International Electronic Journal of Mathematics Education
2022, 17(3), em0696
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article

OPEN ACCESS

The impact of prerequisites for undergraduate calculus I performance

Zachariah Benton Hurdle [1*], Wiktor Mogilski [1]

[1] Utah Valley University, Orem, UT, USA
*Corresponding Author: ZHurdle@uvu.edu

Citation: Hurdle, Z. B., & Mogilski, W. (2022). The impact of prerequisites for undergraduate calculus I performance. *International Electronic Journal of Mathematics Education, 17*(3), em0696. https://doi.org/10.29333/iejme/12146

ARTICLE INFO

Received: 23 Mar. 2022
Accepted: 24 May 2022

ABSTRACT

We conducted a quantitative analysis to determine how the prerequisite path of students taking calculus I impacts their grade performance. We began by investigating the performance of students that took college algebra and trigonometry versus those that took pre-calculus ahead of their credit-bearing calculus I attempt. We concluded that there was a significant difference between the two prerequisite routes. We then performed regression analysis to view the number of credit prerequisite credit hours, including multiple attempts, as a predictor of calculus I GPA and A-proportion. We found a strong negative correlation between these variables. We hope this study can be replicated at other institutions and in other fields to help university policymakers with decision-making regarding course listings.

Keywords: calculus, prerequisite, college mathematics, mathematics performance

INTRODUCTION

Calculus I has often been discussed as a "gateway" course in many recent calculus education research studies (Akbuga et al., 2019; Bressoud, 2014; Bressoud et al., 2013; Wu et al., 2018). By definition, gateway courses prohibit students from successfully completing their primary degree plan (Bloemer et al., 2017). Retention in such a course is then problematic at institutions around the world (Lee et al., 2018). For example, the national passing rate (A, B, or C) of calculus I in the United States was recently found to be only 73%, according to recent studies (Bressoud, 2015), and 12.5% of STEM-seeking students actually change majors entirely (Rasmussen & Ellis, 2013). This is not just a United States issue, but one that has also been documented internationally (Bressoud, 2015; Easey & Gleeson, 2016; Hurdle et al., [under review]).

> "With a growing concern over unproductive rates in mathematics courses and with the widespread effect that mathematics has on other disciplines, it is incredibly important to work to improve student success in mathematics" (Bertrand et al., 2019).

Because of the obvious significance of promoting student success in calculus I, we intend to look at one particular factor in sufficiently preparing students for calculus I. Course readiness and preparedness can at least partially be attributed to the prerequisite pathway students took on their way to taking a subsequent course, along with other factors such as high school GPA and attitude (Eagan et al., 2010; Pyzdrowski et al., 2012). For calculus I, most typical degree plans require either a combination of college algebra and trigonometry, a single iteration of pre-calculus (typically more than a 3-credit hour course), or some testing out of these requirements through placement exams. Many department resources can be spent on these pathways, considering the higher enrollment in STEM degree plans yet decreasing numbers of graduates go on to fill positions in STEM fields (Bergeron & Gordon, 2017).

Calculus I makes a consistent appearance in all STEM-based degree plans. For smaller universities, stretching funds to cover two different paths toward the same ultimate course could require justification that one path perhaps leads to better performance in calculus I than the other. At minimum, the results can be used to determine the cost effectiveness of these pathways. This leads to the following research questions that we intend to explore:

1. **R1:** To what extent is calculus I performance different between students that take college algebra and trigonometry as prerequisites compared to those that take pre-calculus as a prerequisite?
2. **R2:** To what extent does the amount of calculus-related prerequisite credit hours taken by students predict and affect their eventual overall performance in calculus I?

LITERATURE REVIEW

Calculus education is a fairly recent field of research, yet we still have many important findings that have shaped the literature.

> "Since calculus is often seen to be highly symbolic in nature, students often try to get through a first course in calculus by manipulating the symbols without developing a real understanding of the meaning of the symbols" (Berry & Nyman, 2003).

As a result, over the calculus I semester, student attitude toward mathematics tends to decrease (Sonnert et al., 2014). When demographics are more carefully analyzed, however, there are troubling trends. For example, females are 1.5 times more likely than males to change majors due to the calculus sequence (Ellis et al., 2016), and this has resulted in a vast underrepresentation of females in STEM fields (Hagman et al., 2017). On the other hand, female students have been shown to improve performance more than males as calculus I course start times progress later into the day (Hurdle et al., [under review]). Additionally, Latinos and African-Americans have, in the past, earned fewer degrees than other racial groups, and can further be traced specifically to difficulties in retention in STEM (Moreno & Muller, 1999; Riegle-Crumb et al., 2011). When trends are acknowledged for students altogether, Bressoud (2014) summarizes that 31% of students that take a calculus course in high school revert back to pre-calculus in college, and that 13.5% of students that take a calculus course in high school revert even farther to remedial mathematics courses in college.

Beyond demographics, other studies have shown various impactful factors toward successful performance in the calculus sequence. Instructors share their own impact toward successful calculus completion (Bressoud, 2014; Sonnert & Sadler, 2014).

> "The instruction received by calculus students impacts their attitudes through the decisions that their instructor makes about teaching approach (e.g., lecture, discussion, use of technology) and time spent on different aspects of the course (e.g., class time, homework, project work)" (Sonnert et al., 2014, p. 372).

In general, there is a significant relationship between student attitude and teacher attitude when it comes to mathematics courses (Mensah et al., 2013). Of course, there are aspects that instructors typically cannot control, such as the class size. However, studies have shown in some disciplines that class size has no significant impact on student performance, but this can vary depending on the course (Jarvis, 2000). One particularly interesting finding is that

> "the impact of class size upon student achievement decreases as the class size increases" (Gleason, 2010).

Even course start times could have an impact on performance, depending on the research report. For example, Pope (2016) explored

> "results [that] tend to show that students are more productive earlier in the school day, especially in math" (p. 10)

in a K-12 setting (clearly different than undergraduate contexts). However, Carrell et al. (2011) found that earlier course times typically affected students negatively at the university level, but yet this was not a mathematics-focused study; it was later confirmed by Hurdle et al. (in preparation) that students taking calculus I in the spring semester performed better than those that took calculus I in the fall semester, and also that 8:00 calculus I courses tend to lead toward poorer student performance. Some information about college algebra, trigonometry, and pre-calculus has also been individually researched. For example, Sonnert and Sadler (2014) found that students who take pre-calculus do not necessarily perform better in calculus courses. College algebra, when considered as an entry-level course, has been found to have a lower success rate than other freshman courses (Herriott & Dunbar, 2009). Additionally, some researchers have even argued that current trigonometric teaching methods and ideas are not sufficient in preparing for calculus knowledge in the first place, from a curriculum perspective (Orhun, 2004).

The above summary is a fairly large amount of information exploring various factors that impact calculus performance at the university level. However, many of the factors have conflicting conclusions in the literature regarding their specific impact on student performance. Further, many of those factors are difficult to control. For example, some students require certain course times to make room for other limited sections of courses. Class sizes could be decided at a department, college, or even university level over the instructor in charge. However, degree plans at institutions are fairly set, particularly regarding the calculus sequence and what prerequisites are required in order to progress through it. This makes prerequisites an important factor to discuss, one that those in academia can consider when making advising decisions based on the results we will share. Research shows that students who are ready to progress through pre-calculus courses and/or calculus courses in their high school curriculum tend to view mathematics more positively than other students (Wilkins & Ma, 2003). Prerequisites, then, are another factor that should be explored for impact on performance, particularly in early gateway courses (here, calculus I). Prerequisites are defined as

> "the means by which a student can satisfy the institution's requirement to enroll in a course. Prerequisites could include developmental course work, transfer credit from another institution, or a minimum score on a placement test" (Soria & Mumpower, 2012, p. 30).

We use this definition as we progress through this paper to determine the paths that students in our sample took on their way to their placement into a final, credit-bearing calculus I course. Not only can we see if certain course placements have any varying effect on performance in calculus I, but we can also see if having a regression model of course credit quantities can predict performance as well.

Further, we look at Vygotsky's idea of scaffolding to form a basis for the framework of our research on this topic. Vygotsky defined this term, scaffolding, as a

> "role of teachers and others in supporting the learner's development and providing support structures to get to that next stage or level" (Raymond, 2000, p. 176).

While much of this process resides in the student's cognitive development and/or metacognitive structures, we also can claim that prerequisites by nature are an institution's attempt at layering information and knowledge in a way, through an organized degree program, that scaffolds the same student's learning and understanding.

> "In scaffolding instruction a more knowledgeable other provides scaffolds or supports to facilitate the learner's development" (Van Der Stuyf, 2012, p. 2),

and so we believe that a correct use of the prerequisite model would imply a positive impact on overall student performance in the subject at the end of the sequence of prerequisites. It is established that students with higher prior achievement in related courses benefit more in future courses than those with lower prior achievement, as

> "students with higher levels of prior knowledge have a higher chance to identify relevant information, to connect this information to existing schemata, and to integrate new information more easily into their existing knowledge structure in long-term memory" (Kollar et al., 2014, p. 25).

This provides evidence that the structure of degree programs leading to calculus I can be a beneficial measure worth studying, given the potential impact on student success.

METHODOLOGY

Our data set originally included 1,075 data points, including every student that had taken calculus I between the years of 2009 and 2019 at a small public regional university in the southeastern portion of the United States. Some of those data points consisted of repeated iterations of students taking calculus I more than one time, which is both understandable and common; those students would show up in the list as many times as they took the course, so these repeats needed to be removed in the data cleaning process for the *first* research question. Additionally, after categorizing those that took the appropriate prerequisites in the first place, this left 526 data points to consider. We decided, for the purpose of data extraction on the *second* research question, to only include the calculus I grade that was earned in their final attempt as the true output variable, while considering previous attempts to be prerequisites, leaving 949 data points to consider. To fully answer the second research question in this study, it was still important to track previous attempts toward calculus I at the undergraduate level in certain instances, rather than omitting them entirely like we did for the first research question.

The information we obtained on performance was limited to a letter grade scale in calculus I, but the raw data also tracked any college algebra, trigonometry, or pre-calculus courses (along with final letter grade) that each student proceeded through before the calculus sequence, whether the course was taken at the university or as credit transferred in from another institution. This meant that whether a student received an 80 or an 89 overall in any class, for example, our records would only show a B, or a 2.0; there was no plus-minus scale at this university. We would have preferred numerical representations of grades, but this is how the information was collected and stored for a decade. We further filtered the data so that students could be classified as taking a college algebra/trigonometry route (n=404) or a pre-calculus route (n=122), otherwise they were considered exceptions to the prerequisite rule for various reasons, as defined by those categories. We also summarized the data through the total amount of credit hours of relevant prerequisites (defined here as college algebra, trigonometry, pre-calculus, calculus I, and any subsequent combination) that each student took. We only had the final credit-bearing record of any prerequisites, but also had record of all potential retakes of calculus I for each student. We keep in mind that those few students who took only college algebra or only trigonometry at the undergraduate level more than likely gained credit for the other through a high school dual credit model. More analysis about using this information will be provided later as we address each research question individually.

First Research Question

To begin investigating the first research question, we first simply calculated the mean calculus I GPA score for students that took pre-calculus before calculus I (n=122) and calculated the mean the calculus I GPA scores for students that took college algebra and/or trigonometry before calculus I (n=404). **Figure 1** shows how GPA scores for each were distributed through relative frequency. Recall: A=4, B=3, C=2, D=1, and F/W=0.

Figure 1 seemed to immediately indicate that those who took pre-calculus as a prerequisite had a higher performance in calculus I than the other group. Certainly, the failing rate appeared lower in students with the pre-calculus background. Beyond the visual provided in the histogram, pre-calculus students had a mean GPA of 2.303 in calculus I with a standard deviation of 1.291, and college algebra/trigonometry students had a mean GPA of 2.134 in calculus I with a standard deviation of 1.383. Please recall, those who bypassed prerequisites (for any of a variety of reasons) were not included in this portion of the study. Conversely, there were rare occasional instances of students who took all three prerequisites. However, the question remained if the evidently higher mean was statistically significant.

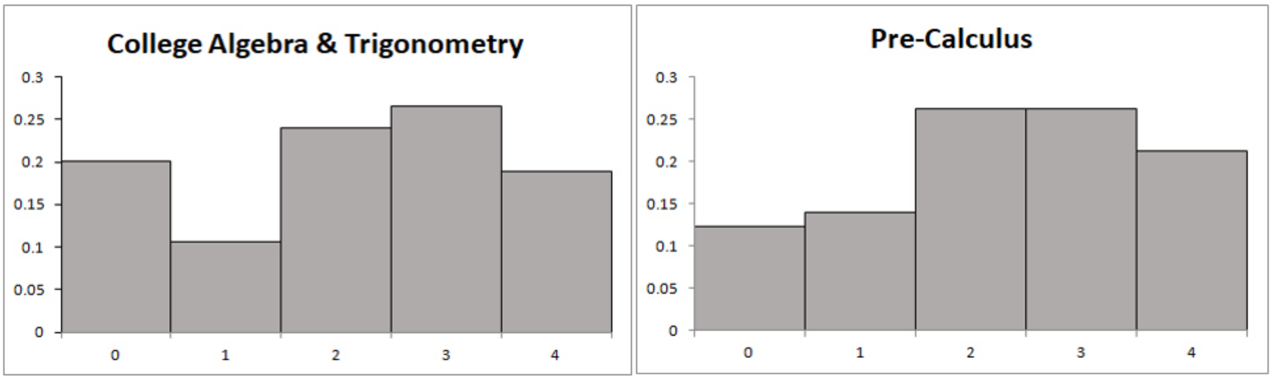

Figure 1. Calculus I performance frequency categorized by prerequisite choice

Table 1. Total number of credit hours before final calculus I attempt

Credit hours	0	3	4	5	6	7	8	9	10	11	12	13	14
Number of students	219	178	5	76	322	13	52	3	9	69	1	1	1

We knew that a two-proportion one-tailed z-test would be the most appropriate considering the data shown in the relative frequency graphs in **Figure 1**. First, we considered the z-test in the context of fail rates, where we tested the proportion of college algebra/trigonometry students that earned an F in calculus I against the proportion of pre-calculus students that earned an F in calculus I (called the F-proportion). We applied the following equation:

$$z = \frac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

Here, $\hat{p}$ denotes the pooled population fail rate proportion; n_1 and n_2, are the sample sizes of college agebra/trigonometry students and pre-calculus students, respectively; the numerator is just the difference of sample fail rate proportions. Our null hypothesis was that the sample proportions would be the same, regardless of prerequisite pathway, while the alternative hypothesis was that college algebra/trigonometry students have a higher fail rate than those taking the pre-calculus pathway. We first computed the F-proportion for college algebra/trigonometry and pre-calculus, respectively: $\hat{p}_1$ =81/404 $\approx$ 0.200 and $\hat{p}_2$=15/122$\approx$ 0.130. The associated p-value for the z-test was 0.026. Fortunately, this means with 95% confidence that the data supports a statistically significant concrete answer to our first research question, so we now claim that if a student takes college algebra and trigonometry as their prerequisites, this could indicate a higher probability of failing in their attempt at calculus I.

Second Research Question

For the second research question, we first looked at a pure count of how many credit hours students had on their path to calculus I. However, this was limited to instances of college algebra, trigonometry, pre-calculus, or calculus I, as defined experience; no other math courses were considered. For example, we believe that a student who had taken calculus I before, and decided to take it again, could have an advantage the next time they enroll, and call this an infrequent prerequisite possibility. Other students could have taken some of the required prerequisites, but still felt inadequately prepared so they took extra courses (thus the occasional appearance of a student that took both prerequisite routes). Regarding prerequisites, the raw data that we obtained only showed course credit for the classes that counted on transcripts. So, for example, if a student failed college algebra and then later passed it, we would only see the passing score. With calculus I, however, we saw all instances of taking that course for each student. This was simply a result of how the data was tracked and reported. We performed a regression analysis on this described data.

We began by running a credit count on all 949 relevant students to see how many calculus-related credit hours students were taking prior to calculus I. By definition, this included all transfer and in-house college algebra, trigonometry, and/or pre-calculus information that counts toward GPA, as well as any previous calculus I attempts (passing or not passing). The results of this running tally are shown in the frequency table (**Table 1**). Note that here we omitted credit hour counts with no students (1, 2, and 15+ credit hours).

Next, we decided that a few of these credit-hour groups did not have enough students to be considered a reliable data set. We decided we should only focus on credit-hour groups that had at least 30 students involved, to assume normal distribution for upcoming statistical tests. This limited us to credit-hour groups of 0, 3, 5, 6, 7, 8, and 11. We then averaged the GPA score of each group, and the results are shown below in Table 2, along with the proportion of those groups who received an A in calculus I, which we call the A-proportion.

We graphed the results from **Table 2** to determine any regression. The value of x=0 (that is, those that took no prerequisites before taking calculus I) immediately appeared as an outlier to the general trend. We interpreted that this would make sense, because in order to take calculus I without the required prerequisites, there would need to be a testing out process or some other justification for strong performance leading to immediate placement into calculus I. We decided to take x=0 out of the data set because we thought it would skew relevant results for all other students, and the resulting scatter plots are shown below in **Figure 2**.

Table 2. Average calculus score, and A-proportion, for select credit-hour groups

Credit hours	0	3	5	6	8	11
Mean calculus score	2.644	2.247	2.303	2.174	2.135	1.812
A-proportion	0.365	0.275	0.224	0.211	0.173	0.072

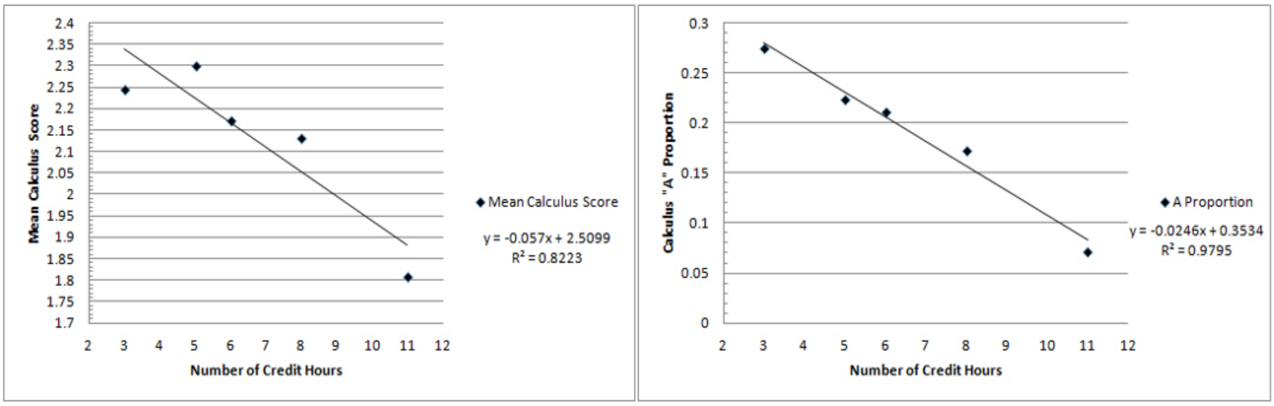

Figure 2. Regression model for prerequisite hours vs calculus performance

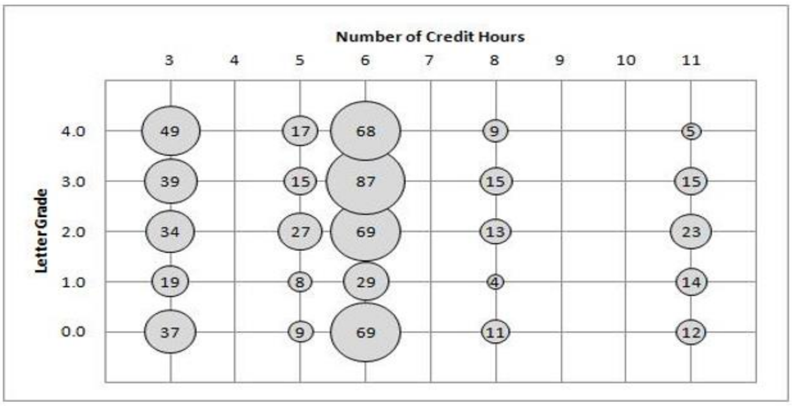

Figure 3. Distribution of calculus I scores per select credit-hour groups

The regression line that results in the first graph shows a downward trend, and thus a negative correlation exists between the number of prerequisite mathematics credit hours and final credit score in calculus I. With a correlation coefficient of R=0.907, this indicates a very strong correlation, with a p-value of 0.0337. The regression line for the second graph also shows a strong negative correlation with a correlation coefficient of R=0.953 and a p-value of 0.0013. Moreover, our correlation coefficients are both much higher than the Pearson correlation threshold of 0.811 that corresponds to df=4 at 95% confidence, further indicating that the results are indeed statistically significant (98% for the first graph, and even 99% for the second graph). Our interpretation from this was the following: because students with no prerequisites were removed from this model, the y-intercept here indicates to us that for students needing some amount of prerequisites before entering calculus I, their starting value was a C+, with a slight decrease in GPA per credit hour necessary before their official calculus I attempt. Such a model could be a rough estimator for expected GPA in calculus I given a known number of math-related prerequisites taken beforehand (including additional attempts to pass calculus I). Also, the portion of students that were able to earn an A in calculus I diminished as the number of prerequisite credits taken ahead of time increased, which follows a similar idea as before.

For the curious reader, we include **Figure 3** to showcase how GPA scores were distributed per number of credit hours taken. Note that the bubbles represent the number of students, with larger bubbles representing a larger number of students in that group.

To further justify the statistical relevance of our results, we ran a two-tailed Welch t-test comparing the Calculus I mean scores between the various credit hour groups. One particular finding stood out from the rest: the t-test found a statistically significant difference of means between those students that took a low number of credit hours versus those that took a high number of credit hours; that is, repeating courses or not before the final calculus I attempt. The p-values of these statistically significant two-tailed t-tests are: 0.0170 for 3 credits vs. 11 credits, 0.0168 for 5 credits vs. 11 credits, and 0.0279 for 6 credits vs. 11 credits. Note that 3, 5, and 6 credits are the lowest amount possible for the appropriate prerequisite routes, and 11 credits is the highest in which we had obtained a large sample size. These credit counts could easily represent the traditional paths of college algebra and/or trigonometry, or pre-calculus, where 3 indicates college algebra or trigonometry, 5 represents pre-calculus, and 6 indicates a college algebra and trigonometry combination; this is obviously not a set standard for every case but is still a reasonably assumed relation. With a one-tailed test, the results become even more significant. Therefore, with 99% confidence we can conclude that students who took a low amount of prerequisite credit hours will have a better chance of passing calculus I than those taking a large amount of prerequisite credit hours. Thus, we can conclude that students who took fewer prerequisite credit hours fared better than those taking larger amounts of those same course credit hours.

RESULTS

We begin by summarizing the results of the tests used to address our first research question. The pre-calculus group had a mean GPA of 2.303 in calculus I and the college algebra/trigonometry group had a mean GPA of 2.134. This led us to hypothesize that students taking pre-calculus fared better in calculus I versus those taking college algebra and/or trigonometry. To test our hypothesis, we first applied a one-tailed z-test, with the null hypothesis assuming no difference in the fail rate proportions. This test resulted in a p-value of 0.026, so we were able to reject the null, and students from pre-calculus had lower fail rates in calculus I than college algebra/trigonometry students.

For the second research question, we began by performing a regression analysis with the number of prerequisite credit hours taken against mean calculus I GPA and A-proportion. The results of the regression analysis were summarized in **Figure 2**. There was a strong negative correlation in both instances, with very high correlation coefficients. With 98% confidence, we can say that the more prerequisite credits taken before calculus I, the lower the expected GPA, and with 99% confidence, the more prerequisite credits taken before calculus I, the lower the A-proportion in calculus I.

3To reaffirm the conclusion of the regression analysis, we performed a Welch t-test comparing the calculus I GPA means between the various prerequisite credit hour groups. Again, we assumed the null hypothesis that the population means are the same. We found highly statistically significant results in comparing the following credit hour groups: 3 vs. 11, 5 vs. 11, and 6 vs. 11. That is, students that took 3, 5, or 6 prerequisite credit hours fared much better than those that took 11 prerequisite credit hours.

CONCLUSIONS, LIMITATIONS AND IMPLICATIONS

Our data was statistically significant enough to provide an answer to our first research question. The fail rate for students who took pre-calculus is significantly lower from those that took college algebra and/or trigonometry. One explanation could be that pre-calculus students are more driven and are on a more definitive path toward a science-heavy degree. Another could be the fact that pre-calculus courses usually meet every day, compared to block scheduling of college algebra and/or trigonometry. There could be other factors that separate those students who would select to take the pre-calculus route, and these could be the foundation of future studies. We were able to provide a statistically significant concrete answer to our second research question as well. We observed a downward trend: the more prerequisite credit hours that a student took prior to calculus I, within the category we define as calculus prerequisites, led to a decrease in earned calculus I GPA and A-proportion. In fact, students that took less prerequisite credit hours fared much better than students that took a large number of credit hours. We found this surprising, as our initial thoughts were that we expected students with more mathematical exposure and background to rely on this experience and perform better. We provide a few possibilities for this. One reason could again be that students taking a low number of prerequisites are often on a driven academic path with a firmly decided major, usually math-intensive (such as engineering, physics, computer science, etc.). This also likely led to a smaller time gap between taking the necessary prerequisites and completing calculus I, and hence the better performance, and stronger students in general. It is likely that students taking a high number of credit hours switched majors or perhaps had a time gap in their higher education that led to the decrease in performance. Another reason could be that students taking a high number of credit hours lacked confidence in their mathematical ability or historically performed poorly in mathematics courses, thus needing to take more and more classes as a result.

When discussing limitations, we should once again point out that we did not track any repeated attempts in college algebra, trigonometry, or pre-calculus. We appropriately note that the collected data did not specify the length of time between prerequisites. Also, our data did not account for student performance in previous mathematics courses outside of the ones we defined as prerequisites for calculus I. We also did not track any other mathematics or math-based science courses. We also make the assumption (which is typically true) that any students who took college algebra or trigonometry, rather than both, still received education about the missing subject, generally in the form of dual-credit high school courses. Rather than tracking those students that took pre-calculus versus those that did not, we recognized that the two specific tracks (that is, college algebra/trigonometry versus pre-calculus) were the important ones to compare. Further, we recognize the problem that comes with running statistics on values that are only between 0 and 4, rather than 0 and 100, which would provide more meaningful variance between values when using numeric scores rather than letter grades. Further, our study did not account for the student majors or specific instructors, and a specific instructor and teaching style could also adversely affect performance in a calculus course. These are additional considerations that could be expanded on in future studies. Finally, while the longevity of the sample is very valuable, it is restricted to one university in the southeastern United States, which clearly has its own limitations in expanding the results to other parts of the country and world.

We believe that there are many implications and future studies that can stem from these findings. For one, department heads can decide the advantages of having multiple streams leading to calculus I if there are any cost issues within a department. At minimum, there can be scheduling implications regarding how often some of these offers can be offered, and in how many sections. We also wonder if similar studies can be done for prerequisites leading to other courses, not solely calculus I. Additionally, this study can easily be replicated at other universities to determine if the trends hold everywhere else. Frankly, the steps listed can be used as a research outline for other fields besides mathematics.

Author contributions: All authors have sufficiently contributed to the study, and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: Utah Valley University Institutional Review Board verified the study as exempt and assigned protocol number 890 on 22 October 2021.

Declaration of interest: No conflict of interest is declared by authors.

REFERENCES

Akbuga, E., Hurdle, Z., Daniel, S., & Laffey, R. (2019). Using calculus writing assignments to foster student motivation. *Mathematics in Schools, 47*(4), 37-39.

Bergeron, L., & Gordon, M. (2017). Establishing a STEM pipeline: Trends in male and Ffemale enrollment and performance in higher level secondary STEM courses. *International Journal of Science and Mathematics Education, 15*, 433-450. https://doi.org/10.1007/s10763-015-9693-7

Berry, J. S., & Nyman, M. A. (2003). Promoting students' graphical understanding of the calculus. *The Journal of Mathematical Behavior, 22*(4), 479-495. https://doi.org/10.1016/j.jmathb.2003.09.006

Bertrand, E. J., McArdle, D. T., Thoma, L., & Wu, L. (2019). Implementing online programs in gateway mathematics courses for students with prerequisite deficiencies, *PRIMUS,* 1-16. https://doi.org/10.1080/10511970.2019.1629556

Bloemer, W., Day, S., & Swan, K. (2017). Gap analysis: An innovative look at gateway courses and student retention. *Online Learning, 21*(3), 5-14. https://doi.org/10.24059/olj.v21i3.1233

Bressoud, D. M. (2014). *Attracting and retaining students to complete two- and four-year undergraduate degrees in STEM: The role of undergraduate mathematics education* [Commissioned paper]. The Committee on Barriers and Opportunities in Completing 2-Year and 4-Year STEM Degrees. National Academy of Sciences.

Bressoud, D. M. (2015). Insights and recommendations from the MAA national study of college calculus. *Mathematics Teacher, 109*(3), 179-183. https://doi.org/10.5951/mathteacher.109.3.0178

Bressoud, D. M., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The calculus student: Insights from the Mathematical Association of America national study. *International Journal of Mathematical Education in Science and Technology, 44*(5), 685-698. https://doi.org/10.1080/0020739X.2013.798874

Carrell, S. E., Maghakian, T., & West, J. E. (2011). A's from Zzzz's? The causal effect of school start time on the academic achievement of adolescents. *American Economic Journal: Economic Policy, 3*(3), 62-81. https://doi.org/10.1257/pol.3.3.62

Eagan, K., Hurtado, S., & Chang, M. (2010). What matters in STEM: Institutional contexts that influence STEM bachelor's degree completion rates? In *Proceedings of the 20th Annual Meeting of the Association for the Study of Higher Education*. National Institute of General Medical Sciences.

Easey, M., & Gleeson, J. (2016). The relevance of mathematics: Leaders and teachers as gatekeeper for queensland senior calculus mathematics. In *Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 198-205).

Ellis, J., Fosdick, B., & Rasmussen, C. (2016). Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PLoS ONE, 11*(7), 1-27. https://doi.org/10.1371/journal.pone.0157447

Gleason, J. (2010). Effect of class size on student outcomes in mathematics course with technology assisted instruction and assessment. In *Proceedings of the 13th Annual Conference on Research in Undergraduate Mathematics Education.*

Hagman, J., Johnson, E., & Fosdick, B. (2017). Factors contributing to students and instructors experiencing a lack of time in college calculus. *International Journal of STEM Education, 4*(1), 12. https://doi.org/10.1186/s40594-017-0070-7

Herriott, S., & Dunbar, S. (2009). Who takes college algebra? *PRIMUS, 19*(1), 74-87. https://doi.org/10.1080/10511970701573441

Hurdle, Z., Akbuga, E., & Schrader, P. (in preparation). Exploring calculus I students' performance between varying course times among other predictive variables.

Jarvis, T. (2000). *Class size and teacher effects on student achievement and dropout rates in university-level calculus*. https://www.math.byu.edu/~jarvis/class-size/class-size-preprint.pdf.

Kollar, I., Ufer, S., Reichersdorfer, E., Vogel, F., Fischer, F., & Reiss, K. (2014). Effects of collaboration scripts and heuristic worked examples on the acquisition of mathematical argumentation skills of teacher students with different levels of prior achievement. *Learning and Instruction, 32*, 22-36. https://doi.org/10.1016/j.learninstruc.2014.01.003

Mensah, J. K., Okyere, M., & Kuranchie, A. (2013). Student attitude towards Mathematics and performance: Does the teacher attitude matter? *Journal of Education and Practice, 4*(3), 132-139.

Moreno, S., & Muller, C. (1999). Success and diversity: The transition through first-year calculus in the university. *American Journal of Education, 108*(1), 30-57. https://doi.org/10.1086/444231

Orhun, N. (2004). Student's mistakes and misconceptions on teaching of trigonometry. *Journal of Curriculum Studies, 32*(6), 797-820.

Pope, N. G. (2016). How the time of day affects productivity: Evidence from school schedules? *The Review of Economics and Statistics, 98*(1), 1-11. https://doi.org/10.1162/REST_a_00525

Pyzdrowski, L. J., Sun, Y., Curtis, R., Miller, D., Winn, G., & Hensel, R. A. M. (2013). Readiness and attitudes as indicators for success in college calculus. *International Journal of Science and Mathematics Education, 11*, 529-554. https://doi.org/10.1007/s10763-012-9352-1

Rasmussen, C., & Ellis, J. (2013). Who is switching out of calculus and why. In *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (pp. 73-80).

Raymond, E. (2000). *Cognitive characteristics*. *Learners with mild disabilities*. Allyn & Bacon.

Riegle-Crumb, C., Moore, C., & Ramose-Wada, A. (2011). Who wants to have a career in science or math? Exploring adolescents' future aspirations by gender and race/ethnicity. *Science Education, 95*(3), 458-475. https://doi.org/10.1002/sce.20431

Sonnert, G., & Sadler, P. (2014). The impact of taking a college pre-calculus course on students' college calculus performance. *International Journal of Mathematics Education in Science and Technology, 45*(8), 1188-1207. https://doi.org/10.1080/0020739X.2014.920532

Sonnert, G., Sadler, P., Sadler, S., & Bressoud, D. (2014). The impact of instructor pedagogy on college calculus students' attitude toward mathematics. *International Journal of Mathematical Education in Science and Technology, 46*(3), 370-387. https://doi.org/10.1080/0020739X.2014.979898

Soria, K., & Mumpower, L. (2012). Critical building blocks: Mandatory prerequisite registration systems and student success. *NACADA Journal, 32*(1), 30-42. https://doi.org/10.12930/0271-9517-32.1.30

Van Der Stuyf, R. (2012). Scaffolding as a teaching strategy. *Adolescent Learning and Development,* 0500A, 2-13.

Wilkins, J. L. M., & Ma, X. (2003). Modeling change in student attitude toward and beliefs about mathematics. *Journal of Educational Research, 97*(1), 52-63. https://doi.org/10.1080/00220670309596628

Wu, X., Deshler, J., & Fuller, E. (2018). The effects of different versions of a gateway STEM course on student attitudes and beliefs. *International Journal of STEM Education, 5*(1), 44. https://doi.org/10.1186/s40594-018-0141-4