

Tracing creative mathematical thinking trajectories of fifth-grade students in open ended geometry tasks: A phenomenographic study

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ABSTRACT

This study aimed to explore the qualitatively different ways fifth-grade students conceptualize perimeter and area through open-ended geometric tasks. By analyzing written work and clinical interviews with five students, the study identified five conceptions ranging from procedural application to flexible spatial reasoning. Findings suggest that mathematical creativity emerges from the dynamic interaction between perceptual engagement, measurement conceptualization, and structural abstraction. The resulting outcome space organizes these conceptions along a continuum of increasing conceptual sophistication. This study offers important insights into task design and formative assessment in mathematics education. The results emphasize the role of open-ended tasks in fostering divergent mathematical thinking and going beyond mere procedural memorization in elementary school settings.

Keywords: mathematical creativity, phenomenography, geometry, open-ended tasks, creative trajectory

INTRODUCTION

Geometry as a cognitive field, where children learn to understand, explain, and reason about spatial environments, plays a fundamental role in elementary mathematics education (Jones & Tzekaki, 2016). As established in foundational constructivist research, children actively construct their own intuitive conceptions when reasoning about mathematical concepts (Confrey, 1990). However, the geometry teaching method common in most elementary schools in Indonesia is still based on memorizing formulas and explaining definitions, so that students have little opportunity to independently create meaning or demonstrate true mathematical creativity (Rachmawati et al., 2025). This aligns with findings by Utami et al. (2025), who highlighted that mathematics instruction in Indonesia heavily emphasizes formal, ready made concepts and procedural skills before applying them to practical problem-solving contexts. This pedagogical approach is clearly seen in learning the concepts of area and perimeter, where students who are able to apply algorithms consistently do not necessarily have a deep understanding of the underlying spatial quantities (Sisman & Aksu, 2016). In this situation, procedural skills are often misinterpreted as indicators of conceptual understanding.

This problem is not isolated or unimportant. According to a growing body of research, students who are able to use area and perimeter formulas correctly at the procedural level often still struggle when comparing shapes with different areas but the same perimeter, explaining the conceptual basis of a formula, or estimating measurements using non-standard units (Smith et al., 2016). Open-ended math exercises highlight these conceptual gaps that are sometimes overlooked by traditional closed-format tests (Kortemeyer et al., 2024). Anugraheni et al. (2025) present strong evidence that problem-based learning strategies that use open (strategy-open) task significantly promote students' creative mathematical thinking because they force students to reach beyond procedural knowledge.

This difference is further strengthened by research on the cognitive basis of problem solving. Rizos and Gkrekas (2023) showed that while closed-ended problem solving relies primarily on mathematical knowledge and general intelligence, performance on open-ended mathematical problems relies heavily on general creativity and verbal ability for elementary school students. This difference implies that open-ended tasks utilize qualitatively different cognitive resources, emphasizing creative and adaptive thinking that is hindered by procedural tasks.

Niyazova et al. (2022) developed a methodological classification of open-ended mathematical problems and explained that the use of tasks with unconventional solution strategies can have a positive influence on academic achievement and student learning motivation. This is complemented by Vlasenko et al. (2021), who showed that cultivating mathematics teachers' and

students' motivational and value oriented readiness to adopt open, innovation-oriented task designs is a precondition for such tasks to be implemented meaningfully in classrooms. Based on these two foundations, the task instrument in this study was prepared by adapting the design concepts proposed by both studies.

To help students develop a deeper conceptual understanding in geometry learning, this study departs from structured classroom observations conducted by the research team prior to the main study. These preliminary observations indicated that some students who appear to understand the concept of geometric measurement through traditional learning still experience conceptual gaps when faced with tasks that require reasoning skills beyond the use of existing formulas. This is in line with the research of Dang et al. (2023), which shows that procedural understanding alone is not enough to support more flexible and creative mathematical thinking skills. This study aims to explore the various ways in which fifth-grade students construct an understanding of the concepts of perimeter and area through open-ended geometry tasks. This study also seeks to identify the forms of students' mathematical thinking, ranging from the use of procedural strategies to more flexible spatial reasoning abilities.

Phenomenography as a Research Framework in Mathematics Education

Phenomenography is known as a second-level research approach that focuses on how individuals experience, understand, and interpret a phenomenon, not on the phenomenon as merely an objective reality. (Hajar, 2021). From a first-level, studies are usually directed at actual mathematical relationships, such as the relationship between the concepts of area and perimeter. In contrast, phenomenography focuses more on how students perceive and interpret these mathematical relationships through their learning experiences. (Guisasola et al., 2023).

The main focus of the phenomenographic approach lies in the variations in “ways of understanding” a phenomenon (Sims, 2024). This form of understanding is generally relatively stable across contexts, but can exhibit qualitative differences between individuals and change within the same individual as learning experiences develop. The results of the phenomenographic analysis are then presented in the form of an outcome space, a conceptual map depicting various categories of understanding arranged hierarchically based on the level of complexity and depth of the concepts possessed by the learners.

Rabab'ah (2025) examined common conceptual response variations in primary school geometry and concluded that many geometric response variations result from deeper conceptual misunderstandings about how shapes and their properties are understood. These findings validate the need for phenomenographically sensitive investigations of elementary geometry in the Indonesian context.

Open-Ended Tasks in Geometric Reasoning

High-level mathematical thinking skills can be developed through open-ended mathematical tasks that accept multiple valid solution techniques or multiple correct answers (Setyowati et al, 2024). In this study, “open-ended” refers specifically to openness in solution process rather than openness in problem formulation (Pehkonen, 1997; Sullivan & Clarke, 1992): the question itself is fixed, but students are free to choose their own strategies, representations, and reasoning pathways to answer it unlike problem-posing traditions (Brown & Walter, 2005) in which the formulation of the question itself is left to students. By applying Van Hiele's framework of geometric thinking levels, Malatjie and Machaba (2019) found that students' geometric understanding often remains at the lower levels of visualization and analysis precisely the levels at which open-ended tasks of this kind can reveal the greatest conceptual variation. This concern is supported by research on geometric thinking in elementary contexts. According to Wu and Ma (as cited in Jupri, 2018), most primary school students remain at Van Hiele levels 0 and 1 and almost never progress to informal deduction during primary education.

Erdem (2023) found that elementary school students commonly confuse perimeter and area, considering them interchangeable, particularly in the case of perimeter and area. Tossavainen et al. (2017) showed that students could reliably calculate area equations despite still having fundamentally incorrect spatial notions of what area means. Sujatha and Vinayakan (2023) found that placing measurement tasks in relevant real-world contexts significantly improved students' conceptual understanding of these quantities. These findings directly informed the task design principles of this study. Mulyono and Hapizah (2018) created student learning trajectories for perimeter and area of rectangles in the context of fish ponds.

Perimeter and Area as Ongoing Conceptual Challenges

Students tend to treat perimeter as a sum of discrete sides rather than as a property of a continuous limit (Battista, 2003). The relationship between perimeter and area is also frequently assumed to be proportional: a common assumption is that a larger area necessarily means a larger perimeter, or that equal perimeters guarantee equal areas (Woodward & Byrd, 1983). Rather than treating this assumption as simply a response variations to be corrected, we note that it is a reasonable generalization from everyday experience with proportional quantities, and can productively be used as a starting point for instruction: presenting students with counterintuitive cases (equal perimeter, different area, or vice versa) invites them to test their own reasoning rather than being told it is wrong, and can help them appreciate one of the more delightful features of mathematics that careful reasoning sometimes overturns reasonable-seeming assumptions. The actual independence, or even inverse variation, of these two quantities is counterintuitive and requires conceptual work that procedural instruction alone does not provide. Gebremeskel et al. (2025), in the *Eurasia Journal*, confirmed that students' conceptual understanding of plane geometry and their problem-solving performance improved substantially when instructional approaches integrated conceptual engagement and relational reasoning, rather than prioritizing procedural execution.

The use of non-standard units of measurement such as paving tiles or counting blocks can either support or complicate the understanding of area, depending on how they are conceptualized. When students treat units as objects to be counted rather than

as measures of two-dimensional space, their understanding of area remains at an enumerative level, unable to support the more flexible spatial reasoning required by perimeter-area ratios.

Mathematical Creativity in Elementary School Mathematics

Mathematical creativity at the elementary school level is increasingly understood not as a rare talent, but as a form of cognitive flexibility that can be developed through appropriate learning design (Al-Sarry & Ghadban, 2020). Key indicators of mathematical creativity include fluency (generating multiple solution strategies), flexibility (switching between different approaches), and originality (generating unexpected or novel relationships) (Kozłowski et al., 2019). Adharini and Herman (2021) showed that an open-ended approach effectively fosters students' creative mathematical thinking skills and self-confidence. The open-ended inquiry approach significantly increases students learning activities and creative thinking skills (Ramdani et al., 2021). This is supported by Ramdani et al. (2021), which demonstrated through a pilot study of lateral thinking problems that semi-structured and open-ended tasks can effectively foster creativity when engaging students in a dynamic cycle of perception, challenge, generation of alternatives, and critical evaluation.

The creative trajectory, as this paper defines it, describes a students path when interacting with an open (strategy-open) task. Specifically, it describes a students unique sequence of representations, strategies, and reasoning steps. An individuals creative trajectory is a record of the process, an entry into the dynamic relationship between understanding and creativity. The creative trajectory can be partially constructed from written artifacts, which justifies the use of task based phenomenographic studies in large group class environments.

METHOD

This study adopted a qualitative research design grounded in the phenomenographic tradition (Marton, 1981; Marton & Booth, 1997). Phenomenography was appropriate for this study because the primary research interest lay in documenting the qualitatively different ways in which students conceptualized geometric measurement, rather than in measuring the frequency or accuracy of specific responses. Data collection took place in two phases: (1) collecting students' written work produced during an open-ended geometric task in a naturalistic classroom setting, and (2) individual clinical interviews conducted within 48 hours of completing the task. This two-phase design reflects established phenomenographic practice, in which learning artifacts are complemented by interview data to access the reasoning processes underlying surface responses (Åkerlind, 2005). We treated students' written work not simply as records of correct or incorrect answers, but as traces of their thinking process how they chose to represent, compute, and explain their reasoning on paper (an "epistemic object," in phenomenographic terms). To ensure qualitative rigor and credibility (Tracy, 2010), this two-phase approach allowed for rich data triangulation between written artifacts and interview responses.

The study participants consisted of 50 fifth-grade students from a public elementary school in Sumedang, West Java, Indonesia. Of these, five students were purposively selected for in-depth analysis using a maximum-variation sampling strategy (Patton, 2002): the classroom teacher's assessments of prior mathematics achievement were used to identify students spanning the full range of achievement levels in the class (from among the highest- to lowest-performing students), on the assumption that this range would be more likely to surface qualitatively distinct conceptions than a random or achievement-homogeneous sample. We acknowledge that a sample of five falls below the range typically recommended for phenomenographic studies aiming for a saturated outcome space (approximately 15–30 participants; Trigwell, 2000); accordingly, the resulting outcome space is presented as an initial, illustrative mapping of conceptual variation within this class rather than an exhaustive account of the variation present in the full cohort of 50 students, a limitation we return to in the Discussion. Written consent was obtained from parents/guardians and all participants prior to data collection. **Figure 1** presents participant demographics and cognitive characteristics identified through structured observations during assignments and interviews.

The four open-ended geometric tasks and their assessment techniques are summarized in **Table 1**.

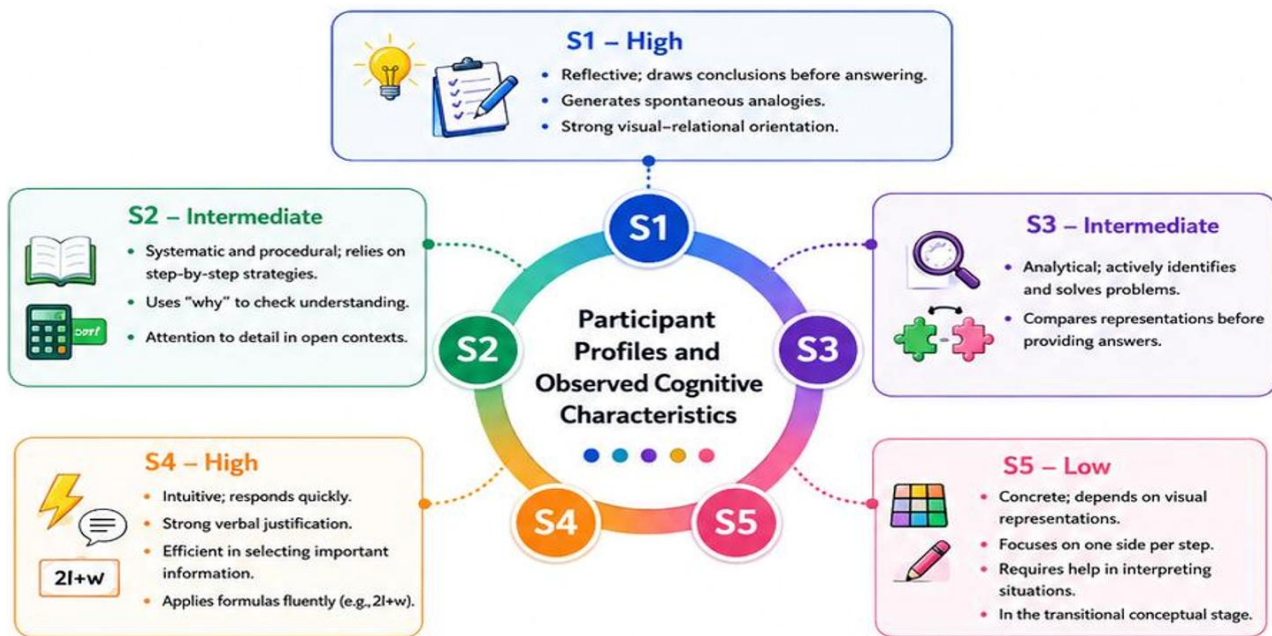


Figure 1. Participant Demographics and Observed Cognitive Characteristics, summarizing demographic information and researcher-observed cognitive characteristics for the five focal participants (Source: Authors' own elaboration)

Table 1. Description of open geometric tasks

No	Indicator	question	Assessment Techniques
1	Decomposition of Shape	A rectangle divided by a diagonal; students identify all the resulting shapes and their possible perimeter values.	Visual classification; multi-form awareness (Van Hiele L1→L2)
2	Perimeter Calculation	5×4 cm rectangle and right triangle (7, 4, 7 cm); students calculate the perimeter and articulate reasoning.	Additive versus boundary concepts; unit attribution.
3	Perimeter Invariance	Combined form (10×6 cm with internal grooves) vs. plain rectangle 10×6 cm; students determine and justify whether the perimeters are the same.	Perimeter invariance; relational awareness; visual vs. quantitative separation
4	Area Estimation (Asphalt Paving)	Two different paving layouts ($2m \times 1m$ grid; $7m \times 1m$ alternating brick layout); students estimate the number of paving blocks and provide rationale for this.	Enumerative conception versus area conservation conception.

Research Instruments

In this study, four open-ended tasks were designed collaboratively by researchers and classroom teachers, based on Sullivan and Clarke's (1992) task design principles and Watson and Mason's (2005) framework for mathematical variation. The tasks were designed to traverse a conceptual continuum from shape decomposition and perimeter computation to perimeter invariance and area conservation, allowing for the emergence of a wide range of conceptions across the outcome space.

Individual interviews were conducted for 20–35 minutes using a semi-structured protocol adapted from Piagetian clinical traditions (Ginsburg, 1997) and phenomenographic interview techniques (Marton & Booth, 1997). Interview questions were designed to explore the 'why' behind written responses, invite elaboration of visual strategies, and reveal conceptual assumptions underlying computational choices. Structured classroom observations during the task phase used a written observation protocol, completed in real time by a member of the research team, that focused on four predefined categories: representational behavior, spontaneous computational strategies, peer interactions, and non-verbal responses (e.g., erasing and redrawing, pausing, cross-referencing the task text). Observer bias was addressed by restricting field notes to descriptive (rather than interpretative) records at the point of data collection, and by triangulating observation notes against the written artifacts and interview transcripts during analysis, consistent with standard trustworthiness procedures for qualitative research (Lincoln & Guba, 1985).

Data analysis

Data analysis followed Sjöström and Dahlgren's (2002) seven-step phenomenographic procedure: (1) familiarization with the dataset through repeated reading of all written work and interview transcripts; (2) condensation of salient meaning units; (3) comparison across cases to identify similarities and differences in how participants experienced the task; (4) grouping of emerging conceptions; (5) articulation of the core meaning of each grouping; (6) naming of conceptions using theoretically grounded labels; and (7) construction of the resulting space as a structured hierarchy of conceptions. The written work was analyzed along with the interview transcripts to allow for triangulation of conceptions. Trustworthiness was established through member checking (returning summaries to participants for verification), discussions with colleagues in mathematics education, and in-depth descriptions of the analytic process (Lincoln & Guba, 1985).

Table 2. Phenomenographic result space: Five qualitatively different conceptions of geometric measurement

Concentration	Material indicators	Description	Student
I	Perimeter as an Additive Side Calculation	Students simply add the explicitly labeled sides without understanding perimeter as a property of spatial boundaries. There is a systematic response variations in associating cm^2 (a unit of area) with perimeter values. Additive logic dominates; there is no relational awareness between perimeter and shape.	S2, S3 [Task 2]
II	Perimeter as a Measurement of Outer Limits	Students understand perimeter as the distance around a shape from the outside. They use the metaphors 'walking around' or 'measuring the fence' spontaneously. The boundary orientation is fixed, but the computational implementation may be inconsistent.	S2 [Task 3], S4 [Task 2], S5
I, I, I	Perimeter as an Invariant Property	Students realize that two geometrically different plane figures can have identical perimeter values. Relational awareness (Åkerlind, 2005): simultaneous focus on visual differences and quantitative identity. Justification through structural equivalence or dimensional compensation arguments.	S1, S3 [Task 3], S4 [Task 3]
IV	Area as a Concrete Unit of Enumeration	Students calculate paving units or squares directly from visual displays. Concepts are contextually constrained by the available grid; linear measurement logic is sometimes applied to the area context (Battista, 2003). Layout-sensitive differentiation is possible at a later stage.	S2, S3, S4, S5 [Task 4]
V	Area as a Spatially Conserved Quantity	Students understand area as a quantity that is conserved across different spatial arrangements, regardless of visual configuration. Analogies of conservation are drawn from everyday experience (clay, cloth). Separation of visual complexity from quantitative quantity.	S1 [Task 4, interview]

RESULTS

Phenomenographic Result Space

Analysis of a combined data set consisting of five sets of written work and five interview transcripts yielded five qualitatively distinct conceptions of geometric measurement. Some participants demonstrated more than one conception depending on the task context, a finding consistent with phenomenographic theory (Marton & Booth, 1997). **Table 2** presents the complete result space.

Detailed Phenomenographic Analysis of Student Responses based on Tasks

The following section presents a micro-level phenomenographic study of each student's written artifacts from each of the four activities. Each artifact is viewed as an epistemic text, and conceptual orientations indexed by its surface features, such as computational techniques, representational choices, language explanations, unit attributions, deletions, and self-evaluative notations, are methodically investigated. To facilitate the reader's connection between interpretive statements and the data, images of the students' original work are included with the analysis.

Task 1: Decomposition of Shapes and Identification of Perimeter

In Task 1, a rectangle is divided into four right triangles by its diagonals. Students are instructed to recognize each shape created and calculate its possible perimeter. This test is designed as a diagnostic of spatial perception, focusing on shifts between Van Hiele's Level 2 (Informal Deduction: understanding relationships between shapes in a configuration) and Level 1 (Analysis: recognizing attributes of individual shapes). This task acts as an open-ended diagnostic rather than a convergent computational task because it allows for a variety of correct answers (different shapes can be found; different perimeter values can be calculated depending on which shape or combination is evaluated).

S1

S1 reproduced the rectangle by carefully observing its diagonal structure, reinforcing the triangle formed by the diagonals with deliberate shading and writing the number '2' in the upper corner, which was later confirmed in the interview as representing "two types of triangles that look different but are actually the same." This spontaneous act of classification has phenomenographic significance: S1 is not simply reproducing the stimulus, but actively applying an organizational scheme to it. He perceives both similarity (the triangles are geometrically identical) and multiplicity (there appear to be four triangles). This dual focus is characteristic of relational awareness (Åkerlind, 2005), that is, the ability to hold two aspects of a phenomenon in mind simultaneously (See **Figure 2**).

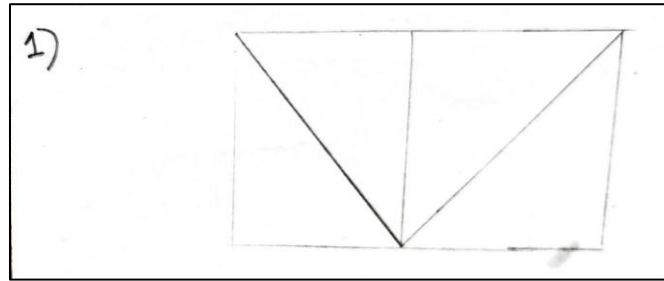


Figure 2. Results of S1 Task 1 Answers, depicting the participant's handwritten decomposition of the rectangle into its diagonal sub-shapes with annotated perimeter values (Source: Participant S1's written response collected during the study; reproduced with consent)

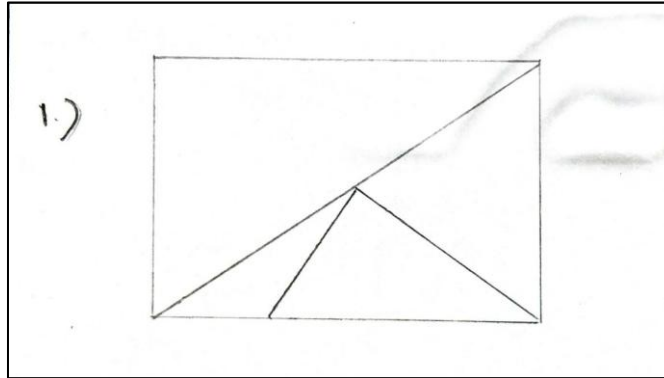


Figure 3. Results of S2 Task 1 Answers, depicting the participant's handwritten decomposition of the rectangle into its diagonal sub-shapes with annotated perimeter values (Source: Participant S2's written response collected during the study; reproduced with consent)

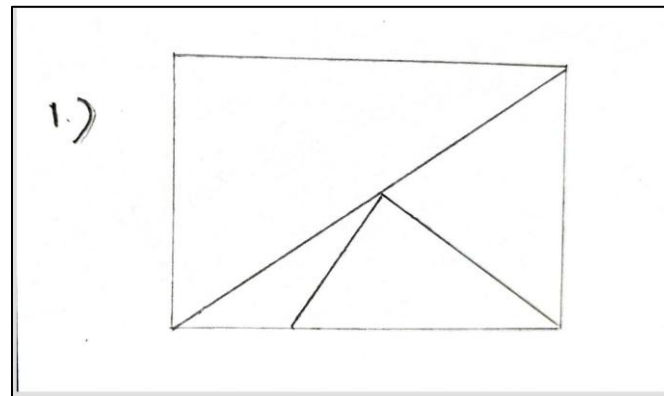


Figure 4. Results of Answers for S3 Task 1, depicting the participant's handwritten decomposition of the rectangle into its diagonal sub-shapes with annotated perimeter values (Source: Participant S3's written response collected during the study; reproduced with consent)

S2

S2 reproduces the rectangle in a minimalist form, labeling its sides (2, 4, 4, 1) without engaging with the diagonal structure. Its attention immediately shifts to the numerical edges of the figure rather than to its internal decomposition. This orientation, which treats the geometric figure as a container for numbers rather than as a spatial object with internal structure, is consistent with Conception I, where perimeter is approached as an additive aggregation of labeled sides rather than as a property of the figure's boundary. The absence of any reference to the triangle formed by the diagonals is no accident: it reflects the perceptual attitude that "triangle" is not a salient category in the given rectangle (See [Figure 3](#)).

S3

S3 reproduces the image with clearly visible diagonals and annotates the numerical value (18) within both triangular regions, indicating that he is calculating the perimeter of the triangular sub-shape. S3 not only observes the triangle but also attempts to quantify it. However, the value of 18 he obtains for the perimeter of the triangle does not correspond to a consistent set of triangle dimensions, suggesting that he is applying a formula or heuristic estimation. The presence of computation in the analytical phase (before the specified computational task) indicates an analytically motivated, though procedurally loose, thinking style (See [Figure 4](#)).

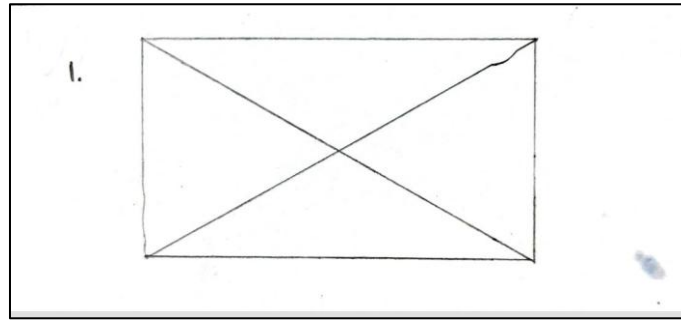


Figure 5. Results of Answers S4 Task 1, depicting the participant's handwritten decomposition of the rectangle into its diagonal sub-shapes with annotated perimeter values (Source: Participant S4's written response collected during the study; reproduced with consent)

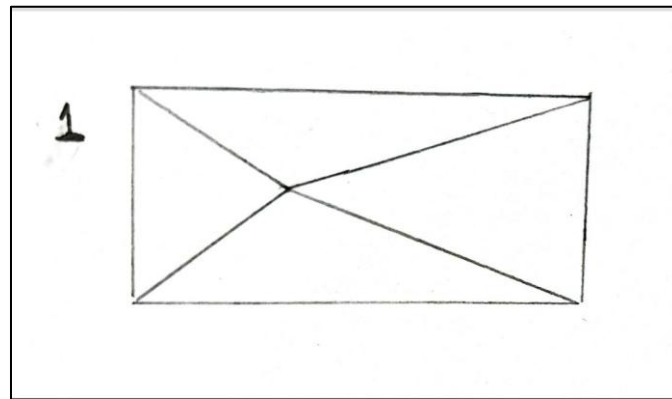


Figure 6. Results of Answers S5 Task 1, depicting the participant's handwritten decomposition of the rectangle into its diagonal sub-shapes with annotated perimeter values (Source: Participant S5's written response collected during the study; reproduced with consent)

S4

S4 produces two specific representations side by side: a triangle and a rectangle, each annotated with a numerical value (18 and 25). The triangle reproduces the internal diagonal division; the rectangle stands independently next to it. This bimodal representational strategy, which produces two separate figures from a single stimulus, demonstrates the awareness that the decomposed rectangle contains distinct geometric entities that can be treated independently. In Van Hiele's terms, this represents Level 2 entry: the ability to reason not only about shapes (their properties) but also about the relationships between shapes within a configuration. S4's spontaneous generation of the value 25 for the rectangle (a variant he invented himself, not given in the task) is a marker of mathematical fluency: the tendency to generate numerical possibilities beyond those strictly required (See **Figure 5**).

S5

S5 reproduced the rectangle with minimal elaboration; diagonal lines were present but not explored, and no sub-shapes were identified or labeled. This response was the most parsimonious among all participants. Rather than being interpreted as inactivity, this should be read through the lens of S5's broader profile as a "concrete-visual" thinker: she engaged deeply when a visual stimulus was presented but had not yet developed the representational ability to generate additional structures beyond what was explicitly given. Task 1 did not provide adequate support for S5 to move from visual reproduction to analytical decomposition (See **Figure 6**).

Based on **Figure 7**, S1 and S4 appear more dominant because they are able to demonstrate a more complex quality of response than the other participants, while S2 and S5 still show a tendency for simple and limited responses. On the other hand, S3 is at an intermediate stage of development which shows the emergence of analytical abilities, although not yet fully consistent. In the visual elaboration dimension, S1 produced the most detailed representation through the use of shading, the addition of shape classifications, and explanatory notes to the drawings created. S4 also demonstrated high visual quality by combining visual and symbolic representations simultaneously. Meanwhile, S3 began to link the image representation with numerical aspects through number annotations on the triangles formed. In contrast to the three participants, S2 and S5 only displayed reproductions of basic shapes without additional details so that their responses tended to be minimalist.

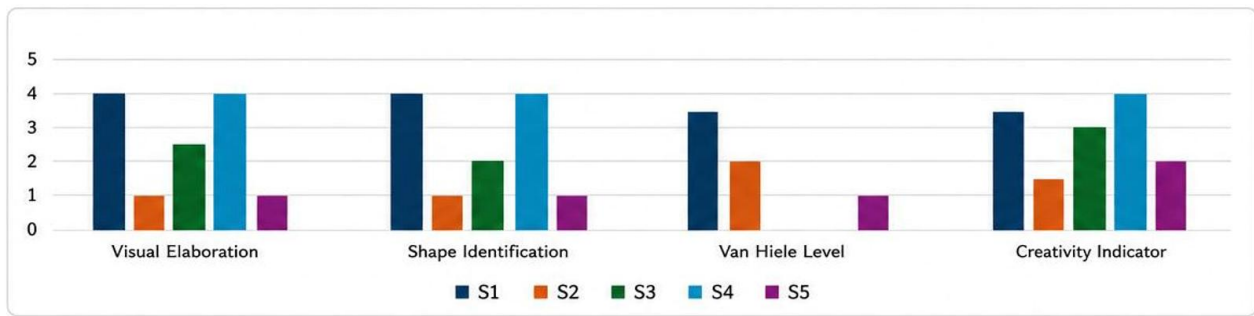


Figure 7. Comparison of participant responses to Task 1 (Source: Authors' own elaboration)

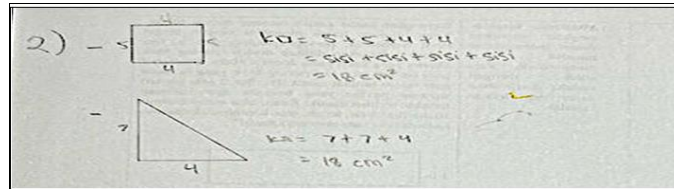


Figure 8. Results of S1 Task 2 answers, depicting the participant's handwritten calculation of the perimeter of the rectangle and right triangle (Source: Participant S1's written response collected during the study; reproduced with consent)

For shape identification, participants in S1 and S4 recognized the presence of a sub-triangular shape in their main shape and demonstrated a more defined level of shape decomposition. Participants S3 recognized the existence of a triangle, though this recognition was more indirect and occurred as a part of the process of computation. Participants S2 and S5 continued to view the shape in its entirety and did not perceive the division of the shape caused by the formation of the diagonals. In line with Van Hiele's level of thinking, participants S1 and S4 operate at Level 2 as they understand the relationship of shapes and express geometric reasoning in a relational manner. Participants S3 are progressing from Level 1 to Level 2, and participants S2 and S5 are still at Level 1. In terms of mathematical creativity, participant S4 demonstrated flexibility of thinking as they provided different numerical variations, participant S1 demonstrated originality as they classified shapes that came spontaneously to them, and participant S3 demonstrated fluency of thinking and switching of representations between visual and numerical. Overall, these results show that geometry learning should incorporate verbal identification, classification of shapes, sub-shapes, and respective reasoning to enable learners progress to a higher level of relational geometric thinking beyond analysis.

Task 2: Calculating the Perimeter of Rectangles and Triangles

In this task, students had to find the perimeter of a rectangle (5 x 4 cm) and a right triangle (sides 7, 4, and 7 cm), and then explain their answer. The goal of this task was to determine how students understood perimeter, as the definition of perimeter changes where it is placed on the boundary of a closed shape (Conception II), as the sum of the segments that comprise the perimeter (Conception I), or the relationship between perimeter and geometric constructs. This task also evaluated the attribution of units: correct calculations alongside incorrect unit attribution (cm² instead of cm) indicated a misalignment of procedure and concept, where the procedure was executed, but the concept of dimension was not understood.

S1

S1 writes: "perimeter = 5+5+4+4 = side+side+side+side = 18 cm²" for the rectangle and "perimeter = 7+7+4 = 18 cm²" for the triangle. Both calculations are arithmetically accurate (18 cm for each). However, S1 consistently applies cm², the unit of area, to both perimeter results. This is one of the most informative findings in the entire dataset. From the perspective of Battista's (2003) measure theory, this constitutes a first-order "procedure-concept gap": the procedure (summing sides) is executed correctly, but the underlying concept (perimeter as a one-dimensional boundary measure) has not been stably distinguished from area (a measure of filling two-dimensional space). S1's annotation "side+side+side+side" is particularly revealing: he has a clear additive model of perimeter, and it is precisely this additive model that makes him prone to dimensionality concatenation. He is thinking about the number of sides, not about the nature of the quantity being measured (See [Figure 8](#)).

S2

Without specifically mentioning the rectangle or triangle provided in the exercise, S2 created two triangles on his answer sheet: one with sides labeled 6, 6, and the other with sides labeled 2, 7, 2. This is an example of task incongruity: instead of calculating the perimeter of the provided shape, S2 appears to have made up his own figures. There are two possible interpretations. First, given the open-ended framework of the instrument, it is possible that S2 misinterpreted the task as an invitation to "draw and calculate" any shape he chose. Second, he may have attempted to work with the provided design but misrepresented its dimensions from memory, creating what he thought was the prescribed shape (See [Figure 9](#)).

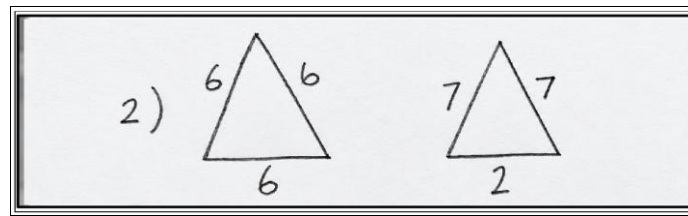


Figure 9. Results of S2 Task 2 Answers, depicting the participant's handwritten calculation of the perimeter of the rectangle and right triangle (Source: Participant S2's written response collected during the study; reproduced with consent)

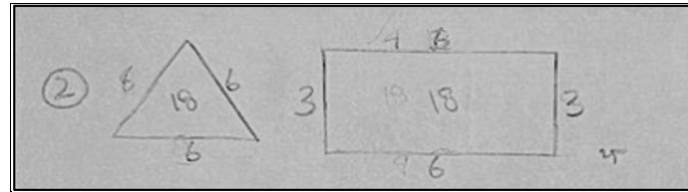


Figure 10. Results of Answers for S3 Task 2, depicting the participant's handwritten calculation of the perimeter of the rectangle and right triangle (Source: Participant S3's written response collected during the study; reproduced with consent)

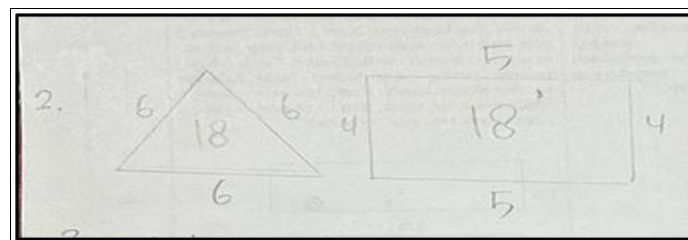


Figure 11. Results of Answers S4 Task 2, depicting the participant's handwritten calculation of the perimeter of the rectangle and right triangle (Source: Participant S4's written response collected during the study; reproduced with consent)

From a phenomenological perspective, both interpretations are informative. The first interpretation suggests that S2 views open-ended geometry tasks as an open-ended area of self-expression, rather than as a limited mathematical inquiry; this notion has both creative potential and real-world constraints. The second interpretation suggests that his working memory for geometric specifications is not robust enough to sustain long-term task engagement without visual references. In both situations, the diagnostic yield of these open-ended tasks is higher than that of closed-ended computational tasks: closed-ended tasks may simply produce inaccurate numbers, but these responses reveal the underlying attitude toward the task itself.

S3

S3 drew two shapes side by side: a rectangle (sides 4, 3, 4, 3, with the numbers 18 and 19 written inside it simultaneously) and a triangle (sides 6, 6, 6, with the number 18 crossed out inside). S3 appears to have calculated the two results, identified a disagreement, and not resolved it at the time of submission, which is why the numbers 18 and 19 appear inside the same rectangle. This is an artifact of constructive cognitive conflict. He actively questions his own calculations, which is a sign of introspective self-monitoring rather than misunderstanding (See [Figure 10](#)).

Flexible mathematical thinking is characterized by a willingness to generate two competing answers before settling on one (Liljedahl & Sriraman, 2006). This demonstrates a focus on mathematical activity as a process of speculation and confirmation, rather than as the provision of a predetermined solution. This is a phenomenological indicator of Conception I in transition: the S3 side's calculation process is active, but he is beginning to acquire metacognitive abilities to challenge the results.

S4

S4 drew a triangle (sides 6, 6, 6, value 18) and a rectangle (sides 4, 5, 4, 5, value 18, with a separate annotation of 25 at the corner). The triangle calculation ($6+6+6=18$) was accurate for the self-selected dimensions. The rectangle calculation ($4+5+4+5=18$) was also arithmetically correct. The appearance of the additional value 25 is significant: confirmed in the interview as a self-generated perimeter variant, the result of imagining a rectangle with different dimensions, this number was not a calculation response variations but a deliberate exploration. S4 was not satisfied with calculating just one perimeter; he spontaneously expanded the task to explore other possible perimeter values (See [Figure 11](#)).

This behavior is a classic example of mathematical fluency (Silver, 1997): the spontaneous generation of multiple examples or cases within a single problem context. It is also diagnostically important in a phenomenographic sense: it indicates that S4 already has an implicit awareness that perimeter is a relational property (depending on the chosen dimensions), not just a fixed numerical property of a particular shape and orientation that precedes Conception III (Perimeter as an Unchanging Property).

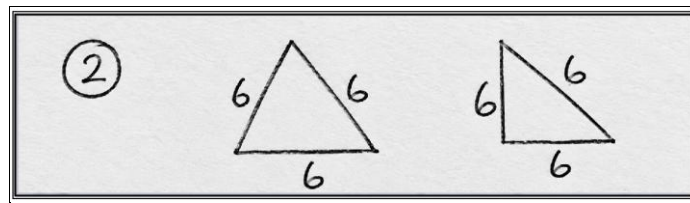


Figure 12. Results of Answers S5 Task 2, depicting the participant's handwritten calculation of the perimeter of the rectangle and right triangle (Source: Participant S5's written response collected during the study; reproduced with consent)

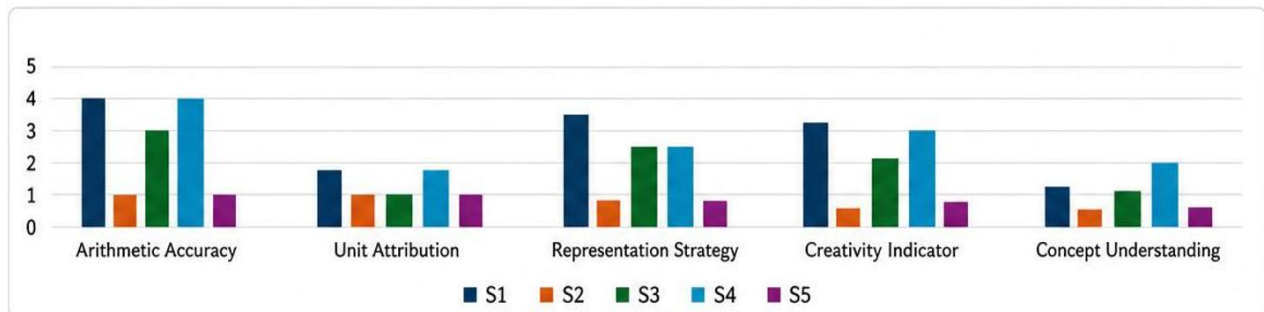


Figure 13. Comparison of participant responses to Task 2 (Source: Authors' own elaboration)

S5

S5 drew a symmetrical triangle with the numbers 6, 6, and 6 on its sides. He then wrote "1.5" and the number 16 above the second attempt at the shape. The notation 1.5 was verified during the interview as S5's own self-assessment score for his response, which was an emotional self-evaluation naturally made during the activity, not a quantitative calculation. This metacognitive activity, which involves simultaneously performing a task and assessing one's own performance, is phenomenographically unique. This suggests that S5 divided his cognitive resources between solving the math problem and monitoring the quality of his own achievement (See [Figure 12](#)).

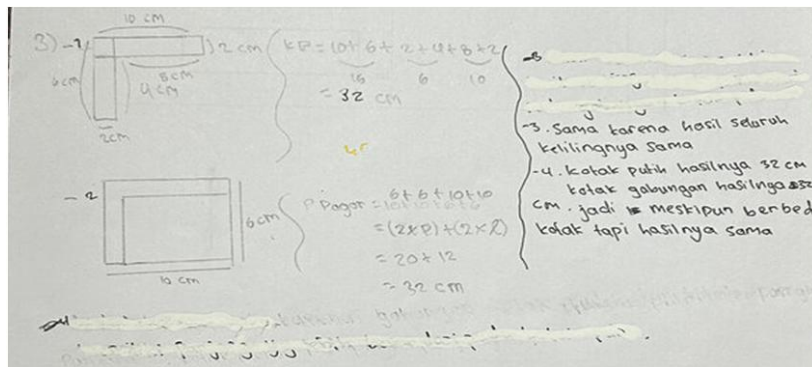
S5's low self-rated score (1.5 of the implied maximum) and incomplete calculation (16 is not equal to the perimeter of a triangle with sides 6, 6, 6) indicate that she is aware of the shortcomings of her solution. However, this knowledge has not yet resulted in corrective action, indicating that she does not yet have the mathematical resources necessary to make corrections at the time of completion of the task. Here, Vygotsky's theory of the zone of proximal development is very useful: There is a useful but unresolved conflict because S5's metacognitive awareness has outpaced her mathematical procedural abilities.

[Figure 13](#) shows differences in ability in aspects of arithmetic accuracy, unit attribution, representation strategies, creativity indicators, and understanding of geometric concepts. In general, S1 and S4 scored higher than the other participants, indicating more developed mathematical representation and reasoning abilities. S3 was in the middle position with analytical characteristics that began to emerge through the process of self-correction and the use of multiple representations. In contrast, S2 and S5 still showed responses that tended to be simple, less consistent, and not fully in accordance with the demands of the task.

In terms of arithmetic accuracy, S1 and S4 were able to produce correct answers, while S3 showed almost correct answers with indications of self-correction during the solution process. In terms of representation strategies, S1 demonstrated explicit additive verbalization, while S3 and S4 began to use bimodal representations that combine visual and numerical. From the perspective of mathematical creativity, S4 demonstrated fluency in thinking through the variety of numerical answers produced, S3 demonstrated flexibility in representation, and S1 displayed originality through metacognitive explanations of the calculation process. Meanwhile, S2 and S5 were still at the pre-conceptual stage because they had not yet demonstrated a relational understanding of the concepts of perimeter and units of measurement. These findings indicate that geometry learning needs to emphasize the use of diverse representations, awareness of unit attribution, and the ability to explain relationships between geometric elements verbally and visually.

Task 3: Perimeter Invariance of Composite Shapes vs. Regular Rectangles

The most conceptually rich tool in this study was Task 3. This task presented two figures: (1) a composite shape (a 10x6 cm rectangle with internal dimensions of 2x4, 2x4, and 8x2) and (2) a simple 10x6 cm rectangle with internal indentations that produce "white" and "red" regions. Students were instructed to determine whether the perimeters of the two figures were identical and to provide evidence to support their conclusions. This task focused on the idea of perimeter invariance, a non-obvious mathematical truth that, as long as the changes are compensatory, a figure can undergo internal geometric modifications (the formation of interior indentations) without changing the size of its outer boundary. The diagnostic power of this result stems from its counterintuitive nature (the composite shape appears more complex and therefore "should" have a larger perimeter). Students who arrived at the correct answer through surface intuition, systematic calculation, or structural reasoning about dimensional compensation demonstrated a qualitatively different form of understanding.

**Translation:**

- First Shape (L-shape/ Composite shape):
 - KP (Combined Perimeter) = $10 + 4 + 2 + 4 + 8 + 2 = 32 \text{ cm}$
 - Second Shape (Rectangle):
 - Bangun 2 (Shape 2) = $6 + 6 + 10 + 10$
 - = $(2 \times p) + (2 \times l) = (2 \times 10) + (2 \times 6)$
 - = $20 + 12 = 32 \text{ cm}$
3. Same, because the total perimeters are the same.
 4. The inner box result is 32 cm, the combined box result is 48 cm. So, even though the boxes are different, the results are the same.

Figure 14. Answer Results S 1 Task 3, depicting the participant's handwritten justification for perimeter invariance between the composite and regular rectangle shapes (Source: Participant S1's written response collected during the study; reproduced with consent)

S1 Relational justification through dual-path computation

S1 produced two separate calculations: (a) "KP = $10+6+2+4+8+2 = 32 \text{ cm}$ " (the outer perimeter of the combined shape, accurately tracing only the outer perimeter and not the inner curved edges) and (b) "P of the box = $(2 \times P) + (2 \times L) = 20+12 = 32 \text{ cm}$ " (using the formula $2(p+l)$ for a plain rectangle). According to his written conclusions, "The white box yields 32 cm; the combined box yields also 32 cm" and "Same because the results of all the perimeters are the same" (Same, because the results of all the perimeters are the same). So even though the boxes are different but the results are the same" (The combined box also yields 32 cm; the white box yields 32 cm). So, even though the boxes are different, the results are still the same (See **Figure 14**).

There are several reasons why this answer is phenomenographically unusual. First, it demonstrates the ability to accurately determine the outer boundaries of a composite image, a challenge that requires distinguishing between internal and external edges, a distinction that often confuses many students. Second, instead of using just one method, this answer uses two different calculation procedures (additive limit exploration and formula application) and compares the results. Third, and perhaps most importantly, structural insight the understanding that visual differences and quantitative identities are logically compatible replaces numerical equality in textual explanations. The notation "before" (although) is a metalinguistic marker of the anticipated paradox: S1 implicitly acknowledges that the normal expectation (different shapes $\rightarrow$ different perimeters) has been replaced by mathematical proof by showing himself and the reader that his claimed result is counterintuitive. Conception III: Perimeter as an Unchanging Property is marked by this.

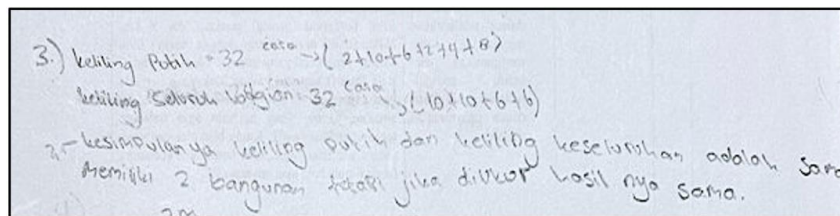
"At first I didn't think of it that way, but after I finished the task and looked at it again, the results turned out to be the same. The shapes are indeed different, but when you look closely, the perimeter really has to be the same, because the 10 cm side has to be the same length as the line parallel to it. Now... this is the 8 plus 2. The 2 comes from here since it's a square, the sides have to be the same length, which is 2. It's the same with the 6 cm width its parallel side is also 6 cm. If you just glance at the picture it looks like the shape should matter, but the real reference point is just the length of the parallel sides. I was surprised. But... um... actually it makes sense once you notice the lines are parallel, so they have to be the same length that's why the perimeters come out the same."

(S1, interview excerpt, Task 3, 26 February 2026, recording 00:06:32; translated by the researcher)

From a creativity perspective, S1's path in completing this task demonstrated all three traditional indicators (Silver, 1997): originality (metalinguistic markers that anticipate paradoxes, spontaneous acts of mathematical communication that go far beyond the task requirements), fluency (two complete calculations), and flexibility (switching between additive and formulaic strategies).

S2: Qualitative reasoning precedes computational consistency

S2 wrote: "White Surround = 32 ways $\rightarrow (2+10+6+12+4+8)$ " and "Whole Building Surround = 32 ways $\rightarrow (6+10+10+6+6)$." The conclusion: "The conclusion is that the white perimeter and the overall perimeter are the same. There are 2 buildings but if you measure them the results are the same" (Conclusion: the white perimeter and the overall perimeter are the same. There are 2 buildings but if you measure them the results are the same) (See **Figure 15**).

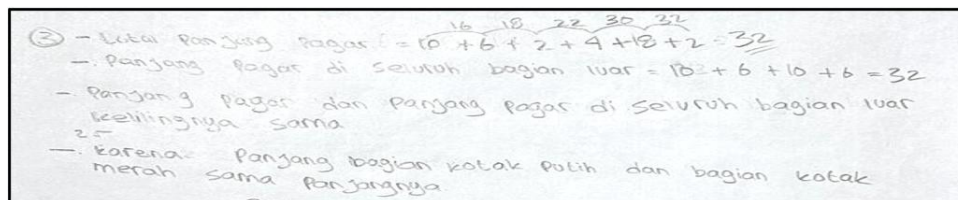
**Translation:****Calculations:**

- White Perimeter = 32 $\rightarrow$ Method: $(2 + 10 + 6 + 2 + 4 + 8)$
- Total Perimeter = 32 $\rightarrow$ Method: $(10 + 10 + 6 + 6)$

Conclusion:

- In conclusion, the white perimeter and the total perimeter are the same. They are 2 different shapes, but when measured, the result is the same.

Figure 15. Answer Results S 2 Task 3, depicting the participant's handwritten justification for perimeter invariance between the composite and regular rectangle shapes (Source: Participant S2's written response collected during the study; reproduced with consent)

**Translation:**

Total fence length = $10 + 6 + 2 + 4 + 8 + 2 = 32$

Fence length along the entire outer section = $10 + 6 + 10 + 6 = 32$

The fence length and the fence length along the entire outer section have the same perimeter.

Because the length of the white box section and the red box section are equal in length.

Figure 16. Results of Answers for S3 Task 3, depicting the participant's handwritten justification for perimeter invariance between the composite and regular rectangle shapes (Source: Participant S3's written response collected during the study; reproduced with consent)

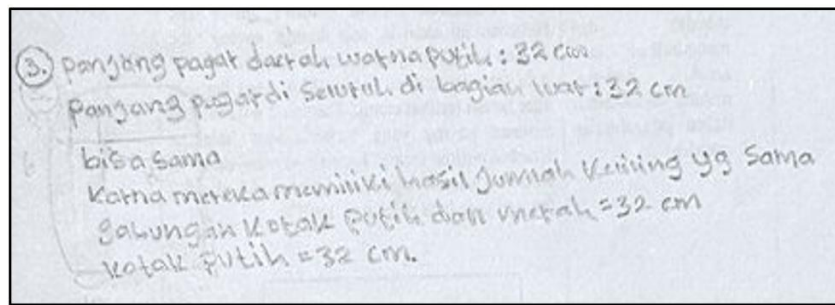
The computational inconsistency here is glaring: the sum " $6+10+10+6+6$ " equals 38, not 32. Yet S2 asserts 32 as the result. This suggests that the answer comes not from computation, but from a prior qualitative belief about what the answer should be, a belief that he then attempted (but failed) to support numerically. This is a sophisticated but unvalidated form of reasoning: S2 reasoned from his conceptual understanding (the outer boundary of any 10×6 region should give the same perimeter) and adapted an internally inconsistent computational justification. The fact that his conceptual conclusion was correct, despite his incorrect computation, is diagnostically important: it places him firmly in Conception II (Perimeter as a Measurement of Outer Boundaries), where spatial concepts precede procedural implementation.

S3 Dimensional compensation as emergent structural reasoning

S3's response was the most systematically structured of the five participants. He wrote: "Total length of the fence = $10+6+2+4+18+2 = 32$ "; "Length of the fence around the entire perimeter = $10+6+10+6 = 32$ "; and his justification: "The length of the fence and the length of the fence around the entire perimeter are the same. Because: the length of the white box section and the length of the red box section are the same length" (Because: the length of the white box section and the length of the red box section are the same length) (See Figure 16).

The final statement in S3's defense of perimeter invariance is the most sophisticated example of structural geometric reasoning in the entire dataset. S3's reasoning is essentially a claim about dimension compensation: to preserve the overall boundary length, the internal curves in a composite shape consist of segments whose combined lengths are equal to the comparable outer dimensions of a plain rectangle. This is a claim he created using data from his two calculations, not a claim he was taught. This is originality in the context of mathematical creativity: the creation of a non-trivial structural relationship not anticipated by the design of the challenge. Furthermore, it places S3 and S1 on Conception III, a conclusion that might not have been anticipated based on their success in previous work.

"Yes, ma'am. I answered by... um, counting the white-colored fence first, and then the whole one that included the red part. I think it was probably counting the perimeter, since the fence problem was asking us to count the total length of the sides on the outside. It comes out the same because... um... the red side, the one labeled 10 cm, has a side across from it that's known to be 8 cm, because next to it there's a square whose sides definitely have to be equal, which is 2 cm, so when you add up the side lengths it also becomes 10 cm. Then the same thing happens with the other one the side that's only known to be 6 must have its opposite side also equal to 6 cm, even though at first only 2 cm of it was known. Yes, it will be the same, because parallel sides facing each other must be the same length. We have to look at the picture more carefully to understand how to answer it. Basically I just added up the lengths of the outermost sides of the white area, like in the

**Translation:**

- Fence length of the white area: 32 cm
- Fence length along the entire outer section: 32 cm

Explanation: They can be the same. Because they have the same total perimeter result. Combined white and red box = 32 cm White box = 32 cm

Figure 17. Results of Answers S4 Task 3, depicting the participant's handwritten justification for perimeter invariance between the composite and regular rectangle shapes (Source: Participant S4's written response collected during the study; reproduced with consent)

problem just adding them up while looking at the picture. If one side is changed to be longer and the other shorter, I think the total perimeter would end up different, because that would affect the sum of the side lengths. I think that's probably how it is, but I'm scared I might be wrong."

(S3, interview excerpt, Task 3, 26 February 2026, recording 00:03:05; translated by the researcher)

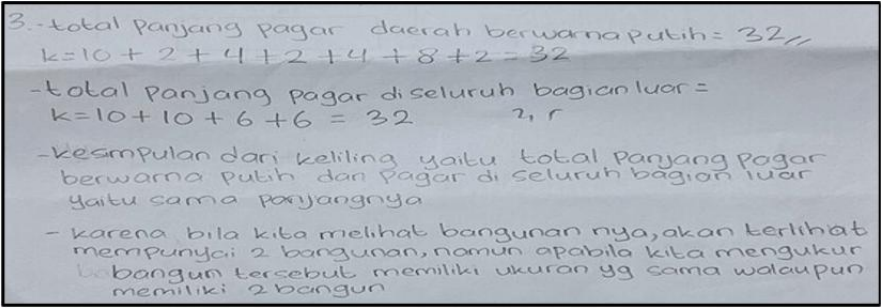
S4 – Visual-quantitative decoupling

Answer S4 : "The total length of the white fence = 32, $k = 10+2+4+2+4+8+2 = 32$ "; "The total length of the fence on the entire outside, $k = 10+10+6+6 = 32$, 2r." Conclusion: "The conclusion from the perimeter is that the total length of the white fence and the fence on the entire outside is the same length. Because if we look at the building, it will look like it has 2 buildings, but if we measure the building it has the same size even though it has 2 shapes." (See **Figure 17**).

The conclusion of the study makes a crucial epistemological distinction between visual appearance ("we see": what we see) and measured reality ("we measure": what we measure). This separation of visual complexity from quantitative magnitude is a defining characteristic of advanced geometric understanding at this level. The annotation "2r" after the calculation of the rectangle denotes the formula $2(p+l)$ in shorthand as a marker of efficiency, indicating that S4 has internalized the formula as a cognitive shortcut rather than simply performing it as a rote procedure. His accurate calculation of both perimeters (each 32 cm) and his explicit verbal recognition of the visual paradox place him firmly at Conception III and, according to Van Hiele's criteria, at Level 3 (Informal Deduction).

S5 – Intuitive invariance without a computational framework

S5 wrote: "The length of the white area fence = 32 cm"; "The length of the fence on all the outside = 32 cm"; "They can be the same. Because they have the same total perimeter. The white and red squares combined = 32 cm. The white squares = 32 cm." S5 reached the correct conclusion through minimal written calculation; he stated that both values were 32 cm without showing the steps. In the interview, he confirmed that he "calculated the outside" for both numbers. This strategy of boundary tracing without explicit numerical decomposition is consistent with Conception II (Perimeter as Measurement of Outer Boundaries): he has a spatial understanding of what perimeter (outer boundary) means, and he applies it directly as an act of measurement rather than as an analytical decomposition. His justification ("because they have the same total perimeter") is tautological but phenomenographically informative: it suggests that S5's conception of invariance is based on the act of measurement (tracing and calculating boundaries) rather than structural reasoning. In Vygotsky's terms, his zone of proximal development for perimeter invariance lies in the direction of structural justification: he can verify the results, but he cannot yet explain why they should be so. The participants' response profiles and geometric conceptions in Task 3 are summarized in **Table 3** and illustrated in **Figure 18**.



Translation:

Calculations:

- Total fence length of the white area = 32K (Perimeter) = $10 + 2 + 4 + 2 + 4 + 8 + 2 = 32$
- Total fence length along the entire outer section: K (Perimeter) = $10 + 10 + 6 + 6 = 32$

Conclusion:

- The conclusion about the perimeter is that the total length of the white fence and the fence along the entire outer section are equal in length. Because if we look at the shape, it appears to have 2 shapes, but when we measure them, the shapes have the same size even though there are 2 shapes.

Figure 18. Results of Answers S5 Task 3, depicting the participant's handwritten justification for perimeter invariance between the composite and regular rectangle shapes (Source: Participant S5's written response collected during the study; reproduced with consent)

Table 3. Participant response profile and geometric conception in Task 3

Participant	Conception and Quality of Task 3 Responses
S1	Conception III (telos). Dual-path computation, metalinguistic paradox markers, relational justification. The most sophisticated responses in the dataset.
S2	Conception II (developing). Qualitatively correct conclusions; computationally inconsistent (sum $\neq$ stated result). Conceptual orientation precedes procedural implementation.
S3	Conception III (emerging). The most systematic dual-path structure; justification of dimensional compensation shows originality.
S4	Conception III (robust). Explicit visual-quantitative separation in language; 2r formula markers; Van Hiele Level 3 indicators present.
S5	Conception II/III Boundaries. Correct through boundary tracing; tautological justification; ZPD for structural explanation of invariance.

Task 4: Area Estimation with Non-Standard Pavement Units

Task 4 presents two different paving configurations, a grid layout (2m×1m, square paving) and an alternating brick layout (7m×1m), and asks students to determine the number of paving blocks required and to justify their reasoning. This task targets the conceptual distinction between area as an enumerative count of discrete units (Conception IV) and area as a spatially conserved quantity independent of layout configuration (Conception V). The non-standard units (paving blocks, not cm²) are a deliberate design choice: they highlight the process of measurement (how many tiles will fit?) rather than the formula (l×l), thus revealing whether students' understanding of area is based on spatial reasoning or on formula derivation.

S1: Visual conservation without explicit quantification

S1 produced two carefully drawn visual representations: a square grid configuration (depicted with uniform square tiles in a 2M×1M format) and an alternating brick pattern (depicted with half-shifted rows in an elongated format). He did not write down explicit numerical quantities for either configuration; the only annotation was “ys 1” (confirmed in the interview as a self-predicted score). In the interview, when asked directly how many paving blocks were needed, S1 spontaneously provided an analogy: “It’s like clay if you flatten it or roll it in a different way, the amount remains the same.” (S1, interview excerpt, Task 4, 26 February 2026, recording 00:04:11; translated by the researcher) (See **Figure 19**).

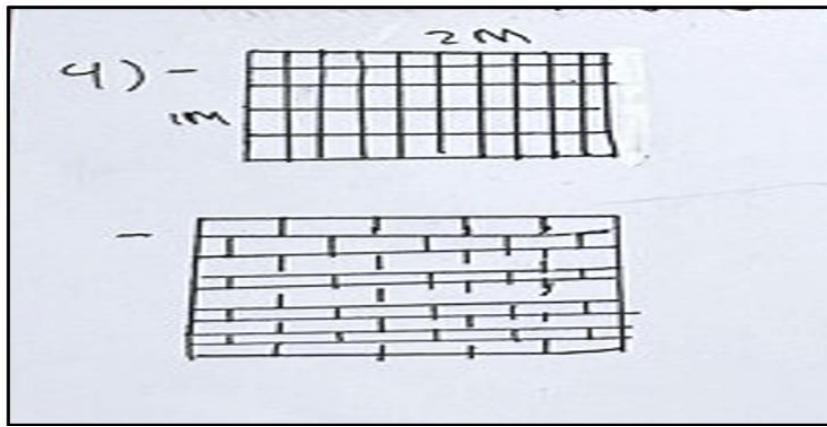


Figure 19. Results of S1 Task 4 Answers, depicting the participant's handwritten area estimation using non-standard paving units (Source: Participant S1's written response collected during the study; reproduced with consent)

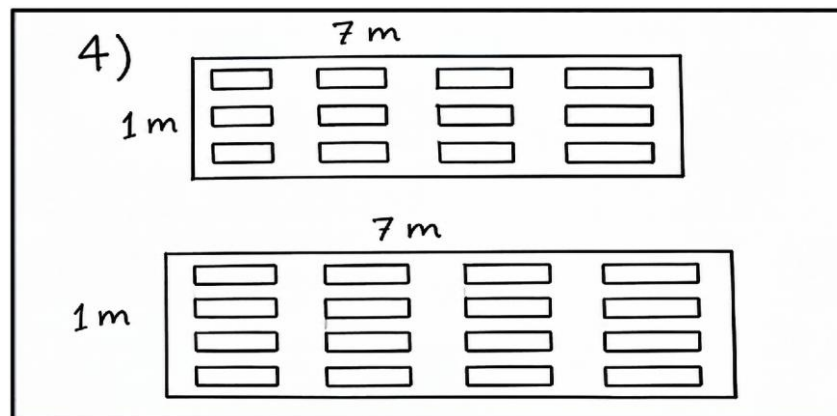


Figure 20. Results of S2 Task 4 Answers, depicting the participant's handwritten area estimation using non-standard paving units (Source: Participant S1's written response collected during the study; reproduced with consent)

This spontaneously emerging conservation analogy, drawn from everyday materials, is a hallmark of Conception V: Area as a Spatially Conserved Quantity. S1 has moved beyond enumerative counting (how many tiles?) to a structural understanding of area as an invariant quantity: regardless of how the paving blocks are arranged, the total area they cover remains unchanged. This is the mathematical equivalent of Piaget's conservation of area, typically acquired only in middle childhood, and here expressed spontaneously through S1's self-generated analogy. Sari et al. (2023), with the conservation of area problem, demonstrated a qualitatively different level of proportional reasoning, confirming that spatial conservation of area is a high-level achievement even among elementary school students. The written artifacts, consisting of two accurate visual representations without a numerical answer, are entirely consistent with this profile: S1's engagement with the task was at the level of spatial structure, not at the level of unit counting. This visual representation is his way of showing an understanding of the equivalence of the two configurations without feeling the need to enumerate them.

S2: Accurate visual representation without numerical translation

S2 drew both paving configurations with considerable accuracy: the square grid was depicted as a uniform $2\text{m} \times 1\text{m}$ square matrix, and the brick pattern as a $7\text{m} \times 1\text{m}$ array with rows shifted accordingly. No numerical calculations were performed. The quality of the visual representations was significantly higher than S2's performance on the previous computational task, indicating that his geometric competence was more strongly developed in the area of visual-spatial representation than in symbolic computation (See **Figure 20**).

The absence of numerical counting has phenomenographic significance: S2 has successfully deciphered the spatial structure of both configurations but has not yet developed the procedural bridge from visual representation to quantitative counting. In Battista's (2003) framework, this places S2 at a transition between "direct visual matching" and "structured unit counting," he can see the structure but cannot yet systematically count it. This is the ZPD for Conception IV: the framework needed is not more visual exposure, but explicit instruction on how to systematize counting from visual arrays.

S3 Dimensional enumeration with linear measurement error

S3 writes: "200 cm = Paving blocks = 10; 100 cm = Paving blocks = 10" (for a square grid) and "200 cm = Paving blocks = 20; 100 cm = Paving blocks = 5" (for a brick layout). He converts the given dimensions (2m, 1m) to centimeters (200 cm, 100 cm) and then calculates the number of paving blocks along each dimension separately (See **Figure 21**).

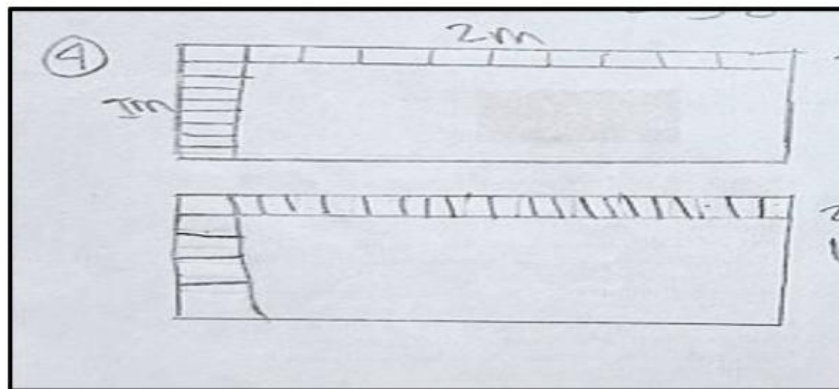


Figure 21. Results of Answers for S3 Task 4, depicting the participant's handwritten area estimation using non-standard paving units (Source: Participant S3's written response collected during the study; reproduced with consent)

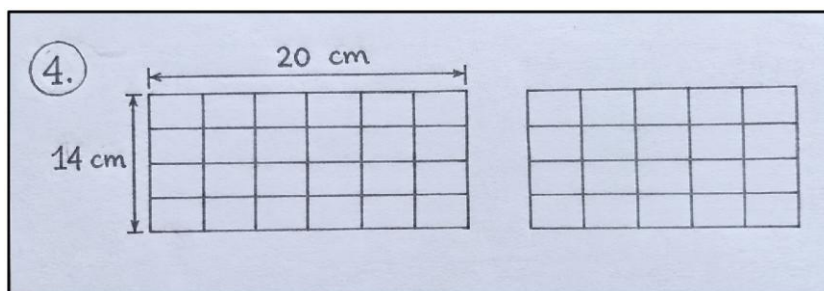


Figure 22. Results of Answers S4 Task 4, depicting the participant's handwritten area estimation using non-standard paving units (Source: Participant S4's written response collected during the study; reproduced with consent)

S3's strategy demonstrates both mathematical sophistication and a reasonable, if incomplete, extension of prior reasoning. Converting to centimeters is a legitimate procedural step that demonstrates dimensional awareness. However, the subsequent step, which treats the number of blocks along each dimension as a measure of area, applies one-dimensional logic (how many blocks fit along this length?) to a two-dimensional context (how many blocks cover this area?). This is precisely what Battista (2003) calls the "linear measurement response variations in the context of area": the yet-to-be-achieved integration of two linear dimensions multiplicatively to obtain area. The different values for the two configurations (10 and 10 for the grid vs. 20 and 5 for the brick layout) indicate that S3 is sensitive to differences in layout geometry; he is not simply copying one answer to the other. His strategy for measuring these differences is one-dimensional rather than two-dimensional a response that is partly aligned with the task (attending to configuration) while still consolidating the dimensional logic of area.

S4 Layout-sensitive enumeration with implicit structural awareness

S4 writes: "200 cm = Paving blocks = 7; 100 cm = Paving blocks = 4" (for grid) and "200 cm = Paving blocks = 6; 100 cm = Paving blocks = 4" (for brick layout). Like S3, S4 treats the two dimensions independently; unlike Citra, he gets different values for the same direction (7 blocks vs. 6 blocks along the 200 cm dimension) depending on the configuration (See **Figure 22**).

The most analytically relevant aspect of the solution to Task 4, S4 is this differential, indicating that he believes the number of blocks that can be accommodated along a given dimension is influenced by the geometric differences between a square grid and an alternating brick pattern. Although the exact number cannot be obtained from the given dimensions using conventional area calculations, this perspective is mathematically correct (the alternating pattern does change the effective packing along one dimension). S4's method is an example of "gestalt-sensitive estimation," a type of spatial reasoning in which local calculations are constrained by the global visual configuration. This is Level 2 behavior, according to Van Hiele: thinking about the relationship between layout characteristics and numerical results, rather than simply using a memorized process.

S5 Visual reproduction with emerging quantitative movement

S5 drew both paving configurations, a square grid and an elongated grid, side by side, with the annotation "1" between them (confirmed in interviews as a reference to the 1m dimension). No numerical paving calculations were given. The visual reproduction is accurate in general structure, although less detailed than the work of S1 or S2 (See **Figure 23**). The complete response profiles and geometric conceptions of all participants in Task 4 are summarized in **Table 4**.

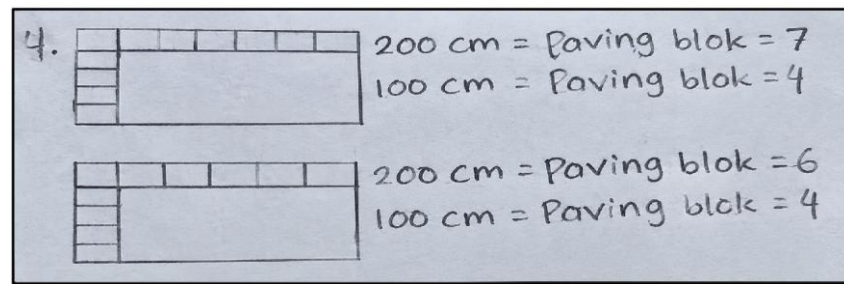


Figure 23. Results of Answers S5 Task 4, depicting the participant's handwritten area estimation using non-standard paving units (Source: Participant S5's written response collected during the study; reproduced with consent)

Table 4. Participant response profile and geometric conception in Task 4

Participant	Conception and Quality of Task 4 Responses
S1	Conception V (confirmed through interview). Two accurate visual representations; conservation analogy (clay metaphor) demonstrates highest level understanding of the area. No written numerical calculations.
S2	Conception IV (threshold). Accurate visual representation of both layouts; no numerical translation. ZPD: systematic enumeration of visual arrangements.
S3	Conception IV (active but limited). Correct dimension conversion; linear measurement response variations: sides are calculated independently and not multiplied. Different strategies per layout (geometry sensitivity).
S4	Conception IV (layout sensitive). Correctly distinguishes the number of blocks in various configurations; estimation is sensitive to gestalt; dimension integration is incomplete.
S5	Conception IV (emerging). Visual reproduction is accurate; quantitative gestures (annotation '1') indicate dimensional awareness; systematic enumeration is not yet accessible.

Table 5. Conceptual maps across four tasks phenomenographic outcome space per participant

Task	S1	S2	S3	S4	S5
Task 1	L2 Van Hiele: spontaneous classification scheme; congruence + multiplicity	L1: numerical peripheral focus; no internal decomposition	L1-2: The perimeter of the triangle is calculated heuristically.	L2: bimodal representation; fluency (variance value 25)	L1 (visual reproduction only);
Task 2	Conception I + cm ² dimensional response variations; explicit additive verbalization	Prejudice: task incongruity; self-selected form	Conception I in transition: self-correction (grades 18 & 19)	Conception I→II: fluency (producing variants); relational prefiguration	affective self-evaluation (ZPD tension);
Task 3	Conception III: dual pathways, metalinguistic paradox markers, relational awareness	Conception II: qualitative conclusion is correct; calculations are inconsistent.	Conception III: dimensional compensation; the most systematic structure	Conception III: visual-quantitative separation; L3 Van Hiele; 2r formula	Conception II (boundary exploration, tautological justification)
Task 4	Conception V: conservation analogy (clay); highest level in yield space.	Conception IV threshold: accurate visual; no numerical translation.	Conception IV: linear measurement response variations; layout-sensitive differentiation	Conception IV-: sensitive to gestalt; distinguishes layout; no multiplicative integration	Conception IV emerges (visual reproduction + quantitative gesture '1')

S5's response to Task 4 is the most numerically parsimonious in the dataset, yet the annotation "1" reveals that she is not entirely detached from the quantitative dimension of the task she has identified and marks the dimension 1m as the relevant parameter. This is a "quantitative gesture": a recognition that number is important in this context, without the procedural resources to operationalize that recognition into a count. In Conception IV terms, S5 is at a very early stage: she recognizes that paving blocks must be counted and that dimension is relevant, but she is not yet able to systematically perform the counting. The transition to Conception IV requires, for S5, structured practice with structured enumeration of visual arrays, starting with small grids where each cell can be counted, and gradually building to the conclusion that dimension determines number. A complete overview of the participants' conceptual patterns across all four tasks is presented in **Table 5**.

Cross-task Synthesis: Creative Trajectories as a Reflection of Conception

Mapping each student's responses across the four tasks reveals qualitatively distinct creative trajectories, individual cognitive paths through the open-ended task space that reflect and refine the conceptual orientations documented in the outcome space. These trajectories are not simply developmental sequences (from lower to higher conceptions) but dynamic patterns in which the same student may exhibit different conceptions depending on the cognitive demands of a particular task context. This intra-individual variability is a hallmark of phenomenographic data (Åkerlind, 2005) and has profound implications for assessment and instruction.

S1 showed the most progressive vertical developmental trajectory: he started from Conception I on Task 2 (additive side counting with dimension confusion) and reached Conception V on Task 4 (spatial area conservation). This development was non-linear; dimension confusion on Task 2 did not predict relational sophistication on Task 3 or conservation understanding on Task 4. S1's trajectory followed a visual-analogical pathway: he built understanding through representational actions (drawing, annotating) and analogical connections (clay metaphor, paradoxical marker) rather than through procedural mastery. This

suggests that visual and analogical scaffolding, rather than procedural practice, was the instructional modality most likely to advance his conceptual development.

S4 demonstrates a relational trajectory grounded in number: from bimodal representation and variant generation in Task 1 to the attainment of Conception III in Task 3, his thinking consistently moves towards relational awareness through numerical exploration. His spontaneous generation of additional values (25 in Task 2; counting different blocks in Task 4) is a consistent marker of mathematical fluency that, when combined with his verbal justification capacity (Task 3), produces a profile of creative mathematical thinking that is analytically grounded and interpersonally communicable.

S3 demonstrated a systematic-analytical creativity trajectory: on each task, he generated two representations before providing an answer, and his justifications were the most explicit and reasoned compared to other participants. His emergence into Conception III on Task 3 was through dimensional compensatory reasoning, a completely self-constructed structural thinking that demonstrates that analytical persistence (refusal to provide a single answer without proof) can yield genuine mathematical insight. The linear measurement response variations on Task 4 is a local limitation in an analytically sophisticated profile: it indicates a specific conceptual gap (as broad as multiplication vs. addition) that can be efficiently addressed by targeted instruction.

S2 and S5 both showed trajectories characterized by contextual dependence: their conceptual engagement was highest when the task provided strong visual support, and weakest when the task demanded independent representational construction. S2's "conceptual islands" of correct qualitative understanding in Task 3. This, coupled with the task mismatch in Task 2, suggests that her geometric knowledge is unevenly distributed across the conceptual landscape, with islands of understanding embedded within a broader region of procedural uncertainty. S5's trajectory is perhaps the most complex and instructive: her affective self-evaluation in Task 2, her boundary tracing in Task 3, and her quantitative gestures in Task 4 all signal a learner keenly aware of the gap between what she intuitively understands and what she can express mathematically in a form of productive conceptual dissonance that well-designed support can transform into accelerated conceptual development.

DISCUSSION

Consistent with broader research on geometric measurement cognition, the five concepts found in the resulting space form a logical developmental path. Both Conceptions I and IV demonstrate what Battista (2003) calls "additive" conceptions of measurement, where students approach measurement tasks more like counting or adding activities than as interactions with continuous or spatially organized quantities. These results confirm that most elementary-aged students function at the visualization or analysis level, consistent with the Van Hiele research tradition. They also support Al-Sarry and Ghadban's (2020) groundbreaking observation that students can perform measurement calculations while having emerging understandings about what they are measuring (Kozłowski et al., 2019).

It is important to acknowledge the limitations of the Van Hiele framework that we draw on descriptively above. The model has been criticized for portraying geometric understanding as a strictly linear, discrete progression, whereas empirical studies show that children's understanding often overlaps between levels and varies by geometric concept, such that the same child may reason at different levels for different shapes or tasks (Burger & Shaughnessy, 1986; Clements & Battista, 1992; Gutiérrez & Jaime, 1998). The model has also been criticized for embedding assumptions drawn from formal, adult mathematical abstraction and logic an emphasis that may not correspond to what is developmentally appropriate or valuable for young children (Clements & Battista, 1992). Aligning with this view, Smith et al. (1993) emphasize that students' naive understanding constitutes a sensible constructive process within their own experience rather than a mere cognitive flaw or misunderstanding. We therefore use Van Hiele's levels only as a descriptive point of reference for situating students' responses within the broader geometry education literature, while our primary analytic framework the five-conception phenomenographic outcome space presented below was specifically designed to capture overlapping and non-linear variation in students' understanding, rather than to assume a single ordered progression.

Qualitative progress is represented by Conceptions II and III. Students who perceive perimeter as a boundary measurement (II) have engaged with the continuous, path-like character of boundaries, going beyond counting discrete sides. Students who demonstrate Conception III, namely perimeter as an unchanging attribute, demonstrate relational awareness (Kozłowski et al., 2019). They simultaneously remember the configuration of the shape and the perimeter value, recognizing that these two features of a figure are logically independent. Research on creative mathematical thinking emphasizes analogical transfer as a sign of flexible conceptual knowledge, which is consistent with S1 and S4's spontaneous use of analogical reasoning to articulate this equivalence (Adharini & Herman, 2021).

The most structurally sophisticated idea in the outcome space is represented by the idea of V area, a spatially conserved variable that appears only in S1's answer. In line with Ramdani et al.'s (2021) finding that open (strategy-open) task performance relies heavily on general creativity and analogical verbal ability, his spontaneous use of conservation reasoning, comparing area conservation to the reshaping of clay, constitutes a sophisticated form of mathematical creativity. Niyazova et al. (2022) further explain that although mathematical creative thinking ability and creative disposition are interrelated, they represent different orientations toward problem engagement. S1's trajectory concretely models this difference: his creative disposition an impulsive desire to create explanations and analogies seems to precede and support his demonstrated creative thinking abilities. The three traditional markers of mathematical creativity fluency, flexibility, and originality are demonstrated by S1's innovative path through this challenge (Kozłowski et al., 2019).

The outcome space directly impacts formative assessment and instructional design from a pedagogical perspective. The discovery of five distinct geometric conceptions in a single class of five purposefully selected children suggests that the existing variation in geometric conceptions is likely much wider in any typical classroom of twenty to thirty students. These results raise doubts about the effectiveness of uniform procedural education across classes as a pedagogical solution to the diversity of geometric knowledge students bring to class. Instead, they support differentiated task design, diagnostic formative assessment techniques, and learning environments that highlight students' diverse ways of thinking and make them accessible for discussion. These ideas align with the Pancasila Learner Profile's emphasis on critical, creative, and collaborative thinking. Niyazova et al.'s (2022) classification of open mathematical problems, Mulyono and Hapizah's (2018) contextual learning trajectory study, and Rabab'ah's (2025) categorization of open geometric response variations all support the usefulness of open-ended task designs for exposing students' full range of mathematical thinking, supporting the need for task design that intentionally elicits qualitatively diverse conceptual engagement.

CONCLUSION

This study makes four key contributions. Open-ended geometry tasks reveal conceptual gaps that remain invisible in traditional, closed-format assessment. Such tasks encourage verbal classification, shape decomposition, and relational reasoning that foster deeper conceptual understanding. They support mathematical creativity by inviting fluency, flexibility, and originality in students' responses. And the creative trajectories students follow reflect their individual reasoning paths, underscoring the pedagogical importance of recognizing diverse solution strategies rather than a single expected route. In more detail: three significant findings emerged from this phenomenographic investigation of fifth-grade students' participation in open-ended geometry problems. First, five conceptions of perimeter and area were found that varied qualitatively across students in a single class. These conceptions ranged from procedural side counting (Conception I) to complex spatial conservation reasoning (Conception V). The existence of this range in a purposive sample of five students suggests that traditional evaluation methods may substantially underestimate the full conceptual diversity of a typical class of 50 students, although confirming the extent of this diversity across the full cohort would require a larger follow-up sample. Second, open-ended geometry tasks served as a highly effective diagnostic tool, revealing aspects of mathematical understanding typically hidden by closed-ended assessments, such as conservation reasoning, dimensional coupling, the procedure-concept gap, and qualitative conceptual beliefs prior to procedural implementation. Third, students' creative trajectories through open (strategy-open) tasks, as traced through written artifacts and clinical interview responses, can be documented and have pedagogical consequences: they suggest that elementary school mathematical creativity is a multifaceted capacity that develops along qualitatively distinct pathways, each of which implies distinct instructional opportunities.

More broadly, this study offers a methodological contribution that is not specific to the Indonesian setting: it demonstrates that a micro-level phenomenographic analysis of written artifacts, treating each student's response as a trace of their reasoning process, can be a viable way of conducting phenomenographic research in large-class settings where lengthy individual interviews for every student are impractical a constraint common to many education systems internationally, not only Indonesia. Within the Indonesian elementary context specifically, our findings additionally point to a continued reliance on procedural, formula-based teaching of geometric measurement, and suggest that task design intentionally encouraging diverse and creative mathematical thinking, together with assessment strategies that consider the full spectrum of conceptual orientations students bring to geometric measurement tasks, would benefit practitioners in this setting. Future studies should examine how conceptual development and creative mathematical engagement are influenced over time by phenomenographically grounded instructional interventions calibrated to the diverse creative trajectories identified in this study. We also see particular value in a complementary line of inquiry suggested by the review process: examining the questions students themselves raise after completing open-ended tasks of this kind, since such student-generated questions may offer a more direct window onto their interests, reasoning, and self-directed strategies than researcher-posed tasks alone.

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