International Electronic Journal of Mathematics Education
2025, 20(4), em0849
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Understanding and strategies for comparing fractions among pre-service teachers: Between procedural rigidity and conceptual flexibility

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Citation: Youkap, P. T., & Nguala, J.-B. (2025). Understanding and strategies for comparing fractions among pre-service teachers: Between procedural rigidity and conceptual flexibility. *International Electronic Journal of Mathematics Education, 2020*(4), em0849. https://doi.org/10.29333/iejme/16834

ARTICLE INFO

Received: 14 Mar. 2025

Accepted: 08 Aug. 2025

ABSTRACT

This study explores the cognitive structures of pre-service primary teachers regarding fraction comparison, using Vinner's concept-image framework to analyze how these pre-service teachers mobilize their knowledge to compare fractions. The mixed-methods approach, combining both quantitative and qualitative analyses (including a questionnaire), highlights significant cognitive challenges related to fraction comparison. The study, conducted with 160 participants, reveals a marked reliance on rigid procedures, with the systematic application of a single comparison procedure, regardless of contextual variations or task-specific demands. The observed errors appear to stem from limited procedural flexibility and underdeveloped pre-service teachers' concept-images on fractions. Furthermore, a significant number of participants do not recognize fraction comparison as a valid mathematical model in problem-solving contexts, which limits their ability to teach this concept effectively. These findings underscore the need to reform teacher education by integrating targeted interventions aimed at increasing procedural diversity and enriching the conceptual understanding of fractions. Such initiatives would enable teachers to perceive fractions not only as static mathematical objects, but as dynamic, interconnected, and evolving entities. This study thus contributes to the enhancement of teacher preparation, particularly in managing foundational arithmetic concepts, to promote a more flexible and conceptually rich approach to teaching fractions in primary education.

Keywords: fraction comparison, concept image, procedures, pre-service teachers, problem-solving

INTRODUCTION

Fractions are an essential concept in arithmetic, introduced in primary school in France and are also part of the primary school teacher training curriculum. They serve as a unifying concept, integrating various aspects of mathematics teaching. In arithmetic, fractions are fundamental for understanding operations and also introduce algebraic concepts such as rational equations. In geometry, they help students grasp ideas like segment division, as well as area and volume calculations. Finally, fractions are valuable for expressing and manipulating quantities (Chambris et al., 2017, 2021). Research in primary-level arithmetic has shown that the concept of fractions presents challenges for both teachers and students (Chambris et al., 2017; Doğan & Yıldırım Sır, 2022; Trivena et al., 2017). For example, the work of Trivena et al. (2017) have shown that students' mastery of the concepts of addition and subtraction of fractions is dominated by misconceptions. These students' difficulties raise questions about teachers' practices in arithmetic. Indeed, what teachers think, say, do and don't do in class about fractions can strongly influence the strategies they implement with students (Robert & Vivier, 2013). The work of Copur-Gencturk (2021) and those of Tastepe (2023) highlight limitations in teachers' and students' understanding of fractions in arithmetic, in particular the division of fractions. According to Graeber et al. (1989) when pre-service teachers have misconceptions about certain concepts, they can contribute to perpetuating students' misconceptions.

Comparing two fractions is just as important as adding and multiplying arithmetic fractions. However, pre-service primary schools' teachers' understanding of the comparison of fractions remains insufficiently known. This research therefore aims to explore how pre-service teachers[1] approach the comparison of fractions. The aim of this study is to understand the specific knowledge mobilized by pre-service teachers when faced with fraction comparison tasks and associated problem-solving.

[1] Future teachers here refer to students enrolled in the MEEF master's program (master's degree in teaching, education, and training professions) and preparing for the CRPE *primary school teaching competitive exam*.

The present study addresses the following research questions:

1. What knowledge do pre-service teachers use when dealing with fraction comparison tasks?
2. What arithmetic models and procedures do pre-service teachers employ to solve problems involving the comparison of fractions?

The hypothesis of this study is as follows: pre-service teachers are more likely to employ the procedure of finding a common denominator to compare two fractions, which may reflect a certain rigidity in their approach to fraction comparison. This study aims to gain a clearer understanding of the procedures used by pre-service teachers when comparing fractions. Additionally, it seeks to identify specific training needs for pre-service teachers to help them develop a stronger understanding of fraction comparison.

Some Previous Work on Fractions in Schools

As mentioned above, learning arithmetic in particular fractions is a real challenge among students as well as teachers in primary schools (Chambris et al., 2017; Doğan & Yıldırım Sır, 2022; Vermette & Martin, 2017). The work of Chambris et al. (2017) highlight the fact that in order to understand the concept of a fraction, we need to grasp its main functions: representing parts of a whole, comparing quantities and arithmetic operations. The work of Doğan and Yıldırım Sır (2022) and Trivena et al. (2017) have shown that learners may encounter obstacles in understanding concepts associated with fractions, such as the representation, operations and framing of rational numbers. According to these authors, the difficulties encountered in learning fractions can be attributed to various factors, including the abstract nature of these concepts, challenges related to the specific vocabulary associated with arithmetic, as well as gaps in mathematical reasoning and metacognition. Adu-Gyamfi et al. (2019) note that both teachers and pre-service teachers express difficulties in dividing fractions. These difficulties can be observed in the procedures they implement in situations and in their ability to analyze their students' productions. In their study, Whitacre et al. (2019) explore the challenges faced by primary pre-service teachers in understanding and teaching fractions. The findings indicate that these difficulties are primarily linked to mathematical anxiety and reliance on standardized procedures rather than actual knowledge gaps. An interview with a pre-service teacher revealed significant progress in her ability to compare fractions, particularly through the use of prior knowledge about fractions and cross-multiplication. The study suggests that affective factors and pre-service teachers' beliefs constrain their exploration of alternative strategies. In a complementary study, Whitacre et al. (2020) observed that pre-service teachers struggle to fully articulate their reasoning when justifying their choice of fraction comparison strategies. However, some pre-service teachers appear to possess potentially productive pedagogical resources, which could be further developed to enhance mathematics instruction. These findings open promising avenues for research on pre-service teachers mathematical thinking and teacher education. This observation underscores the need to strengthen initial mathematics teacher education to ensure a solid grasp of essential didactic concepts. These studies converge in emphasizing the importance of considering not only prospective teachers' mathematical knowledge but also affective factors and their beliefs within the framework of teacher education.

Olanoff et al. (2014) conducted a review of 43 prior studies on fractions and teacher education, categorized into three distinct periods: pre-1998, a contemporary perspective (1998-2011), and a forward-looking phase (2011-2013). Their analysis indicates that pre-service teachers generally possess a solid grasp of procedural knowledge related to fractions, such as performing calculations. However, they often lack the flexibility to move beyond these procedures, apply a "number sense for fractions," and understand the meanings or underlying justifications for these procedures. We argue that this observation is particularly relevant for pre-service teachers in France, especially in the context of fraction comparisons. In this study, participants are likely to demonstrate a lack of flexibility in their approaches. The authors also note a significant shift in research focus over time: from an almost exclusive emphasis on operations (notably multiplication and division) to a more balanced exploration of both operations and conceptual understandings of fractions. However, they highlight that strategies for enhancing pre-service teachers' knowledge of fractions remain insufficiently documented. This underscores the need to strengthen the teaching of fractions in teacher education curricula and to pursue further research on the development of teacher knowledge in this area.

del Mar López-Martín et al. (2022) studies report that pre-service primary teachers demonstrate an insufficient level of foundational mathematical knowledge, particularly regarding fractions and their application in problem-solving. The most common error identified in their responses relates to the understanding of fractions as operators. Pre-service teachers face greater difficulties when tackling multi-step complex problems compared to simpler ones. This highlights a weakness in their ability to apply their knowledge in more sophisticated contexts. These phenomena are likely to be observed in the French context as well. Specifically, the errors made by pre-service teachers in comparing fractions can be attributed to their limited foundational knowledge of fractions.

Recent work by Schwarzmeier and Obersteiner (2025) show that in tasks involving the comparison of fractions using visual representations, pupils use a variety of strategies depending on their level, and that errors stem more from an absolute size bias than from counting or natural number bias; for example, some students judge that 3/8 is larger than 1/2 simply because the bar representing 3/8 is visually longer, without considering the actual proportion.

In the context of fraction instruction, numerous studies highlight the critical role of *pedagogical content knowledge* (PCK) in anticipating students' reasoning processes and common errors. For instance, Zolfaghari et al. (2024) demonstrated that targeted training programs combined with practical field experiences significantly enhance the PCK of pre-service teachers. This is particularly evident through the use of the *PCK-fractions* tool, which assesses their ability to teach fractions effectively. These findings underscore the importance of teacher training programs that integrate both theoretical knowledge and practical experience, fostering teaching competencies that align with students' reasoning patterns.

Comparing Fractions

In his research, Bishop (1996) identified six sub-categories of comparison problems, based on the relationships they establish, in particular the use of the terms 'more' or 'less'. These problems involve comparing two initial numbers, a and b, with a third number c, representing the difference between a and b. The special feature of these problems is that they are formulated around an unknown variable, which can be either one of the initial numbers or the difference c. Several procedures can be used to compare two fractions, depending on their characteristics. One method frequently used in mathematics is to analyze the sign of the difference between the two fractions. The work of Saboya and Rhéaume (2013, 2015) on fraction comparison in Quebec highlight various comparison procedures. These procedures have been organized according to the control that students can exercise over their comparison activity. We present five 5 procedures inspired by their work.

The same denominator procedure

One of the first strategies students learn is the "same denominator" procedure. According to this procedure, when two fractions have the same denominator, the larger fraction is the one with the larger numerator. If the denominators are different, they are transformed to obtain equivalent fractions with a common denominator, and then the numerators are compared. For example, to compare $\frac{2}{5}$ and $\frac{3}{4}$we look for equivalent fractions with the same denominator: $\frac{2}{5}=\frac{8}{20}$ and $\frac{3}{4}=\frac{15}{20}$ then compare $\frac{8}{20}$ and $\frac{15}{20}$.

The same numerator procedure

Another method involves comparing fractions with the same numerator. In this case, the larger fraction is the one with the smaller denominator. For example, $\frac{127}{73}$is greater than $\frac{127}{74}$ because both fractions have the same numerator, but the denominator 73 is smaller than 74.

The quotient procedure

Fractions can also be compared by calculating their decimal values directly (by dividing the numerator by the denominator) and then comparing the results. In their work, Saboya and Rhéaume (2015) calls it transforming fractions into percentages and/or decimal numbers. For example, comparing $\frac{128}{256}$ and $\frac{121}{484}$ is the same as comparing 0.5 and 0.25 after simplification. This category also includes the procedure of simplifying fractions before comparing them. Simplification involves dividing the numerator and denominator by the same number to obtain an equivalent fraction that is easier to handle. For example, comparing $\frac{128}{256}$ and $\frac{9}{27}$ is the same as comparing $\frac{1}{2}$ and $\frac{1}{3}$.

Comparison by reference

For some comparisons, it is useful to use a reference point, such as 1, $\frac{1}{2}$ or 2. For example, to compare $\frac{153}{49}$ and $\frac{79}{69}$ you can choose 2 as the reference point: $\frac{153}{49}$ is greater than 2, while $\frac{79}{69}$ is less than 2, which means that $\frac{153}{49}$ is greater than $\frac{79}{69}$.

The residual procedure

Another approach consists of evaluating the residual, i.e., the quantity that each fraction lacks to reach a reference value, often 1. For example, to compare $\frac{2}{3}$ and $\frac{15}{16}$ we can see that $\frac{1}{3}$ à $\frac{2}{3}$ to reach 1, while only $\frac{1}{16}$ à $\frac{15}{16}$. Since $\frac{1}{3}$ is greater than $\frac{1}{16}$ we conclude that $\frac{15}{16}$ is closer to 1 than $\frac{2}{3}$.

Another example of using this procedure is to compare $\frac{5}{7}$ and $\frac{6}{11}$. For $\frac{5}{7}$ the residual is $\frac{2}{7}$, while for $\frac{6}{11}$ the residual is $\frac{4}{11}$. Comparing these residuals: $\frac{2}{7}\approx 0.2857$ and $\frac{5}{11}\approx 0.4545$. Since $\frac{2}{7}<\frac{5}{11}$is closer to 1 than $\frac{6}{11}$, which leads to the conclusion that $\frac{5}{7}>\frac{6}{11}$. However, although this method may work for some simple comparisons, it is not always reliable, as it relies on estimates rather than rigorous calculations.

The comparison and ordering of fractions are based on their density and size. This task is complex and requires appropriate strategies that should be incorporated into teacher training. Teachers need to be able to draw on a repertoire of alternative resources that enable them to switch from one procedure to another, adapting to the different situations encountered. Ideally, training should enable teachers to develop the skills they need to tackle these tasks more effectively. It should be noted that several procedures can be used to accomplish a comparison task. However, for the purposes of this study, a procedure will be said to be effective when it is optimal in terms of the time, step and resources needed to accomplish the comparison task.

Concept Image Concept Definition

To conduct this research, we chose to adopt the concept image and concept definition theory developed by Tall and Vinner (1981). This theory provides a framework for analyzing the cognitive structure of individuals in relation to a specific concept. According to Vinner (1983), the composition of the cognitive structure of an individual associated with a concept (in this case fractions) is based on two elements: the concept image and the concept definition.

According to Tall and Vinner (1981), the concept image is a term used to describe the entire cognitive structure associated with a concept. It includes all the mental images, properties, rules, processes and procedures associated with the concept. At the same time, the concept definition of an individual associated with a given concept is similar to the notion of definition in mathematics with the distinction that it is personal to an individual. According to Vinner (1983) it is the set of terms used by an individual to define the concept.

An individual's mental image of fractions is formed by any symbols or abstract signs and pictorial representations in their mind associated with fractions, for example, the mental image associated with the concept of fraction can include the image of a pizza that has been divided into equal parts. Properties relating to fractions include the rules of simplification, calculation and comparison associated with fractions. The processes and procedures involved in comparing fractions are clearly defined and ordered sequences of steps to accomplish this task. For example, when two fractions with the same denominator are compared, the numerators are compared and the larger fraction is the one with the larger numerator.

An individual's concept image gradually evolves over time, shaped by various experiences related to the concept. When the components of a person's concept image align with the theoretical understanding of the concept, the concept image is considered coherent. However, it can become incoherent when it contains contradictions with the theory, particularly when the procedures used to accomplish a task are tainted by ambiguity. This ambiguity arises when the foundations are based on informal discourses and rules, thus contributing to the discrepancy between the individual concept image and the established theory (Tchonang Youkap, 2019; Viholainen, 2005). In this study, a concept image will also be considered incoherent if the procedures applied are based on rules that are not accepted by the mathematical community. For instance, an individual might compare two fractions solely by evaluating their numerators and explicitly express this as a rule. Additionally, an individual's concept image is regarded as incomplete when the procedures it includes lack diversity. This incompleteness is evident when the individual consistently relies on a single procedure across all encountered contexts, even when more efficient alternatives exist.

In this research, we use Vinner's (1983) theoretical framework of concept image to analyze the various procedures employed by student teachers when comparing fractions. This approach not only reveals the procedures these pre-service teachers tend to favor but also identifies the limitations within their procedural repertoire and the corresponding opportunities for professional growth. By examining the diversity of the procedures used, we gain deeper insight into how these pre-service teachers perceive fraction comparison, allowing us to propose targeted areas of training to strengthen their instructional practice.

METHOD

This study employs an exploratory research design using a mixed-methods approach, combining qualitative and quantitative methods (Shields & Rangarajan, 2013). Its objective is to gain a deeper understanding of the procedures used by pre-service teachers when comparing fractions.

Population

The study, conducted during the 2023-2024 academic year, involved a sample of 160 pre-service teachers enrolled in the first year of the MEEF[2] master's program, specializing in primary education. The participants come from two institutions: three classrooms from the INSPE[3] of Versailles and three classrooms from the INSPE of la Réunion, located at the Mayotte Site. These classes were taught by five different mathematics professors. These regions were intentionally selected to reflect geographic and sociocultural diversity. Our intention was not to compare the two institutions but to expand our population in order to collect more data. The participants, mostly aged between 23 and 28, had diverse academic backgrounds: the majority (65%) held a bachelor's degree in education sciences, while others had degrees in fields such as literature (20%) or geography (15%). All participants had studied arithmetic during their first four years of secondary education, a foundation further strengthened during their first year of the master's program through mathematics education courses aligned with the lower secondary school curriculum in France. Actively preparing for the competitive CRPE[4], participants demonstrated foundational knowledge of fractions, which is central to this study. The diversity of backgrounds and training contexts provides a rich basis for exploring the cognitive structures employed in fraction comparison.

Data Collection

The data collection instrument for this study consists of a questionnaire specifically designed for our participants. This questionnaire is aligned with their mathematics training curriculum, as the comparison tasks are set at the first cycle of secondary school level, targeting pre-service teachers preparing for the CRPE. The questionnaire is divided into two sections: the first includes six tasks focused on the comparison of two fractions, and the second contains two problems also centered on fraction comparison. For each of the six tasks, it is possible to identify an efficient procedure, which promotes an economical and rapid approach to solving the task (Vergnaud, 2009). **Table 1** presents the six tasks included in the questionnaire for pre-service primary school teachers.

[2] Professions in teaching, education, and training.
[3] It is the French institution responsible for training future teachers and education professionals.
[4] It is the competitive exam required to become a primary school teacher in France.

Table 1. The six questionnaire comparison tasks

Task number	Fractions comparison tasks
Task 1	$\frac{153}{49} \dots \frac{79}{69}$
Task 2	$\frac{225}{200} \dots \frac{725}{610}$
Task 3	$\frac{41}{39} \dots \frac{42}{43}$
Task 4	$\frac{127}{73} \dots \frac{127}{74}$
Task 5	$\frac{354}{317} \dots \frac{426}{317}$
Task 6	$\frac{128}{256} \dots \frac{121}{484}$

As efficient a priori procedures for these six tasks, we can expect the use of the reference point procedure for task 1, as the first fraction is greater than 2 and the second is less than 2. For task 2, the expected procedure is the same denominator. The benchmark 1 procedure is shown for task 3. For task 4, the comparison of denominators is expected, given identical numerators. For task 5, the same denominator procedure with a common denominator is expected. Finally, for task 6, participants should use the quotient procedure by simplifying the fractions to compare them. The statements regarding the two problems in the questionnaire are as follows.

Problem 1: Peter divided a hectare of farmland into 27 equal plots and sold 13 of them. Mary divided a hectare of farmland into 36 equal plots and sold 19 of them. Peter believes he sold a larger area of land, but Mary disagrees. Who do you think sold the larger area of land? Justify your reasoning.

Problem 2: Lisa and Tom need to fill two identical cylindrical barrels with water. Both barrels have the same height, but one is graduated from 0 to 39, while the other is graduated from 0 to 42. Lisa fills her barrel to level 17 out of 39, and Tom fills his to level 23 out of 42. Noémie observes the two results and claims that Tom has poured more water than Lisa. How can we justify Noémie's statement?

To address these two problems, it is essential for pre-service teachers to engage in a process of deep understanding. This involves carefully reading and clearly identifying the given data, followed by a reformulation of the problem statements. Participants should then analyze the problem to recognize that it is a comparison situation. This recognition enables them to select an appropriate procedure for effectively solving the problem. Finally, they must apply this procedure to arrive at a solution. For problem 1, the fractions to be compared are $\frac{13}{27}$ and $\frac{19}{36}$ and for problem 2 they are $\frac{17}{39}$ and $\frac{23}{42}$. They are also expected to adopt an efficient procedure in terms of speed, the number of steps, and spontaneity; in this case, comparing fractions to a reference point, such as $\frac{1}{2}$. It is noted that participants may employ various other strategies, including using diagrams.

To validate our data collection instrument, we consulted four mathematics teachers, who are trainers of primary school teachers with fifteen years of experience in teacher education in France. These experts reviewed the questionnaire and provided specific feedback. They recommended the addition of two additional problems to enhance the relevance and diversity of the scenarios presented. Additionally, they suggested several improvements to refine the clarity and effectiveness of the questionnaire, ensuring its relevance and alignment with the study's objectives. Pre-service teachers are invited to engage actively in these tasks by documenting their procedures on the sheets provided by the researcher. They are encouraged to use these sheets as drafts, noting down any reflections or reasoning that justify their choices as they work. They are informed that their responses will not be evaluated quantitatively but will instead contribute to the research study.

Data Analysis Method

Dey (1993) emphasizes the value of employing mixed-methods approaches, noting that the numerical findings derived from quantitative methods are enriched by the insights provided by qualitative data. In other words, qualitative methods help explain the underlying reasons for the results produced by quantitative techniques. Based on this premise, the present study adopted a combined methodological approach. For the quantitative aspect, we organized the data into categories and calculated the proportion of each category within the sample, allowing us to identify key trends and distributions. Initially, we coded the responses based on their accuracy, using the following categories: correct, incorrect, and no response. We employed descriptive statistical methods, with a primary focus on frequencies, to interpret the results. We analyzed the procedures used by the participants, classifying them into the following categories: by the quotient, reference point, same denominator, same numerator, and informal. Informal procedures refer to those that are unique to the participants, grounded in their concept image, and may or may not yield correct results. For example, when comparing the fractions 11/12 and 4/5, a participant might compare the numerators directly and conclude that 11/12 is greater than 4/5, which is correct. However, this procedure is not mathematically valid in this context. For both the six tasks and two problems, we identified the mathematical models invoked by the participants, as well as the procedures they used to solve the problems. We paid particular attention to categories such as the use of fraction comparison and other models (e.g., division and percentage). After conducting a quantitative analysis of the tasks and problems, we also propose a qualitative analysis of some of the procedures adopted by the participants in order to gain deeper insight into the knowledge they mobilized to complete the tasks.

RESULTS

In this section, we present the results of our research. We begin with a descriptive analysis of the data related to the six tasks, followed by an examination of the results from the analysis of the responses to the two problems.

Analysis of Questionnaire Comparison Tasks

Quantitative data

The data presented in **Table 2** indicate that most participants provided correct answers across all tasks, with accuracy rates ranging from 70% to 95%. Task 5 exhibited the highest percentage of correct responses, at 95%. This suggests a generally strong ability among participants in comparing fractions with the same denominator. However, despite these favorable results, it is important to highlight that some tasks also demonstrate notable error rates. For instance, task 3 (41/39, …, 42/43) showed an error rate of 17.5%, while task 1 had an error rate of 12.5%. These findings point to potential challenges or misunderstandings when comparing fractions with differing numerators and denominators. This suggests the need for further investigation to uncover the underlying causes of these errors and to explore possible areas of conceptual difficulty.

Table 2. Breakdown of participants' responses to the 6 questionnaire tasks

Answers	Task 1		Task 2		Task 3		Task 4		Task 5		Task 6	
	En	%	En	%	En	%	En	%	En	%	En	%
No	28	17.5	12	7.5	4	2.5	0	0.0	0	0.0	4	2.5
Correct	112	70.0	144	90.0	128	80.0	144	90.0	152	95.0	148	92.5
Incorrect	20	12.5	4	2.5	28	17.5	16	10.0	8	5.0	8	5.0
Total	160	100	160	100	160	100	160	100	160	100	160	100

Table 3 presents a comprehensive analysis of the procedures employed in response to each of the six tasks in the questionnaire, measured both by frequency and percentage. In task 1, the most commonly used procedure was the "same denominator" procedure (43%), followed by informal procedures (20%). Task 2 showed a dominant use of the same denominator method (48%), with informal procedures being less frequent (15%). In task 3, the same denominator procedure (43%) and informal procedure (20%) also appeared prominently. Task 4 displayed more diversity, with a relatively balanced use of the various procedures. Task 5 was characterized by a strong preference for the same denominator procedure (70%). Finally, task 6 revealed a notable use of both the same denominator procedure (28%) and informal procedure (18%). It is worth noting that a significant number of participants did not explicitly state their procedure in writing. Overall, the analysis of the totals indicates a fairly uniform distribution of procedures across tasks, with certain procedures more prevalent in specific tasks. Task 5 is particularly notable for the high frequency of the same denominator procedure.

Table 3. Breakdown of participants' procedures for the 6 questionnaire tasks

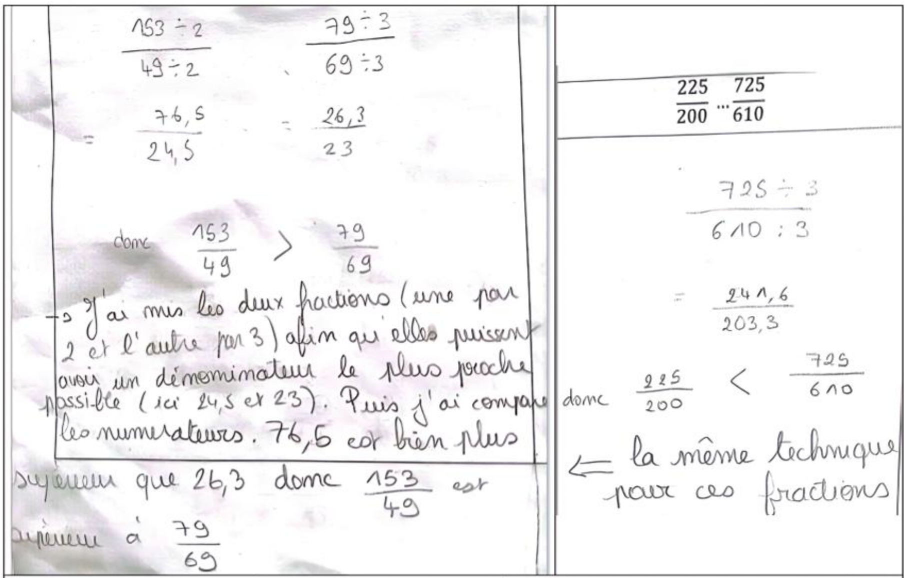
Procedures	Task 1		Task 2		Task 3		Task 4		Task 5		Task 6	
	En	%	En	%	En	%	En	%	En	%	En	%
No procedure	16	10.0	20	12.5	20	12.5	28	17.5	28	17.5	44	27.5
By quotient	44	27.5	36	22.5	28	17.5	24	15.0	12	7.5	24	15.0
Same denominator	68	42.5	76	47.5	76	42.5	52	32.5	112	70.0	60	37.5
Informal	128	20.0	24	15.0	32	20.0	24	15.0	8	5.0	28	17.5
Reference point	0	0.0	4	2.5	8	5.0	4	2.5	0	0.0	4	2.5
Same numerator	0	0.0	0	0.0	4	2.5	28	17.5	0	0.0	0	0.0
Total	160	100	160	100	160	100	160	100	160	100	160	100

The participants' responses reveal that they consistently applied the same procedure across all tasks. Specifically, some participants employed the same denominator procedure and the quotient procedure in each of the six tasks. This pattern suggests a certain rigidity in the procedures within the pre-service teachers' concept image on fraction. It implies that they may be relying on a strategy they consider reliable, yet without necessarily adapting their reasoning to the unique characteristics of each task. This rigidity could indicate that their understanding of fractions comparison is grounded in standardized procedures, rather than in a flexible set of strategies tailored to different types of problems.

Qualitative data

The analysis of the participants' responses highlights several challenges encountered in the comparison of fractions within the questionnaire. Certain participants did not explicitly articulate their procedures, while others employed personal strategies, which we classify as informal procedures. In this study, we present a selection of these procedures that we deem relevant for our analysis. However, it is important to note that the adoption of a specific method does not inherently guarantee its appropriateness for the given task.

Some participants, despite providing correct answers to certain tasks, relied on informal procedures to reach their conclusions. This was evident in the case of participant S39 (see **Figure 1**), who approached the comparison of fractions by first seeking equivalent fractions with denominators that were relatively close in value. Subsequently, he compared the numerators of these fractions. For instance, in task 1, S39 divided the numerator and denominator of the fractions 153/49 and 79/69 by 2 and 3, respectively. However, the resulting quotients from these operations were not whole numbers, suggesting an informal approach rather than adherence to standard fraction equivalence procedures. Once the equivalent fractions had relatively close denominators, he proceeded to compare them. This same informal procedure was observed in his response to task 2, indicating a consistent reliance on heuristic reasoning rather than formal mathematical methods.

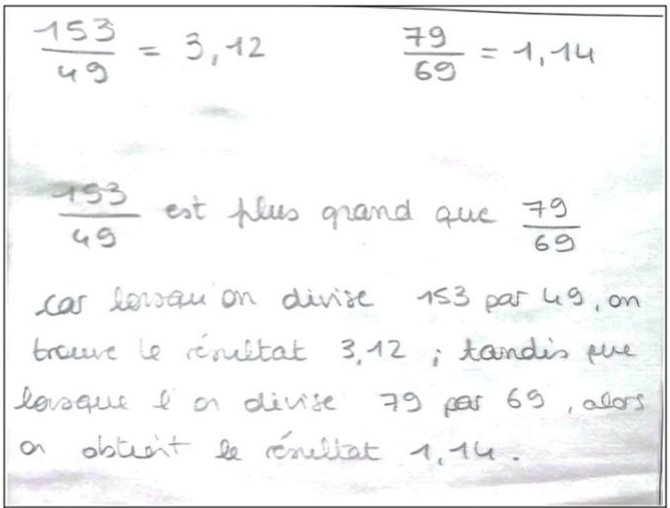


[I divided the two fractions (one by 2 and the other by 3) so that they could have the closest possible denominator (here 24.5 and 23). Then, I compared the numerators. 76.5 is indeed much greater than 26.3, so 153/46 is greater than 79/69.]

Figure 1. Participant S39's output in task 1 and task 2 of the questionnaire (Source: Field study)

The procedure employed by the pre-service teacher reflects a partial and imprecise understanding of the concept of fractions. By simplifying fractions to obtain decimal numbers and approximate denominators, the teacher appears to attempt the traditional procedure of finding a common denominator while relying on decimals, which may be more familiar within their concept image of fractions. However, this approach reveals a lack of understanding of the fundamental properties of fractions, particularly the fact that approximating denominators does not ensure the validity of comparing numerators. Such a procedure may stem from a rigid interpretation of the common denominator method or from a limited concept image regarding fraction comparison. This highlights the need to strengthen the understanding of the definitional properties of fractions and to diversify the procedures taught. Doing so would help enrich the concept image of pre-service teachers and equip them with strategies better suited to various teaching contexts.

The analysis of the procedures used by pre-service teachers shows that some employ the procedure by quotient rather superficially in its implementation. The work of pre-service teacher S1 illustrates this usage (**Figure 2**). Specifically, S1 found decimal numbers corresponding to the fractions 153/49 and 79/69. However, these fractions are not decimal fractions, as they do not have an exact equivalent in the set of decimal numbers. This represents an incorrect approximation of fractions by decimal numbers.



[153/49 is greater than 79/69 because when we divide 153 by 49, we get the result 3.12, whereas when we divide 79 by 69, we get the result 1.14]

Figure 2. Participant S1's output for task 1 of the questionnaire (Source: Field study)

This result highlights a tension between the mental image of fractions and their decimal representation. The pre-service teacher S1 seems to have approached the fractions 153/49 and 79/69 by converting them directly into decimal numbers, which suggests a confusion between their exact representation and their decimal approximation. This approach may reflect a cognitive simplification, where pre-service teachers tend to favor a more familiar form of representation (decimals) at the expense of the accuracy required for exact conversion, sometimes resorting to rounding or using the "approximately equal" symbol. However, this transformation does not respect the intrinsic properties of non-decimal fractions, leading to incorrect approximations. This underscores the need to clarify the relationships between different numerical representations and to foster a deeper understanding among teachers of the limitations of using decimal numbers as approximations. It is crucial for teachers to develop an awareness of the potential conceptual errors when teaching fractions. Furthermore, this comparison procedure does not appear to be the most efficient in this context: a reference point-based approach, which is faster and more accurate, would have been more appropriate.

The pre-service teachers did not diversify the operating properties when comparing fractions. For the most part, they followed the same approach for all six comparison tasks. Participant S32's production is an illustration of this (**Figure 3**). He used the same denominator procedure to compare fractions in the six tasks in the questionnaire. The results were correct, but the procedure used was not the most efficient. Although we might have expected him to use the properties of the same numerators or comparison procedures with a reference point in certain tasks, he preferred to adopt a single procedure.

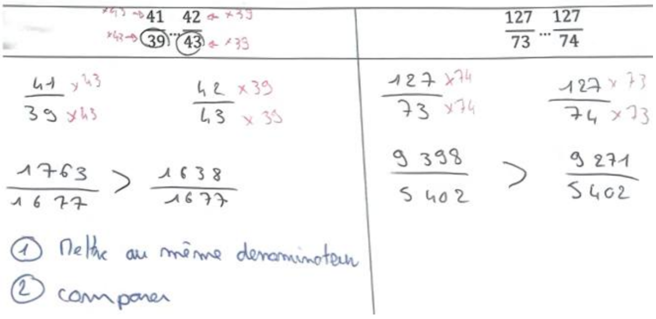


Figure 3. Participant S32's output in task 3 and task 4 of the questionnaire (Source: Field study)

The analysis of the pre-service teacher S32's work highlights a situation where the pre-service teacher seems to maintain a rigid approach to the concept image for comparing fractions. Indeed, they consistently refer to the single procedure of finding a common denominator throughout the six comparison tasks, despite the diversity of contexts that could have led them to use other strategies, such as utilizing equal numerators or comparing via a reference point. This uniform application of procedures, although leading to correct results, reflects a partial understanding of fractions, which appears reductive. The lack of variation in the use of procedures suggests that the student has not yet developed sufficient conceptual flexibility to adapt their choices based on the specifics of the tasks at hand. This situation raises the question of developing a richer and more dynamic representation of fractions, where various procedures could be appropriately employed depending on the demands of each situation.

Problem-Solving Analysis

The analysis of the data presented in **Table 4** reveals notable patterns in the responses to the two problems in the questionnaire. For problem 1, 22.5% of pre-service teachers did not provide an answer, while the majority, 77.5%, gave a correct response. It is important to note that the correctness of the answer was determined solely by the final solution, excluding the method used by participants. For problem 2, a larger proportion, 30%, did not answer, and 70% provided a correct answer. Interestingly, problem 1 generated a higher rate of correct responses compared to problem 2. These findings suggest that problem-solving, particularly tasks involving the comparison of fractions, remains challenging for pre-service teachers. Although some pre-service teachers were able to provide correct answers, the differing success rates between the two problems emphasize that the skill of comparing fractions is not yet fully mastered by this group.

Table 4. Participants' responses to the two questionnaire problems

Answers	Problem 1		Problem 2	
	En	%	En	%
No	36	22.5	48	30.0
Correct	124	77.5	112	70.0
Total	160	100	160	100

Modeling: Mathematical Tools Used

Regarding the "modelling" skill, **Table 5** illustrates that 17.5% and 25% of the participants, for problem 1 and problem 2, respectively, did not establish a connection with a mathematical model, meaning they did not propose any solution. Approximately 15.0% of participants opted for fraction comparison as the mathematical model to solve problem 1, while a significant majority of 67.5% employed other mathematical models for the same problem. For problem 2, 10.0% selected fraction comparison, while 65.0% relied on other mathematical models. These results underscore the diversity of approaches adopted by the participants and indicate that fraction comparison is not the most frequently used model to solve these problems.

Table 5. Mathematical models used to solve the questionnaire problems

Mathematical models	Problem 1		Problem 2	
	En	%	En	%
Comparison of fractions	24	15.0	16	10.0
Other	108	67.5	104	65.0
No	28	17.5	40	25.0
Total	160	100	160	100

Procedures used to solve the two problems in the questionnaire

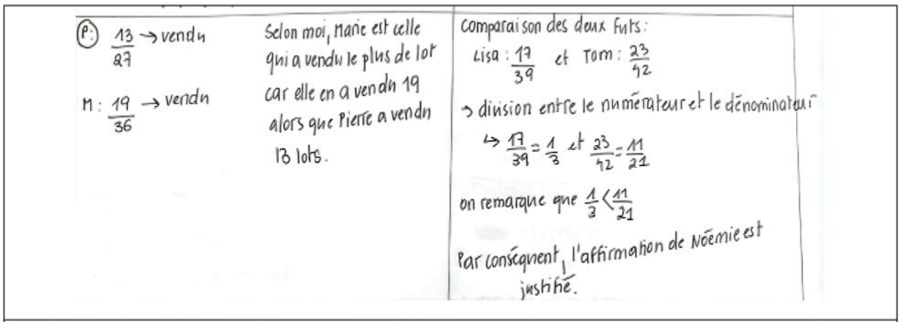
Table 6 presents the various procedures employed by the participants to solve the given problems. A closer analysis of the data reveals intriguing trends in the strategies used for the two problems. For problem 1, the predominant approach was the same denominator procedure, adopted by 42.5% of respondents, followed by a notable 22.5% who did not employ any identifiable procedure. In contrast, problem 2 demonstrates a distinct preference for the absence of a defined procedure, reported by 35.0% of participants, although the same denominator strategy still accounted for a significant 40.0%. It is noteworthy that informal procedures were less frequently observed in responses to problem 1 (17.5%) compared to problem 2 (12.5%). These findings suggest that participants' choice of procedures varied significantly depending on the nature of the problem, underscoring the diversity in cognitive approaches to problem-solving. This variability highlights the need to further explore the factors influencing procedural choices and their implications for mathematical reasoning and instruction.

Table 6. Participants' procedures for the two problems

Procedures	Problem 1		Problem 2	
	En	%	En	%
No procedure	36	22.5	56	35.0
Quotient	24	15.0	16	10.0
Same denominator	68	42.5	64	40.0
Informal	28	17.5	20	12.5
Diagram	4	2.5	4	2.5
Total	160	100	160	100

Qualitative data

The analysis of the pre-service teachers' work reveals that some approached the comparison of fractions as a mathematical model to solve the given problems. However, their comparison procedures often lacked formal rigor. This is exemplified by the production of participant S18 (see **Figure 4**). In both tasks, the participant identified the fractions associated with the problems but employed two distinct procedures for comparison, depending on the problem context. For problem 1, the participant reached the solution by directly comparing the numerators of the two fractions. For problem 2, they determined what they believed to be equivalent fractions, rewriting 17/39 as 1/3 and 23/42 as 11/21, subsequently concluding that 1/3<11/21. However, these so-called equivalent fractions do not align mathematically with the original fractions, as the transformations applied were inconsistent. Upon inquiry, the participants explained their approach: they had "simplified" the first fraction by dividing both numerator and denominator by 17 and the second fraction by dividing by 2, before rounding the results. They then compared the resulting numerators. This procedure is ambiguous and lacks a coherent mathematical justification, highlighting the need for further emphasis on conceptual understanding of fraction equivalence and comparison methods in teacher training.

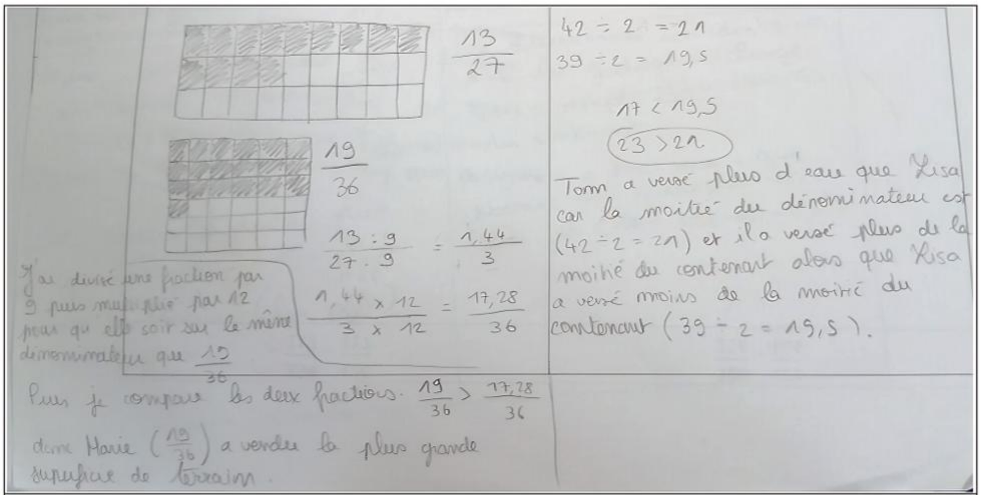


[(P1): In my opinion, Marie is the one who sold the most lots because she sold 19, whereas Pierre sold 13.
(P2): Dividing the numerator by the denominator, we notice that 1/3<11/21. Thus, Noémie's statement is justified.]

Figure 4. Participant S18's output for the two questionnaire problems (Source: Field study)

The analysis of this participant's responses to problem 1 highlights the use of an incorrect procedure: comparing numerators is only valid when the fractions share a common denominator, which is not the case here. For Problem 2, the participant references dividing the numerator by the denominator to simplify the fractions. However, it is important to note that 17 and 39 are coprime, as are 23 and 42, meaning that these fractions are irreducible. This reveals a misunderstanding of the principles of fraction simplification. Furthermore, the comparison strategy employed in problem 2 appears to be the same as the procedure used in problem 1. The participant's reliance on informal procedure within their *concept image* of fractions leads to incorrect outcomes. This inconsistency suggests gaps in their conceptual understanding of fractions and highlights the need for interventions to address misconceptions in fraction comparison and simplification.

The analysis of participant S33's response to problem 1 reveals a schematic approach to representing the problem. This is evidenced by his hand-drawn depiction of two rectangles, which he partitioned into 27 and 36 sub-rectangles, respectively. It appears that S33 employed the comparison of fractions as a mathematical model to address the tasks. In his work (refer to **Figure 5**), S33 simplified the fraction 13/27 by dividing both the numerator and the denominator by 9 and subsequently multiplying the resulting fraction's numerator and denominator by 12. This process led to two fractions with the same denominator. Interestingly, the participant used a fraction where the numerator was expressed as a decimal number. After obtaining fractions with identical denominators, S33 applied the standard "same denominator" procedure to compare them.



[I divided a fraction by 9 and then multiplied it by 12 to make it have the same denominator as 19/36. Then, I compared the two fractions: 19/36 > 17.28/36. Therefore, Marie (19/36) sold the larger area of land.]

Figure 5. Participant 33's output for the two questionnaire problems (Source: Field study)

Participant S33's response reveals a partial grasp of fraction comparison procedures. While the ability to simplify fractions accurately and employ strategies such as finding common denominators reflects foundational competence, the specific implementation in this instance highlights a limit of procedural efficiency. Rather than leveraging a more efficient strategy, such as the reference point method, S33 consistently applies the common denominator procedure across all tasks in the questionnaire. This consistent reliance results in the use of fractions with decimal numerators, which may indicate limited procedural flexibility. This rigidity suggests that S33's concept image of fraction comparison is insufficiently developed to adapt strategies effectively to the demands of varying contexts. Consequently, this constrained approach restricts the overall efficacy and elegance of the solutions provided.

The solving strategy employed by participant S24, as illustrated in **Figure 6**, showcases a particularly innovative approach. For the first problem, the participant utilized a 50% benchmark to compare the number of batches prepared by Peter and Mary. This reasoning was similarly applied in addressing the second problem. Notably, in this instance, the participant compared the volume of water in the tank to its midpoint capacity by doubling it, thereby determining whether it surpassed or fell short of the tank's total capacity. While this method is broadly valid, it poses potential challenges in cases where the resulting doubled value exceeds or falls short of the actual total capacity of the tank.

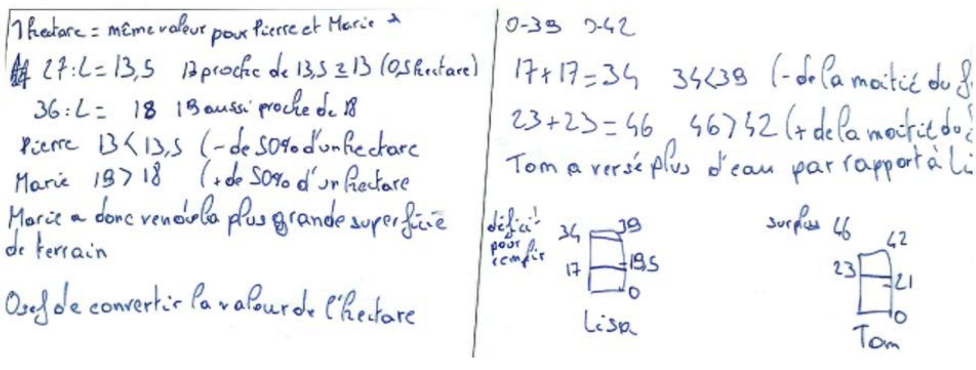
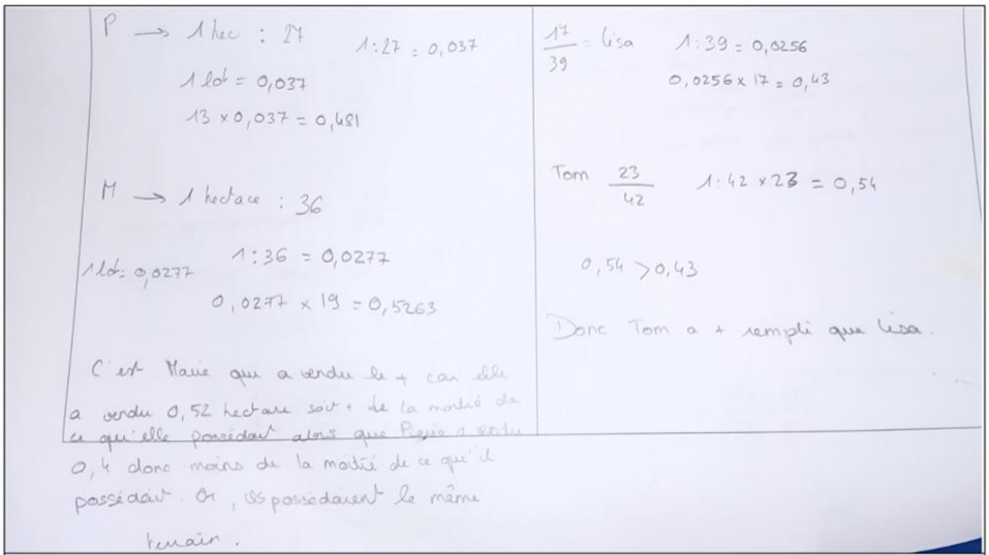


Figure 6. Participant S24's output for the two problems in the questionnaire (Source: Field study)

The analysis of this participant's output reveals the use of a personal strategy for comparing fractions, which does not align with any of the formalized procedures outlined in the theoretical framework. However, this approach bears similarities to the logic underlying the "reference point" strategy. Specifically, the participant compares the quantities of land by anchoring his reasoning to a reference unit in this case, half of the original parcel of land. This represents an effective and contextually appropriate solution to the given problem, showcasing the pre-service teacher's capacity for creativity and innovation. That said, some limitations of this procedure warrant consideration. For instance, how might the participant have approached a scenario where both shares exceeded half the initial land? While the strategy employed here demonstrates a coherent concept-image quality, it also highlights a potential challenge: establishing a clear connection between the contextual problem and the formalized mathematical notion of fractions. This observation suggests a need for further development in linking contextual reasoning with mathematical abstractions in the training of pre-service teachers.

Let us examine the procedure employed by participant S37, who did not rely on fraction comparison as the mathematical model for solving the two problems (**Figure 7**). The process begins with the calculation of the area of a lot for Peter, where one hectare is divided by 27, yielding an area of approximately "0.037" per lot. This value is then multiplied by 13, resulting in an area of approximately "0.481" for the total land sold by Peter. In a similar fashion, for Mary, one hectare is divided by 36, which gives "0.0277" per plot, and this is multiplied by 19, yielding a total area of approximately "5.263" for the land sold by Mary. The pre-service teacher concludes by comparing the two areas sold and accurately determines that Mary sold a larger area than Peter. This approach is methodically executed, demonstrating a clear structure of division and multiplication operations to derive the correct solution.



[Marie sold the most because she sold 0.52 hectares, which is more than half of what she owned, whereas Pierre sold 0.4 hectares, which is less than half of what he owned. Yet, they had the same plot of land.]

Figure 7. Participant S37's output for the two problems in the questionnaire (Source: Field study)

The procedure employed by S37 demonstrates a competent application of fundamental mathematical operations such as division and multiplication, ultimately leading to a correct conclusion. This approach aligns closely with the "quotient method." However, it is less efficient in terms of the number of steps and the time required to reach the solution. A more streamlined strategy would have involved directly dividing the individual share by the total quantity. The analysis of responses from pre-service teacher's procedure reveals significant difficulties in their mastery of the concepts of equality and decimal representation of fractions. The identified errors, such as 1/36 = 0.0277 and 1/27 = 0.0372, indicate a misinterpretation of the rules of approximation and rounding. These errors may reflect a static or incomplete conceptualization of equality, wherein numerically close decimals are perceived as equivalent, without a clear distinction between mathematical exactness and practical approximation. Moreover, these responses seem to highlight fragmented concept image related to the notions of fractions and decimal numbers, potentially influenced by an overly segmented approach in their prior education. For instance, pre-service teachers in initial teacher training may not have developed a precise understanding of the distinction between "decimal" fractions (convertible into exact decimal numbers) and their complements within the set of rational numbers.

DISCUSSION

This study investigates the concept image of pre-service primary school teachers with respect to their approaches to comparing fractions and solving problems involve fraction comparison. The findings reveal that this task poses a significant challenge for these teachers, primarily due to the procedural strategies they employ during the comparison process.

The errors observed among pre-service teachers when comparing fractions can largely be explained by the use of inappropriate procedures. However, it is worth noting that some informal procedures adopted by these teachers occasionally lead to correct answers (see **Figure 1**). Nevertheless, an analysis of their work shows that these procedures often rely on a limited understanding, or a restricted concept image, of the notion of fractions. This observation provides additional insight into the findings of previous studies regarding the knowledge of pre-service teachers on the teaching of fractions (Chambris et al., 2017; del Mar López-Martín et al., 2022; Olanoff et al., 2014; Trivena et al., 2017; Whitacre et al., 2019). Indeed, this research has highlighted recurring difficulties both in the application of operations on fractions and in the fundamental understanding of this concept. As Sveider et al. (2023) emphasize, rethinking teachers training is crucial to promote the use of diverse and rigorous methods for fraction comparison. Such efforts are necessary to address these challenges and support the development of a deeper conceptual understanding of fractions. Teachers' awareness of the cognitive mechanisms involved in fraction comparison particularly students' reliance on misleading visual cues such as absolute size is essential for designing instructional approaches that reduce these biases and promote a deeper conceptual understanding of fractions (Schwarzmeier & Obersteiner, 2025).

The analysis of pre-service teachers' responses indicates a strong tendency to systematically rely on a single procedure for comparing fractions across the six tasks presented in the questionnaire (refer to **Table 3**). This consistent application of one method highlights challenges in contextualizing fractions and selecting the most appropriate comparison procedure based on the nuances of each problem. The teachers' concept image of fractions, as observed in their comparative reasoning, appears constrained, primarily revolving around procedures such as finding a common denominator or calculating the quotient. While effective in some cases, these approaches can be less efficient or intuitive in certain contexts due to the complexity and number of steps involved (see **Figure 3**). This rigidity in procedural application suggests a lack of flexibility and a somewhat narrow conceptualization of fractions among the pre-service teachers. These findings align with the observations of Olanoff et al. (2014) who identifies a lack of "number sense for fractions" among pre-service teachers, meaning the ability to handle and interpret fractions in a flexible and intuitive manner, beyond the mechanical application of learned procedures. This result is also consistent with those obtained by Copur-Gencturk (2021) and del Mar López-Martín et al. (2022) who report that teachers often exhibit a limited understanding of fractions within arithmetic. This study underscores the importance of diversifying the strategies and methods emphasized during teacher training. Equipping pre-service teachers with a broader repertoire of tools could foster a more adaptive and context-sensitive approach to fraction comparison, as advocated by Adu-Gyamfi et al. (2019).

Pre-service teachers often encounter substantial challenges in understanding tasks related to the comparison of fractions, presenting a significant barrier to their professional skill development. An analysis of their responses to problem-solving tasks (refer to **Table 4**) underscores the perceived difficulty of such activities. On the one hand, some pre-service teachers exhibit limited engagement in the problem-solving process. On the other hand, those who attempt to solve these problems frequently rely on informal procedures, including abusive approximations of fractions (see **Figure 4** and **Figure 7**). This reliance on informal procedure may hinder their capacity to effectively guide students through similar tasks. Such initial difficulties have implications for the quality of mathematics instruction, as they can impact the clarity of explanations provided to students and the appropriateness of examples selected to illustrate key concepts. These findings align with those of Barham and Barham (2020), who highlighted notable gaps in pre-service teachers' understanding of problem-solving situations, particularly in contexts involving fractions.

Pre-service teachers often encounter difficulties when solving problems involving fractions comparison, particularly in terms of the models they use. The models employed are not always conducive to effective problem-solving, with more appropriate strategies potentially leading to better solutions. Furthermore, there are noticeable gaps in the application of procedures, especially regarding the concepts of equality and rounding (see **Figure 2** and **Figure 7**). Additionally, their reasoning tends to be flawed, exhibiting limited depth and lacking the necessary rigor for teaching purposes. These findings align with those of Barham and Barham (2020), who identified similar deficiencies specifically in data interpretation, modelling, and logical reasoning among pre-service teachers when addressing arithmetic problems.

Based on the findings of this research, some recommendations can be made for pre-service teachers training. Our recommendations align with those of previous research by focusing on the training of pre-service primary teachers in fractions (Zolfaghari et al., 2024). It is recommended that the curriculum incorporate didactic approaches that focus on the comparison of fractions, combining practical workshops with theoretical insights. It is essential to expose pre-service teachers to a diversity of procedures (same denominator, same numerator, reference point, remainder, quotient) while guiding them to analyze their advantages and limitations depending on context. Common student errors should be used as pedagogical tools, encouraging pre-service teachers to identify, correct, and discuss these errors based on authentic or hypothetical examples. The implementation of iterative and reflective learning sequences, supported by classroom simulations and case studies, allows pre-service teachers to experiment with different didactic strategies, confront their theoretical conceptions with concrete practices, and broaden their concept image of fractions through the use of visual or manipulative representations. Additionally, regular diagnostic assessments and interactive digital tools should be incorporated to strengthen their mastery of fractions and develop their ability to adapt procedures to meet the needs of students. Finally, collaboration and action research should be promoted to foster an innovative and reflective approach to teaching fractions.

The sample size represents one of the key limitations of this study, with only 160 participants involved. While this number is sufficient for exploratory analysis, it remains relatively small for making broad generalizations. Future research could address this by expanding the sample to include participants from a wider range of institutions, regions, or socio-cultural contexts. This would provide greater representativeness and strengthen the validity of the findings. Beyond sample size, this study is also limited by its reliance on written responses to a questionnaire. Further studies might explore pre-service teachers' reasoning in real classroom settings or through interviews. Future research should also investigate the effects of targeted training interventions aimed at fostering flexibility in fraction comparison strategies. Furthermore, potential biases also limit the scope of this study. For instance, if the majority of participants come from the same institution or share similar educational backgrounds, this could reduce the diversity of experiences and influence the results. To mitigate this, future studies could aim to recruit a more diverse group of participants, considering geographical, institutional, and professional variation. It is crucial to further explore the cognitive structures of primary school teachers, particularly with respect to their comprehension and mastery of fraction operations. This can be achieved through an analysis of the arguments they construct in their discourse during problem-solving situations involving fractions. Future studies might also focus on the development of a training program that encourages pre-service teachers to adopt a flexible approach in selecting problem-solving strategies related to fractions, using an action-research methodology. Although this study was conducted with a small sample of 160 pre-service teachers, it presents an opportunity to extend the research to a broader population, thereby enabling the generalization of the findings.

CONCLUSION

This study offers critical insight into the conceptual and procedural orientations that shape pre-service teachers' understanding of fraction comparison. Beyond identifying specific procedural preferences, our analysis reveals underlying epistemological tensions in how fractions are conceived as static entities manipulated through fixed algorithms, rather than as dynamic quantities situated within broader mathematical structures. This rigidity underscores the need for teacher education to engage more deliberately with the epistemic complexity of fractions, making explicit the interpretive and representational choices involved in comparison tasks.

To address these issues, teacher education must move beyond corrective approaches that focus solely on procedural refinement. Instead, it should foster epistemic reflexivity, inviting pre-service teachers to examine the origins, affordances, and limitations of the strategies they employ. Such an orientation implies integrating mathematical, didactical, and meta-cognitive perspectives within training programs, and designing tasks that destabilize overlearned procedures to prompt conceptual reorganization. Ultimately, cultivating a richer concept image of fractions is not merely a cognitive goal it is a professional imperative for those tasked with fostering mathematical thinking in the early years of education.

Author contributions: TYP: conceptualization, data curation, formal analysis, investigation, methodology, project administration, resources, software, supervision, validation, visualization, writing – original draft, writing – review & editing; **NJ-B**: resources, software, validation, visualization. Both authors have agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors confirm that the study was approved by the local ethics committee of the University of Mayotte on November 2023, although no formal approval code was issued. Written informed consents were obtained from the participants.

AI statement: The authors stated that no generative artificial intelligence or AI-based tools were used for the writing, editing, or data analysis of this article. All analyses, interpretations, and wording were entirely performed by the authors.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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