köhler, 1998; Radatz & Rickemeyer, 1991), and is also suitable for developing process-related skills (e.g., in partner and group work) (Franke & Reinhold, 2016; Nührenbörger & Verboom, 2005; Vollrath & Roth, 2012). Such practical and social activities allow a more student-directed mode of instruction in which the interaction between students and students' interaction with the materials are at the forefront of the lesson. That good geometry lessons are associated with practical activities was reported in

the participant-produced drawings by either illustrating or reporting a variety of geometric work and visualization materials (e.g., in the form of drawing instruments, body models, (tangram) tiles).

The results of this study confirmed earlier work (Calderhead & Robson, 1991) that the understanding of good (geometry) lessons could be traced back to school experience, where both positive and negative experiences in geometry lessons serve as a model for good teaching, and consequently influence beliefs about good geometry lessons. The complex interplay between the students' own school experiences and their (future) teaching style is also evident in the descriptions of the students surveyed regarding the question of why they chose their drawing. One student explained:

> My primary school teacher in Grades 1-3 is a role model for me in many ways. Back then, she often introduced new topics with station work, group work, or project weeks. That's how I initially came to present the situation in a similar manner.
>
> (JU03W)

Another student pointed out that the experiences during field placements were key:

> I chose this drawing because I was able to experience such geometry lessons during my field placements.
>
> (CL12W)

Consequently, not only prospective teachers' personal school experiences but also teaching experiences in the context of internships appear to have an effect on their belief system.

With regard to the second research question, it could be shown that university teacher training has a positive influence on the prospective teachers' belief system and can contribute to a more modern understanding of teaching and learning, which supports the findings of Swars Auslander (2007), Safrudiannur et al. (2021), and Kelkel and Peschel (2023). The results of the present study indicated that 44.5% of the prospective primary school mathematics teachers had a more student-directed understanding of good geometry lessons after attending the two-semester geometry course than prior to it. Similar to the results of Swars Auslander (2007), they developed new or changed beliefs concerning the mode of instruction in good geometry lessons. Compared to only 2% of prospective primary school mathematics teachers who drew a more teacher-directed geometry lesson in their post-drawings, this reflects a very positive development, taking into account the 47.5% who showed no change.

Possible explanations for this positive change are provided by the previously presented research findings of Kelkel and Peschel (2023), Rumpf and Schöps (2013), Steinmann and Oser (2012), and Swars Auslander (2007), who considered active engagement with one's belief system to be central to changing established beliefs. As part of their homework, the prospective primary school mathematics teachers were given the opportunity to engage creatively with their own beliefs. In particular, the last homework assignment after completing the two-semester geometry course encouraged the students to reflect on their impressions, experiences, and the geometric content they had learned on the course. Based on the research findings, it can be assumed that this active discussion contributed to the positive development. In addition, the role of the teacher educators provides a further explanation for the positive development of the prospective mathematics teachers' images of good geometry lessons. Given the research findings of Steinmann and Oser (2012), the role model function of the lecturers could have made a decisive contribution to this positive development.

However, despite this positive development, 47.5% of the prospective primary school mathematics teachers showed no change in their belief system. Therefore, the two-semester geometry course cannot be regarded as guaranteed to have an influence on the belief systems of prospective primary school mathematics teachers. It can be assumed that some students who did not show any change in their beliefs or moved towards a more teacher-directed mode of instruction already had very entrenched beliefs that are difficult to change. As Pajares (1992) pointed out, beliefs that have existed for a very long time are more difficult to change.

The effect of the interplay of the presented favorable and unfavorable factors on the development or change of the prospective primary school mathematics teachers' understanding of teaching and learning could also be seen in the degree of change shown in the individual pairs of drawings. The majority of the pairs showed a simple onefold (30.2%) or twofold shift (37.7%) toward the poles of teacher or student-directedness. Major changes in teachers' belief systems are not to be expected, particularly due to the stability of perceptions and beliefs (Kuhl et al., 2013). However, the 12 changes (22.6%) shown around a threefold shift as well as five changes (9.4%) around a fourfold shift toward the pole of student-directedness indicate that major (positive) changes are certainly possible in the context of teacher training targeting a modern, student-directed approach to teaching geometry.

Limitations of the Study and Future Research Directions

With regard to the suitability of the method for the objectives of the study, drawings and short descriptive texts were used to visualize individual and subjective impressions of the teaching-learning experiences (Beeli-Zimmermann, 2014; Gulek, 1999), and to capture unconscious beliefs (Hatisaru, 2021). However, the short descriptive texts highlighted some limitations in the use of drawings. Individual students referred to difficulties in drawing:

> I didn't know exactly how to draw certain things [...].
>
> (SU06WJ)

or

> However, I find it difficult to represent [activity-based learning] in drawings.
>
> (MA07WJ)

For these students, it was helpful to be able to record their images of good geometry lessons in writing as well as in a drawing. In addition, the drawings proved to be insufficient for some students to visualize the complexity of good geometry lessons. One student pointed out that she could not combine all aspects in one picture:

> It doesn't generally represent good geometry teaching. Personally, I wouldn't know how to include different things here.
>
> (MA01W)

Two other students reported that:

> There are many more ways to organize [good] lessons.
>
> (TA02WJ)

Despite these limitations, the combination of drawings and short descriptive texts proved to be a suitable method to capture prospective mathematics teachers' images of good geometry lessons. Hatisaru (2021) showed that the additional use of short descriptions counteracts the limitations of drawings as a single method and thus increases the data reliability. Despite the drawing difficulties encountered by individual students, the majority of the students showed good to very good drawing skills in line with the initial assumption, which contributed to the data analysis.

The study results showed that personal school and student teaching experiences had an influence on the images student teachers have of a good geometry lesson. In order to gain a better understanding of the development of the students' images and beliefs and to draw conclusions about the respective prevailing belief system, the following questions could complement the first homework assignment:

(a) Thinking back to your school days, what were your geometry lessons like? Describe a situation that you particularly remember.

(b) Have you ever taught geometry yourself? Describe your experiences.

(c) Have you ever observed another teacher teaching geometry? Describe your experiences.

With regard to the impressions and experiences gained from university teaching, the particularly memorable experiences also proved to be relevant for impacting established beliefs (Kelkel & Peschel, 2023; Rumpf & Schöps, 2013). As there was no information on experiences that were of personal relevance to the students on the course, no statements could be made about the extent to which a specific situation from the two-semester course caused "a kind of emotional switch for the students and thus stimulated changes in their beliefs" (Kelkel & Peschel, 2023, p. 299). In order to gain a better understanding of the causes of changes in students' images of and beliefs about university teaching and thus open up new starting points for improving teacher training, the following questions could complement the final homework assignment:

(a) How often did you attend lectures and exercises?

(b) What type of instruction was used in the lectures? Describe it.

(c) What type of instruction was used in the exercises? Describe it.

With a view to future research in the area of prospective mathematics teachers' belief systems, the results make it clear that their images are not only subject-specific (Kuhl et al., 2013) but also depend on the respective sub-discipline or sub-area. In order to establish comparability and confirm or refute the findings shown here, further research is needed in the field of geometry on similar questions with a larger sample. Furthermore, conducting a large-scale longitudinal study would open up new research opportunities with regard to the developments and changes in the prospective primary school mathematics teachers' perceptions during the teacher training program. Another important aspect that should be addressed in future research is the discrepancy between the preferred and actual mode of instruction in good geometry lessons. A comparison of the studies that recorded the preferred type of instruction (Murphy et al., 2004) with those that reflected the actual mode of instruction shown (Hatisaru, 2019; Picker & Berry, 2000) showed that although (prospective) teachers preferred student-directed instruction, they rarely implemented it in their lessons. The extent to which this discrepancy also exists in geometry lessons needs to be examined as part of further research.

Implications for Teacher Education

By relating the study results to teaching practice, some implications for geometry teaching can be drawn. In order to design good geometry lessons, prospective and practicing (primary school) mathematics teachers should become aware of their beliefs and reflect on their actions with regard to the learning success of their students. Here, it may be helpful for them to reflect on their own teaching experience from school days and internships during their university studies. In addition, the prospective primary school mathematics teachers not only had the opportunity to deal with their own beliefs but also to gain a variety of ideas for the implementation of student-directed geometry lessons (e.g., suitable teaching and learning materials) and to understand what makes a good geometry lesson (e.g., discovery learning) during the two-semester geometry course. Thus, actively reflecting on, being immersed in and experiencing a reform-oriented approach to teaching and learning geometry over a period of two semesters contributed to a shift in their understanding of what constitutes 'good' geometry lessons.

More generally, in line with the findings of Kuhl et al. (2013) and Pajares (1992), it is paramount that prospective primary teachers discuss and experience what constitutes a 'good' lesson in all subjects studied. This would prevent prospective primary teachers from having different images about what constitutes good teaching, thus preventing them from thinking that good teaching is subject-specific. In initial teacher education, general pedagogy courses could involve prospective primary teachers reflecting on their school experiences individually followed by group conversations. The latter could prompt reflection on what good teaching should look like and how good lessons should be conducted. These ideas align with literature (Kelkel & Peschel, 2023; Rumpf & Schöps, 2013; Steinmann & Oser, 2012; Swars Auslander, 2007) emphasizing that active engagement with one's belief system is central to changing established beliefs. Prospective teachers' ideas could be then discussed in relation to the current, theory-based understanding of good teaching. However, these ideas alone will not suffice if the notion of good lessons is not thoroughly discussed and practised in subject-specific courses. Thus, subject-specific reflections would be necessary. Subject-specific courses should also reflect aspects of good teaching (e.g., lecturers modelling good practices) and provide prospective primary teachers with opportunities to experience and implement these practices in their practical work, as suggested by Swars Auslander (2007).

CONCLUSION

From the perspective of prospective primary school mathematics teachers, the study findings demonstrated that good lessons in primary school geometry are characterized by student-directed mode of instruction, in line with Wygotski's (1987) social constructivist perspective. Specifically, they considered active learning, social learning, and individual learning processes to be integral to good geometry lessons. Geometry itself offers many opportunities for active learning; for example, students can build cube structures or lay patterns (Franke & Reinhold, 2016). This was particularly evident by illustrating primary school students being engaged with different learning materials and manipulatives, and social activities. Frequent depictions of partner and group work, and station work also illustrated the importance of social learning for success in geometry lessons, as well as the new, changed role of teachers in this context. Relating the study results to teaching practice, the prevailing understanding of teaching and learning according to the social constructivist perspective suggests a potential positive shift in the mode of instruction in geometry lessons. Since prevailing understandings of teaching and learning influence professional teaching practice (Steinmann & Oser, 2012), this represents an important step towards enabling a potential change in geometry lessons in school mathematics. Consequently, such lessons would reflect modern teaching practices as explicated by OECD (2016).

Acknowledgements: The author would like to thank the student research assistant who served as an additional coder.

Funding: The author has no relevant financial or non-financial interests to disclose. No funding was received to assist with the preparation of this manuscript.

Ethical statement: The author stated that this study used anonymized data from non-vulnerable adults, with written informed consent, and followed APA (2017) and GDPR (2025) ethical guidelines.

Declaration of interest: The author declares no competing interest.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

REFERENCES

Aktas, B. C. (2010). Investigating primary school students' perceptions regarding "teacher" through their drawings. *The International Journal of Learning, 17*(8), 409-425.

Alfieri, L., Brooks, P. J., Aldrich, N. J., & Tenenbaum, H. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology, 103*(1), 1-18. https://doi.org/10.1037/a0021017

Beeli-Zimmermann, S. (2014). Beyond questionnaires – exploring adult education teachers' mathematical beliefs with pictures and interviews. *Adults Learning Mathematics: An International Journal, 9*(2), 3-53.

Bohl, T., & Kucharz, D. (2010). *Offener Unterricht heute. Konzeptionelle und didaktische Weiterentwicklung* [Open lessons today. Conceptual and didactic development]. Beltz.

Brunner, M., Kunter, M., Krauss, S., Klusmann, U., Baumert, J., Blum, W., Neubrand, M., Dubberke, T., Jordan, A., Loewen, K., & Tsai, Y.-M. (2006). Die professionelle Kompetenz von Mathematiklehrkräften: Konzeptualisierung, Erfassung und Bedeutung für den Unterricht. Eine Zwischenbilanz des COACTIV-Projekts [The professional competence of mathematics teachers: Conceptualization, recording and significance for teaching. An interim assessment of the COACTIV project]. In M. Prenzel & L. Allolio-Näcke (Eds.), *Untersuchungen zur Bildungsqualität von Schule. Abschlussbericht des DFG-Schwerpunktprogramms* (pp. 54-82). Waxmann Verlag.

Calderhead, J., & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching and Teacher Education, 7*, 1-8. https://doi.org/10.1016/0742-051X(91)90053-R

Corbett, D., & Wilson, B. (2002). What urban students say about good teaching. *Educational Leadership, 60*(1), 18-22.

Crain, W. C. (2000). *Theories of development: Concepts and applications* (4th ed.). Prentice Hall.

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory Into Practice, 39*(3), 124-130. https://doi.org/10.1207/s15430421tip3903_2

Debreli, E. (2012). Change in beliefs of pre-service teachers about teaching and learning English as a foreign language throughout an undergraduate pre-service training program. *Procedia – Social and Behavioral Sciences, 46*(1), 367-373. https://doi.org/10.1016/j.sbspro.2012.05.124

Dole, S., Bloom, L., & Kowalske, K. (2016). Transforming pedagogy: Changing perspectives from teacher-centered to learner-centered. *Interdisciplinary Journal of Problem-Based Learning, 10*(1), 3-18. https://doi.org/10.7771/1541-5015.1538

Domović, V. (2003). *Školsko ozračje i učinkovitost škole* [School atmosphere and school efficiency]. Naklada Slap.

Einarsdóttir, J. (2007). Research with children: Methodological and ethical challenges. *European Early Childhood Education Research Journal, 15*(2), 197-211. https://doi.org/10.1080/13502930701321477

Evans, I. M., Harvey, S. T., Buckley, L., & Yan, E. (2009). Differentiating classroom climate concepts: Academic, management, and emotional environments. *Kotuitui: New Zealand Journal of Social Sciences Online, 4*(2), 131-146. https://doi.org/10.1080/1177083x.2009.9522449

Franke, M., & Reinhold, S. (2016). *Didaktik der Geometrie in der Grundschule* [Didactics of geometry in elementary education] (3rd ed.). Springer.

Fromm, M. (2017). *Lernen und Lehren. Psychologische Grundlagen für Lehramtsstudierende* [Learning and teaching. Psychological basics for preservice teachers]. Waxmann. https://doi.org/10.25656/01:20864

Gulek, C. (1999). *Using multiple means of inquiry to gain insight into classrooms: A multi-trait multimethod approach* [Unpublished doctoral dissertation]. Boston College.

Hatisaru, V. (2019). Putting the spotlight on mathematics classrooms. In J. Novotná & H. Moraová (Eds.), *Proceedings of the International Symposium Elementary Mathematics Teaching: Opportunities in Learning and Teaching Elementary Mathematics* (pp. 182-192). Department of mathematics and mathematical education, Charles University, Faculty of Education.

Hatisaru, V. (2020). School students' depictions of mathematics teaching and learning practices. *International Electronic Journal of Elementary Education, 13*(2), 199-214. https://doi.org/10.26822/iejee.2021.184

Hatisaru, V. (2021). Draw a mathematics classroom! Teaching and learning practices through the eyes of students. *Mathematics in School, 50*(2), 4-8.

Hatisaru, V. (2022). The knowledge produced through student drawings. *Frontiers in Psychology, 13*. https://doi.org/10.3389/fpsyg.2022.1042383

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* Routledge.

Helmke, A. (2009). *Unterrichtsqualität und Lehrerprofessionalität. Diagnose, Evaluation und Verbesserung des Unterrichts* [Teaching quality and teacher professionalism. Diagnosis, evaluation and improvement of teaching]. Klett Kallmeyer.

Kearney, K., & Hyle, A. (2004). Drawing out emotions: The use of participant-produced drawings in qualitative inquiry. *Qualitative Research, 4*(3), 361-383. https://doi.org/10.1177/1468794104047234

Kelkel, M., & Peschel, M. (2023). ‚Was willst du lernen?' – Teil II. Irritationen ändern. Beliefs in Hochschullernwerkstattseminaren ['What do you want to learn?' – Part II: Changing irritations. Beliefs in university learning workshop seminars]. In P. Kihm, M. Kelkel, & M. Peschel (Eds.), *Interaktionen und Kommunikationen in Hochschullernwerkstätten. Theorien, Praktiken, Utopien* (pp. 287-302). Verlag Julius Klinkhardt. https://doi.org/10.35468/6009-21

Köhler, E. (1998). *Geometrie und Geometrieunterricht Teil 1* [Geometry and school geometry]. Institut für Grundschulpädagogik der Universität Potsdam.

Kuhl, J., Moser, V., Schäfer, L., & Redlich, H. (2013). Zur empirischen Erfassung von Beliefs von Förderschullehrerinnen und -lehrern [Empirical acquisition of teachers' beliefs about special education]. *Empirische Sonderpädagogik, 5*(1), 3-24. https://doi.org/10.25656/01:8907

Kuzle, A. (2021). Drawing out emotions in primary grade geometry: An analysis of participant-produced drawings of Grade 3–6 students. *LUMAT: International Journal on Math, Science and Technology Education, 9*(1), 844-872. https://doi.org/10.31129/lumat.9.1.1620

Kuzle, A. (2023). Geometry teaching in transition: An investigation on the importance of school geometry in primary education. *CEPS – Center for Educational Policy Studies Journal, 13*(2), 97-123. https://doi.org/10.26529/cepsj.13.2

Kuzle, A., Glasnović Gracin, D., & Krišto, A. (2023). Depicting classroom social climate: Using drawings to examine primary students' perceptions of geometry teaching and learning practices. *International Electronic Journal of Mathematics Education, 18*(4), Article em0757. https://doi.org/10.29333/iejme/13743

Lin, Y.-C. (2018). Are drawings better than questionnaires? Comparing two methods of studying mathematics teachers' beliefs. In F.-J. Hsieh (Ed.), *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education* (Vol. 2, pp. 93-99). EARCOME.

Meyer, H. (2019). *Was ist guter Unterricht?* [What is good teaching?] (14th ed.). Cornelsen.

Murphy, P. K., Delli, L. A. M., & Edwards, M. N. (2004). The good teacher and good teaching: Comparing beliefs of second-grade students, preservice teachers, and inservice teachers. *The Journal of Experimental Education, 72*(2), 69-92. https://doi.org/10.3200/JEXE.72.2.69-92

National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all.* NCTM.

Nührenbörger, M., & Verboom, L. (2005). *Eigenständig lernen – Gemeinsam lernen. Mathematikunterricht in heterogenen Klassen im Kontext gemeinsamer Lernsituationen* [Learning independently - learning together. Teaching mathematics in heterogeneous classes in the context of shared learning situations]. SINUS-Transfer Grundschule.

OECD. (2010). *Mathematics teaching and learning strategies in PISA*. OECD Publishing.

OECD. (2016). Ten questions for mathematics teachers ... and how PISA can help answer them. *OECD Publishing*. https://doi.org/10.1787/9789264265387-en

Pajares, F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332. https://doi.org/10.3102/00346543062003307

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Sage.

Picker, S. H., & Berry, J. S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics, 43*(1), 65-95. https://doi.org/10.1023/A:1017523230758

Radatz, H., & Rickmeyer, K. (1991). *Handbuch für den Geometrieunterricht an Grundschulen* [Handbook for teaching geometry in elementary education]. Schroedel.

Rathgeb-Schnierer, E., Schuler, S., & Schütte, S. (2023). *Mathematikunterricht in der Grundschule. Lernangebote fachorientiert, kindorientiert und differenziert gestalten* [Teaching mathematics in elementary school. Designing learning opportunities in a subject-oriented, child-oriented and differentiated way]. Springer Spektrum. https://doi.org/10.1007/978-3-662-65856-7

Reinmann, G., & Mandl, H. (2006). Unterrichten und Lernumgebungen gestalten. In A. Krapp & B. Weidenmann (Eds.), *Pädagogische Psychologie* (5th ed., pp. 601-646). Beltz.

Reiss, K., & Hammer, C. (2013). *Grundlagen der Mathematikdidaktik. Eine Einführung für den Unterricht in der Sekundarstufe* [Fundamentals of mathematics didactics. An introduction for teaching at secondary level]. Birkhäuser. https://doi.org/10.1007/978-3-0346-0647-9

Reusser, K. (2016). Jenseits der Beliebigkeit. ‚Konstruktivistische Didaktik' auf dem Prüfstand der empirischen Unterrichtsforschung [Beyond arbitrariness. 'Constructivist didactics' on the test bench of empirical teaching research]. *Journal für LehrerInnenbildung, 40*(2), 40-48.

Rott, B., Barton, L., & Hatisaru, V. (2023). What do mathematics lessons look like? Analyses of primary students' drawings. *Frontiers in Psychology, 14*. https://doi.org/10.3389/fpsyg.2023.1019299

Rumpf, D., & Schöps, M. (2013). Hochschulwerkstätten als Raum für Kooperation [University workshops as a space for cooperation]. In H. Coelen & B. Müller-Naendrup (Eds.), *Studieren in Lernwerkstätten* (pp. 31-39). Springer. https://doi.org/10.1007/978-3-658-00315-9_3

Safrudiannur, S., Belke, L., & Rott, B. (2021). A pseudo-longitudinal approach for investigating pre-service teachers' beliefs during their university education. *International Journal of Science and Mathematics Education, 20*(1), 1099-1122. https://doi.org/10.1007/s10763-021-10194-x

Segall, W., & Wilson, A. (1998). *Introduction to education: Teaching in a diverse society*. Prentice Hall.

Sinclair, B. B., Szabo, S., Redmond-Sanogo, A., & Sennette, J. D. (2013). Investigating perceptions of teachers and teaching using the draw-a-teacher checklist. *Issues in Teacher Education, 22*(1), 105-123.

Sitter, K. (2019). *Geometrische Körper an inner- und außerschulischen Lernorten: Der Einfluss des Protokollierens auf eine sichere Begriffsbildung* [Geometric solids in in-school and out-of-school learning settings: The impact of logging on confident concept formation]. Springer Spektrum. https://doi.org/10.1007/978-3-658-27999-8

Steinmann, S., & Oser, F. (2012). Prägen Lehrerauszubildende die Beliefs der angehenden Primarlehrpersonen? Shared Beliefs als Wirkungsgröße in der Lehrerausbildung [Are the beliefs of future primary teachers determined by teachers in teacher training? Shared beliefs as a quantity in teacher training]. *Zeitschrift für Pädagogik, 58*(4), 441-459. https://doi.org/10.25656/01:10388

Streit, C., & Royar, T. (2014). Lernen zwischen Instruktion und Konstruktion. Wie Instruktionen konstruktive Prozesse beim frühen Lernen von Mathematik unterstützen können [Learning between instruction and construction. How instruction can support constructive processes in the early learning of mathematics]. In E. Hildebrandt, M. Peschel, & M. Weißhaupt (Eds.), *Lernen zwischen freiem und instruiertem Tätigsein* (pp. 32-42). Verlag Julius Klinkhardt. https://doi.org/10.35468/5375_03

Strickland, J. F., Page, J. A., & Page, F. M. (1986). *Preservice teachers' perceptions teaching as a career.* American Educational Research Association.

Swars Auslander, S. L. (2007). *The development of mathematics beliefs of elementary school teachers* [Unpublished study]. Georgia State University.

Thomas, J. A. (1998). On becoming a good teacher: Reflective practice with regard to children's voices. *Journal of Teacher Education, 49*(5), 372-380. https://doi.org/10.1177/002248719804900507

Trigwell, K., Prosser, M., & Waterhouse, F. (1999). Relations between teachers' approaches to teaching and students' approaches to learning. *Higher Education, 37*, 57-70. https://doi.org/10.1023/A:1003548313194

Utley, J., Reeder, S., & Redmond-Sanogo, A. (2020). Envisioning my mathematics classroom: Validating the draw-a-mathematics-teacher-test rubric. *School Science and Mathematics, 120*(6), 345-355. https://doi.org/10.1111/ssm.12426

Vollrath, H., & Roth, J. (2012). *Grundlagen des Mathematikunterrichts in der Sekundarstufe* [Fundamentals of mathematics teaching at secondary level] (2nd ed.). Spektrum. https://doi.org/10.1007/978-3-8274-2855-4

Voss, T., Kunina-Habenicht, O., Hoehne, V., & Kunter, M. (2015). Stichwort Pädagogisches Wissen von Lehrkräften: Empirische Zugänge und Befunde [Keyword pedagogical knowledge of teachers: Empirical approaches and findings]. *Zeitschrift für Erziehungswissenschaft, 18*(2), 187-223. https://doi.org/10.1007/s11618-015-0626-6

Weinstein, C. (1989). Teacher education students' perceptions of teaching. *Journal of Teacher Education, 40*(2), 53-60. https://doi.org/10.1177/002248718904000210

Wiese, I. (2016). *Kognitive Anforderungen in geometrischen Aufgaben des vierten Schuljahres* [Cognitive demands in geometric tasks of the fourth school year] (Doctoral dissertation). Universität Koblenz-Landau, OPUS.

Winter, H. (1991). *Entdeckendes Lernen im Mathematikunterricht. Einblicke in die Ideengeschichte und ihre Bedeutung für die Pädagogik* [Discovery-based learning in maths lessons. Insights into the history of ideas and their significance for pedagogy] (2nd ed.). Vieweg.

Wygotski, L. S. (1987). *Arbeiten zur psychischen Entwicklung der Persönlichkeit* [Work on the psychological development of the personality] (Vol. 2). Pahl-Rugenstein.