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The Influence of Proof Understanding Strategies and Negative Self-concept on Undergraduate Afghan Students' Achievement in Modern Algebra

Ahmad Khalid Mowahed [1*], Naiqing Song [1], Yang Xinrong [1], Pei Changgen [1]

[1] *School of Mathematics and Statistics, Southwest University, Chongqing, CHINA*

* CORRESPONDENCE: khalid.mowahed@gmail.com

ABSTRACT

This study explored the influence of proof understanding strategies and negative self-concept on undergraduate Afghan students' achievement in modern algebra1. To examine the relationships among proof understanding strategies, negative self-concept and achievement in modern algebra1, we used structural equation modeling on data collected from three classes of students taking modern algebra in two consecutive years of 2016 and 2017. Participants of this study included 139 sophomore Afghan students, 40.29% male and 59.71% female. Conducting SEM analysis resulted the following findings: firstly, negative self-concept had significant negative influence over students' achievement in modern algebra1; secondly, proof understanding strategies had significant positive influence over students' achievement in modern algebra1; thirdly, negative self-concept had significant negative influence over proof understanding strategies; finally, the data fitted very well the SEM model, implying that proof understanding strategies and negative self-concept significantly influence students' achievement in modern algebra1.

Keywords: achievement in modern algebra, negative self-concept, proof understanding strategies

INTRODUCTION

Proof Understanding Strategies

Advanced mathematics typically consist of definition, theorem, proof (Davis & Hersh, 1981). Poofs are given in textbooks of undergraduate mathematics to explain why a mathematical result is true and how one can achieve the result following the process of mathematical reasoning. Indeed, modern algebra is an advanced mathematics course in which the definitions are solely made of a collection of axioms or self-evident statements that derive the whole body of concepts, theorems, and proofs. A typical introductory modern algebra course will start off by preliminaries such as sets, mappings, relations, and continues to discuss groups, group properties, subgroups, cyclic groups, cosets, factor groups, and a brief of rings and fields. Usually the course of action in teaching such a course is to follow the sequence of "theorem-proof-application" (Dreyfus, 1991). The frequently practiced way of sharing and discussing these complex ideas of modern algebra is that the professor of the course presents proofs of theorems to students who are observing the process. Professor of the course expects students understand the theorem and every step in its proof so that they will be able to reproduce or produce proofs for theorems and problems posed throughout the assessment process as a main contributing factor to their achievement in modern algebra.

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Teaching experience as well as literature (Conradie & Frith 2000; Porteous 1986; Rowland 2001) shows that students and their educators struggle to comprehend as they read proofs of theorems. As a supporting evidence, it has been observed that students are struggling to comprehend the statement of some theorems in modern algebra, let alone to understand its proof. In many instances of teaching theorems of group theory in the class of modern algebra, before starting to state the theorem and its proof, we usually ask students to read the theorem in at least three minutes and elaborate what they understood after reading the theorem. It has been observed that there was an 'aha' moment when we explained every part of the theorem to students when they could not grasp it at the first reading. This is not only the case in the context of Afghanistan, several other empirical researches (Alcock & Weber, 2005; Ko & Knuth 2012) also show that undergraduate mathematics students have difficulty while reading theorems and proofs and are not able to distinguish true and false claims.

Weber (2015) contends that little research has been carried out to know "how students should read proofs to foster comprehension" (p. 290). With aim finding answer to how one should read proofs and understand them in advanced mathematics, Weber (2015) carried out an extensive study of finding effective proof reading strategies for comprehending mathematical proof. His search on the reading of mathematical proof was based on the findings of some researchers who found that some students thought a mathematical claim is true when its verified by several specific examples (e.g., Harel & Soweder, 1998; Healy & Hoyles 2000; Martin & Harel 1989; Segal 2000), and some researchers found that students have difficulty in validating proof tasks because they focus mainly on the calculation aspect within a proof instead of paying attention to overall structure of the proof (Inglis & Alcock 2012; Selden & Selden 2003).

Weber (2015) adopted the proof comprehension model of Mejia-Ramos et al. (2012) to know about students' understanding of proofs. In order to construct the model, Mejia-Ramos et al. conducted interviews with nine mathematicians about "what they hoped their students would gain from the proofs that they represented in their classrooms" (Weber, 2015, p. 292). Based on the shared aspects of the responses, Mejia-Ramos et al. found seven "facets of what it means for a mathematician major to understand a proof" (ibid, 292). Weber (2015) summarizes the Mejia-Ramos et al. (2012) model as:

> [they] distinguished between *local understanding* – where the understanding can be gleaned by studying a small number of statements (perhaps a single inference) within the proof – and a *holistic understanding* based upon the ideas or methods that motivate the proof in its entirety. At a local level, understanding a proof would be comprised of (a) knowing the meaning of the terms and statements within the proof, including the meaning of the claim being proven, (b) being able to justify how new assertions in a proof followed from previous ones, and (c) identify the proof framework being used, such as a direct proof or proof by contradiction, and seeing how the assumptions and conclusions of the proof fit within this framework. [And] the holistic understanding of a proof consists of being able to (a) provide a summary of the proof that emphasizes its high level goals, (b) apply the methods of the proof in other situations to prove new theorems, (c) break the proof into its main parts or sub-proofs, and (d) apply the methods of the general proof to a specific example. (p.292)

Therefore, according to this model, the indicator of understanding a proof is, knowing the meaning terms and statements in a proof, being able to justify how new assertions followed from the previous ones, being able to identify the proof framework being used, being able to provide a summary of the proof, apply the proof method in other situations to prove a new theorem, break the statement of the proof in main parts or sub-proofs, and apply the general method of proof on a specific example. In order to extend indicators of knowing about one's holistic understanding of the proof, Weber (2015) also incorporated some other researchers' findings such as stating that students did not understand the theorem statement before reading its proof (Conradie & Frith, 2000; Selden, 2012), and mathematics students often do not justify how new assertion in a proof was derived logically from the previous statement(s) in the proof (Alcock, 2009). After conducting an extensive analysis of the theoretical and empirical studies on proof comprehension strategies in order to identify six strategies that successful students use to comprehend proofs and mathematicians' desire that students used these strategies, Weber (2015) proposed the following six effective proof reading strategies for understanding mathematical proof:

Strategy#1: understanding the theorem statement before reading its proof.

Strategy#2: trying to prove a theorem before reading its proof.

Strategey#3: considering the proof framework being used in the proof.

Strategy#4: Breaking a long proof into parts or sub-proofs.

Strategy#5: checking assertions within the proof with examples.

Strategy#6: comparing the method in the proof to one's own methods.

However, we have changed the measuring items of each strategy from a mathematician or a professor's desire to a student's use of such strategy to understand the theorem and its proof in modern algebra.

Modern algebra concepts, theorems and proofs are mainly based on set-theoretic definitions and axioms (Tall, 2013), and students' success in the subject is usually measured by giving a combination of proving and computation, it can be hypothesized that students' achievement in modern algebra is influenced by how much successful students are in solving problems that require proof in modern algebra. So our first hypothesis to be tested in this research is that proof understanding strategies positively influence students' achievement in modern algebra.

Academic Self-Concept

Self-concept (self-image) develops as a result of one's evaluation of the experiences with environment in which he/she lives (Shavelson et al. 1976). According to Möller et al. (2009), good opinions and constructive feedback of others have important role in the process of self-concept development; and among the various types of self-concept, academic self-concept is of most interest in educational contexts. Academic self-concept is generally believed to be a student's "image of themselves with respect to how she or he is perceived and valued in a mathematics learning context" and it is to be conceived as a substructure derived from beliefs structure (Ignacio et al., 2006, p. 18; McLeod, 1992). A self-concept, or the set of beliefs we hold about who we are, is believed to be a mediating variable that affects academic achievement (Marsh et al., 2005; Skaalvik & Valas, 1999). The past researches conducted by a number of researchers, (see for example, Byrne, 1996; Helmke and Van Aken, 1995; Marsh, 1990a, 1993; Marsh & Hattie, 1996; Möller & Köller, 2001a; Shavelson & Bolus, 1982; Valentine et al., 2004) has shown that academic self-concept that is mediated by motivational variables can "foster learning processes at school" (Möller et al., 2009, p. 114). Previous researches has also shown that academic self-concept has "significant impact on students' coursework selection in American high schools" (ibid, p. 114). But non-academic self-concept at elementary and middle-school levels is almost unrelated with academic achievement (Byrne, 1996). Many studies have been shown that positive mathematics self-concept has positive relationship with academic achievement (Byrne, 1996; Casey et al., 2001; Ercikan et al., 2005; Kung, 2009; Marsh et al. 2005; Marsh & Yeung, 1997; Ross, Scott, & Bruce, 2012; Sarouphim & Chartouny, 2017).

However, the influence of negative self-concept on academic achievement, such as in modern algebra, is not extensively explored as to the best knowledge of the researchers. As modern algebra instructors, we often hear from undergraduate students that modern algebra is boring, difficult to learn, has no applications in real life, concepts are very abstract, proofs are very different from high school mathematics and is less comprehensible. Though learning modern algebra would need some time and effort to be understood well, students generate positive and negative sets of beliefs toward learning modern algebra. These sets of beliefs will definitely affect their learning, for some it would be a source of joy and satisfaction, while others would be frustrated and discouraged to learn modern algebra (Tall, 2013). It has also been heard from so many students that they are not made for learning this subject, resulting either student quit the learning of the subject or be less successful in the subject, and this will definitely affect students' overall achievement in the subject. The research literature shows that having negative set of beliefs or negative self-concept about learning mathematics will prevent students to improve their mathematics performance and achievement (Chapman, 1988). Taking into consideration the affective nature of negative self-concept on mathematics achievement, it can be hypothesized that negative self-concept has influence over students' achievement in modern algebra.

Research Objectives and Hypotheses

Since the influence of proof understanding strategies and negative self-concept on students' achievement in modern algebra has not been well researched in the context of Afghanistan and in the literature, more research is needed to be carried out on the hypotheses emerged from the literature and actual phenomenon, both of which make sense to us based on our experience which is not empirically tested yet, it will be interesting to test the hypotheses of the model (see **Figure 1**) based on data collected via a set of questions from Afghan undergraduate students.

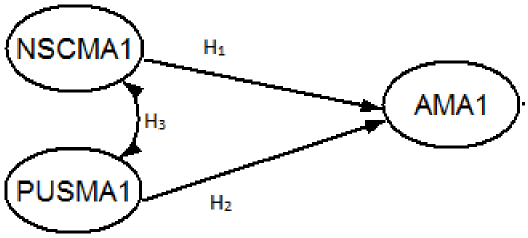

Figure 1. The hypothetical developed model to be tested with the data

In current research, we try to understand the relationship between the latent variables in our hypothetical model, so we believe that applying SEM analysis on the developed model can analyze the stated variables more accurately and reliably. Based on the discussions on the nature of the variables concerned in this study, proof understanding strategies and negative self-concept are exogenous latent variables measured by 18-item questionnaire, and final exam scores is observed endogenous variable believed to be a substantial contributor to students' achievement (endogenous latent variable) in modern algebra1.

In **Figure 1**, AMA1 stands for achievement in modern algebra1, PUSMA1 stands for proof understanding strategies in modern algebra1, NSCMA1 stands for negative self-concept in modern algebra1, Hs indicated the hypotheses to be tested about the assumed or developed model explaining the phenomenon.

By setting the model given in **Figure 1**, whether the model fits collected data was tested and the effects of proof understanding strategies and negative self-concept on students' achievement were examined in the model. The model is mainly based on the following main hypotheses:

Hypthesis#1: Proof understanding strategies positively influence undergraduate Afghan students' achievement in modern algebra.

Hypothesis#2: Negative self-concept negatively influences undergraduate Afghan students' achievement in modern algebra.

Hypothesis#3: Negative self-concept negatively influences proof understand strategies employed by undergraduate Afghan students to comprehend proofs in modern algebra.

METHOD AND MATERIALS

Research Method

The most important feature of any scientific study is to measure and relate the variables that would reveal a causality, if exists and can be showed. However, some variables such as students' scores can be directly observed and measured either by taking it from students' records or administering a test, while latent variables such as achievement, negative self-concept, and proof understanding strategies cannot be directly measured because it may have several other dimensions, such as errors in items of a survey questionnaire or a test. In such cases, according to Yilmaz et al. (2006), it is important to establish regression models that will show how endogenous structures (predicted-endogenous) are linked with exogenous structure (predictive-exogenous) and get benefit from a multivariate statistical approach that is being used frequently to combine measurement principles like structural equation modeling (Hair et al., 1998). Structural equation modeling (SEM) is a very powerful statistical technique that is used commonly in educational sciences. SEM can allow one to test a previously developed theoretical model focusing to explain the relationship between theoretical constructs embedded in the model by testing mixed hypotheses related to the models based on statistical dependence. The main objective in SEM analysis is to test a model of the relations among the variables that the researcher has in his/her mind before the research is made and conducted via data collection (Hair et al., 1998). SEM is a combination of factor analysis, regression analysis and is useful in complex relations in a theoretical structure that is represented by latent variables. SEM is of a nature of confirmatory structure, whereas other multivariate statistical techniques are of exploratory and descriptive nature. The most important aspect of SEM is that it can determine and correct measurement errors (Hox & Bechger, 1995).

Participants

Sophomore undergraduate students studying at the mathematics department of school of education of a public university, located in the north of Afghanistan, are the participants of this study. The sample of the study included 139 undergraduate Afghan students, 40.29% male and 59.71% female, who already took modern algebra1 course in their third semester of undergraduate education during the years 2016 and 2107. The participants varied in terms of socio-economic situation, majority were coming from low income families and rural areas. Most of them do not have access to good housing and computer facilities where they study and most of them cannot afford to buy textbooks and lecture notes required for their courses, some unable to borrow textbooks from vary hardly accessible public or private libraries. Most of them come to class worrying about their families living in villages under attack either by Afghan, US or Taliban forces and being under constant fear of war and bombing.

Instruments

Negative self-concept and proof understanding strategies questionnaire

Not having a particular instrument previously developed to measure negative self-concept and proof understanding strategies in the literature specifically for modern algebra1 subject in tertiary level mathematics, we adapted all the items on measuring proof understanding strategies form Weber (2015) and changed the measuring items of each strategy from a mathematician or professor desire to a student's use of such strategy to understand a theorem and its proof in modern algebra, but we tried to keep intact the overall measuring item goal in each strategy and translated from English to Farsi Dari language which is one of the official languages in Afghanistan and is also the main medium of instruction in the context. There were 14 items measuring a student's frequent use of those six categories of proof understanding strategies in the 6-point Likert scale of 1=never to 6=always. There were four items to some extent measuring a student's negative self-concept levels of agreement or disagreement with the statement of the item in the 6-point Likert scale of 1=strongly disagree to 6=strongly agree (neutrality was controlled by omitting it from both scales). In addition, we were not interested in determining how many factors are in each category, rather we preferred to find the overall influence of these two factors on students' achievement in modern algebra1.

Modern algebra achievement questionnaire

Students' final exam sores from the official records of department of mathematics was used as students' achievement in modern algebra1. The final exam assesses how much students learned in overall topics discussed in modern algebra1. The final exam usually consists up to 30 questions, of which 50% are multiple choice and the rest are questions that require written response for proving theorems, defining concepts and solving conceptual problems. The correct response for each multiple choice question is worth of one point, while other questions that require written response is worth up to 5 points each, depending on the degree of difficulty of the questions and the amount of work it requires; the distribution of scores, type of questions and the ceiling for the final exam is determined by the ministry of higher education bylaw of examinations; also a maximum of 70 points are allocated for the final exam for modern algebra1 subject. In addition, we have used the findings of the research conducted by Weber (2015) to relate the influence of proof understanding strategies on modern algebra1 achievement and Marsh et al. (2005) to relate the influence of negative self-concept on undergraduate students' achievement in modern algebra1, though there is no such explicit study conducted before on relating the influence of proof understanding strategies and negative self-concept on undergraduate students' achievement in modern algebra1.

Data Collection and Procedure

To collect data, the questionnaire was developed. The questionnaire was sent to an instructor teaching at department of mathematics to collect data from students. The objectives of research as well as the questionnaire was explained to the instructor via WeChat communication app commonly used in China. The instructor then got permission from the department and sought participants consent to respond to the questionnaire. The questionnaire was then distributed to the existing three classes of students who already took modern algebra1 course in the years of 2016 and 2017. The questionnaire was administered during one-week period, and most of the students responded to questionnaire in the class setting. Students who were absent on the day of questionnaire administration, they were asked to complete the survey questionnaire in the following week. After cross checking, few missing values was observed in the completed survey questionnaire. To deal with those missing values, either those students who did not respond to item(s) were asked to complete it or filled the missing value with the overall average score of the item(s). Finally, a request was made for the students' affaire office to provide a copy of participants' final exam scores from students' grade records. After collecting all the data via questionnaire administration and final exam scores, the data

Table 1. Mean, Standard Deviation(SD), Skewness and Kurtosis of all items

	#	Items	Mean	SD	Skewness	Kurtosis
NSCMA1	1	Modern algebra is more difficult for me compared to my other classmates	3.32	1.56	0.02	1.85
	2	My knowledge of modern algebra is not that much strong	3.78	1.51	-0.29	1.92
	3	Learning modern algebra makes me confused	3.17	1.55	0.33	2.00
	4	Modern algebra is harder for me than any other subject	3.56	1.58	-0.06	1.80
PUSMA1	5	Before reading its proof, I prefer to understand the theorem itself	4.28	1.54	-0.69	2.57
	6	Before reading its proof, I prefer to describe the theorem in my own words	4.58	1.28	-0.79	2.74
	7	Before reading its proof, I prefer to express the theorem in logical notation in order to understand what is required to be proved	4.82	1.21	-1.00	3.5
	8	I do not express the theorem in logical notation	2.86	1.61	0.32	1.91
	9	I prefer to prove the theorem before reading its proof	3.74	1.54	-0.33	2.17
	10	I prefer to prove the theorem after reading its proof	4.81	1.30	-1.06	3.56
	11	I prefer to know the hypothesis, conclusion and method of proof while reading a theorem	5.34	1.00	-1.83	6.5
	12	I prefer to know in a proof that how each new statement is derived from previous statements	5.15	1.11	-1.49	5.03
	13	I prefer to break a long proof into sub-proofs while reading it	4.66	1.29	-0.98	3.58
	14	I prefer not to break a long proof into sub-proofs while reading it	2.33	1.46	0.77	2.45
	15	I prefer to exemplify with numerical examples the confusing assertions in a proof while reading it	5.12	1.25	-1.60	5.16
	16	I prefer to see how the new assertion in a proof is a logical consequence of previous statements	4.93	1.03	-1.07	4.71
	17	While reading a proof, I prefer to prove numerically each statement of the proof	5.02	1.23	-1.40	4.58
	18	While reading a proof, I prefer to compare the utilized method of proof in the proof of theorem with the method I wanted to prove the theorem with	4.42	1.24	-0.74	3.07

were entered into Stata14 for further exploratory factor analysis and structural equation modeling (SEM) analysis.

RESULTS

The results of data analysis with Stata14 are presented as descriptive statistics of items, tests of reliability, exploratory factor analysis, test of the measurement model, and test of hypotheses. The research model given in **Figure 1** hypothesizes that proof understanding strategies and negative self-concept have influence with final exam scores as students' achievement in modern algebra1.

Descriptive Analysis

Though there is a disagreement in whether to use means, standard deviations, or other descriptive measures for Likert scale data, many researchers today accepted to calculate such measures and we are also following that path because otherwise we would be unable to understand about the phenomenon under investigation. The means, standard deviations, skewness and kurtosis of each item under its respective latent variable are given in **Table 1**.

The mean scores range between 3 and 4 for negative self-concept, indicating that the participants somewhat disagree to somewhat agree to the factor items that were measured in the study. The standard deviations in table for negative self-concept indicate that the responses were narrowly spread around the mean. The skewness and kurtosis values were not far from zero, indicating that respective distributions of each subscale do not differ substantially from a normal distribution (Tabacknick & Fidell, 2013). In addition, the mean for proof understanding strategies range between 3 to 5, indicating that participants were somewhat disagree to agree to the proof understanding strategies factor measured in the study; though the skewness for this factor were somehow close to zero, the kurtoses were located far away from zero indicated cusps in the distribution. Although not a robust method for normality, skewness and kurtosis at least can give us how close or different each item is to normality condition. Though SEM requires multi-normality that is difficult to meet with Likert scale data, this condition is usually relaxed in SEM analysis.

Structural Equation Model Analysis

A requirement in SEM analysis is the assumption that the data are multivariate normal (McDonald & Ho, 2002). Preliminary descriptive analysis identified no multivariate outliers, and the assumption of normality was never severely violated for any variable, considering the guideline of normality (i.e., skewness < 2; kurtosis < 7) as is proposed by Curran, West, and Finch (1996). In addition, the multivariate normality of data was further evaluated using Mardia's (1970) multivariate skewness and kurtosis coefficient. The value of Mardia's multivariate skewness and kurtosis coefficients obtained in our study was 62.89 ($\chi^2 = 1492.06, p < .000$) and 325.21 ($\chi^2 = 83.55, p < .000$), respectively. As suggested by Raykov and Marcoulides (2008) the value for Mardia's s multivariate skewness coefficient 62.89 was less than $p(p+2)$ = 16 (18) = 288 where p = total number of observed indicators or items = 16, so the requirement of multivariate normality was therefore satisfied and the data considered normal for SEM analysis.

Before using SEM to test the model, we first analyzed the data using principle-component factor analysis with Varimax rotation in order to make sure there are two factors measuring students' proof understanding strategies and negative self-concept. Before factor analysis, Kaiser-Meyer-Olkin and Bartlett's sphericity tests were performed to determine the adequacy of the sample and to check whether or not the data fitted for factor analysis. Kaiser-Meyer-Olkin ratio should be above 0.50 (Temel, Sen, & Yilmaz, 2015). The KMO value of 0.74 showed that the data set was appropriate for a factor analysis and the value of Bartlett's test of sphericity was $\chi^2 = 560.05$ ($df = 153$, $p < 0.000$) that indicated a high level of correlation was evident between the variables, and the determinant of the correlation matrix was 0.014, implying that the correlation matrix is positive definite. Since we were interested in two main factors of proof understanding strategies and negative self-concept, Stata14 was programed to report only two factors retained, so the items were divided by stata14 into two dimensions based on the SEM model: (1) Negative self-concept (4 items, items 1-4), (2) proof understanding strategies (14 items, items 5-18). These dimensions explained 34.13% of total variance, and the most variance was explained by the first dimension (20.75%). The critical value for inclusion of an item to a dimension was 0.40. Items #8 and #14 rotated factor loadings were lower than 0.40, so these two items were deleted. The reliability of the scale was determined by Cronbach's alpha coefficient for all items and the two dimensions, the overall 16 items reliability had acceptable value of 0.77; the reliability of negative self-concept construct had acceptable value 0.75 and the reliability for proof understanding strategies construct had acceptable value 0.79 (Cronbach, 1951).

The results found in factor analysis indicated that all item loadings found to be above the recommended cut-off point 0.40, except for item #8 and item #14 which were deleted from further analysis also the communalities of these two items were less than 0.30. Also reliability obtained for all 16 items, proof understanding strategies and negative self-concept latent variables showed that they met the suggested minimum value of 0.70 (Cronbach, 1951). Therefore, we decided to include 16 items for further SEM analysis because the previous analysis results indicated that the items in negative self-concept and proof understanding strategies were highly correlated and reliable.

To conduct SEM analysis, the hypothesized model was then evaluated using SEM in stata14 to test whether and to what extent the model fits the data. The hypothesized model is shown in **Figure 1**, the correlation matrix for negative self-concept, proof understanding strategies and final exam scores (as indicator of achievement) in modern algebra1 given in **Table 2** and the fit index is shown in **Table 3**. The model evaluation criteria, i.e. the index of assessing the extent to which a model fits analyzed data set, used to test the fit of the models included the chi-square statistic (χ^2), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), Root-Mean-Square Error of Approximation (RMSEA), Standardized Root Mean Square Residual (SRMR) and Coefficient of Determination (CD). It should be clarified that the χ^2 statistic is sensitive to sample size; therefore, alternative goodness-of-fit indices were used for the study. Values of .90 and above for CFI, and TLI were regarded as indicating a reasonable fit. The index SRMR has an acceptable level when it is less than .05 (Schumacker & Lomax, 2015). RMSEA value of .05 indicate a close fit and values below .08 indicate a fair and adequate fit (Brown & Cudeck, 1993; Hair et al., 2010). Finally, a value of CD close to 1 indicates a good fit.

Table 2. Correlation Matrix of items and Final Exam Score(FES)

items	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***9***	***10***	***11***	***12***	***13***	***15***	***16***	***17***	***18***	***FES***
1	1																
2	.48	1															
3	.47	.43	1														
4	.40	.37	.42	1													
5	-.05	.01	-.01	-.10	1												
6	-.09	-.14	-.03	.00	.38	1											
7	-.16	-.19	-.07	-.21	.28	.37	1										
9	-.07	-.15	-.07	-.11	.18	.28	.30	1									
10	-.08	-.18	-.20	-.18	.29	.16	.29	.22	1								
11	-.05	-.19	-.03	-.13	.00	.16	.29	.27	.16	1							
12	-.04	-.05	-.10	-.20	.20	.22	.37	.29	.26	.47	1						
13	.03	-.01	.16	.11	.20	.14	.36	.10	.23	.23	.30	1					
15	-.12	-.22	-.06	-.03	.02	.05	.28	.14	.23	.38	.40	.29	1				
16	-.09	-.04	-.03	-.18	.19	.31	.22	.07	.14	.27	.26	.08	.17	1			
17	-.09	-.10	-.01	-.07	.13	.18	.30	.38	.23	.35	.33	.21	.43	.22	1		
18	-.07	-.11	-.04	-.07	.17	.19	.23	.26	.26	.31	.15	.17	.30	.26	.43	1	
FES	-.31	-.21	-.32	-.14	.14	.10	.11	.02	.34	.22	.19	.11	.17	.15	.12	.15	1

Table 3. Goodness-of-fit indices for hypothesized model

Model fit indices	***Values***	***Acceptable range***
χ^2	177.19 (df = 117, p = .000)	p > .05
RMSEA(90% CI)	.061 (.042, .078)	<.08(adequate fit)
CFI	.88	≥ .90
TLI	.85	≥ .90
SRMR	.07	< .08
CD	.95	Close to 1.00

Table 4. Model Hypotheses Paths' Coefficients and Significance level

Path	***Path Coefficient***	***T-statistic***
NSCMA1 → AMA1	-3.4	-3.72**
PUSMA1 → AMA1	1	2.43*
NSCMA1 ↔ PUSMA1	-.69	-2.37*

* p < .05 ** p < .01

Looking at correlations in **Table 2**, it seems that the relationship between negative self-concept (items1-4) with proof understanding strategies and final exam scores is negative, which in turn implies that if students have negative self-concept while learning modern algebra1, it can substantially influence negatively their proof understanding strategies as well as their achievement. In addition, it can also be seen that employing proof understanding strategies, while learning abstract algebra, can positively influence students' achievement in modern algebra1. From **Table 3**, we can conclude that the established structural equation model gave results compatible with hypotheses of the study; and the date fits the model, implying that the model adequately explains the phenomenon.

The path coefficients and *T-statistic* for each of the hypothesized relationships in the model are given in **Table 4**. It was observed that employing proof understanding strategies while learning modern algebra1 can positively influence students' achievement in modern algebra1 ($\beta = 1, p < .05$), whereas having negative self-concept while learning modern algebra1 can negatively influence students' achievement ($\beta = -3.4, p < .01$), and having negative self-concept can have negative influence while employing proof understanding strategies to learn modern algebra1 ($\beta_{cov} = -.69, p < .05$).

DISCUSSION AND CONCLUSION

This study examined the influence of employing proof understanding strategies and negative self-concept on undergraduate Afghan students' achievement in modern algebra1. Searching for affecting factors on modern algebra1 is an important issue in undergraduate mathematics education, where the focus is to make the subject meaningful and understandable for students. Though underlying influence of these variables on modern algebra achievement may not be well established in current study as well as in the literature, the results of it can be a starting point in finding candidates of influential variables such as negative self-concept as well as proof understanding strategies on modern algebra achievement among the vast space of influential variables domain influencing students' achievement in modern algebra. On the other hand, testing the model once and with the small sample size and the scarcity of the literature on the topic cannot allow to draw solid conclusions, but relying on the results of current analysis, one may consider to open a discussion about such potential variables so that students can employ proof understanding strategies and reduce negative self-concept in order to increase their achievement in modern algebra. To support the argument, the current study confirmed the hypotheses regarding the influence employing proof understand strategies and having negative self-concept on modern algebra achievement by developing the model, fitting data into the model, examining the goodness-of-fit indices by SEM statistical analysis, and confirming each path in the model to be correctly based on SEM results, confirming the model fits the reality hypothesized in current study. Jöreskog and Sorbom (1993) suggest that an appropriate model has a small *Mean Square Errors of Approximation* (RMSEA). The results of present study given in **Table 3** suggest that the model adequately fits the data. In fact, the results for the path between negative self-concept and achievement clearly suggest that this path coefficient is stronger than the weak relationship (0.03 on a scale of 0 to 1) between mathematics achievement and self-concept found by Wang (2007) when studying the trend of self-concept and mathematics achievement in a cross-cultural context. In addition, Weber (2015) believes that "if students could be led to apply these strategies effectively, the comprehension of the proofs that they read might be improved" (p. 307).

Thus, the results of this study confirmed that (1) employing proof understanding strategies significantly and positively influenced undergraduate students' final exam scores (regarded as an indicator of achievement) in modern algebra, (2) having negative self-concept significantly and negatively influenced undergraduate students' final exam scores in modern algebra1 and (3) having negative self-concept significantly and negatively influenced undergraduate students' employment of proof understanding strategies. The implications for learners is that they should employ these proof understanding strategies in order to better understand the proofs in modern algebra, and at the same time, they should try to not to allow negative feelings come into their mind while trying to make meaning of concepts, theorems and proofs in modern algebra.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Ahmad Khalid Mowahed – School of Mathematics and Statistics, Southwest University, Chongqing, China.

Naiqing Song – School of Mathematics and Statistics, Southwest University, Chongqing, China.

Yang Xinrong – School of Mathematics and Statistics, Southwest University, Chongqing, China.

Pei Changgen – School of Mathematics and Statistics, Southwest University, Chongqing, China.

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