

Probabilistic Ideas and Methods in Undergraduate Mathematics: Axiological Aspects

Elena Kuznetsova ^{1*}

¹ *Lipetsk State Technical University, Moskovskaya, Lipetsk, RUSSIA*

* CORRESPONDENCE: ✉ eva351@yandex.ru

ABSTRACT

The goal the article is to answer the questions: "Why the probability theory and its applications should be studied?" and "What is the attitude of students to the study of probabilistic branches of mathematics?". The values of probabilistic branches of mathematics (stochastics) are revealed on the analysis of scientific publications on the philosophy and history of probability theory, as well as pedagogical works of mathematicians known for their achievements in this area. Further, to examine the attitude of students to study probability theory and its applications, a survey of 76 mathematics sophomores was conducted. The results of the survey were investigated using cluster analysis. It has been found that students, who have a positive attitude and motivation to study probabilistic branches of mathematics, realize the value of probabilistic ideas and methods at the philosophical level and recognize the practical utility of this. Therefore, the scientific publications on the philosophy and history of probability theory, as well as the pedagogical works of well-known mathematicians, can be the valuable resource that could be of use in the educational process to improve the motivation of students in learning probability theory.

Keywords: probability, purposes, values, training of Bachelors of Mathematics

INTRODUCTION

Recent changes in the training of specialists with different majors make it clear that the probabilistic branches of Mathematics (probability theory, mathematical statistics, stochastic processes, applied data analysis, and so on) attract the increasing attention of educators. Further, following the tradition of some Russian and international studies (see, for example, Garfield & Ahlgren, 1988; Kuznetsova, 2018c), we will use the term "stochastics" to combine branches of Mathematics, where probability models of random phenomena are built and studied.

As a rule, during the development of the course of stochastics, much attention is paid to the choice of topics, which constitute the course content, methods, and resources of training, organizational forms of the lessons. However, it is well known that motivation, personal interest, and enthusiasm are significant for learning. That is why many teachers adopt different resources and pedagogical practices, which increase students' participation and involvement in the study of probability theory and its applications. First of all, this is aiming at the future profession and application of knowledge gained in the everyday life, and using real data in the learning process (Brown, 2017; David & Brown, 2012; Libman, 2010; Neumann et al., 2013; Woltman, 2017), as well as exploring real-life situations (Byun & Croucher, 2018; Leech, 2008; Shanks, 2007). Using the history of the probability theory (Lacoma, 2000) and exploration of paradoxes and counterexamples (Klymchuk & Kachapova, 2012; Lesser, 1998) allow understanding the meaning of terms and random phenomena. Applying information technology for modeling and simulations opens up new possibilities in the study of all subjects of

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the probabilistic cycle (Biehler, 1991; Jiang & Potter, 1994; Maltby, 2001; Mills, 2002; Prodromou, 2014). Involvement of students in research and studies (Bailey et al., 2013, Knypstra, 2009), writing interpretations of analysis outcomes (Radke-Sharpe, 1991), a person-oriented approach in education (Prins, 2009), creating the atmosphere of cooperation (Roseth et al., 2008) also should be mentioned. Every time we proceed with development or planning of the improvement of the course in stochastics, it seems necessary to define the goals and answer the following questions: “Why should the probability theory and its applications be studied?” and “What is the attitude of students to the study of probabilistic branches of mathematics?”

REVIEW OF LITERATURE

When searching for the answers to the questions raised, one needs to refer to the literature on the historical and philosophical aspects of stochastics.

There are many studies, proving the effectiveness of inclusion of the history of mathematics in the teaching process. For example, Ernest (1998) wrote: “A historical approach can help to improve perceptions of mathematics and attitudes to it, by making it interesting, alive and part of human history and culture.” Furthermore, knowledge of the history of the development of a particular area in mathematics helps teachers to understand better the difficulties, faced by students and helps students to “derive comfort from knowing that they are not the only ones with problems” (Fauvel, 1991). It is very important in the teaching of stochastics since a study of probabilistic ideas connect with considerable difficulties (Borovcnik & Bentz, 1991; Cobb & Moore, 1997; Garfield & Ahlgren, 1988).

Also, be noted that the probability theory occupies a unique position in many other mathematical disciplines. On the one hand, it is characterized by the unity of formally structural features of the mathematical apparatus, and on the other hand, the notion of causality, randomness, and probability are philosophical categories. Indeed, the definition of the basic concepts of probability theory is impossible without revealing interrelations between philosophical views as to the necessity, randomness, possibility, and probability. Consequently, available studies in history and philosophy of probability are intimately connected and, also, many specialized, scientific and pedagogical works of eminent scientists in the field of probability theory and its applications are methodologically loaded. As noted by Grigorian (2004), a Russian philosopher, the need to address the actual scientific problems forces the scientists to search for the answers to questions about the nature of the random things, about the place of probability theory in the system of the scientific knowledge.

Thinking about the history of development of probabilistic branches of mathematics, Swedish mathematician, actuary, and statistician Cramer (1979, p. 38) wrote: “If in 1920, the probability theory hardly deserved the name of the mathematical theory, in 1945 it entered the postwar world as the well-organized branch of pure (abstract) mathematics with its objectives and methods, and constantly expanding areas of applications in other sciences, as well as in various types of practical activities”. In the post-war period, essential works on history and methodology of probability theory were written (for example, Kolmogorov, 1947). Since the 1960's, almost simultaneously in Russia and abroad, books devoted to history and philosophy of probability theory were published, which is an indication of some results of the development in this branch of mathematics. In Russia, those are monographs of Maistrov (1967, 1980), Pyatnitsyn (1976), Sachkov (1971, 1999), Grigorian (2004). Among the international publications, there are following works can be mentioned: David (1962), Fine (1973), Heitele, (1975), Hacking, (1977, 1990), Gigerenzer and Porter (1990), Kriiger, et al. (1987a, 1987b), Stigler (1986), Tryfos (2004). Among the publications of recent years there is the article by Debnath and Basu (2015). The studies describe the development of probabilistic ideas, observe the occurrence of probabilistic models and methods in the natural and human sciences, and provide examples of its application in the stock market, insurance, and sports.

Publications of scientists known for their achievements in the field of stochastics should be mentioned separately. These are historical essays (e.g. Kolmogorov 1947, Gnedenko 1954, 2013; Shiryaev 1998), pedagogical works, which contain the examples of the studies on the probability theory and valuable comments on the way of its teaching (e.g., Gnedenko, 1982, 1985; Kolmogorov, 1988; Khinchin, 1980) and popular articles and essays, clearly outlining certain aspects of stochastics (e.g. Gnedenko, 1978, 2010a, 2010b; Gnedenko and Khinchin, 1970; Kolmogorov et al., 1982). Many of these works, because of their relevance, have been republished in recent years.

PROBABILISTIC IDEAS AND METHODS IN THE SYSEM OF HUMAN VALUES

Thinking about the problem of the definition of the role and place of probabilistic ideas in the training of Bachelors of Mathematics and making sense of the sources reviewed, one can draw the following conclusions.

First, the study of stochastics is required for the training of modern specialists, capable of adequately addressing the challenges in their professional activity, which have a probabilistic framework. It is because randomness in natural processes, technology, economics and other fields of human activity, production development, and informatization of the society all together put the humankind to some tasks and challenges, which can be solved only by a specialist, who has a sound probability-theoretical education. As Gnedenko (1982), a Russian mathematician, rightly pointed out: "The situation is that many people, including those who have no relation to scientific researches, need elements of statistical knowledge in development of views on the patterns of nature, social phenomena, and technological processes, which are broader than those, which have been developed by mankind for thousands of years and clearly reflected in the mechanical determinism". In addition, our analysis of the literature confirms that scientists-theoreticians (for example, the famous Russian mathematicians: Gnedenko, Kolmogorov, Khinchin) paid great attention to the practical applications of the probability theory, and the practicing educators paid great attention to formation of skills in their students to build and analyze the stochastic models in their future professional engagement. This point of view is consistent with the view of international scientists. For instance, Cobb and Moore (1997) in their article on methodological issues of the teaching of probability branches of Mathematics, noted: "Probability is an essential part of any mathematical education. It is an elegant and powerful field of mathematics that enriches the subject as a whole through its interactions with other fields of mathematics. ... The probability is also essential to the serious study of applied mathematics and mathematical modeling".

Second, the study of stochastics is critical due to the fact, that the probabilistic ideas and methods play an essential role in shaping human culture of the future specialist, have a substantial humanitarian worldview potential, the revelation of which is impossible without understanding its philosophical aspects, the role, and place in scientific cognition, in culture and the system of human values. Thus, Rényi, the Hungarian mathematician, wrote: "My experience in teaching probability theory ... and my attempts to apply it in practice, allowed me to make the following conclusion. To deepen into the mathematical theory of probability and its application, it is not enough (but necessary) to grasp its essence; it is necessary to understand and independently think out the key issues, associated with the notion of probability" (Rényi, 1980, p. 191). Based on the studies, having been previously reviewed by us, let us analyze the role of probability in modern scientific cognition.

First of all, inclusion of the idea of probability into the cognition has led to the radical transformations in scientific perception of the world, the style of scientific thinking and in the basic models of the universe and its cognition, which allows saying about the presence of the probabilistic revolution in science (Kriiger et al., 1987a, 1987b; Sachkov, 1999). It is enough to compare two models of the world. The first model (classical one) is based on the ideas of classical mechanics: all connections in the material world are similar to mechanical one, i.e., are straightforward and entirely defined by equations which describe the motion, and by initial conditions. The systems are not subject to the qualitative changes over time. Randomness is the result of partial or complete lack of knowledge about the subjects of the study; statistical regularities are not independently valuable and complete. It is evident that this model of the world is a substantial simplification and does not describe the entire diversity of the real systems and processes. The second model based on the recognition of randomness as an objectively existing independent start of the world. Along with this, statistical regularities are recognized to be complete, independently valuable and non-reducible to the laws of rigid determination. Thus, the foundations of the world can be considered as something, which is in constant change. There is no doubt that the second model more closely matches the image of the modern information society with its rapid development, instability, and risks. "In all cases, when the science faces a difficulty, the study of complex and elaborate systems, the probability is crucial" (Sachkov, 1998).

Further development of probabilistic ideas is seen in the dialectics of randomness and necessity: there is no perfect randomness; any real process contains the necessary and random features. The new scientific worldview most adequately reflects the current state of development of the society, which is characterized by a complex arrangement of the technology and production cycles, providing thus a basis for the formation of personal qualities, required in the information society, consistency and variability in thinking, width of views, the ability to generalize and understand in general. Not coincidentally, Blalock (1987), as the general goals in teaching statistics, formulates: "(1) overcoming fears, resistances, and over memorizing; (2) stressing the

importance of intellectual honesty and integrity; (3) understanding the relationship between deductive and inductive inferences; (4) learning to play the role of reasonable critic; and (5) handling complexities systematically". Thus, under the conditions of the dominance of information and knowledge, the training is essential for a modern person for self-realization in the professional sphere.

The concept of probability is non-separable from the study of complex self-organized systems, including the sciences of humans and society. The Russian philosopher Sachkov (1996) noted that there are two ways of occurrence of probability in the social science.

The first way (direct) is that the idea of probability is included in the statistics, the science of quantitative relationships in the social phenomena, which is the empirical basis of economic and social sciences. In other words, due to development of stochastic methods and models, application of mathematics expanded its borders from learning of precise and straightforward dependencies to modeling complex phenomena in economics, sociology, psychology, and other sciences. As was noted by Cobb and Moore (1997) in their article, "The domain of determinism in natural and social phenomena is limited so that the description of random behavior must play a large role in describing the world." Due to this fact, the study of stochastics provides a basis for mutual understanding among specialists of various profiles, promotes professional mobility in the information society, successful and harmonious interaction with other people.

The second way of occurrence of probability in the social science is mediated. Its essence is that "... during the analysis of the reasons for the entrance by the human in various social structures, we proceed from the recognition of the presence in each person of own independent beginning... Similar initial orientation, when it is recognized that the system components have their independent beginning, practically express the fundamental idea of probability" (Sachkov, 1998). The conclusion follows that the concept of probability is non-separable from the notions of randomness, independence, necessity, which are philosophical categories, associated with the fundamentals of understanding by a human of its existence, as well as of the history of culture. The concepts of the random event and the independence, in its turn, are non-separable from the problems of definition of the human free will, its responsibility for its own choices and actions. Thus, Panarin (Russian philosopher and publicist) believes that the modern historical consciousness is characterized by a transition from the view of history as a process, predetermined by previous events (causes), to the risky and alternative-oriented history: the modern stochastic pattern of the world is approved in the social and historical sciences. "Instead of dogmatic prophecies about the only possible future, the stochastic pattern of the world in social sciences reveals that, in the diversity and alternativeness of the historical process, there are not only risks but also opportunities to affect the real scenarios and improve the chances of those, which are more in line with our human dignity and interests" (Panarin, 1999, p. 11). These ideas partly coincide with some reasoning of Joosten (2013) as to the role of uncertainty in the modern life and as to the training of "professionals who can stand their ground in an uncertain and changing (professional) world."

Values and objectives of the course of stochastics, in their turn, determine the choice of content, tools, techniques, forms of control, the practical application of the techniques, listed in the introduction.

THE RESEARCH ON THE STUDENTS' ATTITUDE TO THE STUDY STOCHASTICS

Both Russian and international researchers recognise the fact that students' attitudes and beliefs about mathematics and statistics are critical in mathematics education (Kuznetsova, 2018c; Gal & Ginsburg, 1994; Lipnevich et al., 2016; Ma & Kishor, 1997; Shaw & Shaw, 1997). However, in Russia, there is no widespread practice of interviewing students in order to study their attitude toward learning mathematics or statistics. It, in turn, is the reason for the lack of survey tools like ATMI (Tapia & Marsh, 2004), the Mathematics Attitude Inventory (Sandman, 1980), SAS, ATS, SATS-28 (Nolan et al., 2012).

The experience of our university has shown that it is not enough to take a translation into Russian of a well-known international questionnaire and conduct a student survey. A large number of answers skipped by the students did not allow us to interpret the results of the survey. It became clear that international questionnaires require adaptation and approbation.

To study the attitude of students to the probabilistic branches of mathematics (stochastics), we compile a questionnaire Students' Attitude toward Stochastics (SatS). The theoretical basis of the questionnaire is a hierarchical model of personality, combining three levels:

- the top floor: personality orientation (worldview, value orientations, beliefs, aspirations);
- the middle floor: human experience (knowledge, skills, habits);

Table 1. V-scale: Value of Probability

Variable	Questionnaire Item
V1	Probabilistic ideas and methods have a significant impact on the development of society.
V2	Probabilistic ideas and methods are not needed in everyday life
V3	The study of probability theory and its applications develops thinking and broadens the horizon.
V4	The study of probability theory and its applications is necessary for a modern specialist.
V5	Knowledge of probability theory and its applications will be useful in further studies.
V6	Scientific research requires knowledge in the field of probability theory and its applications.
V7	Probabilistic ideas and methods play an important role in modern scientific knowledge.
V8	I think that probability theory and its applications are unlikely to be useful to me in professional work.

Table 2. E scale: Enjoy of Probability Learning

Variable	Questionnaire Item
E1	I'm interested in studying probabilistic branches of mathematics.
E2	I was bored when studying these branches of mathematics.
E3	I easily cope with the tasks from these branches of mathematics.
E4	Tasks, where there is fortuity, throw me into confusion.
E5	The problems of probability and chance annoy me.
E6	Tasks, where there is uncertainty, fortuity, probability, I decide easily and dispassionately.
E7	The study of probability theory and its applications is a waste of time
E8	I would like to know more and be more able in this field.

- the lower floor: higher mental processes (intelligence (thinking, memory, attention), will, emotions) (Platonov, 1986).

The purpose of the survey was to study two aspects of the attitude of students to stochastics: worldview-value belief (the top floor of a hierarchical personality model) and emotional perception (the lower floor of a hierarchical personality model). The fact that these two factors affect the students' achievements in the field of mathematics education is manifested in both international and Russian studies (Kuznetsova, 2018c; Sherman and Wither, 2003). Therefore, the questionnaire "Attitude of students to stochastics" developed by us contains two scales "Values" (V-scale) and "Emotional perception" (E-scale). Each scale consists of eight items. Some of them (about half) reflect a positive attitude towards stochastics, the rest - a negative attitude. Students could express their agreement with the proposed statements using 5-points Likert's scale: 1 - "Strongly Disagree," 2 - "Disagree," 3 - "Undecided," 4 - "Agree," 5 - "Strongly Agree."

The items of the V-scale were formulated based on the analysis of the literature presented above. Items V1-V8 reflect relevance and importance of probability ideas and methods to society and an individual (see **Table 1**).

Studies Pekrun et al. (2017) and Mega, et al. (2014) show that such emotions as pleasure, interest, anger, anxiety, boredom, hopelessness affect motivation, choice of cognitive strategies and educational achievements. Following this, the items of the E-scale presented in **Table 2** were formulated. Items E1-E8 reflect students' enjoyment of learning of probability mathematics branches (see **Table 2**).

Then sixteen items of the questionnaire were randomly arranged together. The article Kuznetsova (2018c) considered the reliability and validity of the questionnaire. The research also revealed that the variables of this questionnaire have statistically significant correlations with each other. For example, the variable V1, which reflects the degree of students' understanding the value of probabilistic ideas and methods at a philosophical and methodological level, has significant correlations ($p < 0.05$) with all other variables of the V-scale, as well as with variables E6 (ease in solving problems and lack of anxiety) and E8 (motivation to study stochastics). This fact proves the importance of forming a worldview when studying the theory of probability and its applications (Kuznetsova, 2018c).

Since this article is the part of the research into the problems of preparing future mathematicians at a university (see also Kuznetsova, 2018a, Kuznetsova, 2018b), we invited undergraduates, majoring in mathematics and informational technologies, to participate in the survey, to find out their attitude to the study of probability theory. Seventy-nine sophomores studying the discipline Theory of Probability and Mathematical Statistics took part in the survey. Of these, 33 students are majoring in Applied Mathematics, 19 students are majoring Informatics and Information Technology at Lipetsk State Technical University, and 27 students are majoring Mathematics Education at Lipetsk State Pedagogical University (Russia). In total, there are 79 people, including 37 men and 42 women. The students were chosen to participate in our study, since all these specialties are related to mathematics and require in-depth study of mathematics disciplines, including probability theory and its applications. Also, students of these specialties have, as a rule, the same

Table 3. Converted variables

Variable	Questionnaire Item
V2*	Probabilistic ideas and methods are needed in everyday life.
E2*	I was not bored when studying these branches of mathematics.
E4*	Tasks, where there is fortuity, do not cast me into confusion.
E5*	The problems of probability and chance do not annoy me.

Table 4. Average means of variables for each cluster

Variable	Cluster1	Cluster2	Cluster3	Cluster4
V1	4.4	3.8	3.4	4.4
V2*	4.0	3.1	3.6	3.6
V3	4.7	4.5	4.1	2.8
V4	4.4	4.2	3.8	2.8
V5	4.5	4.4	3.8	4.8
V6	4.7	4.6	3.6	3.4
E1	4.4	2.4	3.8	3.2
E2*	4.7	2.6	3.9	2.2
E3	3.7	1.9	3.0	4.4
E4*	3.7	2.6	3.6	3.4
E5*	4.4	2.5	3.8	4.0

level of training at school and about the same curriculum in mathematics at university. Taking into account the contradictory answers of students to the questions *V4* and *V8*, *V6* and *V7*, *E3* and *E6*, *E7* and *E8*, it was decided to remove three samples from the survey. By the remaining 76 questionnaires cluster analysis was carried out.

During the study, we excluded the variables *V7*, *V8*, *E6*, *E7*, and *E8* because they make an insignificant contribution to clustering. Next, for the convenience of interpretation, we have turned to the negation of the content of the items *V2*, *E2*, *E4*, and *E5*, reflecting a negative attitude towards the learning of stochastics. To do this, we have made a replacement according to the following rule: five replaced by one, four replaced by two, three remained unchanged, two replaced by four, and one replaced by five. We have got the statements formulated in a positive form. They correspond to the variables *V2**, *E2**, *E4** and *E5** (see **Table 3**).

The most informative was the division into four clusters. The mean values of the variables for each cluster are shown in **Table 4**. Based on **Table 4**, we can give a characteristic for each cluster.

Cluster 1 (30 members) - students who are aware of the value of stochastics (average of the variables of V-scale not less than 4), interested in the subject (the average of the variables *E1*, *E2** more significant than 4) and constructively overcome arising difficulties (the average of the variable *E5** more than 4).

Cluster 2 (14 members) - students with a practical approach to learning. However, they feel anxiety and boredom. The features of the cluster are low enough values of variables *V1* and *V2** (the value of probabilistic ideas and methods for social and everyday life). At the same time the values of variables *V3-V6* reflecting utility, are high. Low average values of the E-scale variables for Cluster 2 members show that the students experience negative emotions when studying the Theory of Probability: lack of interest, confusion, irritation, anxiety.

Cluster 3 (27 members) - students are stable in their assessments: almost all the average values of variables of V-scale and E-scale are approximately the same and are between 3 and 4. The values of the learning difficulties are similar to those in Cluster1. Perhaps these students are merely more restrained in their judgments.

Cluster 4 (5 members) - the students learn without difficulty (the average of the variable *E3* is 4.4) and do not feel anxiety (the average of the variable *E5** is 4.0). However, they feel bored (the average of the variable *E2** is 2.2). Also, they believe that stochastics is necessary for studying at the university (the average of the variable *V5* is 4.8 - this is the highest value in **Table 4**), but it is not necessary for the profession and the development of thinking (the mean variables *V3* and *V4* are 2.8). That is, a formal approach to learning characterises these students. The understanding of the value of the stochastics at the philosophical level (the average *V1* is 4.4) as well as the realizing of the usefulness of these branches of mathematics in further studying at university does not compensate for the lack of comprehension of the importance of probabilistic ideas and methods for personal and professional development. Therefore, despite the easiness in knowledge acquisition, they do not have the opportunity to enjoy learning.

Thus, the analysis allows us to conclude that in order to be able to enjoy the study of stochastics, it is essential to understand both its methodological importance and its usefulness for learning and future work. It is necessary to understand why the theory of probability and its applications should be studied, why it is important and what gives every person professionally and personally. It is no accident that many scientists, for example, Kolmogorov (1947, 1988), Gnedenko (1982, 1985, 2013), paid much attention to the philosophical aspects of probability in their scientific and pedagogical publications.

CONCLUSION

Therefore, we can conclude that the value of the course of stochastics lies in the fact that the idea of probability plays an important role both in scientific cognition, the building of the scientific pattern of the world, and in the determination of the place of a human in the world, the formation of professional culture.

Studying the attitude of bachelors of mathematics to the study of stochastics showed that in order to experience the pleasure of learning, it is essential to have a holistic view of the value of the subject matter. It is not enough to recognize only the usefulness of stochastics (such as the students, united in Cluster2) or to realize the significance of probability only on a philosophical level, but to deny its usefulness in its practical activities (such as the students, united in Cluster4). Therefore, when studying probabilistic branches of mathematics, the instructor should not only demonstrate practical importance of the subject but also disclose the value of probabilistic ideas and methods on the philosophical level.

Knowledge of the history of development, understanding of the methodological and axiological fundamentals of probability theory is useful to both teachers and students. Teachers receive the base for the development of the course of stochastics, corresponding to the modern requirements for training of specialists of mathematical sciences; students have the incentive to overcome difficulties which occur in the process of learning and development of scientific and cultural potential.

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Notes on contributors

Elena Kuznetsova – Lipetsk State Technical University, Moskovskaya, Lipetsk, Russia.

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