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MODESTUM

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Using the history of mathematics as a methodological strategy to understand Gaspar Nicolas' "errors"

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ABSTRACT

Two problems, both from Gaspar Nicolas' *Tratado da Prática d'Arismética*, published in 1519 are considered in this study. The tasks were carried out using pencil, paper and students from Portuguese schools. The aim was to explore the following students' performances: 1. The students were able to recognize the "error", explaining it by using a coherent procedure. 2. The students argued why this answer is wrong/incomplete. 3. The solutions of some students present a good concept and clarity of procedure, compared to aspects omitted in historical literature. 4. The students understood the importance of knowing the "errors" made by famous mathematicians in two related aspects: mathematics is a human activity prone to errors and, consequently, the fear of making errors should not be an obstacle to learning math at school.

Keywords: history of mathematics (HM), learning from "errors"

> Learning Mathematics is often perceived by many as a difficult task; but so, can activities such as learning the violin or Chinese. (…) there is a certain tendency (…) to retreat or create blocks when faced with difficulties in Mathematics (…). We assume that it is perfectly natural that learning the violin involves many, many hours of horrible screeching sounds, and that learning to speak Chinese means making mistake after mistake (…) when it comes to Mathematics, we are afraid of making mistakes.
>
> (Gazeta da Matemática nº200 Editorial por Paulo Saraiva, UC)

INTRODUCTION

The OECD Future of Education and Skills 2030 project (Schmidt et al., 2022) proposes looking to the future of societies with school curricula in mind. Given the technological advances and other changes that societies are facing, education needs to provide students not only with solid knowledge, but also with the skills, attitudes and values to become active, responsible and enterprising citizens. Mathematics is considered a relevant subject for achieving these competences, which presupposes a commitment to mathematical literacy. Mathematics should also play the role of cultural endeavor, and, in this sense, a constantly evolving approach is pertinent, closely linked to other sciences, culture and society, bearing in mind a detailed study of historical examples. The ICMI study (Fauvel & van Maanen, 2000) recognizes the benefits that history of mathematics (HM) can play in education through five main areas: the learning of mathematics, the nature of mathematics and mathematical activity, the didactic training of teachers, the affective predisposition towards mathematics, and the appreciation of mathematics as a cultural enterprise. Other recent studies, (Barbin, 2022; Bütüner, 2024, Demattè & Furinghetti, 2022; Tzanakis et al., 2002), have investigated the implementation of a historical perspective in classroom settings, analyzing the impact on student learning. These experiments demonstrate the effectiveness of history in mathematics education.

Primary source projects have been used to improve students' understanding of mathematics. Empirical studies with students have shown interest in cultural and scientific issues. Innovative approaches have been applied in teacher training.

To reinforce and enhance the presence of HM in the teaching-learning process, Salas-García and collaborators described a tool for evaluating historical resources in math textbooks (MARH) published in the British Journal for the History of Mathematics (Salas-García et al., 2022). This tool encompasses ten dimensions (content, connections, activities, methodology, language, illustrations, motivation, ICT, emphasis, general resources) with the respective indicators that we recommend you consult. In this work we have kept in mind the grid recommended by Salas-García et al. (2022). Dimensions/indicators are used, such as (1.11) Inclusion of fragments of original mathematical texts; 4. Methodologies: 4.5. Use of the history of math as a methodological strategy to understand the errors, doubts and controversies surrounding it, as well as the diversity of approaches to problems; 5. Language: 5.1. Translating passages from mathematical texts into modern language; 5.2. Application of mathematics to familiarize students with the languages and notations used to express concepts) and the teacher appeared as a mediator in the construction of knowledge.

Since the 1990s, school programs in Portugal for secondary education (15-18 years) have provided for the recreation of historical settings, using mathematical knowledge from different eras and including original texts in problem solving. In an article published in 1993, the coordinator of the national mathematics programs for secondary education, Jaime Carvalho e Silva, asked about the possibility of using HM in teaching, presenting suggestions for use in the classroom (Carvalho e Silva, 1993). At the same time, Fernanda Estrada (1993, p. 17-20), in a published work, considers that “The role of the history of mathematics is fundamental (...) for the development of a critical spirit and for the student to feel and realize the ideas behind the theories and theorems they have just learnt”. Thirty years on, all the questions of the past have reached the present and continue to arouse the interest of researchers in the inclusion of HM in Portuguese school education. Rodrigues and Moura (2023) wrote about the potential of HM for developing the creative process, promoting transdisciplinary competences and grasping disciplinary content through tasks that include HM problems. In the guiding documents for secondary school curricula, HM is one of the nine key ideas advocated in Essential Learning:

> History of Mathematics valuing the importance of Mathematics in the evolution of society: The use of outstanding episodes and problems from the History of Mathematics should motivate research, studies or debates, not of an encyclopedic nature, but contributing to the appreciation and understanding of the progress of Mathematics. In addition to its intrinsic value as a cultural heritage that should be valued, there are numerous facts, particular aspects and episodes from the History of Mathematics that, due to their pedagogical potential, should be explored in tasks both inside and outside the classroom. Teachers should take advantage of contemporary facts from the history of math to help students understand the decisive role of math today. For example, they could mention the first Fields Medals awarded to female mathematicians, the importance of mathematical models in understanding the climate crisis, the evolution of epidemics or space exploration.
>
> (Carvalho e Silva, 2023, p. 7)

There are no specific HM items in Portuguese secondary school curricula. Portuguese school curricula often prioritize technical content and practical skills. The syllabus guidelines give teachers freedom over the topics/activities to be taught using HM. For example, in school programs in France (Programmes d'enseignement de seconde générale et technologique à partir de 2019, https://www.data.gouv.fr/fr/) (accessed on 2 September 2024), the suggestions for HM themes to be mentioned in each subject are explicit in the guidelines for teachers. The French programs, like the Portuguese ones, in addition to the scientific component, propose a humanistic side which, as Kuntz describes. So, to introduce a mathematical culture within mathematical concepts (Kuntz, 2022).

MAIN AIM OF THE STUDY AND HISTORICAL CONTEXT

In general, Portuguese educators express insecurity and a lack of training in a historical approach to the classroom when asked about the use of HM in their classes. On the other hand, Moyon (2022, p. 1614) describes the resistance of teachers who attended his training to using HM in their practices. Siu (2006) collected teachers' opinions on the use of the history of mathematics in their classes. The data show that although teachers recognize the value of history, its practical application is low. In Martins et al. (2021) some results were presented on the conceptions and practices of Portuguese mathematics teachers participating in a training workshop on the use of HM in teaching the subject. The study allowed us to conclude that when teachers participate in training that prioritizes tasks involving HM framed in specific mathematical programmatic contents, in a reflective and collaborative environment and with experimentation in their practices, they tend to recognize the potential of using HM in teaching and to value it, introducing it into their practices. A significant number of teachers in the training (83%) said that they initially used HM as a teaching resource, increasing this level to 100% after the training. They admitted a similar increase in the frequency of this use (sometimes/often): initially, 60%; after the training, 92%. Nevertheless, it is feasible to view the situation as a collection of educators who are eager to implement HM in the classroom.

In the HM training courses that we provide at the Research and Training Laboratory in the History of Mathematics and Education (LIFHIMATE) of the Mathematics Department of the University of Aveiro (https://www.ua.pt/pt/lifhimate/), It is our conviction that the training courses will serve as an ideal opportunity for Portuguese educators to disseminate information regarding the educational potential of HM. However, we also have testimonies from teachers who rarely or never use HM. Their justifications are as follows: they often feel pressure to cover curricular content, and schools pressure them to succeed in end-of-year exams. As such, they avoid topics that are not directly assessed. The history of mathematics is frequently mentioned as a subject that may necessitate additional class time, which is a challenge in curricula that are already overloaded. At the same time, they feel helpless in terms of resources and training opportunities. As for the students, they are not used to math lessons in a historical environment. Our focus in training is to show teachers ways to integrate history without compromising other essential content. In our opinion, clear guidelines and practical examples are needed to guide educators.

The presence of errors in a solution is another issue that we address in this paper. The role that "errors" have played in the history of mathematics is comprehended. Far from being just an obstacle, errors have often driven discoveries, reformulated theories and enriched mathematical thought. Students are trained to produce "clean" and error-free results. The evolution of mathematical knowledge has not been exactly this way.

As Paulo Saraiva (2023, p. 2) writes in the introduction to this study "(…) when it (error) comes to Mathematics, we are afraid of making mistakes." Students usually perceive the error as a definitive failure. We advocate practices that lead students to understand error as something natural and not a deviation from the mathematical path, but often the path itself.

Two issues from Gaspar Nicolas' work were presented in this study. His book is rich in problems/content for use in the classroom, in any year of schooling. The topics cover the commercial practices of the 1500s and mathematics topics of no apparent interest to merchants. The main aim of the study is to ascertain the impact of the author's 'errors' on students' learning.

The author

There is little information about Gaspar Nicolas (https://dicionario.ciuhct.org/nicolas-gaspar/) (accessed on 20 May 2024). The work of Gaspar Nicolas is linked to information about him. In the prologue, the author mentions his connections to the city of Guimarães, where he met D. Rodrigo, Count of Tentúgal. When Gaspar Nicolas clarified his doubts about arithmetic with D. Rodrigo, he said he felt motivated to write about arithmetic. This apparently isolated purpose gave rise to the first Portuguese treatise on practical arithmetic. This treatise belongs to a group of books that came at a time marked by a desire to perpetuate knowledge that had previously been transmitted orally in written form. The emancipation of the vernacular languages from Latin, the advent of the printing press and the use of Indo-Arabic numerals all also contributed to this change.

The author in the socioeconomic context of his time

The Tratado da Pratica d'Arismetica is contemporary with the Portuguese Discoveries. For the first time in history, the world would be totally interconnected. Overseas expansion ushered into a revolution in big business, and with it the search for new lands that would bring gold and silver that would revitalize European economies, which had been in crisis during the 14th century. Portugal was a key player in this change, partly due to its development in shipbuilding and navigation techniques (Leitão, 2009). The great social changes felt did not go unnoticed by Gaspar Nicolas, who saw the need for training in business and navigation. The treatise includes algorithms based on the vulgarization of calculation with Indo-Arabic numbers, as well as so-called 'classical' topics such as progressions, square and cube roots, problems with numbers, among others which, although disconnected from the world of commerce, took on a prominent place in the mathematical knowledge of the time. According to Nicolas, some merchants came to him at the India House to ask questions about the arithmetic rules of taxation. The training of officials was also a task. The development of foreign trade meant that voyages had to be planned: the costs of setting up ships, investment in cargo and an agreement between the parties involved. It also became necessary to implement an insurance system to cover possible damage during the voyages. The period between the 13th and 16th centuries saw an evolution in the professionalization of mercantile activity. We have no concrete data on the training of national merchants. It is thought that part of their training took place within the family. However, it is recognized that national merchants were poorly trained in business compared to Italians and Flemish, hence the strong motivation to write books that would even allow for self-training, as Nicolas states in his treatise (Clain, 2015).

About numbers and quantities

Commercial arithmetic is based on calculation techniques, numbers become "measurement" entities, and their previous speculative "figure" was lost in the mercantile world. "Whole", "broken" and "mixed" numbers (Spiesser, 2008) it is precisely this typology that we found in the national arithmetic of the 16th century. That is, whole numbers except zero, represented in the Indo-Arabic decimal system; fractional (broken) numbers, of the form a / b , such that a and b are natural integers and $0 < a < b$; the mixed numbers, represented by $a\ b / c$ such that a, b and c are natural integers and $a > 0$ and $0 < b < c$. These numbers written in this way correspond to $a + b / c$. The treatises on practical arithmetic written in Portugal followed a tendency, already manifested in other previous works, to frame themes linked to «doing mathematics for the sake of doing it» through subjects initially disconnected from the commercial world. Practical geometry, within commercial arithmetic, between the 14th and 16th centuries, is a rarely present topic. Spiesser (2008) carried out a study which showed that few arithmetic works produced in France and Spain contain geometry problems. This theme is present in Portuguese arithmetic by the hand of Gaspar Nicolas.

METHODOLOGY

Problem 1

Participants

Problem 1 (P1) was given to secondary school students from Escola Secundária José Falcão and Escola Secundária Jaime Cortesão. The first school has a total of 30 students, with 20 girls and 10 boys. In the second school, there are 20 students, with 8 girls and 12 boys. Students in both schools range in age from 15 to 17 (See **Figure 1**).

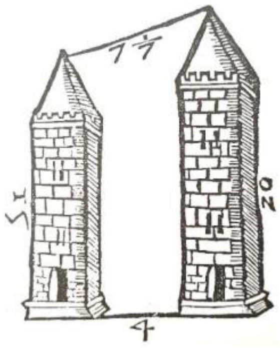


Figure 1. The drawing included Problem 1 (Nicolas, 1963, f.88v)

The students from the first school were in the tenth year of Mathematics A and the second in the eleventh year of Mathematics Applied to the Social Sciences.

In both schools, for the activity, we used pencils, paper and the school manual, in case it was necessary to review any concepts. Each student received a worksheet. The work was carried out in groups of four students. This activity was carried out at the time of a complete class (100 minutes).

Activity implementation strategy for problem 1 by Gaspar Nicolas

In the introduction to the activity (P1), our objective was to provide the students with a context of the 16th century in terms of maritime and commercial expansions. The institutions, such as the House of India, and other historical elements of the society of the time in which the treaty is set, were considered. A document was given in the first part of the class to introduce the students to a historical setting. To show the importance of calculating with Indo-Arabic numerals, in relation to other older systems with limitations, we referred to Babylonian numerals and Roman numerals, already familiar to these students from the dates inscribed on monuments. They were also shown a Roman calculation table for addition. The aim was for the students to feel how easy it was to work with the ‘new’ numbers. The arrival of ‘Indian calculus’ in Europe and particularly in Portugal was mentioned: it was Prince Pedro who, in his Livro da Virtuosa Bem feitoria, used Indo-Arabic numerals for the first time (around 1415), while Duarte Pacheco Pereira's Esmeraldo de Situ Orbis (written between 1505 and 1508) was the first text by an author born and educated in Portugal in which the percentage used of these numbers is higher than that of Roman numerals (Carvalho, 1993).

The only textbook on practical arithmetic written in the 1500s in Portugal that deals with geometry is that of Gaspar Nicolas. In the historical approach, we chose a geometry problem to present to students for the following reasons: importance of recognizing cultural knowledge given that it is an old text from the first mathematics book written in Portugal, topicality of the subject presented in Portuguese curricula, opportunity to analyze statements and solutions considering knowledge already acquired. The implementation of the task comprised several stages, which we will now describe. The formulation adapted to spoken Portuguese of a problem taken from an old math’s book, Tratado da Pratica d'Arismetica by Gaspar Nicolas, facsimile edition by Livraria Civilização, Porto 1963, with its original figure (Nicolas, 1963, f.88v):

Problem 1 There are two towers that are not equal in height. One is 20 fathoms high, and the other is 15 (fathoms). And they are separated from each other by 4 fathoms. I ask, casting a line from end to end, how long is this line? (Nicolas, 1963, f.88v).

Interpretation and questions

The students read the statement, observed the figure and were asked some questions about the measurements, namely about the 4 between the towers:

> Is 4 between the bases center of the towers? Or just the distance to the edge of one tower and the edge of another?

At this point we clarify that, in ancient authors, it is common to find answers to questions of this nature in the resolution and, thus, the author's solution, modified for modern Portuguese, is the next step.

> The tower that has 20 has 5 more than the other. Now, do 5 times 5 which are 25 and do 4 times 4 which are 16. Add 16 to 25 and they are 41. Take its square root which is 7 and a seventh. And this root is *sorda* (irrational number). And you will say that it goes from one tower to another, that is, that it goes from end to end.

The students analyzed Gaspar Nicolas' answer and identified the application of the Pythagorean theorem to the right triangle with side’s measures 4 and 5.

Activity orientation and questions

Inquiries regarding the outcome were posed to the students, and they were requested to document their responses.

> Is the $7\frac{1}{7}$ a solution $\left(7\frac{1}{7}=7+\frac{1}{7}\right)$ presented by Gaspar Nicolas, correct? How would Gaspar Nicolas have arrived at this answer?

Let's look at one answer:

Gaspar Nicolas used the Pythagorean theorem $4^2 + (20 - 5)^2 = x^2$

$\Leftrightarrow x = \sqrt{41}, x > 0$ because it's a measure. The reasoning is correct; the result is not, as shown in **Figure 2**.

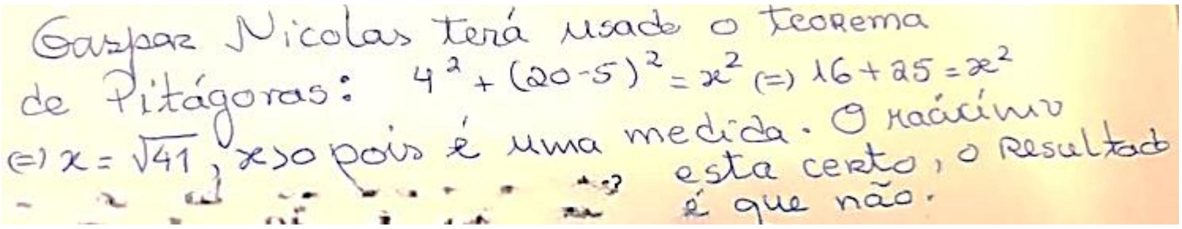


Figure 2. Reaction from a class of pupils (Source: Field study)

Why is Gaspar Nicolas' answer incorrect?

$7\frac{1}{7} = 7 + \frac{1}{7} = \frac{50}{7} \approx 7{,}142 \ldots \neq 6{,}403 \ldots = \sqrt{41}$ and $\sqrt{36} = 6, \sqrt{49} = 7$, so $6 < \sqrt{41} < 7$. In other words, it would be between 6 and 7, which doesn't correspond to Nicolas' result, as shown in **Figure 3**.

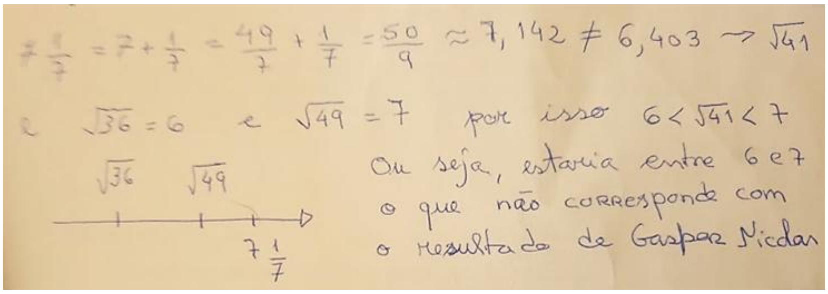


Figure 3. Reaction from a class of pupils (Source: Field study)

Without using mathematical language, describe a plan to find a good solution.

We can consider the right-angled triangle between the two towers. The two sides are the distance between the towers and their difference in height. Obtaining the hypotenuse that corresponds to the desired value, as shown in **Figure 4**.

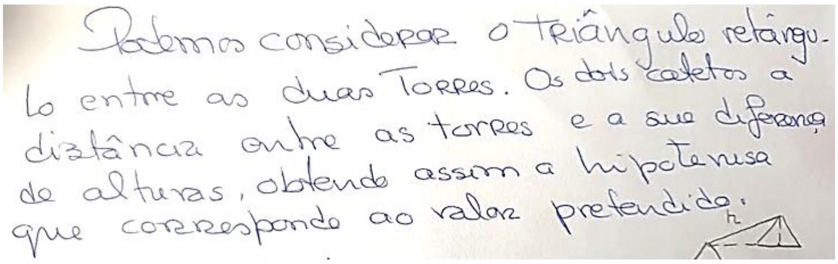


Figure 4. Reaction from a class of pupils (Source: Field study)

Students' perceptions of sixteenth-century mathematical language compared to current mathematical language varied greatly, but there were trends toward common difficulties. Poorly standardized mathematical notation, the problems and calculations were described in running text, compared to current Mathematics with highly symbolic language and globally standardized conventions. There was some initial confusion with the figure, as mentioned above. However, this was overcome with the interpretation of the text, which required some effort but was also a challenge for the students.

Correction of the statement in relation to the knowledge already acquired

At this juncture, it was necessary to revise the language and ensure that it aligned with the students' comprehension of geometry questions, while also considering the mathematics curriculum. Thus, the authors composed the following: As you can see in the initial figure, each tower can be made up of a prism and a pyramid, both of which are regular quadrangles. Consider that the front faces of the two towers are in the same plane. Also consider that the pyramids are geometrically equal. The distance between the centers of the bases of the towers is equal to 4 fathoms (answer to the observation made by the students). Look at the diagram (**Figure 5**), which is not too scale, and solve the problem posed by Gaspar Nicolas using your knowledge of analytical geometry in the plane.

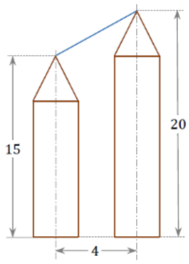


Figure 5. Corrected figure (Source: Field study)

Analyzing the answers

Two answers were chosen (**Figure 6** and **Figure 7**).

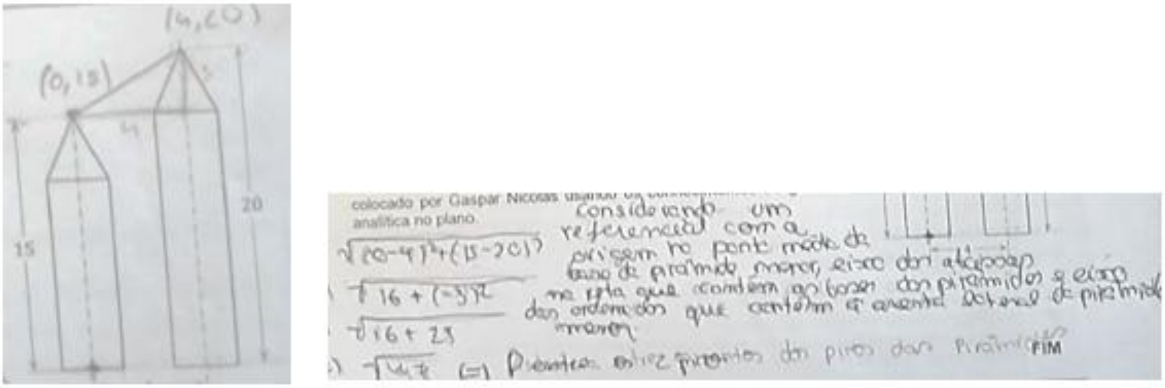
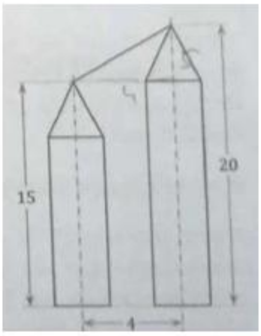
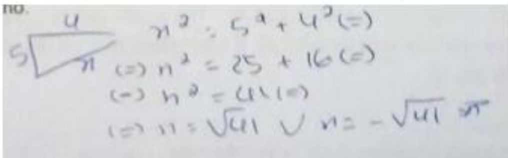


Figure 6. Using the values of coordinates for finding the distance between the tops of the towers (Source: Field study)



$$x^2 = 5^2 + 4^2 \Leftrightarrow$$
$$\Leftrightarrow x^2 = 25 + 16 \Leftrightarrow$$
$$\Leftrightarrow x^2 = 41 \Leftrightarrow$$
$$\Leftrightarrow x = \sqrt{41} \vee x = -\sqrt{41}$$



$$x > 0, x = \sqrt{41}$$

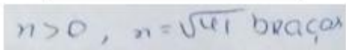


Figure 7. Using the Pythagorean theorem to find the distance between the tops of the towers (Source: Field study)

Calculating the distance between two points on the plane

> We consider a reference point with the origin at the midpoint of the smaller pyramid, the abscissa axis on the line containing the bases of the pyramids and the ordinate axis containing the side edge of the smaller pyramid (calculating the distance between the points (0,15) and (4,20)).

This answer was given by a student who used a frame of reference identifying the important points, (0,15) and (4,20), to calculate the distance between them. The definition of distance between two points on the plane is a subject of the curriculum. The problem of the past was solved with knowledge of the present. At this point, we discussed the structure of Nicolas's mathematics book, which was intended to teach but did not have a prior definition of concepts. When you read math in school textbooks, it's presented as a product with definitions and properties listed before problems are solved.

Application of the Pythagorean theorem

In this answer, the student applied the Pythagorean theorem to the right triangle with sides 4 and 5 and gave the exact value of the hypotenuse. Note that these students are used to working with exact values. They only use approximate values when asked to do it. The students compared and evaluated the two strategies (their own and the original) and drew their own conclusions. (Note: The four steps were first carried out in groups of four students. Then the groups shared their solutions and conclusions with the whole class).

About making errors

In order to conclude the assignment, the authors requested:

> Do you think it's important to know that even famous mathematicians have made mistakes? Tick one of the options below.

(A) Yes ......

(B) No.....

(C) I don't know what to say....

In one line, justify your choice.

RESULTS

The answers of 50 students involved in the activity were the following:

(A) Yes - 46

(B) No - 0

(C) I don't know what to say - 4.

In most of the responses, students emphasize the human side of mathematics:

> Humans do mathematics and make mistakes like humans, so the presence of errors is natural.

Others focus on making mistakes, correcting them, and moving on. Knowledge evolves with mistakes if they are corrected. During the discussion the teacher asked:

> Can we succeed in mathematics if we are persistent?

The answer was yes.

The teacher points out the difficulty some students have in understanding statements in general, stating that:

> In these students the difficulty is related to a general deficiency in reading and interpreting texts.

Although a mathematical approach that considers its history is not a miracle to guarantee immediate success, when presenting the Gaspar Nicolas problem, the authors consider historical statements that motivate students to read with attention, underline key words, reread and reformulate the problem in their own words. And with these guidelines we observe very engaged groups.

Some considerations

In the responses given by students from both schools involved in P1, there is a feeling that making mistakes when solving problems is part of the process. But is this always the students' feeling? Charles Pépin, a French philosopher, in an interview with El Pais, said that he knows young people who have been traumatized by a school system that doesn't encourage uniqueness, forcing them to adapt to what has been defined as the norm. And he believes that any success can be considered, in this sense, a corrected failure (https://brasil.elpais.com/brasil/2018/01/03/cultura/1514978576_244946.html#, accessed on 1 September 2024). In certain philosophical theories, for instance, error has been a significant factor. Karl Popper, a 20th century philosopher, argued in the philosophy of science that error is the engine of scientific progress (https://plato.stanford.edu/entries/popper/, accessed on 1 September 2024). Mathematics teachers face many situations in class where students say:

> I don't answer so I don't make a mistake, and I don't look bad in class because 'some teachers don't like it when we make mistakes.

They get angry. Both teachers and students should bear in mind that learning is a process that takes time and involves many mistakes in between, as Paulo Saraiva (Saraiva, 2023, p.2) also said in the introduction to this study. It's the "errors" that take us further and make us want to improve!

Gaspar Nicolas' distractions and "errors" did not make him "give up", and he was recognized as a determining figure in the mathematical knowledge of his time. The *Tratado da pratica d'arismetica* (Nicolas, 1963) was widely used to teach practical arithmetic in Portugal. The 1519 edition came from the workshop of Germão Galharde. There is a copy of this edition at the Faculty of Sciences of the University of Porto which, in 1963, was published in a facsimile edition by *Livraria Civilização* in Porto and prefaced by Luís de Albuquerque. Apart from these editions, they were printed in the sixteenth, seventeenth and eighteenth centuries (only one edition) and were printed again, under the coordination of Jorge Nuno Silva and Pedro Jorge Freitas, in an edition published by the Calouste Gulbenkian Foundation in 2022. In the editions consulted (see **Appendix 1**), it is in the latter that the 'error' in problem 1 is corrected. In notes 363 and 364 we read 'This result is wrong since $\sqrt{41} = 6{,}4$ ...'; 'Assuming the towers have no thickness, it's a question of using the Pythagorean Theorem to determine the hypotenuse of a right triangle with sides 4 and 5. The author makes a mistake when calculating the square root of 41, perhaps because he has confused it with the root of 51, since $\left(7\frac{1}{7}\right)^2 = 51{,}020$ ......' (Silva, 2022, p. 212).

Problem 2

Some additional considerations

Problem 2 is a very old problem. It is thought that a version was set out in the *Problems for developing the minds of the young people*, attributed to Alcuino of York (Nogueira & Marques, 2020, p. 16). The original text, in Latin, *Propositiones ad Acuendos Juvenes*, had versions in the 9th, 10th and 11th centuries and has been a way of transmitting mathematical knowledge with playful characteristics over the centuries. Nogueira (Nogueira & Marques, 2020) carried out a survey of manuscripts and printed editions up to the 21th century. Some authors have divided the problems into mathematical themes, and the problem that Nogueira numbers 42 falls under the Succession theme. It's not about picking oranges but about counting doves on a ladder with 100 rungs (Nogueira & Marques, 2020, p. 55). In the 15th century, the friar Luca Pacioli, referred to by Gaspar Nicolas, presented his version for picking up 100 stones (Nogueira & Marques, 2020, p. 61). This would have inspired the Portuguese author to include the problem of oranges in his treatise a century later.

Nicolas' treatise does not give a general rule for adding consecutive terms of an arithmetic progression (Nicolas, 1963, f. 39). Some cases are worked out, such as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and the sum are done like this: divide the last term by the first, add 1 to this result and this sum is multiplied by half of the last term, i.e.,

$$\left(\frac{a_n}{a_1}+1\right)\times\frac{a_n}{2} \tag{1}$$

The result of calculating the number of steps to pick the 100 oranges, if the sequence 2, 4, 8, ..., 198, 200 is taken into account

$$\left(\frac{200}{2}+1\right)\times\frac{200}{2}=(100+1)\times 100=100\times 100+100 \tag{2}$$

The rule changes for the example of adding odd numbers from 1 to 21. According to Nicolas, add the first term to the last, calculate half of this value and multiply by itself, i.e,

$$\left(\frac{a_1+a_n}{2}\right)^2 \tag{3}$$

These 'rules' work in very specific cases. The sequence 1, 5, 9, 13, and the reasoning behind it can be interpreted as follows:

$$\left(\frac{1+13}{2}\right)^2=49\neq 1+5+9+13=28 \tag{4}$$

$$\left(\frac{13}{1}+1\right)\times\frac{13}{2}=14\times 6{,}5=91\neq 28 \tag{5}$$

In any case, Gaspar Nicolas must have noticed the property of arithmetic that the sum of two equidistant terms is always equal to the same number, which makes it possible to deduce the formula for consecutive terms of arithmetic progression. However, he didn't mention it and limited himself to very specific examples in his 1519 edition.

Formulae (1) and (3) are corrected in Gaspar Nicolas' treatise from 1594. The author writes that expression (1) is only valid if the first term is equal to 1, considering that the ratio is also 1. If this is not the case, the general rule is used:

> We can start wherever we like. We add the largest to the smallest and this sum is multiplied by half the number of numbers you take.

(Nicolas, 1594, p.48) (**Appendix 1**)

As a result, the students are now acquainted with the formula for the sum of n consecutive terms. On the other hand, given the enthusiasm and involvement we saw when proposing the activities, we could return to the work of Gaspar Nicolas where we will find a lot of content to explore.

Participants

The problem 2 (P2) was part of an activity organized at Escola Secundária D. Maria II with a group of eleventh year Math A students (N = 39). The group consisted of 19 boys and 20 girls aged between 16 and 18. The task was completed in 120 minutes. It was an activity where we used pencils and paper and the school manual.

Activity implementation strategy (problem 2 by Gaspar Nicolas)

No changes were made to the introduction to problem 2 from the one we used in problem 1. That is, a framework of the author and his time in the sixteenth century. The authors emphasize the importance of the subject matter, which remains a subject in the 11th grade school curriculum, to students. Students know how to write the general term of arithmetic progression and determine the sum of terms. In this activity we highlight contact with sources, in addition to promoting the development of problem-solving skills and students' cultural awareness. Students were encouraged to solve problems and record their impressions.

The 11th graders learnt about problem 2 (P2) from a statement transcribed from the original text in 16th century Portuguese as was said previously. The choice of the 'original' text considered the linguistic issues addressed previously in Portuguese lessons.

It's always a challenge to present the text as it was written by the author at the time. And here, there was some hesitation at the beginning, such as:

> Am I reading this right?

which was used to motivate us to continue reading in stages and to translate the ideas present into current Portuguese. The statement is as follows:

> (P2) A man has spread out 100 oranges and tells another man to pick them up one by one in a pile. You'll know that the friar puts it in these same terms and says that the oranges will be picked in 10100 steps and because the steps of 100 oranges aren't that many, some people want to say that he wrote falsely and that he didn't feel it and I say no, it's hard to believe that anyone who took such subtle rules as he did understood all of Euclid's works, both geometry and arithmetic, and scolded Bernardino in geometry, as you can see in his work. I can't believe that the work was written falsely unless those who make such reprimands don't understand the rules, because they say that 100 oranges are picked in 9900 steps. It's true that this would be the case if he started picking from the first orange, but if you're old enough to understand 100 steps of space, you should know that when he starts picking, he'll have to move one step away to pick all 100. And that starting from the first, without breaking stride, he only caught 99, and that the other, the first, was caught. Now I want to give you a general rule as to where those 10100 steps came from, and you'll do it this way. You always multiply the oranges as many as you want and with this multiplication you add the same oranges together and the sum you make in as many steps will be caught. So, you say 100 times 100 is 10000, add the same 100 as you said and it's 10100 and you're right.

(see Problem 2, transcription in Portuguese in the **Appendix 2**)

Interpretation and questions

Following the reading, the students posed the subsequent inquiries:

a) Who is the friar? (Luca Pacioli) (https://en.wikipedia.org/wiki/Luca_Pacioli)

b) We know Euclid, but who is Bernardyno? (Thomas Bradwardine (ca 1300 - 26 August 1349) (https://catholicscientists.org/scientists-of-the-past/thomas-bradwardine/)

To answer these questions, the students were asked to search for the authors mentioned on the internet, with the results being presented in the following lesson. The websites given in a) and b) were found by the students.

Activity orientation and questions

The students were questioned about the text as we continued to work on the issue.

> What is the proposed question?

Only one student mentions that it's a question of calculating the number of steps to catch the 100 oranges, as well as discussing the right solution and the authors who accuse the friar of making a mistake. The rest only mentioned the first aspect.

This was followed by an analysis of the arguments put forward by the author regarding the solutions of those who were right and those who, according to him, were wrong:

> Gaspar Nicolas says there are two solutions. One is right and the other is wrong. Identify the solutions. Is what Gaspar Nicolas saying correct?

Table 1 shows the data collected from the answers to these questions from 39 students: In this question, ten students think that the only correct solution is 9900, three students think that only the solution 10100 is correct, seventeen students say that both solutions are correct, and the rest say nothing about the subject or didn't understand what was asked and give an answer out of context.

Table 1. Identifying the solution/solutions

Identifying	Is what Gaspar Nicolas saying correct?			
	9900 rights	10100 rights	Both correct	No answer
39 students	10 students (A)	3 students (B)	students (C)	9 students (D)

The next step is to look at the resolution. We started by drawing up a plan to solve the question.

> Describes verbally (without using mathematical language) a plan to find the solution to the problem, starting by translating the reasoning into actual mathematical language, using knowledge already acquired.

> Translate your reasoning into 3. in mathematical language. Check the validity of the solution.

And to conclude question 4, the statement was reworded if it was a current problem. He also mentioned that, at that time, mathematics texts were like a 'conversation' between the reader and the author, which is why the subjects appeared little by little. However, Nicolas points the finger at those who doubt Pacioli and sees the subject as an affront to the great master. We found it enriching to comment on the author's words.

At the end of his answer/comment, Gaspar Nicolas says that in this statement there is no possibility of being next to the first orange, hence the error of those who say there are fewer steps... Do we have this information in advance? Did Gaspar Nicolas make an error, or did he simply omit important information?

The importance of knowing that even famous mathematicians have made errors:

Do you think it's important to know that even famous mathematicians have made errors?

35 students answered "Yes", and 4 students answered, "I don´t know what to say".

Analyzing the answers

Let's go back to **Table 1** to analyze the students' answers. For this purpose, the reasoning and contemplation were taken into account for the students in groups (A), (B), (C) and (D).

In group (A), ten students think the solution 9900 is correct. In their answers, 5 students consider that there are 99 equal spaces between the oranges (by the quotient of 9900 by 100). They say that the expression $u_n = 100 \times (n-1)$ gives the total number of steps travelled, so $u_{100} = 9900$. The remaining 5 didn't explain how they arrived at the answer 9900 steps. Regarding question 6, 8 students think it's important to know that you make mistakes, the rest didn't answer. Some of the answers were presented as justifications:

I liked the problem, we humanized mathematics and showed that making mistakes is part of creating knowledge.

We learn from mistakes to try good solutions and to read statements carefully.

Sometimes we make mistakes because we misinterpret the situation, that's what happened... They weren't wrong. It wasn't all written down...

This group didn't show much effort in exploring the situation or explaining their reasoning properly. However, in their own words, they believe that dealing with mistakes is essential.

Regarding the 3 students in **group (B)**, one of them refers to the expression $u_n = 100 \times (n+1)$ for the total number of steps, making $u_{100} = 10100$. Another student gives the sequence $u_n = 2 + (n-1) \times 2 = 2n$, calculating:

$$S_{100} = \frac{2+200}{2} \times 100 = 10100 \tag{6}$$

The latter: It's important to make mistakes in order to improve

The former: To understand the reason for math: To find problems to find solutions. All three consider it important to recognize that even famous people make mistakes.

In group (C), the students start by drawing diagrams to arrive at their answers and consider that both solutions are correct, both are divisible by 100. ("You take enough steps to catch all the oranges, and it doesn't matter how long the steps are."). They also consider the oranges to be equally spaced and two hypotheses: The person starts on top of the first orange or one step away from the first orange. Then they find the general expression of the successions that represent the two situations, determining the total number of steps. There were two possibilities (see **Figure 8**): You can start on the first orange; You can start one step away from the first orange.

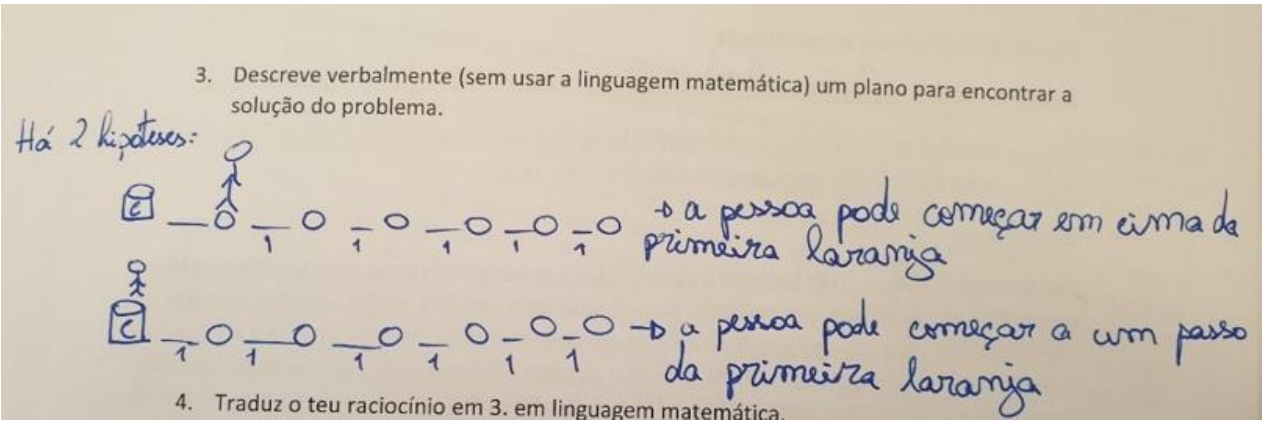


Figure 8. Expressing two different plans for solving the problem (Source: Field study)

There are 2 possibilities:

→ 1. the person can start at the first orange;

→ 2 the person can start one step away from the first orange.

First possibility

Orange 1 : 0 steps

Orange 2 : 2 steps (round trip)

Orange 3 : 4 steps (two to get there and two to get back)

And so on...

Second possibility

Orange 1 : 2 steps (round trip)

Orange 2 : 4 steps (two to get there and two to get back)

Orange 3 : 6 steps (three to get there and three to get back)

And so on...

They conclude that: Gaspar Nicolas states that in this statement the possibility of being next to the first orange is not assumed, hence the error of those who say there are fewer steps...

The man is next to the first orange: If *n* is the number of steps, $u_1 = 0$. Considering the sequence of the number of steps, with the steps equal, $u_n = 0 + (n-1) \times 2 = 2n - 2$. The man must take one step to reach the first orange: Let *n* be the number of steps yields $u_1 = 2$. Considering the sequence of the number of steps, with the steps equal, $u_n = 2 + (n-1) \times 2 = 2n$. This is how 17 students responded. Remember that succession is taught in the 11th grade. Adding up the total number of steps (sum of the n consecutive terms of an arithmetic progression), yields in the first case:

$$S_{100} = \frac{0 + 198}{2} \times 100 = 9900 \tag{7}$$

In the second case:

$$S_{100} = \frac{2 + 200}{2} \times 100 = 10100 \tag{8}$$

This group showed a strong perception of the humanization of mathematics: seeing that mathematicians of the past also made mistakes, experimented and performed with limitations can make mathematics more accessible.

The students in **group (D)** left the proposed questions blank and simply transcribed some of the author's statements.

Considering the students' statements, it is understood that learning is a process of trying to get it right and getting it wrong, conjecturing, deducing and arriving at valid results. Bearing in mind that famous mathematicians also made mistakes motivated the development of strategies aimed at overcoming error, promoted self-esteem, and aroused curiosity to 'dive' into previous eras where mathematical knowledge itself knew its 'limitations.

CONCLUSION

Historical problems introduce a narrative that is not found in textbooks and an "old" language, forcing students to leave their comfort zone and immerse themselves in a distant era. This provides an experience of putting themselves in the time and place of the author, in this case, Gaspar Nicolas. In this way, students are encouraged to reflect on the historical and cultural context of the problems. The activities described provided students with enriched learning experience, based on different dimensions of mathematics teaching. The inclusion of fragments of original mathematical texts allowed direct contact with the language and thinking of mathematicians of the past, promoting the appreciation of the history of mathematics as a pedagogical tool. The students were encouraged to translate and reinterpret old mathematical texts, facilitating the understanding of concepts considering modern language and current notations. This practice contributed to developing mathematical communication skills and familiarization with different forms of symbolic and conceptual expression.

The use of the history of mathematics not only contributed to contextualizing concepts and procedures but also served as a strategy to reflect on the errors, doubts, and controversies present throughout the construction of mathematical knowledge. This approach favored the recognition of mathematics as a science in constant evolution, the result of multiple approaches and diverse cultural visions. In the activities described the act of "making errors" is linked to a result and not to a resolution process. Errors appear in medieval texts (Slisko, 2020). But can we talk about serious, punitive errors? If we think that errors can appear in a multitude of contexts, then here we need to value errors as an integral part of the learning process. The concept of error should emphasize the desire to self-evaluate to learn how to do things correctly. However, many students perceive mistakes as something serious. Faced with this attitude, the teacher should analyze and explain that by living positively with the mistakes they make, students will be motivated to overcome them. In this way, their learning will evolve. Errors will then become a source of knowledge and motivation to learn.

The activities that were implemented enabled us to observe the points 1 through 4 that were specified in the Summary. Discussions between pairs, arguments arose about the errors and inaccuracies in the statements, and coherent results were achieved. The students-built bridges between Nicolas' ideas and their knowledge of mathematics. And, in their own words, they showed sensitivity to the fact that mathematics is a human activity and therefore prone to errors, contrary to what they might believe when looking at the textbook. In the textbook, mathematics is clean, omitting the long road travelled and full of obstacles, which is no reason to 'give up'. Likewise, the fear of making mistakes should not be an obstacle to learning. In these classes, the teacher acted as a mediator of knowledge, promoting an environment of dialogue, reflection and discovery. Instead of providing ready-made answers, he encouraged students to explore ideas, question interpretations and collaboratively construct meanings for the concepts covered. This methodology favored more meaningful, critical and contextualized learning, respecting the different rhythms and ways of understanding mathematics, and reinforced the active role of students in the teaching-learning process. As a result, greater student engagement was observed, as well as a deeper understanding of the concepts addressed, articulating language, history and methodology in an integrated and formative perspective.

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Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX 1

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Tratado da Pratica d’Arismetica ordenada per Gaspar Nicolas e agora a tercera vez impressa e enmendada, Luís Rodrigues, Lisboa, 1541 (Biblioteca Pública de Évora, cot. 82)

Tratado da Pratica d’Arismetica composta e ordenada por Gaspar Nicolas. Agora de novo emendada nesta quinta impressam. Impressa com licença do Supremo conselho da santa geral Inquisiçam, edição de Domingo Miz, Lisboa, 1594

(Biblioteca Nacional de Portugal, Cot. Res. 4355 P)

Tratado e arte de Arismetica Para fazer um Perfeyto Cayxeyro, Seu autor Gaspar Nicolas. E emendada e acrescentada Por Manoel de Figueyredo, Cosmografo Mor que soy das Conquiatas destes Reynos de Portugal, E no fim com várias curiosidades de Arismetica. Offerecida A’Inclyta Doutora Sta. Catarina, Bernardo da Costa de Carvalho, edição da Irmandade de Santa Catarina, 1716

(https://books.google.pt/books/about/Tratado_e_arte_de_arismetica_para_fazer.html?hl=pt-PT&id=MTe0vXrq_rMC&redir_esc=y) (accessed on 20 July 2015)

APPENDIX 2

A1. Problem 2 transcription in Portuguese: “Huũ homem espalhou .100. laranjas e dyz ha outro que as apanha huũa a huũa todas em huũa pinha. Ora eu demando em quantos passos apanhara aquella pinha aquellas laranjas tu saberás que ho frade poem esta questam per estes mesmos termos e diz que has laranjas seram apanhadas em .10100. passos e por que hos passos de .100. laranjas nam sam tantos querem dizer alguũs que escreveu falso. E que ho nam sentio e eu diguo que nã que nã he de creer que quẽ tyrou tã sotys regras como elle que entendia todas as obras de Euclydes assy de gyometrya como de arismetyca e reprendeo a Bernardyno na gyometrya como podes veer na sua obra. Nam posso creer que obra escrepvesse falsa salvo nam entendem as regras hos que taes reprensoões fazem por que dyzem que .100. laranjas sam apanhadas em .9900. passos verdade he que assy seria se começasse apanhar da primeyra laranja mas ha vello d’entender que estee .100. passos d’espaço conven asaber que quando elle começar a apanhar que se ha de faltar huũ passo pera as apanhar todas .100.” (Nicolas, 1963, f. 72 f)

A1. Problem 2 transcription in English: A man has spread out 100 oranges and tells another man to pick them up one by one in a pile. You'll know that the friar puts it in these same terms and says that the oranges will be picked in 10100 steps and because the steps of 100 oranges aren't that many, some people want to say that he wrote falsely and that he didn't feel it and I say no, it's hard to believe that anyone who took such subtle rules as he did understood all of Euclid's works, both geometry and arithmetic, and scolded Bernardino in geometry, as you can see in his work. I can't believe that the work was written falsely unless those who make such reprimands don't understand the rules, because they say that 100 oranges are picked in 9900 steps. It's true that this would be the case if he started picking from the first orange, but if you're old enough to understand 100 steps of space, you should know that when he starts picking, he'll have to move one step away to pick all 100. And that starting from the first, without breaking stride, he only caught 99, and that the other, the first, was caught. Now I want to give you a general rule as to where those 10100 steps came from, and you'll do it this way. You always multiply the oranges as many as you want and with this multiplication you add the same oranges together and the sum you make in as many steps will be caught. So, you say 100 times 100 is 10000, add the same 100 as you said and it's 10100 and you're right. (Nicolas, 1963, f. 72 f)