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# Preservice Teachers Engage in a Project-based Task: Elucidate Mathematical Literacy in a Reformed Teacher Education Program

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**ABSTRACT**

To meet the demand of the incoming school curriculum reform focusing on competency-based learning in Vietnam, this paper reports on an innovation project on developing secondary mathematics preservice teachers' (PSTs) mathematical literacy and preparing them to teach mathematics contextually. We developed a curriculum and studied the effectiveness of the implementation to a secondary mathematics PST education program that integrates mathematical literacy (ML) in methods courses. The courses offer PSTs opportunities to experience ML as active learners and prepare them to teach ML. In this paper, we discuss the results on a project-based modeling task. The results showed that the PSTs begin to develop an understanding of ML when they engaged with the phases of the modeling cycle at different levels of sophistication. The PSTs did not take advantage of visual representations to support the analysis of the project-based task and to communicate their work. Discussion about the tension between simplifying models and reflecting the real problem and directions for future study are suggested.

**Keywords:** mathematical literacy, opportunities to learn, pedagogical content knowledge

## INTRODUCTION

The Vietnam current school curriculum, introduced in 2002, indicates knowledge and skills to develop for students. This curriculum does not emphasize the relationship between mathematics and the real world, nor it mentions mathematical modeling. To meet the demands of societal development, the reformed school curriculum follows a competency-based learning model, which will be implemented beginning of 2020 (Vietnam Department of Education, 2018). In this reformed curriculum, mathematics is specified as a subject to help students develop mathematical competencies, which include mathematical thinking and reasoning, mathematical modeling, mathematical problem solving, communication, and using mathematical tools and software. It also underscores the close relationship between mathematics and the real world, helping students see the meaning and applications of mathematics, and experience mathematical applications in their lives. This curriculum will create a challenge for mathematics teachers including preservice teachers (PSTs) because the teachers lack the knowledge to teach mathematics contextually. Therefore, we implement a two-year project focusing on developing PSTs' mathematical literacy, in turn, preparing them to teach mathematics contextually to their future students. The purposes of this project are twofold (a) to explore models to implement mathematical literacy into the current teacher education program, and (b) to research the

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effectiveness of the program. The model should integrate knowledge, skills, and capacity for the PSTs to experience mathematical literacy as active learners, and to teach mathematics contextually as future teachers. Part of this project, this study focuses on PSTs working on a project-based task implemented in the second semester during the project.

# CONCEPTUAL FRAMEWORKS

## Mathematical Literacy

Mathematical literacy (ML) is an individual's ability to understand and use mathematics in a variety of contexts, including everyday life, professional, and scientific settings. Mathematics serves as a tool to describe, explain, and predict phenomena (The Organization for Economic Cooperation and Development [OECD], 2013). In turn, individuals appreciate the role mathematics plays in the world and how it prepares them to be constructive citizens and make well-founded judgments and decision. Additionally, OECD (2013) utilizes the mathematical modeling cycle (cf. Kaiser & Stender, 2013) to describe students' actions when facing challenges. In this cycle, students are required to apply mathematics and perform actions, such as formulate, employ, and interpret mathematics in a variety of contexts.

Several researchers and mathematics educators highlight the difference between mathematics and ML and argue that some people who are good at mathematics are not necessarily good at ML (e.g., Steen, 2001). The teaching focusing on developing ML might be different from that of developing mathematical understanding. For example, whereas the aim of developing school mathematical understanding is to help students climb the ladder of abstract structure, ML is anchored in data that are derived from the empirical world. In addition, school mathematics tends to develop school-based knowledge, but ML involves mathematics acting in the world (Steen, 2001). We take this difference as a lens to guide our designing and researching in this project as arguably our PSTs experience (advanced) mathematics in the teacher education program, but not necessarily ML. It is important to note that ML used in this context is not limited to understanding and applying arithmetic but the abilities to use different mathematical knowledge, which might include advanced mathematics. Moreover, ML includes not only the skills and knowledge but also the beliefs, dispositions, and habits of mind people need to engage effectively in quantitative situations in life and work (International Life Skills Survey, 2000). The ML concept serves as a foundation to help PSTs make the connection between mathematics and real life. One of the foci in the project is to implement activities embedded in situations in different contexts in real life, professions, and science, which offer the PSTs with the opportunities to use mathematics in solving them and in turn to appreciate the roles of the subject and to develop the utility aspect of the subject, not merely as a platonic view.

## Knowledge for Teaching Mathematics

Teacher knowledge is an important predictor of student achievement because a mathematics teacher's decision-making in class is a function, in part, of her/his knowledge (Schoenfeld, 2010). Educational researchers have conceptualized knowledge for teaching to include subject matter knowledge and pedagogical content knowledge (Shulman, 1987). In particular, pedagogical content knowledge (PCK) refers to:

> The most powerful analogies, illustrations, examples, explanations, and demonstrations—[...] the most useful ways of representing and formulating the subject that make it comprehensible to others... Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them. (Shulman, 1987, p. 7)

This conceptualization of teacher mathematical knowledge informs us to provide the PSTs with opportunities to learn (OTL) ML and to teach ML to their future students. Therefore, we create opportunities for the PSTs to experience ML as active learners and engage in developing their knowledge/skills to teach ML through teaching tasks such as analyzing curriculum, selecting, adapting tasks, and using appropriate approaches to teach. In this paper, we focus on content knowledge (subject matter knowledge) that PSTs exhibit when they engage in a project-based task. This is crucial as project-based learning is sparse in school mathematics. This also serves as a stepping stone to prepare the PSTs on how to teach them in the future. During this experience as active learners, the PSTs appreciate the challenges their future students might face when involving in such tasks and ways to address students' difficulties. The study addresses the research questions:

(1) How does a model of secondary mathematics teacher education that implements ML look like? And

(2) What aspects of ML understanding is evident in PSTs' project works as part of the implementation of the project?

# RESEARCH DESIGN

## Research and Data Collection

This ongoing two-year project has been implemented with a cohort of 120 PSTs. The cohort started participating in the project in September 2017, at the beginning of their third year in the program. We adopted a design-based research methodology (Cobb et al., 2003) that involves continuing data collection and data analysis, and curriculum development and implementation. First, we identified gaps related to ML in the current mathematics methods courses in the program. The current program indicates limited opportunities for PSTs to experience ML as learners and to develop PCK for teaching ML. In mathematics methods courses, the opportunity to learn in ML is limited (1.2 % of total training time) to an introduction to mathematical modeling and the Program for International Student Assessment (PISA). In 2017, we collected empirical data on the PSTs' opportunities to learn ML and their beliefs about mathematics and mathematics teaching and learning. The initial data analysis sheds light on our curriculum development, focusing on PSTs' mathematics methods courses and PSTs' field experience (school placement), and how to change PSTs' disposition/attitudes towards the subject.

In addition to measuring OTL and beliefs, we assessed PSTs' modeling competencies as a proxy of their ML by using both a multiple-choice test and open-ended word problems. We adopted a research-based multiple-choice test (Haines et al., 2002) to measure the PSTs' understanding of ML when they started the methods courses in the program. This tool was developed measuring aspects of modeling competencies, which was administered to the PSTs individually within 60 minutes. We also got the PSTs to work in pairs for 120 minutes on open-ended tasks focusing on four content areas: shapes, quantity, data and chance, and change. The data provided us with information about the PSTs' current content knowledge related to ML and the weakness and strength the PSTs have prior to the methods courses. All the data were used to incorporate the opportunities to learn ML.

We have conducted interviews (task-based and stimulated recall). Other data sources include notes from classroom observations, PSTs' reflection on their placement related to ML, and their lesson plans. In 2019, the project will finish with a post measure of PSTs' ML OTL, beliefs, and modeling competencies. Post interviews will be conducted on some participants (cases). **Table 1** summarizes the timeline of data collection.

**Table 1.** Project timeline and data collection

| Content | Pre | Curriculum Implementation and Data collection | Post |
|---|---|---|---|
| | • OTL ML measures (Individual)<br>• Beliefs about mathematics and mathematics teaching and learning (Individual)<br>• Multiple choice modeling test – Open-ended modeling tasks (Pairs)<br>• Stimulated recall interviews about their OTL and beliefs<br>• Stimulated recall interviews on modeling competencies | • Curriculum Implementation: Mathematics methods courses<br>• School Placement<br>• Artefacts collections – Student works and presentations on ML tasks<br>• Classroom observations | • OTL ML measures (Individual)<br>• Beliefs about mathematics and mathematics teaching and learning (Individual)<br>• Multiple choice modeling test – Open-ended modeling tasks (Pairs)<br>• Special-cased interviews on their experience of the program |
| **Timeline** | 09/2017 | 09/2017-05/2019 | 05/2019 |

## Curriculum Development and Implementation

Opportunities to learn and teach ML were incorporated into four methods courses: Mathematics Teaching Methods and Assessment of Mathematics Learning (Semester 1 of 2017-18), Mathematics Curriculum Development and New Trends in Mathematics Teaching and Learning (Semester 2 of 2017-18). Additionally, we asked the PSTs to reflect on their experience of ML when they were at their school placements. The first placement was mainly focusing on observing real classrooms and planning mathematics lessons but not implementing the lessons. In this instance, the PSTs were asked to reflect on how the observed lessons offer OTL ML and nominate their one best lesson plan that incorporates ML. In the second placement when PSTs will plan and implement their lessons in real classes, they will report on how they incorporate ML into the classes as well as reflect on the challenges/success they have when teaching ML.

First, we exposed PSTs to tasks that offer rich opportunities to engage in ML as active learners. ML tasks have been integrated into mathematics methods courses, which range from standard applications to true (authentic) modeling problems (Tran & Dougherty, 2014). These tasks were adapted from researches (e.g., PISA) to fit in the context of Vietnam. Some tasks were created based on the project team's experience of the training program and understanding of the local context, such as designing birthday cake boxes and designing Hue University of Education parking lots for staff and students. We scaffolded PSTs' experience of ML tasks by introducing them with increasing levels of authenticity tasks (Palm, 2009; Tran et al., 2016) that were solved within different time periods, such as several tasks in one session (Semester 1 of 2017-18 academic year), one task in a session (Semester 1 of 2017-18), and project-based tasks that last for several weeks (Semester 2 of 2017-18 school year). These tasks necessitate the use of realistic considerations, not merely mathematical tools. We aimed to help the PSTs to experience revising model and validating process as they went through the modeling cycle.

Second, we prepared PSTs with PCK to teach ML. In their third year of the program, PSTs were introduced to the modeling cycle (OECD, 2013) to inform phases that students generally go through when solving modeling problems and to reflect on the process of solving ML tasks. In Semester 2 of 2017-18, the PSTs were exposed to knowledge about ML and how to incorporate ML into the current curriculum. PSTs analyzed current school curricula to investigate how ML was introduced in the documents and contrast them with the reformed curriculum. They also explored how ML was emphasized in curricula from other countries. PSTs were asked to plan a lesson that integrates ML into the content specified in the curriculum. In Semester 1 of their fourth year, PSTs will be asked to analyze tasks based on the modeling cycle and the level of authenticity and then adapt them to incorporate into real lessons. In addition, they will analyze student works on modeling tasks and how to evaluate them as an assessment practice.

## Project-based Task and Data Collection

In Semester 2 of 2017-18, PSTs were asked to work on the following project-based task: "Currently, on our university campus, there are five parking regions that are close together that look quite messy. Can you design a parking lot for the university to solve the current issue so that it looks neat?" This task was similar to tasks found in literature, yet the uniqueness is that vehicles include cars, bikes, electric bikes, and motorbikes, not just cars or motorbikes. PSTs were asked to work on this project for four weeks in groups of 4-5 and report to the class in Week 4. Students presented weekly on their progress of the project to get feedback / questions from peers (not in their groups) and the lecturers to improve their reports. They submitted their written report and gave a presentation to the class. The data for this task included their written reports. A total of eight (8) written reports were collected.

## Data Analysis

To evaluate the initial success of the implementation, we focused on preliminary results related to different ways the PSTs approached an authentic project-based task of designing a parking lot for the university. As adopting OECD's (2012) conceptualization of ML, we are interested in how the PSTs carried out the modeling process when they engaged in the project-based task. Especially, we looked for how the PSTs: (a) formed mathematics problems from the real-world situation, (b) took into account the assumptions, the estimations when simplifying the problem, (c) dealt with mathematical tools, (d) interpreted the results, and (e) critiqued the model they built and revised their model. In addition, the situation was provided in real life, and the PSTs had the freedom to pose a question and in turn utilize mathematical tools to solve their own problem. There would be tension about the formation of complex problems that call for complicated mathematical tools to accomplish vs. of simple problems with straightforward tools. Therefore, we looked into how such mathematical tools were utilized when solving problems: are they straightforward arithmetic or advanced mathematical topics. In addition, we look for PSTs' evidence of justification in their written work and their mathematical representations. A third aspect was their communication including their writing, their use of mathematical conventions and their reference. **Table 2** summarizes the coding framework used in this study.

A group of five researchers met to discuss the coding framework and refine the criteria until consensus. Two meetings of 1.5 hours each were conducted on two written projects to help understand the framework and to try it out on the projects. A discussion was conducted to reconcile the coding. Then, each researcher coded

**Table 2.** Coding Scheme

| Criterion | | Level 4 | Level 3 | Level 2 | Level 1 |
|---|---|---|---|---|---|
| **Demonstration of Mathematical Understanding and Application (MA)** | **(a)** | Appropriate selection and use of definitions, results and rules in more than one mathematical topic | Appropriate selection and use of definitions, results and rules in more than one mathematical topic | Appropriate selection and use of most definitions, results and rules in more than one mathematical topic | Recall and use of definitions, results and rules in only one mathematical topic |
| | **(b)** | Error free and proficient use of complex mathematical procedures | Attempt to use of some complex mathematical procedures (which could be unaccomplished) | Accurate and proficient use of most simple mathematical procedures | Mainly use of simple mathematical procedures |
| | **(c)** | Mathematical representations are used in a systematic manner | Most mathematical representations are used in a systematic manner | Some mathematical representations are used in a systematic manner | Very few mathematical representations are used |
| | **(d)** | Justification are appropriately documented | Evidence of justification of most decision making | Evidence of justification of some decision making | Little or no evidence of justification of decision making |
| **Undertaking mathematical modelling (MM)** | **(a)** | Poses a complex problem from a real-world situation | Poses a complex problem from a real-world situation | Poses a simple problem from a real-world situation | Poses a highly simplistic problem from a real-world situation or the problem is not realistic |
| | **(b)** | All necessary assumptions in modelling are identified and all estimations and sources of values for quantities in modelling are clearly justified. | Most valid assumptions are identified, and most estimations are identified and justified | Identifies at least one valid assumption and some estimation are identified | No valid assumption is identified, and no estimation is identified |
| | **(c)** | Interprets the results of solving the mathematical model(s) within the real-world situation to answer the question posed | Interprets the results of solving the mathematical model(s) within the real-world situation to answer the question posed | Attempt to interpret the results of solving the mathematical model(s) within the real-world situation to answer the question posed | No interpretation of the results of mathematical activities within the real-world situation |
| | **(d)** | Critiques the strengths and limitations of the models developed suggesting refinements and checks the validity of model(s) used. | Critiques the strengths and limitations of the model(s) sourced. | No critiques the strengths and limitations of the models | No critiques the strengths and limitations of the models |
| | **(e)** | A range of resources is used in interfacing with the real world to generate/collect data and to perform mathematical analysis. | A range of resources is used in interfacing with the real world to generate/collect data and/or to perform mathematical analysis | Some resources are used in interfacing with the real world to generate/collect data and/or to perform mathematical analysis | Very few or no resources are used |
| **Communication (MO)** | **(a)** | Correct use of mathematical language and terminology, and conventions. | Correct use of mathematical language and terminology, and conventions. | Generally correct use of mathematical language and terminology and conventions (might have a few errors). | Errors found in the use of mathematical language, terminology and conventions |
| | **(b)** | Writing is concise, well-structured and error free | Writing is clear and coherent, with very few errors | Writing is largely free of errors that affect readability | Errors in writing hinder communication |
| | **(c)** | Excellent use of visual representations for illustration, display and facilitation of mathematical analysis. | Evidence of use of visual representations to facilitate mathematical analysis. | Visual representations purely for illustrative & display purposes. | Poor use of visual representations or no visual representation |
| | **(d)** | All reliable sources used acknowledged in correct style | Most reliable sources used acknowledged in correct style | Some reliable sources used acknowledged | Few or none reliable sources used acknowledged |

the remaining written projects independently and regrouped on succeeding meetings to discuss their coding until consensus. Therefore, the results presented show the final coding that was agreed upon the group.

We looked for (a) evidence PSTs took realistic concerns into account (data, information, technical considerations) when designing the university parking lot and (b) their experience of different phases of the modeling cycle when working on the task. We identified how the PSTs transferred from real life to mathematical problems and what variables they took into account to formulate mathematical models. We examined how they solved the problems and interpreted them back to real-life issues. We chose two sample projects to elucidate the two different ways the PSTs approach the project-based task.

**Table 3.** Coding for eight projects

| | ID1 | ID2 | ID3 | ID4 | ID5 | ID6 | ID7 | ID8 |
|---|---|---|---|---|---|---|---|---|
| MAa | 3 | 3 | 3 | 3 | 2 | 1 | 1 | 1 |
| MAb | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 |
| MAc | 3 | 3 | 2 | 3 | 1 | 2 | 1 | 2 |
| MAd | 3 | 3 | 1 | 1 | 1 | 3 | 1 | 1 |
| MMa | 3 | 3 | 2 | 3 | 2 | 2 | 1 | 1 |
| MMb | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| MMc | 2 | 2 | 2 | 1 | 3 | 3 | 2 | 2 |
| MMd | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| MMe | 2 | 3 | 1 | 1 | 1 | 3 | 1 | 1 |
| MOa | 3 | 3 | 3 | 3 | 3 | 2 | 1 | 1 |
| MOb | 2 | 3 | 2 | 2 | 3 | 3 | 3 | 3 |
| MOc | 3 | 2 | 1 | 3 | 2 | 2 | 2 | 2 |
| MOd | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |

## RESULTS

**Table 3** summarizes the coding for eight projects. In half of the projects (4), the PSTs used appropriate definitions, results, and rules in more than one mathematical topic, whereas three (3) projects utilized only one appropriate mathematical topic (arithmetic). For the four projects that addressed more than one mathematical topic, they attempted to use complex mathematical procedures when solving their own problem, but they did not accomplish their problem. For the remaining four projects, they used simple mathematical procedures accurately and proficiently. Three written reports included most mathematical representations in a systematic manner, three used some, whereas two used very few mathematical representations. Regarding their justification, only three projects provided appropriate evidence of justification for most of their decisions, whereas the other five provided very little or no evidence of justification.

Regarding the modeling stages, three problems posed were quite complex as they tried to address the optimal problems, whereas three identified simple mathematical problems, and two specified a problem that is too simplistic. More than half (5 out of 8) of the written projects indicated at least one valid assumption and estimation when solving the problem. Likewise, the majority of the projects attempted to interpret the results of solving the mathematical models to real life. In two projects, PSTs attempted to critique their model without revising their model whereas the remaining took the model as is. The majority of the projects (5) used very few resources when solving their problem, whereas only two used them to collect data for their solution.

The majority of the projects (5) used correct mathematical language, conventions in their written projects, whereas two (2) projects had errors in mathematical conventions. Their writing was comprehensible when five reached the third level of coding indicating that their writing was clear, coherent that might have very few grammatical errors. The majority of the projects used a visual representation for display and illustrative purposes, whereas only two projects used the representation to facilitate mathematical projects. Almost all (7) the projects did not cite proper resources on their written product.

The analysis showed that the PSTs formulated three mathematical problems or a combination of them: (a) design parking lots based on the information about the number of vehicles, (b) find the cost to build the parking lots, and (c) maximize the utility of the parking lots. The analysis revealed that the PSTs used a combination of arithmetic and proportions as main tools on this task. Some used sampling and data collection techniques to estimate the number of vehicles and used direct measurement and area formulas. Some built regression models to predict the cost.

Two samples from PSTs were chosen to (a) highlight the PSTs' considerations of real-life issues and the collecting of empirical data (measurement of the parking lot, surveying numbers of each of the vehicles) and (b) represent different mathematical tools the PSTs used to solve relevant mathematics problems formulated from real-world problems (e.g., arithmetic, advanced mathematical tools such as linear programming).

### Surveying the Number of Vehicles and Designing Parking Lots

Group 1 specified real-life problems to address the issues messiness of the parking in Hue University of Education. They evaluated the quality of Hue University of Education's current parking and provided a plan for building the new facility with given funding. They found information about the number of vehicles present

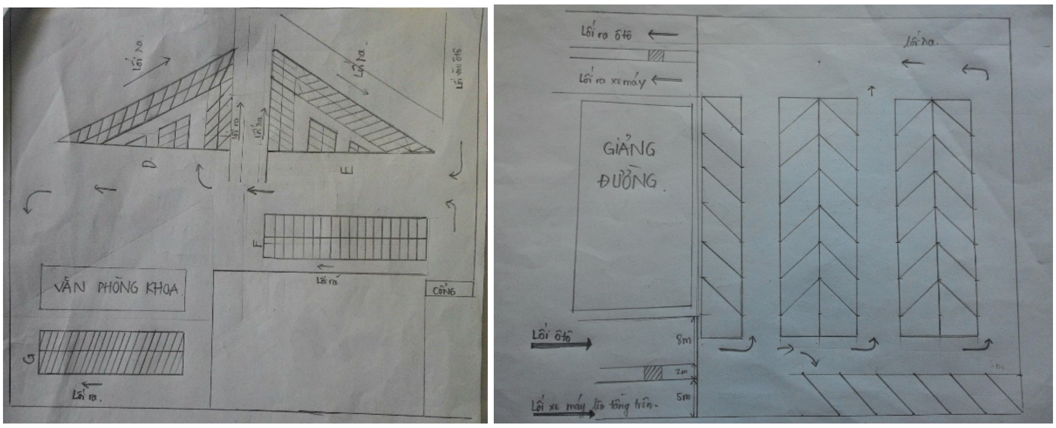


**Figure 1.** Layout of vehicles into roles in two parking lots

daily at the university and areas available for parking. These were the two sub-mathematical/statistical problems formulated from the real issue.

## Finding the Number of Vehicles

They searched the university website (http://www.dhsphue.edu.vn) for information about the number of staff members and students. However, the data might not reflect the exact number of vehicles, which was the main variable to consider when solving the problem (validating). The group then surveyed the number of vehicles of each type on four random weekdays. When collecting the data from the four days, on average, they found the percentages of vehicles in each of the parking lots, H (50%), DEG (40%) and GV (10%). In addition, they estimated the number of vehicles of each type motorbikes (60%), electric-bikes (15-17%), regular bikes (20-22%), and cars (2-3%). They estimated 1500 vehicles on the student parking lot and 180 on the staff parking lot. Therefore, they decided to design two parking spaces, one for students and one for staff. The student parking lot can hold 900 motorbikes, 250 electric-bicycles, and 350 bicycles. The staff parking lot can hold 135 motorbikes and 45 cars. During this process, the students used sampling and surveying as mathematical/statistical tools to collect data. Moreover, they utilized knowledge about proportion and percentages to estimate the capacity for each type of vehicles in each parking lot.

## Designing Parking Lots

On the second sub-problem, the PSTs measured the sizes of current parking lots by applying their knowledge about the area. They drew a floor plan with specified dimensions for the parking lots.

The PSTs then searched for dimensions of each vehicle type to decide the appropriate space for them using a rectangular model and compared the area of the models to those of the real parking lots proportionally. They found that one-story parking lots would not be sufficient to meet the demand of the space for all vehicles; therefore, they needed to look for an alternative design. The PSTs investigated parking lots in other universities and those of a supermarket in the city to look for parking designs and how to operate the parking. As they found that no students travel to the university by cars, they decided to create one parking lot for staff and two for students. After collecting all relevant data, they designed two-story parking that reserved one story for 45 cars and the other for motorbikes. Particularly, with their survey of car dimensions and spaces between two cars (length: 5,5 m, width: 2,3-4 m and the gap: 4-6 m), they figured out that the area of the parking should be about 36*30 (m^2).

For the students' parking lots, this group revamped the model of current parking lots by including specified dimensions for each row, taking into account the dimension of bikes, motorbikes, and electric bikes with the length of 2m and width of 0.8m. The distance between two consecutive rows is 1.8m. Therefore, they used a 2-meter square for each vehicle in these parking lots. They worked out the number of vehicles for each of the parking lots in the university and checked if the lots meet the demands of student vehicles from their survey (see **Figure 1**). After finishing these sub-problems, they determined the cost to build such parking lots. They then submitted their findings and presented their plans to the class.

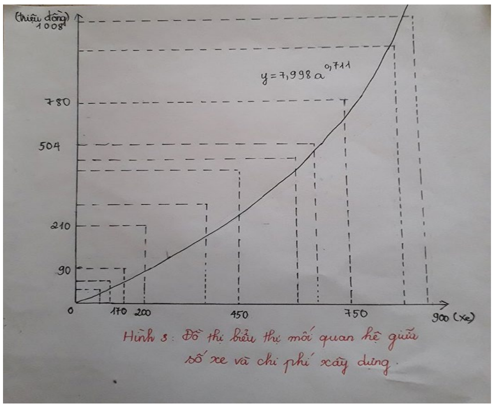


**Figure 2.** Power functions to predict the cost for one parking

## Focusing on Predicting the Cost of Building Parking Lots on the Number of Vehicles

Group 2 surveyed the number of vehicles on three random days and found 1600 vehicles per day (motorbikes, electric-bikes, regular bikes) for both staff and students. This group measured the sizes of parking lots and calculated areas. They also decided that one of the parking lots needed to be two-story. They decided to build three parking lots: one two-story and two one-story that would connect to the three entrances into the University: G32, G34, G36. Additionally, they formulated a mathematical problem to predict the cost of the parking when knowing the number of vehicles.

Based on the information about the cost of materials and relevant equipment needed to operate the parking lots (e.g., camera) and the cost to demolish the current parking lots in the University, they recorded the data on a table. The data were based on the following variables: the money to demolish the current parking lot, how much of the old infrastructure could be reused, the area of the parking lots, the number of stories, and the number of vehicles in each of the parking lots. They then graphed the data in a coordinate plane with one axis used for the number of vehicles and the other for the cost (in Vietnamese dong). They created a power function as an approximation for the collected data to come up with a model. The coefficients were an estimation based on the data, without checking if the models were good for prediction, or a regression model to minimize the total sum to minimize the total sum of square deviations (**Figure 2**). The two models are shown in **Figure 2**.

A two-story parking lot at G34 for staff:

Model 1: C = $\boldsymbol{C_{32}} + \boldsymbol{C_{34}} + \boldsymbol{C_{36}} = 0{,}\mathbf{918}.\, \boldsymbol{x_{32}}^{\mathbf{0{,}95}} + 2{,}05.\, \boldsymbol{x_{34}}^{\mathbf{0{,}95}} + 1{,}22.\, \boldsymbol{x_{36}}^{\mathbf{0{,}83}}$

(x … is the number of vehicles in the parking …, and C… the cost to build parking lot ...)

Two-story parking lot at G32 for students and keep the staff G34 parking lot as is:

Model 2: C = $\boldsymbol{C_{32}} + \boldsymbol{C_{36}} = 1{,}28.\, \boldsymbol{x_{32}}^{\mathbf{0{,}98}} + 1{,}22.\, \boldsymbol{x_{36}}^{\mathbf{0{,}83}}$

# DISCUSSION

The analysis showed that the PSTs used more than one mathematical topic appropriately and attempted to use complex mathematical procedures. What they need to work on are mathematical representations and justification criteria. This might be due to their rare opportunities in writing in mathematics classes where they need to present their whole work. Related to this issue is the PSTs' use of visual representations. Most did not attend to the role of the visual form to help facilitate their analysis, instead, they are familiar and comfortable with the symbolic representation. Regarding the modeling process, the PSTs need to explicitly make assumptions in simplifying their real-life problems. In addition, they should experience more opportunities to critique and revise their models to improve it. This again can be explained by their lack of opportunities to engage in this behavior in mathematics classes. Most notably, the PSTs displayed their lack of attention to using resources when solving problems and acknowledge them.

This ongoing project is in the process of implementing the innovative curriculum focusing on the developing PCK for the PSTs to teach ML. We have not collected the post data to investigate the effectiveness of the program. However, at this stage, the data suggest that the PSTs started to experience mathematics in a different way--not merely considering real-world contexts as a cover, which is easily stripped out to reveal the mathematics. Additionally, the PSTs experienced uncertainty when using mathematics to solve problems they encounter in their lives. However, opportunities to discuss the difference between their estimations of vehicles were not taken, which could be powerful for validation. When predicting the cost to build parking lots, the PSTs need to balance how much they simplify the model so that they can formulate a problem that is solvable versus how to develop a model that is sophisticated enough to capture the real-world yet challenging -to-solve problems with their current mathematical knowledge. The PSTs were not familiar with regression models in prediction and unsure how to evaluate the goodness of their model; such findings call for possibly having collaborations between mathematics educators and mathematicians who are responsible for training the students. A question emerged is what the program would look like if the mathematicians take an ML perspective when teaching their courses: how could PSTs' mathematical knowledge be strengthened?

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## Disclosure statement

No potential conflict of interest was reported by the authors.

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