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Synthetic and analytic geometry: A meta-analysis of their didactic relationship in the 21st century

Aitor Alfonso-Castelló [1], Ismael Cabero-Fayos [2*]

[1] Universitat Politècnica de València, SPAIN
[2] Jaume I university, SPAIN
***Corresponding Author:** icabero@uji.es

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ABSTRACT

This study examines how 21st-century quantitative research addresses the relationship between synthetic and analytic geometry in secondary education using Dynamic Geometry Systems, particularly GeoGebra. Following PRISMA guidelines, a meta-analysis of 38 experimental and quasi-experimental studies was conducted. The random-effects model yielded a large aggregated effect size (Hedges' g = 1.24), indicating strong associations between GeoGebra-supported instruction and learning outcomes, albeit with substantial heterogeneity. The analysis also explores temporal effect size patterns and identifies geometric topics most frequently associated with coordinating visual-synthetic and algebraic-analytic perspectives. The findings show that most interventions emphasise representational coordination, although only a limited number of studies explicitly foster bidirectional translation between geometric constructions and algebraic reasoning. Ultimately, this review structures how the integration of synthetic and analytic geometry has been operationalised, providing a solid foundation for more fine-grained future investigations.

Keywords: analytical geometry, dynamic geometry systems, GeoGebra, meta-analysis, secondary education, synthetic geometry

INTRODUCTION

The distinction between synthetic and analytic geometry has traditionally been framed in terms of representational emphasis rather than epistemological opposition. From a historical perspective, synthetic geometry prioritises geometric constructions and visual reasoning, whereas analytic geometry relies on coordinate systems and algebraic representations. In educational contexts, however, this distinction has often been institutionalised as a relatively rigid separation, despite long-standing theoretical arguments in favour of their complementarity. Articulating both approaches is not only conceptually coherent but also pedagogically relevant for supporting students' understanding of geometric structures and relationships.

This separation has generated increasing interest in approaches that connect synthetic and analytic reasoning within geometry education. Dynamic Geometry Systems (DGS) have been proposed in the literature as technological environments that may support such articulation under specific instructional conditions. Guerrero-Ortiz (2015) showed that the use of a DGS can facilitate the coordination of synthetic and analytic reasoning in geometric problem solving when accompanied by appropriate instructional guidance. Their work shows how visual constructions can be connected with symbolic interpretation during geometric problem solving.

Through dynamic manipulation, students can explore invariant geometric relationships while observing related algebraic representations. This creates opportunities to connect visual exploration with formal reasoning. However, the authors also emphasise the role of specific heuristics, such as controlled movement and the construction of loci, in supporting meaningful connections between visual behaviour and algebraic relationships, rather than assuming such integration occurs automatically.

Recent studies in geometry education increasingly emphasise the pedagogical value of connecting geometric exploration with symbolic analysis through dynamic technological environments. Several studies indicate that students benefit from instructional designs that integrate geometric constructions with algebraic representations, allowing them to coordinate multiple forms of reasoning within problem-solving activities. (see Jablonski et al., 2023; Medina Herrera et al., 2024; Weigand et al., 2025). These studies suggest that geometric learning benefits from connecting visual, deductive, and algebraic problem solving.

Dynamic Geometry Systems (DGS) have been proposed as technological environments capable of supporting the articulation between synthetic and analytic geometry. Among these systems, GeoGebra has received particular attention due to its capacity to integrate geometric constructions, algebraic expressions, and graphical representations within a single digital environment.

Under well-designed instructional conditions, GeoGebra allows students to dynamically manipulate geometric objects while simultaneously observing corresponding algebraic relationships, thereby opening up opportunities for connections between visual exploration and formal analysis (Ishartono et al., 2022).

Although previous meta-analyses on GeoGebra, such as Juandi et al. (2021), have examined the effects of GeoGebra on mathematics achievement, most have explicitly treated geometry interventions as a relatively homogeneous category without analysing how dynamic geometry environments mediate the relationship between synthetic and analytic forms of reasoning. This perspective also informs the analytical categories used in the study, particularly the classification of GeoGebra-based interventions according to the degree of explicit integration between synthetic and analytic representations. The theoretical framework of this study draws on perspectives that emphasise the coordination of multiple mathematical representations. Duval (2006) argues that mathematical understanding depends on coordinating different semiotic registers, particularly visual, symbolic, and algebraic representations. Similarly, Kaput and Schorr's (2007) notion of representational infrastructures highlights how digital environments can support connections between symbolic, graphical, and geometric forms of reasoning. Some of these ideas also align with situated abstraction perspectives (Denton, 2017), which describe mathematical understanding as emerging through interactions between representations and problem-solving activity.

This study examines how quantitative research has reported the use of GeoGebra to connect synthetic and analytic geometry in secondary education. Rather than assuming that dynamic geometry environments automatically foster such integration, the meta-analysis analyses reported effect sizes, instructional approaches, and the geometric topics most frequently associated with representational coordination. GeoGebra is designed to integrate geometric constructions, algebraic expressions, and graphical representations within a single environment, making it a relevant case for examining how visual exploration and analytic reasoning are reported to be coordinated in instructional contexts. Rather than assuming that all uses of GeoGebra achieve such integration, this meta-analysis synthesises reported effect sizes, examines their variability across studies and over time, and identifies the geometric topics and task types most frequently associated with reported connections between synthetic and analytic geometry. Existing meta-analyses have largely treated geometry interventions as homogeneous, leaving the synthetic–analytic relationship underexplored on the extent to which GeoGebra-based interventions explicitly coordinate geometric constructions, algebraic representations, and multiple forms of mathematical reasoning within secondary geometry education. In this sense, the study contributes not only an updated quantitative synthesis, but also a theoretically informed classification of representational integration in DGS-based instruction. Based on this perspective, the study examines how synthetic–analytic coordination is reported across GeoGebra-based interventions. From this perspective, the relationship between synthetic and analytic geometry is understood not as the coexistence of visual and symbolic forms, but as the explicit links between geometric constructions and algebraic expressions, and deductive reasoning during problem solving. This distinction is particularly relevant when analysing how GeoGebra-based interventions operationalise representational integration in classroom settings.

RESEARCH OBJECTIVES

The present study is designed to summarise and quantify research articles that employ Dynamic Geometry Systems (DGS) to connect analytic and synthetic geometry within the context of secondary education. The following research questions guide the investigation:

- **RQ1.** To what extent do quantitative studies explicitly report the integration of synthetic and analytic geometry in GeoGebra-based interventions at the secondary education level?
- **RQ2.** What patterns of magnitude and variability are observed in the reported effect sizes of GeoGebra-based geometry interventions over time?
- **RQ3.** Which geometric topics and types of tasks are most frequently addressed in studies that report connections between synthetic and analytic geometry through GeoGebra, and how are these connected to other mathematical domains?

METHODOLOGY

This study adopts a meta-analytic approach to synthesise quantitative research on the use of GeoGebra in secondary geometry education. Meta-analysis enables the systematic aggregation of effect sizes across independent studies, providing a structured means of examining patterns, variability, and reported outcomes within a heterogeneous body of literature. This approach is particularly appropriate given the diversity of research designs, educational contexts, and outcome measures present in studies addressing the integration of synthetic and analytic geometry.

Exclusion and Inclusion Criteria

Inclusion criteria

The following studies were included:

1. Type of research: studies with an experimental or quasi-experimental design analysing the effects of using *GeoGebra* on variables such as academic performance, conceptual understanding, geometric visualisation or mathematical reasoning ability.

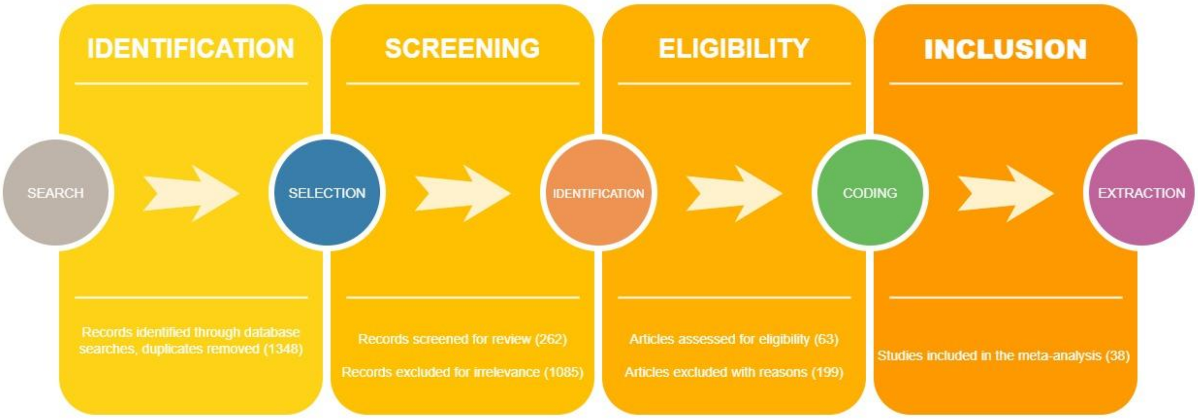

Figure 1. Development of the PRISMA model used in the research (Source: Authors' own elaboration)

2. Geometric approach: research that explicitly addresses the relationship between synthetic and analytical geometry in problem solving, representing geometric objects or transitioning between these two ways of thinking. Studies in which *GeoGebra* is used as a tool to connect geometric constructions with their algebraic or analytical expressions were included.
3. Participants: secondary school or sixth form students, with no restrictions on age or geographical context.
4. Intervention: The intervention involved the explicit use of *GeoGebra* as a primary or complementary teaching or learning tool for geometric content, in both face-to-face and virtual environments.
5. Quantifiable results: Studies presenting sufficient numerical data (e.g. means, standard deviations, sample sizes or statistical values) to enable calculation of the effect size.
6. Language: publications in English or Spanish, regardless of the country of origin.
7. Source and quality: peer-reviewed articles, theses or academic conference proceedings that are available in recognised databases such as Science Direct, Scopus, Google Scholar and ResearchGate.
8. Publication period: research published between 2000 and 2025.

Exclusion criteria

Studies with any of the following characteristics were excluded:

1. Non-experimental designs (e.g. theoretical studies, essays, descriptions of teaching experiences or literature reviews).
2. Research focused on the use of other educational technologies without the direct involvement of *GeoGebra* or similar software.
3. Articles without sufficient statistical information to calculate the effect size, or without a comparison group.
4. Duplicate publications, incomplete publications, and publications without access to the full text.
5. Studies that were not peer-reviewed or written in languages other than English or Spanish.

Study Selection

The systematic search initially identified 1,348 records across all databases. After removing duplicates and screening titles and abstracts for relevance, 262 studies were assessed for eligibility. Of these, 38 met all inclusion criteria and provided sufficient statistical data to be included in the meta-analysis. The selection process is summarised in **Figure 1**.

Study Coding and Reliability Testing

To ensure reliability in the coding process, two independent coders analysed all selected studies. Inter-rater agreement was assessed using Cohen's Kappa, yielding a value of 0.89, which indicates a high level of consistency in the coding decisions.

RESULTS AND STATISTICAL ANALYSIS OF THE DATA

Effect Size Calculation

Effect sizes were calculated for all included studies to enable comparison and synthesis across diverse research designs. Hedges' *g* was used as the standardised measure of effect size, with calculations performed using Comprehensive Meta-Analysis (CMA) software. This approach allows both the magnitude of reported effects and variability across studies to be examined systematically.

Selection of the Effect Model

As shown in **Table 1**, Cochran's *Q* statistic revealed substantial heterogeneity among the included studies ($Q = 326.66$, $df = 37$, $p < .001$). The I^2 statistic was 88.7%, indicating that the observed variability in effect sizes exceeds what would be expected by

Table 1. Overall effect size estimation and heterogeneity

Model	N. of studies	Effect Size	Heterogeneity			Null Hypothesis (Two-Tailed)		
		Point Est.	Std. Error	*Q*-Value	df (*Q*)	*Z*-Value	I^2 (%)	*p*-Value
Fixed Effects	38	0.9087	0.03744	326.663	37	24.2667	88.7%	0.000
Random Effects	38	1.2437	0.11752	—	—	10.5824	88.7%	0.000

Note. Effect size expressed as Hedges' *g*. Heterogeneity assessed using Cochran's *Q* statistic. The random-effects model incorporates between-study variability.

Table 2. Moderator Analysis by degree of synthetic–analytic integration

Integration category	k	Mean effect size	Interpretation
High	5	1.46	Largest descriptive effect
Partial	14	~1.14	Moderate-to-large
Minimal	19	~1.30	Positive but heterogeneous

Note. Differences between categories were not statistically significant, Q(2) = 0.79, p = .673.

chance alone (Botella & Zamora, 2017). Given this heterogeneity, a fixed-effects model— which assumes a single true effect—was deemed inappropriate. Accordingly, a random-effects model was adopted as the primary analytical approach, as it accounts for both within-study sampling error and between-study variability. This model yielded a statistically significant aggregated effect size (g = 1.24), which corresponds to a large magnitude according to conventional benchmarks but must be interpreted in light of the substantial heterogeneity observed across studies. **Table 1** presents the Meta-analysis results: overall effect size estimation and heterogeneity.

Sensitivity Analysis

Additional sensitivity procedures were conducted through the sequential exclusion of highly influential studies in order to evaluate the robustness and stability of the aggregated random-effects estimate. To evaluate the influence of extreme effect sizes on the aggregated results, a sensitivity analysis was conducted through the sequential exclusion of the most influential studies. Given the unusually large effect size reported by Subroto (2011) (g = 8.66), this study was first removed independently from the random-effects model. After its exclusion, the pooled effect size decreased from g = 1.24 to g = 1.14, while remaining statistically significant. This result indicates that, although the study exerted a notable influence on the magnitude of the estimate, the overall pattern of positive associations remained stable.

A second sensitivity analysis was then carried out by excluding all studies reporting effect sizes greater than 4. Four studies met this criterion: Supriadi et al. (2014), Kado and Dem (2020), Hidayat et al. (2023), and Subroto (2011). After removing these studies, the pooled random-effects estimate decreased to g = 0.94 (SE = 0.075, 95% CI [0.797, 1.091]), while remaining statistically significant.

Taken together, these analyses indicate that extreme studies influenced the magnitude of the aggregated effect size. However, the overall pattern of moderate-to-large positive associations remained relatively stable across the analysed literature. These findings suggest that the results of the meta-analysis were not driven exclusively by a small number of highly influential studies. At the same time, the reduction in the pooled estimate highlights the need for cautious interpretation, particularly given the substantial heterogeneity across interventions. Importantly, none of the sensitivity procedures altered the direction of the pooled effect, which remained positive across all model specifications.

Moderator Analysis

To further examine the substantial heterogeneity identified across studies, a moderator analysis was conducted based on the degree of explicit integration between synthetic and analytic geometry. Studies were classified into three theoretically informed groups according to the pedagogical role assigned to GeoGebra: (a) high integration, involving explicit bidirectional coordination between geometric constructions and algebraic representations; (b) partial integration, involving graphical and algebraic perspectives without deep coordination between paradigms; and (c) minimal or implicit integration, where GeoGebra functioned mainly as a visualisation or construction tool.

A random-effects meta-regression with categorical predictors was performed using Comprehensive Meta-Analysis (CMA) software. In addition, a sensitivity analysis excluding studies with unusually large effect sizes was conducted to evaluate the robustness of the findings. The absence of statistically significant differences requires careful interpretation, as it may partly reflect limited statistical power, particularly given the relatively small number of studies in the high-integration category (k = 5).

Results of the Moderator Analysis

Table 2 summarises the moderator analysis by degree of synthetic–analytic integration. A random-effects meta-regression using categorical predictors did not reveal statistically significant subgroup differences, Q(2) = 0.79, p = .673.

The moderator analysis did not explain a substantial proportion of the heterogeneity. However, studies that explicitly coordinated algebraic and geometric representations tended to report stronger outcomes.

Sensitivity analyses excluding studies with extreme effect sizes yielded substantively similar conclusions, indicating that the general pattern of results was not driven exclusively by highly influential studies.

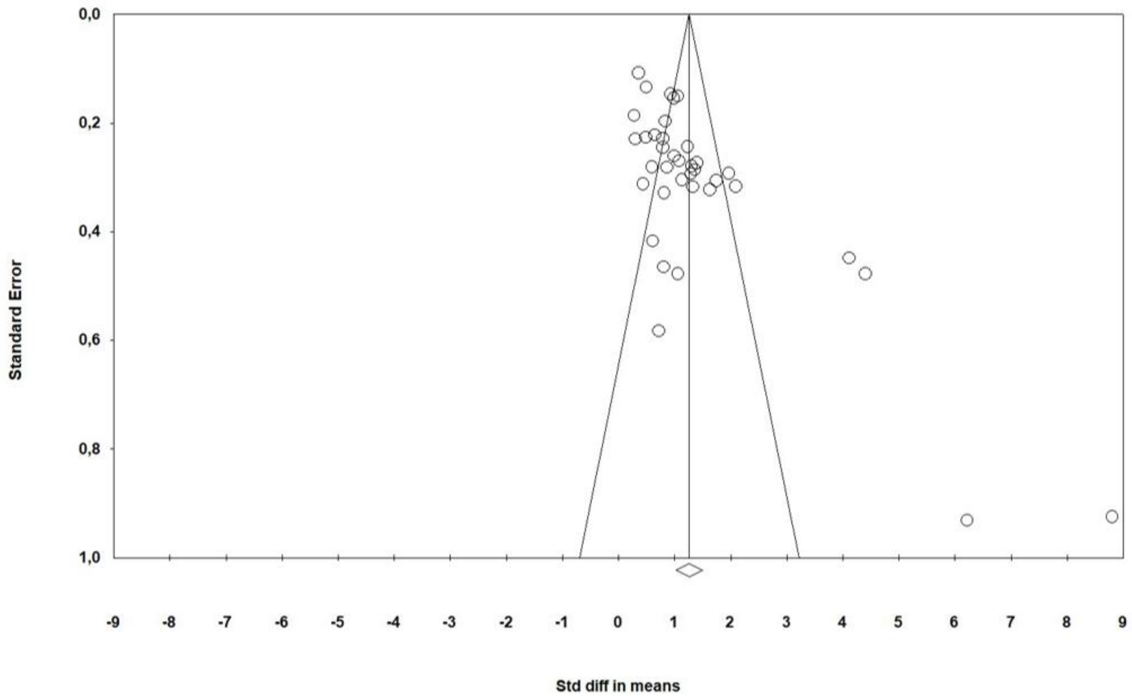

Figure 2. Funnel plot (Source: Authors' own elaboration)

Table 3. Trim and fill analysis

Analysis		Fixed Effects			Random Effects			*Q*-Value
	Trimmed Studies	Value	Lower Limit	Upper Limit	Value	Lower Limit	Upper Limit	
Observed		0.91744	0.84326	0.99163	1.26127	1.02773	1.49481	327.98097
Adjusted	0	0.91744	0.84326	0.99163	1.26127	1.02773	1.49481	327.98097

Operationalisation of Integration Categories

Studies were classified according to the degree of explicit coordination between synthetic and analytic representations present in the instructional design. "High integration" referred to interventions requiring bidirectional translation between geometric constructions and algebraic expressions during problem solving. "Partial integration" referred to studies employing geometric and algebraic expressions without systematic coordination between them. "Minimal integration" described interventions in which GeoGebra functioned primarily as a visualisation or construction tool, with limited explicit connection to algebraic reasoning. Coding decisions were based on the reported instructional tasks, learning objectives, and assessment procedures described in each study.

Publication Bias

Publication bias was examined using both a funnel plot and the trim-and-fill procedure. The analysis did not indicate substantial asymmetry, and no studies were imputed by the trim-and-fill method. These results suggest that publication bias is unlikely to have significantly distorted the aggregated effect size, although the observed heterogeneity warrants cautious interpretation.

The bias analysis was conducted using two analytical approaches: the trim and fill method and a funnel plot (**Figure 2**).

Table 3 presents the results obtained using the trim and fill method proposed by Duval and Tweedie (2000). It can be observed that the reported and adjusted values are identical, indicating that no studies were trimmed or filled, as shown by the row labelled "0 studies trimmed". Therefore, the procedure did not detect sufficient asymmetry to impute any potentially missing studies.

Minor differences between reported effect sizes reflect rounding conventions and distinct analytical outputs generated by the CMA software. The value reported in **Table 1** corresponds to the primary random-effects model, whereas **Table 3** reports the trim-and-fill estimation. Accordingly, $g = 1.24$ is used throughout the text as a rounded reference value for interpretation.

Results Addressing the First Research Question

The results of the data extraction process using CMA are presented in **Table 4** and **Figure 3**. Effect sizes and their corresponding confidence interval limits were obtained.

Table 4. Effect sizes with Confidence Intervals (CI)

Study	Author(s) and Year	*Hedges' g*	Standard Error	Lower Limit	Upper Limit
Study 1	Hutkemri and Zakaria (2017)	0.357	0.108	0.144	0.569
Study 2	Supriadi et al. (2014)	6.140	0.920	4.337	7.943
Study 3	Juandi and Priatna (2018)	0.492	0.133	0.232	0.753
Study 4	Ljajko (2016)	0.301	0.229	-0.148	0.750
Study 5	Birgin and Topuz (2021)	1.291	0.276	0.749	1.833
Study 6	Zulnaidi et al. (2019)	0.484	0.225	0.044	0.925
Study 7	Joshi and Singh (2020)	0.799	0.323	0.167	1.431
Study 8	Kado and Dem (2020)	4.066	0.443	3.197	4.934
Study 9	Em and Roman (2020)	0.986	0.258	0.480	1.492
Study 10	Bakar et al. (2015)	0.855	0.278	0.310	1.400
Study 11	Kurniawan and Ulfah (2023)	0.789	0.243	0.313	1.264
Study 12	Misrom et al. (2020)	1.069	0.266	0.547	1.591
Study 13	Diaz-Nunja et al. (2018)	0.710	0.573	-0.413	1.833
Study 14	Mushipe and Ogbonnaya (2019)	2.068	0.313	1.456	2.681
Study 15	Murni et al. (2017)	1.347	0.283	0.792	1.901
Study 16	Mthethwa et al. (2020)	1.274	0.290	0.707	1.842
Study 17	Choco Coronel et al. (2024)	1.016	0.457	0.119	1.912
Study 18	Bayaga et al. (2020)	0.830	0.196	0.447	1.214
Study 19	Bhagat and Chang (2015)	1.117	0.300	0.529	1.705
Study 20	Pumacallahui Salcedo et al. (2021)	1.385	0.270	0.856	1.915
Study 21	Onaifoh and Ekwueme (2017)	1.722	0.302	1.130	2.315
Study 22	Hashem and Arman (2013)	0.590	0.403	-0.201	1.380
Study 23	Kohen et al. (2022)	0.279	0.185	-0.084	0.643
Study 24	Denbel (2015)	0.961	0.150	0.667	1.254
Study 25	Tamur et al. (2022)	1.221	0.242	0.748	1.695
Study 26	Zengin et al. (2012)	1.606	0.318	0.982	2.229
Study 27	Kusumah et al. (2020)	0.789	0.227	0.343	1.234
Study 28	Hidayat et al. (2023)	4.350	0.472	3.425	5.275
Study 29	Shehayeb et al. (2018)	0.773	0.445	-0.100	1.646
Study 30	Saha et al. (2010)	0.593	0.277	0.051	1.136
Study 31	Ljajko et al. (2010)	0.929	0.147	0.641	1.217
Study 32	Jurdak and Nakhal (2008)	0.432	0.306	-0.168	1.033
Study 33	Adelabu et al. (2019)	0.646	0.220	0.214	1.078
Study 34	Lekitoo et al. (2021)	1.945	0.290	1.377	2.514
Study 35	Doğan and Içel (2011)	1.058	0.150	0.763	1.353
Study 36	Subroto (2011)	8.664	0.910	6.880	10.448
Study 37	Reis and Ozdemir (2010)	0.929	0.147	0.641	1.217
Study 38	Thohirudin et al. (2017)	1.308	0.313	0.695	1.921

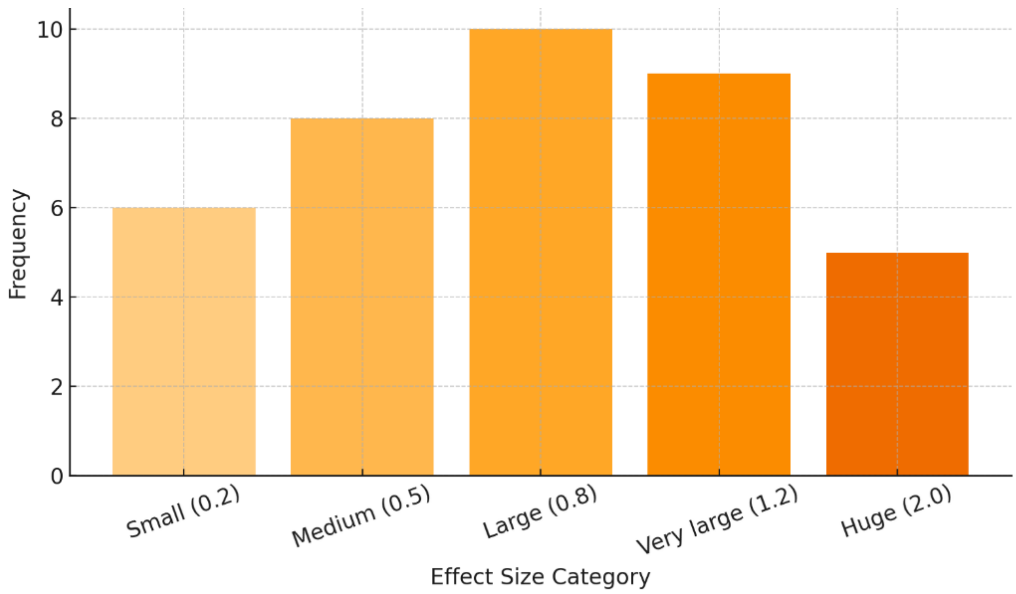

Figure 3. Distribution of the effect sizes of the analysed articles (Source: Authors' own elaboration)

Several extreme effect sizes may be attributable to contextual variability, reflecting highly specific settings, limited sample sizes, or methodological particularities rather than typical instructional effects.

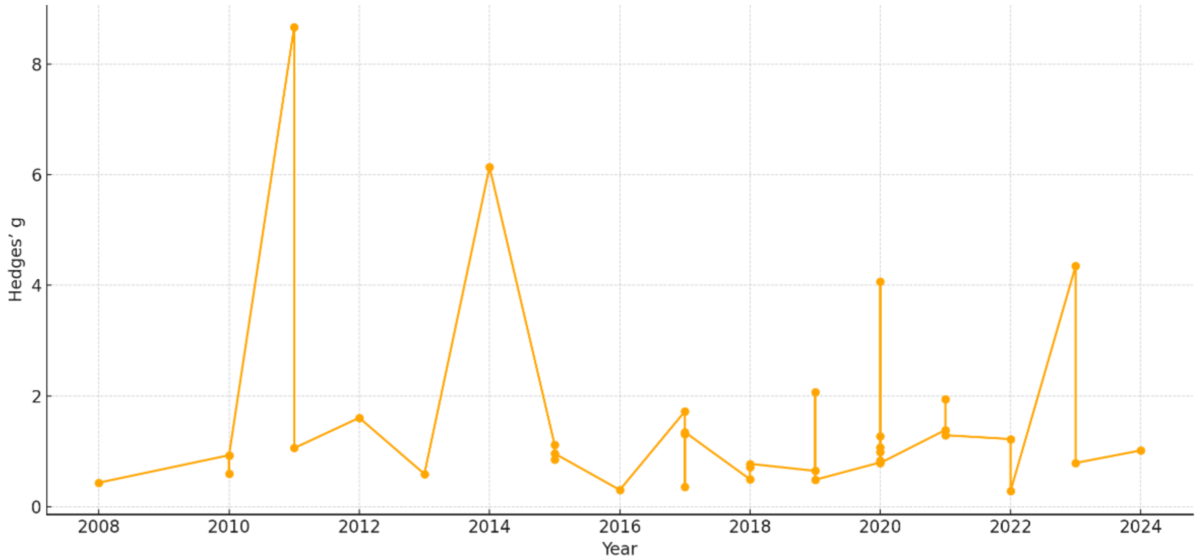

Figure 4. Effect size (Hedges' *g*) over the years (Source: Authors' own elaboration)
Note. The scatter plot shows effect sizes across years. Outliers are indicated. The line is for visual guidance only and does not imply a temporal trend or causal relationship

To provide a more precise answer to RQ1, the degree to which the included studies explicitly operationalised the integration between synthetic and analytic geometry was systematically coded. Of the 38 studies analysed, 21 (55%) designed instructional tasks that explicitly required coordination between geometric constructions and algebraic representations. These studies typically involved activities such as expressing loci through equations, verifying invariants algebraically after dynamic manipulation, or translating between coordinate-based and construction-based representations.

Twelve studies (32%) employed GeoGebra primarily as a dynamic visualisation tool to support geometric exploration, without systematically requiring students to connect constructions to formal algebraic expressions. In these cases, although both representational registers were technically available within the software environment, their coordination was not always made explicit in the instructional design.

The remaining five studies (13%) focused directly on analytic geometry topics—such as coordinate geometry, conic sections, or linear functions—while incorporating synthetic visual exploration to varying degrees. However, the explicit bidirectional articulation between synthetic and analytic reasoning was not consistently foregrounded.

These distributions indicate that, while the articulation between synthetic and analytic geometry is a recurring theme in the literature, the depth and explicitness of this integration vary considerably across instructional designs.

Results Addressing the Second Research Question

Across publication years, reported effect sizes show substantial variability, with no clear linear trend over time. While studies published from 2017 onwards tend to cluster within a moderate-to-large range (approximately 0.7–1.9), earlier years include several extreme outliers (notably 2011 and 2014). These patterns suggest changes in methodological practices rather than a progressive increase in effectiveness (see **Figure 4**).

Results Addressing the Third Research Question

Mapping of geometric concepts applied with GeoGebra

To address RQ3, a descriptive frequency analysis was conducted to identify the geometric topics and task types represented across the 38 included studies. Frequencies are reported to indicate prevalence within the reviewed corpus (N = 38), without implying statistical generalization.

The analysis revealed that the most frequently addressed topics were Basic Geometric Elements (n = 5; 13.2%), followed by studies employing GeoGebra primarily as a Dynamic Geometry System (DGS) and those focusing on Visual and Spatial Thinking tasks (n = 4; 10.5% each). The detailed mapping of the identified concepts is presented below.

Pure Geometry and Fundamental Elements

- *GeoGebra* as a Dynamic Geometry System (DGS) (Adelabu et al., 2019; Diaz-Nunja et al., 2018; Kusumah et al., 2020; Pumacallahui Salcedo et al., 2021).
- Euclidean Geometry (Bayaga et al., 2020; Mthethwa et al., 2020).
- Basic Geometric Elements (Points, Vectors, Segments, Lines, Polygons, Angles) (Ljajko, 2016; Ljajko et al., 2010; Shehayeb et al., 2018; Supriadi et al., 2014).
- Geometric Constructions (Denbel, 2015; Diaz-Nunja et al., 2018; Shehayeb et al., 2018).

- Theorems and Properties (Verification, Proof, Exploration) (Doğan & Içel, 2011; Jurdak & Nakhal, 2008; Mthethwa et al., 2020).
- Visual and Spatial Thinking / Visualization (Diaz-Nunja et al., 2018; Juandi & Priatna, 2018; Subroto, 2011; Thohirudin et al., 2017).

Specific Geometric Figures

- Triangles (including perpendicular bisectors, angle bisectors, altitudes, Pythagoras, congruence, and similarity) (Adelabu et al., 2019; Birgin & Topuz, 2021; Doğan & Içel, 2011; Joshi & Singh, 2020; Pumacallahui Salcedo et al., 2021; Shehayeb et al., 2018).
- Quadrilaterals and Polygons (Birgin & Topuz, 2021; Denbel, 2015; Jurdak & Nakhal, 2008; Pumacallahui Salcedo et al., 2021; Shehayeb et al., 2018)
- Circle and Circumference (circle theorems, parts, etc.) (Bayaga et al., 2020; Birgin & Topuz, 2021; Diaz-Nunja et al., 2018; Kusumah et al., 2020; Ljajko, 2016; Ljajko et al., 2010; Pumacallahui Salcedo et al., 2021; Shehayeb et al., 2018).
- Areas and Perimeters (Plane Figures) (Birgin & Topuz, 2021; Choco Coronel et al., 2024; Pumacallahui Salcedo et al., 2021).

How different studies have shown that GeoGebra connects synthetic and analytic geometry

In many of the reviewed studies, the relationship between synthetic (or Euclidean geometry, based on visualisation and constructions) and analytic geometry (based on algebra, equations and coordinates) is facilitated through the use of GeoGebra, a software environment designed to support the coordinated use of graphical, algebraic, and geometric representations for selected mathematical topics.

The following section delineates the aforementioned relationship, accompanied by the designation of the source from which the information has been obtained.

***GeoGebra* as a Unifier of Synthetic and Analytic (Algebraic) Geometry**

- *GeoGebra* combines geometry, algebra, and calculus: *GeoGebra* is software that dynamically unifies these three areas, as evidenced by studies (Adelabu et al., 2019; Kado & Dem, 2020; Pumacallahui Salcedo et al., 2021).
- The Dual Nature of the Software: *GeoGebra* can be defined as a Computer Algebra System (CAS) due to its symbolic and visualisation features (coordinates and equations), and as Dynamic Geometry Software (DGS) due to its focus on constructions with points, segments and lines (Zengin et al., 2012).
- Linkage as an Essential Feature: The capacity of *GeoGebra* to interweave geometry and algebra is a pivotal facet within the mathematics curriculum (Adelabu et al., 2019; Bakar et al., 2015; Zengin et al., 2012). The appellation *'GeoGebra'* is precisely the conjunction of 'geometry' and 'algebra' (Thohirudin et al., 2017).

Dynamism and Multiple Representations

- Dynamic Connection of Representations: *GeoGebra* simultaneously links geometry and algebra (Zengin et al., 2012).
- Dual Visualisation: The software comprises two main components: one for entering algebraic expressions or equations (the analytical aspect) and another for graphical representations (the geometric/synthetic aspect) (Em & Roman, 2020).
- Improved Conceptual Understanding: Typical forms of mathematical representation include graphical, numerical, algebraic, and verbal formats. Translation between these representations, facilitated by *GeoGebra*, enhances students' conceptual understanding (Kohen et al., 2022).
- Connection between Disciplines: Within a dynamic geometry context, *GeoGebra* helps establish links between geometry, measurement, and algebra, thereby promoting the development of mathematical connection skills (Adelabu et al., 2019; Birgin & Topuz, 2021).

Direct Application in Analytic Geometry

- Educational Importance: Analytic geometry is considered a highly relevant field in secondary education (Ljajko et al., 2010).
- Case Studies and Achievements: *GeoGebra* has been used to facilitate the learning of coordinate geometry. Evidence indicates that teaching supported by dynamic geometry software improves student performance in topics related to analytic geometry (Birgin & Topuz, 2021; Joshi & Singh, 2020; Saha et al., 2010).
- Specific Example – Parabola: The teaching of the parabola benefits from *GeoGebra*, as it enables connections between geometric, algebraic, and graphical representations (Mushipe & Ogbonnaya, 2019; Reis & Ozdemir, 2010).

Across the reviewed studies, this articulation is most commonly achieved through tasks involving geometric loci, dynamic transformations, and the algebraic validation of constructed properties, such as verifying invariants or expressing trajectories through equations.

DISCUSSION

The findings of this meta-analysis suggest that the articulation between synthetic and analytic geometry appears as a recurring theme in quantitative studies that employ GeoGebra; however, the aggregated effect size obtained under the random-effects model ($g = 1.24$, rounded from 1.244) should be interpreted with caution. Although this value corresponds to a large magnitude according to conventional benchmarks, it summarises highly heterogeneous and context-dependent results rather than a single, stable instructional effect. The very high level of heterogeneity observed among the studies ($Q = 326.66$; $p < 0.001$), together with the presence of extreme outliers (e.g., $g > 6$ in some small-sample interventions), indicates that reported outcomes are far from uniform and may be influenced by specific research designs, participant characteristics, and task configurations. Consequently, the aggregated effect size is better understood as a descriptive indicator of patterns in the literature—namely, the frequent reporting of positive associations between GeoGebra-supported instruction and proximal learning outcomes—than as a precise estimate of the causal impact of integrating synthetic and analytic perspectives.

Sensitivity analyses further contribute to the robustness of these conclusions. While the exclusion of highly influential studies reduced the magnitude of the pooled effect size, the overall direction of the effect remained consistently positive across all model specifications. These results suggest that the general pattern identified in the meta-analysis is not driven exclusively by a small number of extreme cases. At the same time, the reduction in effect size after excluding studies with very large values highlights the importance of cautious interpretation, particularly in light of the methodological variability that characterises this field.

Although sensitivity analyses and moderator analyses were conducted to examine the robustness of the findings and the role of synthetic–analytic integration, several limitations should still be acknowledged. First, the moderator analysis was constrained by the relatively small number of studies within some categories, particularly the high-integration subgroup ($k = 5$), which may have limited statistical power to detect significant subgroup differences. Second, the sensitivity procedures were based on the exclusion of highly influential studies rather than on more advanced approaches such as robust variance estimation. Finally, substantial heterogeneity remained across interventions, suggesting that additional moderating variables — such as intervention duration, assessment instruments, or national educational contexts — should be explored in future research.

Given the high heterogeneity observed across studies, the moderator analysis examined whether the degree of explicit synthetic–analytic integration was associated with differences in reported outcomes. Although no statistically significant differences were identified between the three integration categories, descriptive patterns suggest that instructional approaches explicitly coordinating geometric constructions with algebraic representations tend to be associated with stronger learning outcomes. A plausible interpretation is that such approaches better support the coordination of different mathematical forms. Previous research has consistently identified this coordination as important for conceptual understanding in geometry learning.

At the same time, the absence of statistically significant subgroup differences is difficult to generalise. It does not necessarily indicate the absence of meaningful instructional effects, but may instead reflect limitations related to statistical power, particularly given the relatively small number of studies in the high-integration category ($k = 5$). In addition, variability in study design, differences in intervention duration, and the diverse ways in which representational integration is operationalised across studies likely contributed to the observed results. These factors limit the extent to which the moderator can fully account for the substantial between-study heterogeneity.

A notable finding emerging from this analysis is that relatively few studies explicitly treat the coordination between synthetic and analytic geometry as a central instructional objective. In many cases, GeoGebra is used primarily as a tool for visualisation or procedural support rather than as a means of systematically connecting geometric constructions with algebraic problem solving. This pattern indicates that, despite longstanding theoretical arguments emphasising the pedagogical importance of integrating multiple representations, many instructional implementations do not fully exploit the affordances of dynamic geometry environments.

This observation can also be interpreted in light of the moderator analysis, which, although not yielding statistically significant differences, indicated a descriptive tendency for studies with more explicit synthetic–analytic integration to report stronger effect sizes.

This observation is consistent with prior research highlighting the importance of task design and instructional guidance in supporting representational coordination. The effectiveness of GeoGebra does not reside in the technology itself, but rather in how it is embedded within pedagogical practices. Instructional approaches that encourage students to relate geometric invariants, dynamic transformations, and algebraic expressions are more consistent with theoretical perspectives that emphasise the coordination of semiotic systems.

From a temporal perspective, the analysis does not support the identification of a clear linear trend in effect sizes over time. Although studies published from 2017 onwards tend to show greater clustering of effect sizes within a moderate-to-large range (approximately 0.7 to 1.9), this pattern requires careful interpretation. Rather than indicating a progressive increase in effectiveness, it may reflect changes in methodological practices, more consistent reporting standards, or greater alignment between instructional interventions and assessment instruments. The presence of extreme outliers in earlier years (notably in 2011 and 2014, with $g = 8.66$ and $g = 6.14$) raises methodological concerns, suggesting that some studies may have been conducted under highly specific conditions or with limited sample sizes, thereby restricting the generalisability of their findings.

Visual inspection of the distribution of effect sizes indicates that the majority of studies cluster within a moderate-to-large range (approximately 0.7–1.9), while only four studies report values exceeding $g = 4$. Although these extreme values contribute to overall heterogeneity, they represent a small proportion of the included studies (4/38). The use of a random-effects model partially mitigates this influence by incorporating between-study variability into the estimation of the aggregated effect size.

With regard to educational implications, the results do not allow for strong normative claims about instructional effectiveness. However, they do point to the prominence of research approaches that seek to connect synthetic and analytic geometry through the use of dynamic environments. In this sense, previous work cited in the literature provides valuable contextualisation. However, they do point to the prominence of research approaches that seek to connect synthetic and analytic geometry through the use of dynamic environments. In this sense, previous work cited in the literature provides valuable contextualisation. Guerrero-Ortiz et al. (2016) described how DGS can function as a mediating tool between visual exploration characteristic of synthetic geometry and symbolic formalisation associated with analytic geometry, enabling students to move between these perspectives when solving geometric problems. These studies illustrate specific instructional designs rather than generalisable effects, and their findings should be interpreted within the contexts in which they were conducted.

Beyond their instrumental role, dynamic geometry environments have been discussed in the literature as potential epistemological mediators in mathematics learning. From the perspective of situated abstraction, DGS may be conceptualised as microworlds in which learners can explore mathematically coherent situations and progressively generalise from particular cases (Denton, 2017). Such environments can support the formulation and testing of conjectures, as well as the coordination of visual and symbolic reasoning. Nevertheless, the extent to which these epistemological functions are realised depends critically on task design and instructional guidance. The promotion of deductive reasoning and the validation of conjectures, emphasised by Ayalon and Even (2010), Joachin-Arizmendi et al. (2024), and Valenta et al. (2024), cannot be assumed to emerge automatically from the use of technology, but must be intentionally embedded within learning experiences.

The broader literature also highlights the cross-curricular potential of GeoGebra within secondary mathematics education. As noted by Hohenwarter et al. (2009), GeoGebra has been used in studies that aim to support connections across domains such as algebra, geometry, and introductory calculus, facilitated by a wide range of digital resources and applets. Drijvers et al. (2016) similarly emphasise that the integration of multiple representations within a single environment may contribute to more coherent mathematical experiences. However, the present meta-analysis does not evaluate these curricular ambitions directly. Instead, it highlights the diversity of ways in which GeoGebra has been used and studied, reinforcing the need for cautious interpretation of aggregated outcomes and for more fine-grained analyses of how specific instructional designs leverage the affordances of dynamic geometry environments.

CONCLUSION

As a meta-analytic review, this study does not evaluate classroom effectiveness directly, but rather synthesises patterns reported in quantitative research. This meta-analysis provides a descriptive synthesis of empirical studies that report quantitative outcomes associated with the use of GeoGebra in secondary geometry education. The findings indicate that many studies that explicitly integrate synthetic and analytic perspectives report positive associations with learning outcomes closely aligned to the interventions. However, these results should be interpreted as patterns observed in the literature rather than as evidence of a general or uniform enhancement of mathematical learning.

Across many of the reviewed studies, students worked with geometric, algebraic, and numerical representations simultaneously, allowing them to move between different forms of mathematical reasoning during problem-solving activities. Several studies point to the possibility that this coordination between representations can enrich students' mathematical understanding, although its impact depends strongly on task design and classroom implementation.

In this sense, DGS environments may contribute to broader mathematical understanding by supporting problem solving, reasoning, and the exploration of mathematical relationships across topics.

The moderator and sensitivity analyses conducted in this study suggest that the observed heterogeneity cannot be explained solely by the degree of explicit integration between synthetic and analytic geometry. Although studies with more explicit representational integration tended to report larger effects, substantial variability remained within all categories. This suggests that other factors — including intervention duration, teacher guidance, assessment practices, and sociocultural context — likely influenced the reported outcomes.

Over the past two decades, this issue has become a recurring focus in empirical research on geometry education. Examining how these connections have been explored in quantitative studies helps clarify current research trends and the different ways GeoGebra may support geometric reasoning in secondary education.

Rather than demonstrating a transformative shift in geometry pedagogy, the present study highlights the ways in which dynamic digital environments are frequently used to support the coordination of visual constructions and algebraic representations. Across the reviewed studies, GeoGebra is commonly employed to enable students to work with multiple representations—geometric, algebraic, and numerical—within a single environment. This representational coordination is described in the literature as having the potential to enrich learning experiences by allowing learners to explore relationships between different semiotic registers, although the extent to which such potential is realised depends on instructional design, task selection, and assessment practices.

The synthesis also suggests that dynamic geometry environments are often situated within broader curricular contexts, where geometric concepts are connected to other mathematical domains such as algebra, functions, or introductory calculus. In this sense, DGS may support cross-curricular connections and the development of transversal competencies, including problem solving, reasoning, and the validation of conjectures.

From a methodological perspective, the substantial heterogeneity observed across studies and the presence of extreme outliers underscore the need for caution when interpreting aggregated effect sizes. Several studies reported unusually large effects, particularly in small-sample interventions. Future research would benefit from more rigorous and transparent reporting, controlled experimental designs, and differentiated analyses that take into account moderating variables such as educational level, duration of interventions, teacher preparation, and the explicitness with which synthetic and analytic perspectives are coordinated.

The implications for curriculum and teacher education should therefore be framed tentatively. Rather than prescribing specific instructional reforms, the findings suggest that greater attention should be given to how digital tools are used to articulate different geometric perspectives in classroom practice. In this regard, Gascón's (2002) reflections remain highly relevant. His critique of the curricular separation between synthetic and analytic geometry as a "false alternative" anticipated the need for continuity and complementarity between these approaches, while also recognising the limitations of purely theoretical proposals in the absence of appropriate instructional tools.

Future research would benefit from more targeted methodological approaches capable of capturing the complexity of representational coordination in geometry learning. In this regard, mixed-methods studies could provide a more comprehensive understanding by combining quantitative effect measures with qualitative analyses of students' reasoning processes. Similarly, design-based research approaches may offer valuable insights into how specific instructional designs can effectively support the integration of synthetic and analytic perspectives over time.

Contemporary digital environments, such as GeoGebra, provide practical means through which such complementarity may be explored in classroom settings. By clarifying how this integration has been operationalised and reported in quantitative studies, the present meta-analysis offers a more systematic and transparent overview of the current state of research and provides a foundation for future investigations into the conditions under which dynamic geometry environments can meaningfully support students' geometric reasoning.

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