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TEACHING NUMBER SENSE FOR 6TH GRADERS IN TAIWAN

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ABSTRACT. This study reports on two excerpts from a 6th grade Taiwanese class, describing how a teacher examined and promoted his students' development of number sense. It illustrates an effort to integrate number sense activities into the mathematics curriculum in ways that encourage exploration, discussion, thinking and reasoning. The results indicate that number sense can be developed through well-designed number sense activities, effective teaching, and a good learning environment. It also demonstrates that students' number sense and mathematical thinking can be promoted through the use of multiple representations.

KEYWORDS. Benchmark, Estimation, Learning environments, Number sense.

RATIONALE AND PURPOSE

The teaching and learning of number sense has been considered to be an important topic in mathematics education internationally (Anghileri, 2000; Dunphy, 2007; Ministry of Education in Taiwan, 2003; McIntosh, Reys, Reys, Bana, & Farrrell, 1997; NCTM, 2000; Verschaffel, Greer, & De Corte, 2007; Yang, 2005). Due to its importance, number sense has recently evoked a growing amount of attention and research in Taiwan. In addition, there is a major mathematics curricula reform effort underway and the new guidelines (Ministry of Education in Taiwan, 2003) highlight that mathematics instruction should help children develop number sense. Moreover, several studies showed that children in Taiwan are highly skilled in written computation and are not accompanied with the development of number sense (Reys & Yang, 1998; Yang, 2005; Yang & Li, 2008; Yang & Reys, 2002). For example, when children were asked to find the answer 32×75÷(8×25), they usually need to calculate 32×75= 2400, 8×25=200, and then 2400 ÷ 200= 12.

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It seems difficult for them to flexibly find the relationships among these numbers and know 32÷8=4 and 75÷25=3. Therefore, the answer should be 12. This encourages the researcher to do this teaching experiment. Furthermore, when children were asked to compare $\frac{8}{9}$ and $\frac{11}{12}$, they usually are taught to use the rule-based method to find the answer. If they were not allowed to use the written method, many different misconceptions were found, such as 11 > 8 and 12 > 9, so $\frac{11}{12} > \frac{8}{9}$; the larger the numerator is, the larger the fraction is; or the less the denominator is, the larger the fraction is.

Based on the above descriptions, many different misconceptions were found. This encourages to proceed this teaching experiment. Therefore, the purpose of this study is to report on two particular instances from a 6th grade Taiwanese class and describe how the teacher created a learning environment that encouraged exploration, discussion, thinking, and reasoning. This study also illustrates how the teacher integrated number sense activities into his mathematics classes and effectively promoted his students' number sense.

BACKGROUND

Number sense and its' components

Number sense refers to a person's general understanding of numbers and operations and his ability to handle daily-life situations that include numbers. This ability entails the usage of useful, flexible, and efficient strategies, such as mental computation and estimation, to handle numerical problems (McIntosh, Reys, & Reys, 1992; Reys & Yang 1998 ; Sowder, 1992; Yang, Hsu, & Huang, 2004).

Number sense implies meaningful learning and understanding, therefore, it has been extensively discussed and is widely accepted in mathematics education (Anghileri, 2000; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Yang, 2005). However, no two researchers in the research fields "define number sense in exactly the same way" (Berch, 2005, p. 333). Due to its importance, number sense has produced many discussions and studies from different fields, such as mathematics educators, researchers, educational psychologists, and so on (Greeno, 1991; Markovits & Sowder, 1994; McIntosh, et al., 1992; Siegler & Booth, 2005; Verschaffel et al., 2007). Based on a review of the number sense literature, the activities used in this study specifically narrow their focus to include:

(1)Developing and using benchmarks appropriately

This implies a person can use benchmarks, such as 1, $\frac{1}{2}$, 100, and so on, to solve problems flexibly and appropriately under different situations (McIntosh, et al., 1992). For example, when children were asked to select the best estimation for $\frac{9}{19} \text{x} \frac{17}{23}$, they knew $\frac{17}{23}$ is less than 1 and $\frac{9}{19}$ is less than $\frac{1}{2}$, so the result should be less than $\frac{1}{2}$.

(2)Developing estimation strategies and judging the reasonableness of computational results.

This implies that individuals can mentally apply estimation strategies to problems without using the written method and be able to judge the reasonableness of the result (McIntosh, et al., 1992; Sowder, 1992). For example, when children were asked to place the decimal point using this estimation: 49.05×6.044=2964582, they should not need to rely on the rule-based method to determine the answer. They should know that 50 (49.05 is about 50) multiplied by 6 is about 300, and that the answer 29.64582 is unreasonable.

The importance of teaching and learning number sense

Why is the teaching and learning of number sense for elementary and middle grade students so important? Firstly, number sense is often characterized as "flexibility", "inventiveness" (Dunphy, 2007, p. 2) and "reasonableness". It should play a key role for helping children develop holistic understanding of quantitative concepts. Secondly, number sense should "be a holistic concept related to everyday use of number and to encompass skills, understanding, disposition, and flexibility" (Dunphy, 2007, p. 8). Thirdly, overemphasis on written computation often hinders the children's mathematical thinking and understanding (Burn, 1994; Kilpatrick, Swafford, & Findell, 2001; Reys & Yang, 1998). Fourthly, human should posses an intuitive number sense and it should be nurtured to support future development of mathematical thinking and application (Dehaene, 1997; Berch, 2005). Finally, several number sense related research studies internationally show that children in elementary and middle grade levels are lacking of number sense (Alajmi, 2004; Markovits & Sowder, 1994; Menon, 2004; Yang, 2005; Yang, Hsu, & Huang, 2004; Yang & Li, 2008). Therefore, teaching and learning number sense should be highlighted in elementary and middle schools mathematics classrooms.

Research studies related to teaching and learning number sense

Several studies and documents suggested that human's use of number representation and mathematical thinking partly depend on number sense (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Lipton & Spelke, 2003). However, the study of Reys & Yang (1998) found that children skilled in written computation do not necessarily accompany with the development of number sense. In fact, some studies and reports (Burns, 1994; Reys & Yang, 1998; Yang, 2005; Yang & Li, 2008) documented that excessive reliance on rule-based methods to solve problems which might suppress this intuitive ability and leaded children to produce erroneous results and unreasonable answer. Moreover, several studies (Anghileri, 2000; Warrington & Kamii, 1998; Yang, 2006) suggested that students are more likely to develop number sense after participating in well-designed number sense activities than they are through standard instruction that encourages the use of written algorithms. Furthermore, the number sense related studies (Markovits & Sowder, 1994; Yang, et al., 2004) had confirmed that activities and effective teaching not only promote students' number sense, but also advance mathematical thinking and learning.

THIS STUDY

This study illustrates how one Taiwanese 6th grade teacher integrated number sense activities into his mathematics class to promote the development of number sense. More specifically, it discusses the impact of two activities used by this teacher and describes some of the dynamics within this particular classroom environment.

The setting. The classroom teacher, Mr. H, is a veteran mathematics teacher and has a strong background in number sense. His sixth grade class has 29 students (16 boys and 13 girls). Mr. H agrees in the importance of developing number sense for his students. He also realizes that the overemphasis on computational proficiency does not necessarily develop the meaningful understanding of numbers and operations that characterizes number sense. The activities illustrated here are selected from a semester-long series of lessons that were designed to focus on specific components of number sense to promote the students' development.

Procedures. Mr. H. divided his students into small groups and encouraged them to become actively involved in the learning environment. He provided challenging questions for the groups to solve and encouraged every student to engage each other and share their thoughts. He later led a class discussion, allowing the students to explain their ideas and strategies. This study reports the instructional activities conducted and the resulting group discussions as they occurred in the classroom. The researcher recorded whole-class discussions and conversations between the students and their teacher. The recorded data was transcribed, and excerpts are used to illustrate the dialogue between the students and their teacher.

THE EPISODES

Introduction. It is a new and challenging experience when Taiwanese students are asked to estimate an answer without being able to do exact calculations. This is due to the emphasis on written computation in traditional Taiwanese mathematics instruction, which has always rewarded the obtainment of an exact answer. However, benchmarks are powerful tools in making comparisons or estimates. Thus, appropriate development of the use of benchmarks, such as 1 or $\frac{1}{2}$, greatly facilitates comparisons among fractions or estimations involving fractions. Yet benchmarks and estimation are not emphasized in the Taiwanese mathematics curricula. The following two episodes show how a teacher posed a question and used it to help students to develop number sense.

Episode 1: Benchmark, estimation, and deciding the reasonableness of results.

The question: Without calculating an exact answer, circle the best estimate for:

$$\frac{15}{16}+\frac{11}{12}$$

(1)26 (2) 1 (3) 2 (4) 28 (5) Without calculating cannot find the answer

The question was used to jumpstart a lesson focusing on the use of benchmarks. Mr. H. knew that his students tend to use written methods to find the sum of two fractions, as they had been taught through Taiwanese mathematics textbooks. Mr. H. posed the question, then asked each small group to determine their answer and be ready to explain their reasoning. At this time, one student asked: "**Can we use a paper and pencil to find the answer?**" Mr. H. responded: "**I encourage all of you to develop different methods to solve this problem.**" He then moved from group to group, listening to their discussions, probing responses from students for more details, and checking on their progress. Here are excerpts of the entire class discussion:

First group

S1: Our answer is $1\frac{31}{48}$. So we think the answer is (3) 2.

Mr. H.: Can you tell us your reasons?

S1: We used the paper-and-pencil method to find the answer. [They showed their worksheet (Figure 1) to the whole class.]

Mr. H.: Does anyone have a question?

S4: The question said "without calculating an exact answer." So you could not use written computation.

S1: This is the only way we could solve the problem in our group.

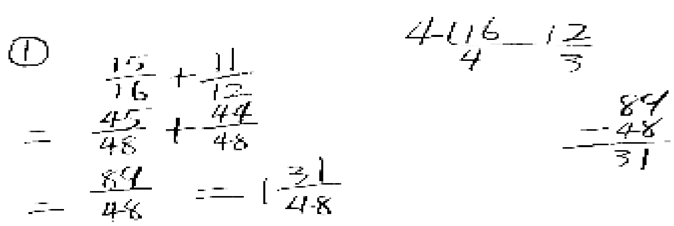


Figure 1. First group's working sheet

Second group

S2: Our answer is (3) 2.

Mr. H.: Please tell us your reason?

S2: [After some hesitation] Well… We think the answer is 2, but we don't know how to explain it.

The students of Group 1 used the written method to solve this question rather than any computation related to number sense. They could not develop a different strategy. No students in Group 2 were able to support their answer with a convincing argument. Even though Group 2 students were unable to explain their answer,, Mr. H still encouraged them to continue their line of thought. Rather than settle the debate, Mr. H. moved on to the remaining groups' answers and explanations.

Third group

S3: We think (3) 2 is the answer.

Mr. H.: Please justify your answer.

S3: Because $\frac{15}{16}$ and $\frac{11}{12}$ are each almost 1, and the question asks us to find the sum. So the answer is $1+1=2$. Hence, the answer is about 2.

S5: How do you know $\frac{15}{16}$ and $\frac{11}{12}$ are almost 1?

S3: Because both $\frac{15}{16}$ and $\frac{11}{12}$ are proper fractions, and they are a little less than 1. They are very close to 1. Therefore, the sum of $\frac{15}{16}$ and $\frac{11}{12}$ is almost 2 and less than 2.

S4: Why did you say $1+1=2$ is the answer?

S3: They are almost 1. The sum of $\frac{15}{16}$ and $\frac{11}{12}$ is close to $1+1=2$. So the answer is 2. For example, you can see two circles on our worksheet with the same size. [This group presents their worksheet, **Figure 2**, to the class] **The shaded area of left side is $\frac{15}{16}$ and the right side is $\frac{11}{12}$. As you can see, both of them are less than 1 circle. So their sum is almost $1+1=2$.**

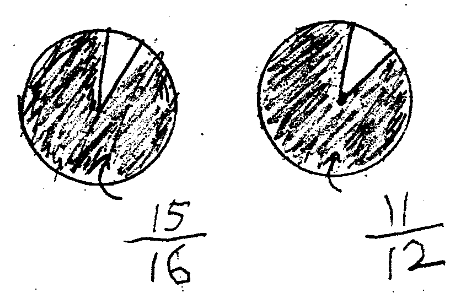


Figure 2. Third group's working sheet

This group showed the class with their graph (Figure 2). They knew that $\frac{15}{16}$ and $\frac{11}{12}$ are each almost 1 through the help of graph; however they failed to point out the key issue—why the numbers are almost 1. Even though the students of this group are strong in graphic representation (a way of good number sense on a semi-concrete level), Mr. H. believes they can do better. He also knows that high-level mathematical thinking relies on the flexible use of different mathematical representations.

Mr. H. returned the group to the classroom discussion, and the children of other groups also shared their answers and ideas.

Fourth group

S4: The answer is (3) 2.

Mr. H.: Please tell us why?

S4: Because $\frac{15}{16}$ plus $\frac{1}{16}$ equals 1, and **$\frac{11}{12}$ plus $\frac{1}{12}$ also equals 1。So we think the sum is about 1＋1＝2. The answer is (3).** [They also present their graphs, **Figure 3**].

Mr. H.: Does anyone have any questions?

S3: Why did you say $\frac{15}{16}$ plus $\frac{1}{16}$ equals 1?

S4: Because 1 is equal to $\frac{16}{16}$, $\frac{16}{16}-\frac{15}{16}$ is equal to $\frac{1}{16}$.

S2: Why did you say your answer was 1+1, but it was not 1+2?

S4: You can see our graphs here [Figure 3]. Both of the circles are the same size. The shaded area ($\frac{15}{16}$) plus the white area ($\frac{1}{16}$) is 1. The shaded area ($\frac{11}{12}$) plus $\frac{1}{12}$ is 1. The $\frac{1}{16}$ and $\frac{1}{12}$ are small. So the answer is about 1 (circle) + 1 (circle), which is equal to 2 (circles). It cannot possibly be 3 (circles).

Mr. H.: Very good!

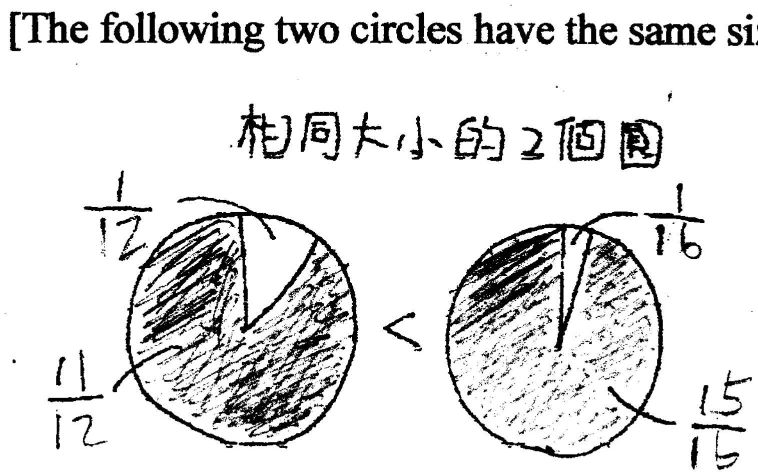


Figure 3. Fourth group's working sheet

The students in Group 4 supported their answer with clear explanations and graphic representation. They used the graph (Figure 3) to explain that the un-shaded parts ($\frac{1}{16}$ and $\frac{1}{12}$) on the circle are small, so $\frac{15}{16}$ and $\frac{11}{12}$ are close to 1. They also recognized that the sum of $\frac{15}{16}$ and $\frac{1}{16}$ was one. They were not only able to move fluently in their exploration of the relationships of fractions and their complements, but also knew how this relationship helped solve the question. Through graphic representation, Group 4 made sense of the meaning of fractions.

Fifth group

S5: Our answer is 2.

Mr. H.: Please tell us your reason.

S5: Because $\frac{1}{16}$ more than $\frac{15}{16}$ is 1, and $\frac{1}{12}$ more than $\frac{11}{12}$ is also 1. Since $\frac{1}{16}$ and $\frac{1}{12}$ are very small, we considered both $\frac{15}{16}$ and $\frac{11}{12}$ to be 1. Their sum is $1+1=2$. So the answer is 2.

Mr. H.: Does anyone have any questions?

****S1: Why is $\frac{1}{16}$ more than $\frac{15}{16}$?**

S5: Because $\frac{16}{16} - \frac{15}{16}$ is equal to $\frac{1}{16}$.

S2: Where does $\frac{16}{16}$ come from?

S5: $\frac{16}{16}$ equals 1.

Mr. H.: Do all of you agree?

S((Many students answered at the same time): Yes.

Mr. H.: You all did a good job. Group 2 and 4 presented us with very good graphs. They used the circle to represent 1. What term can be used to name the number "1?"

S (Many students answered at the same time): Benchmark.

Mr. H.: Great! We use 1 as the benchmark. As you can see, it helps us to efficiently determine the answer when we do estimation. Correct?

S: Yes!

Mr. H: This question asks us to find the best estimate. If you use the paper-and-pencil method, would you spend a lot of time finding the common denominator and computing the numerators?

S: Yes!

Mr. H: Why did we use "1" as a benchmark in this question?

S: Because $\frac{15}{16}$ plus $\frac{1}{16}$ is 1, and $\frac{11}{12}$ plus $\frac{1}{12}$ is also 1. $\frac{1}{16}$ and $\frac{1}{12}$ are small, so both $\frac{15}{16}$ and $\frac{11}{12}$ are almost 1. Therefore, the best estimate is 1+1=2.

The final group was able to switch back and forth between the fractions ($\frac{15}{16}$ and $\frac{11}{12}$) and their complements ($\frac{1}{16}$ and $\frac{1}{12}$) without using the graph. They understood that the sum of these fractions is one and made sense of the meaning of the fractions with formal symbolism. They were also able to fluently compare the relationships of fractions with their complements and know how this relationship helps to find the best estimate.

Later, Mr. H posed a similar question to test the students' understanding of the use of benchmarks and estimation.

Episode 2: Benchmark, estimation, and determining the reasonableness of results.

The question: Without using calculation, which total is more than 1?

(1) $\frac{5}{11}+\frac{3}{7}$ (2) $\frac{7}{15}+\frac{5}{12}$ (3) $\frac{1}{2}+\frac{4}{9}$ (4) $\frac{5}{9}+\frac{8}{15}$

Mr. H. found that students in each group were able to flexibly use different benchmarks (such as $\frac{1}{2}$ or 1) in different situations. For example:

One group

S1: In $\frac{1}{2}+\frac{4}{9}$, one is equal to a half, the other one is less than a half, so the answer is less than 1. In $\frac{5}{9}+\frac{8}{15}$, the half of 9 is 4.5 and 5 in the $\frac{5}{9}$ is greater than 4.5, which means it is greater than $\frac{1}{2}$. The half of 15 is 7.5 and 8 in the $\frac{8}{15}$ is greater than 7.5, so it is also greater than $\frac{1}{2}$. Therefore, the answer is (4).

Mr. H: Does anyone have any questions?

S3: Why did you say "$\frac{5}{9}$ is greater than 4.5, but not $\frac{4.5}{9}$?"

S1: I am sorry! I wrote it in the wrong way. It should be $\frac{5}{9}$ is greater than $\frac{4.5}{9}$.

S4: Which one is a half and which one is less than a half in $\frac{1}{2}+\frac{4}{9}$?

S1: $\frac{1}{2}$ **is a half of 1 and** $\frac{4}{9}$ **is less than half of 1.**

S4: How do you know $\frac{1}{2}$ **is a half of 1 and** $\frac{4}{9}$ **is less than half of 1?**

S1: Because $\frac{2}{2}$ **is equal to 1, a half of** $\frac{2}{2}$ **is** $\frac{1}{2}$**.** $\frac{9}{9}$ **is equal to 1, and a half of** $\frac{9}{9}$ **is** $\frac{4.5}{9}$**.** $\frac{4}{9}$ **is less than** $\frac{4.5}{9}$**, so it is less than** $\frac{1}{2}$**.**

During the discussions, students in Group 1 chose the correct answer, yet some of their explanations were incorrect ($\frac{5}{9}$ **is greater than 4.5, so it is greater than** $\frac{1}{2}$. $\frac{8}{15}$ **is greater than 7.5)**. However, students from a different group pointed out this mistake immediately. This error gave the students a chance to debate and discuss their reasons.

S2: The answer is (4) $\frac{5}{9}+\frac{8}{15}$**.**

Mr. H.: Please give us your reason.

S2: Each fraction in $\frac{5}{11}+\frac{3}{7}$ **and** $\frac{7}{15}+\frac{5}{12}$ **isn't over a half of 1. In** $\frac{1}{2}+\frac{4}{9}$ **only** $\frac{1}{2}$ **is half of 1, and** $\frac{4}{9}$ **is less than** $\frac{1}{2}$**. Both** $\frac{5}{9}$ **and** $\frac{8}{15}$ **are greater than** $\frac{1}{2}$**. So the answer is (4).**

Mr. H.: Does anyone have any questions?

S5: Half of $\frac{15}{15}$ **is** $\frac{7.5}{15}$ **and** $\frac{8}{15}$ **is greater than** $\frac{7.5}{15}$**, so** $\frac{8}{15}$ **is greater than** $\frac{1}{2}$**. Half of** $\frac{9}{9}$ **is** $\frac{4.5}{9}$ **and** $\frac{5}{9}$ **is greater than** $\frac{4.5}{9}$**, so** $\frac{5}{9}$ **is also greater than** $\frac{1}{2}$**. Their sum is over 1.**

S4: How do you know $\frac{4}{9}$ **isn't greater than** $\frac{1}{2}$**?**

S2: As I said earlier, half of $\frac{9}{9}$ is $\frac{4.5}{9}$ and $\frac{4}{9}$ is less than $\frac{4.5}{9}$. So $\frac{4}{9}$ is not greater than $\frac{1}{2}$.

Students in this group flexibly used $\frac{1}{2}$ as the benchmark to obtain the correct answer, and were also able to support their answer with symbolic representation by language. Their explanations were clear and correct.

Mr. H.: All of you did a good job. In this question, you used $\frac{1}{2}$ as a …

S: Benchmark.

Mr. H.: Great! Let me ask you one more question: Without calculating, find the best estimate for $\frac{10}{9}+\frac{12}{11}$

(1) 1 (2) 2 (3) 20 (4) 22 (5) I cannot find the answer

S (many students answered at the same time): 2

Mr. H.: Why?

S: $\frac{10}{9}$ is a little over 1 and $\frac{12}{11}$ is also a little over 1, so their sum should be over 2. In reviewing the answers, 2 is the best estimate.

Mr. H.: Great! 1 is the…

S: Benchmark.

These students demonstrated that they have developed the ability to flexibly use different benchmarks (such 1 or $\frac{1}{2}$) in multiple situations. They also applied their previous knowledge to find the halves of fractions (such as **half of** $\frac{9}{9}=\frac{4.5}{9}$, **half of** $\frac{15}{15}=\frac{7.5}{15}$, or **half of** $\frac{3}{7}=\frac{3.5}{7}$), even though the concept of fractions including decimals is not taught in school. Their number sense and mathematical thinking greatly improved through their participation in these activities and discussions. The students were able to transfer and generalize their knowledge—which is mathematical power in action. This demonstrates the fundamental goal of number sense in the

PSSM of NCTM (2000) to "learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (p. 20).

DISCUSSIONS

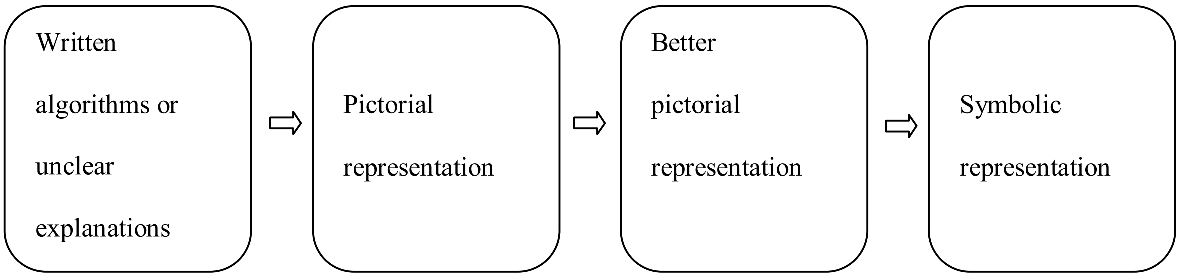
Presented here are only two activities from one class. Yet this study shows how challenging questions can lead to a rich mathematical learning experience. In the learning and teaching process, this activity displays:

1. Multiple problem solving strategies were created and used by these sixth graders. In the rich classroom discussions, different representations were developed and shared by the students. For example, Group 1 supported their first response the only way they could—via the written method. Group 3 solved the problem by using graphic representation. The students of Group 3 knew that $\frac{15}{16}$ and $\frac{11}{12}$ were almost 1 by using their graphs, but they could not support their answer with clear explanations of why the fractions totaled almost 1. Group 4 applied graphic representation with clearer explanations (**The un-shaded area [$\frac{1}{16}$ and $\frac{1}{12}$] of $\frac{15}{16}$ and $\frac{11}{12}$ are small, so $\frac{15}{16}$ and $\frac{11}{12}$ are close to 1**). Group 5 created and utilized the symbolic representations with conceptual understanding. The students' number sense was developed and promoted through the flexible use of multiple representation.

2. The teacher plays a key role in helping children develop number sense. Mr. H. knew how to create a positive learning environment that encourages exploration, discussion, thinking, and reasoning. He knew his role was not only to pose challenging problems and encourage discussion, but also provide opportunities for students to explain and share their thinking process with classmates. He encouraged his students to create different problem-solving strategies, to utilize more efficient solution methods, and to develop higher-level mathematical thinking. His instructional strategies reflect the statement in the PSSM Teaching Principle that "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p. 16).

3. Teachers must have different and skillful strategies to help students learn special mathematical concepts (NCTM, 2000). During the small group discussion, the teacher moved among groups to observe students and listen to their ideas and explanations. Mr. H. knew what

strategies and methods were used by each group. When the class discussions began, he skillfully led each group in sharing their explanations, first from the lower-level method then on to the higher-level method as shown in the following flow chart:



Mr. H used the information he had gleaned from each group to decide in which order to present the groups' findings. This helped students reflect on their thinking process and learn from their mistakes. He knew how to manage the students' work, how to lead discussions among groups with varied levels, "and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2000, p. 19).

4. The teacher needs to pose worthwhile mathematical tasks and help students develop conceptual understanding. Questions that focus on using benchmarks and estimation are not emphasized by the traditional Taiwanese mathematics curricula. Challenging mathematical problems are not always integrated in the lesson plan. Mr. H. not only posed worthwhile mathematical tasks, but also knew how to promote the students' level of thinking. He understood that his students were skilled in standard written computation, but that they did not necessarily have a strong development of number sense. Mr. H. knows that "conceptual understanding is an important component of proficiency" and that "learning with understanding is essential" (NCTM, 2000, p. 20-21).

CONCLUSIONS

This study only presented two teaching examples. This limited its generalization. However, these examples show how the teacher encouraged students to develop alternate ways to solve fractional problems, encouraged students to share their ideas, and supported students in developing meaningful understanding on fractions. These episodes demonstrated that teacher can apply different approach to help children develop fractional number sense without reliance on rule-based method**.** They also illustrate that the teacher plays a key role in designing and asking questions, listening carefully to student explanations, and providing guidance that stimulates students' understanding on fractions**.** Though only two activities were presented here, they clearly showed how a teacher could include interesting, meaningful, and challenging fraction questions into his/her teaching to create beneficial learning experiences for the students. Through use of interactive activities and discussions, teachers can promote their students' understanding, and also refine their own teaching skills.

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