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Preparing, leading, and reflecting about whole-class discussions in a lesson study in initial mathematics teacher education

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ABSTRACT

Based on the analysis of a lesson study carried out in an initial mathematics teacher education program, we aim to know how a prospective teacher prepares, leads, and reflects on the whole-class discussion, during her participation in a lesson study based on the framework of a model that we developed for this purpose. We follow a qualitative approach, and the data was collected through observation and recording the sessions and document collection. The data is analyzed using that model that presents the main practices and actions of the teacher in preparing and leading the whole-class discussion. The results show that the prospective teacher managed to promote a productive discussion, inviting the pupils to participate, intervening according to the situation, and analyzing the pupils' work with a focus on mathematical ideas. This lesson study seems to have contributed to the development of the prospective teacher's knowledge of how to prepare and lead a whole-class discussion.

Keywords: whole-class discussion, lesson study, mathematics teaching, initial teacher education

INTRODUCTION

Lesson study is a formative process known to promote the development of various aspects of teachers' knowledge, but also of prospective teachers (Ponte, 2017). This teacher education process focuses on pupils' learning and promotes moments in which participants plan a research lesson in detail, in a reflective and collaborative environment. Then, the lesson is taught, and the participants discuss and reflect on the pupils' learning, considering the connection between theory and the moments of teaching practice, which promotes the development of their knowledge.

In Japan, the country of origin of the lesson study, the structured problem-solving curricular approach (Fujii, 2018) is usually used in the research lesson. This approach is similar to the exploratory approach (Ponte, 2005), often used in lesson studies carried out in Portugal, in which pupils work on tasks to support their learning. However, conducting exploratory lessons has been pointed out as a great challenge for teachers and, particularly, for prospective teachers, since it requires them to have extensive knowledge in several domains regarding the teaching of mathematics. Much of this complexity is associated with leading the whole-class discussion, and it requires prospective teachers to relate several dimensions such as knowledge of the content, didactics and pupils and the way they learn (Fujii, 2018; Stein et al., 2008). In addition, and given their limited experience in the classroom, prospective teachers are often surprised by the pupils' work and interventions during the discussion, making it difficult for them to know what and how to respond to their interventions, in order to best guide the discussion (Fujii, 2018; Stein et al., 2008).

Lesson study has been recognized as a formative process capable of promoting the development of participants' knowledge, namely on whole-class discussions (Fujii, 2018; Ni Shuilleabhain & Bjuland, 2019). However, there is still a lack of research on the contribution of this teacher education process to the development of prospective teachers' knowledge about the preparation, leading and reflecting on these discussions. And, although there are several models of analysis of whole-class discussions, they tend to focus on specific aspects, which is why we developed a comprehensive model that considers all the fundamental aspects of that moment of the lesson and on which we relied on analyzing the practices and actions of a prospective teacher in the preparation and leading of the discussion. We use this model both to prepare and to analyze the whole-class discussion. So, our research question is to know how a prospective teacher prepares, leads, and reflects on a whole-class discussion, within the framework of the model, during her participation in a lesson study.

PREPARING, LEADING, AND REFLECTING ON WHOLE-CLASS DISCUSSION THROUGH LESSON STUDY

Lesson study is a collaborative and reflective teacher education process in which a group of teachers, based on their experience and on preparatory study, seeks to find ways to promote better pupil learning (Fujii, 2018). Although usually carried out with in-service teachers, lesson study has been adapted to initial teacher education. With their participation, prospective teachers have the opportunity to plan, lead and observe, discuss and reflect on real classroom situations, promoting a close relationship between the theory they learn during the initial teacher education courses and teaching practice and, therefore, develop their knowledge about teaching and learning (Larssen et al., 2018; Ni Shuilleabhain & Bjuland, 2019).

Preparing the research lesson, the participants consider in detail various aspects such as the selection of the mathematical topic and the learning aim to be developed in the lesson. The teachers begin by clearly identifying the mathematical content to be worked on and define the learning aim that they want to develop with the pupils (Boerst et al., 2011). Then, they select, adapt or elaborate the tasks to propose to the pupils, which must meet a certain degree of challenge and allow them to build or deepen their knowledge with regard to important mathematical concepts, procedures, representations, ideas and connections. Once the task is defined, the teachers solve it, anticipating the strategies that pupils can use, as well as possible difficulties, both in relation to the interpretation of the statement and its solution (Kooloos et al., 2023). To this end, it is important that teachers consult the curriculum documents and other pertinent literature on strategies and difficulties that pupils usually depict in learning the content in question (Stein et al., 2008). Consulting these documents also helps the teachers understand what ideas, knowledge and skills pupils are expected to possess to solve the proposed task (Boerst et al., 2011). Then, they prepare the undertaking of the task in class, predicting how they may promote the establishment of connections between the different responses of the pupils and between these and the aim they have set for the lesson. Starting from the anticipation of the pupils' answers–regardless of their degree of correctness and sophistication–the teachers consider which ones should be selected to take to the whole-class discussion and by what sequence, considering the aim of the lesson (Fujii, 2018). The teacher's interventions during the discussion are also prepared, seeking to take advantage of possible situations of disagreement, to understand the pupils' difficulties and to interpret the reasoning involved in their answers (Duarte et al., 2024).

Conducting the research lesson, by a teacher or prospective teacher, is a fundamental phase of the lesson study and allows the other participants to observe it and record important information to be later discussed. Conducting the lesson planned in the previous phase and observing it, are important actions for the development of their knowledge that are motivated by the direct relationship that is created between theoretical aspects and teaching practice (Leavy & Hourigan, 2016). In the research lesson, when the exploratory approach is adopted, the teacher begins by presenting the task to the pupils and clarifies aspects of the statement that may raise questions. Teachers should pay attention to ensure that their interventions do not condition the solution of the task and the whole-class discussion, avoiding influencing pupils or inducing them to respond, explicitly suggesting relevant information (Duarte et al., 2024; Stein & Smith, 2009). In the next phase, the teacher monitors the pupils' autonomous work, identifying their difficulties and errors, strategies and reasoning. The teacher also decides which solution strategies should be shared in the discussion, and sequences them according to criteria thought out during the preparation of the discussion (Fujii, 2018; Kooloos et al., 2023; Stein et al., 2008).

At the end of the pupils' autonomous work phase, the teacher initiates the whole-class discussion and guides it, ensuring that it follows the intended course through interventions appropriate to the situation and the aim (Ponte et al., 2013). With his/her interventions, the teacher motivates the pupils to participate in the discussion, involving most of the pupils and not just one group, so as not to lose the attention of the others or run the risk of making this moment unproductive. Then, when the pupils selected present their response to the task, the colleagues must listen, analyze and respond to the reasoning presented, critically arguing their interventions (Boerst et al., 2011). The orientation of the discussion and the development of the pupils' learning depend strongly on the way the teacher conducts this moment of the lesson and allows the pupils to have an active and central role in the explanation of their reasoning and strategies.

Also, during the whole-class discussion, the teacher analyzes the pupils' work focusing on their mathematical ideas and asks them questions in order to support them to express and clarify their reasoning, highlighting important mathematical ideas. Also based on this analysis, the teacher considers how to establish connections between the strategies presented by the pupils and their relationship with the aim outlined for the lesson, and strives to establish connections with relevant mathematical concepts, representations and procedures (Stein et al., 2008). The teacher also considers the answers that pupils present in an incorrect or incomplete way, in order to allow them to transform these ideas into complete, correct and concise ideas (Author et al., 2024). With the presentation of these ideas, situations of disagreement may arise among pupils, which is an excellent opportunity for them to argue, criticize and explore the errors presented (Stein et al., 2008; Wood, 1999).

The whole-class discussion ends with a final synthesis that may be enriched with the collaboration of the pupils. This moment of systematization is intended to highlight and summarize the main mathematical ideas explored throughout the lesson and to highlight connections between the established aim and other ideas previously known by the pupils (Duarte et al., 2024).

The last sessions of the lesson study, which follow the research lesson, concern the reflection on pupils' learning. In this phase, the participants review their practice and think about how to improve it through the analysis of situations of the research lesson conducted. Such analysis is based on the data collected during the lesson and through which the prospective teachers have the opportunity to critically reflect on their practice, looking for solutions for aspects to be improved, and evaluating the results (Bjuland & Mosvold, 2015).

Several studies have explored the formative potential of lesson study as a means for teachers' and prospective teachers' professional growth (Fujii, 2018; Larssen et al., 2018; Ni Shuilleabhain & Bjuland, 2019; Ponte, 2017). While lesson study has proven effective in promoting collaboration, reflective practice, and connections between theory and practice, its specific contribution to prospective teachers' abilities to prepare and lead whole-class discussions remains less clearly understood. Much of the previous research focuses either on describing the process of lesson study or on participants' general reflections on pedagogical content knowledge (PCK) rather than the specific practices involved in leading mathematical discussions (Bjuland & Mosvold, 2015). Parallel to this, extensive work has examined teacher support for preparing whole-class mathematical discussion. Frameworks such as Stein et al.'s (2008) five practices model has underlined the importance of anticipating pupil responses, monitoring strategies, selecting and sequencing them, and connecting them to mathematical goals. However, this model focuses mainly on teachers' preparation for the discussion than and does not address in dept teachers' actions during the discussion.

Aitjanov et al. (2025) indicate that recent research also suggests that the extent of teachers' professional growth during lesson study may be related to the preparation they receive during their initial teacher education. In their view, teachers who engage more deeply with pedagogical and content knowledge during their preparation tend to assume greater responsibility for instructional decisions and demonstrate higher levels of professional development when participating in collaborative professional learning contexts. From this perspective, lesson study can function not only as a collaborative planning structure, but also as a context in which preservice teachers mobilize and extend the knowledge already developed during their teacher education programs.

Two interrelated limitations may be identified in the existing literature. First, it often isolates the lesson leading stage from the preparatory and reflective phases of teachers' work. Yet, as Fujii (2018) argues, teachers' ability to orchestrate classroom discourse is inherently tied to how they prepare tasks and how they later analyze the discussion. Second, prospective teachers' voices are rarely foregrounded in these models, as most frameworks were developed from research with experienced teachers (e.g., Stein et al., 2008). Consequently, little is known about how prospective teachers mobilize and internalize these practices as they learn to enact discussions within structured teacher education programs like lesson study. To address these limitations, our study draws on a comprehensive model of whole-class discussion developed by the authors. This model integrates the preparation, leading, and reflection phases of discussion into a single analytical structure. Unlike previous frameworks (e.g., Ponte et al., 2013; Stein et al., 2008), it treats preparation and reflection not as adjacent supports but as integral components of the discussion process. The model defines three main practices (initiate the whole-class discussion and intervene according to the situation and the goal, analyze pupils' work with a focus on mathematical ideas, and promote pupils' participation) each including actions that can guide teachers across all moments of planning, enactment, and reflection. This study therefore positions itself at the intersection of two strands of research–lesson study in initial teacher education and models of mathematical discussions–to investigate how a prospective teacher mobilizes and develops these practices within a lesson study cycle. By connecting a structured model of discussion to the collaborative and reflective processes of lesson study, this work contributes to understanding how prospective teachers may construct knowledge for leading mathematical discussions. In addition, previous research emphasizes the role of lesson study in supporting the development of preservice teachers' PCK. PCK refers to the knowledge that teachers mobilize to transform mathematical content into forms that are understandable to pupils, including knowledge of typical misconceptions, representations, and instructional strategies. Studies show that participation in lesson study enables preservice teachers to analyze pupils' reasoning, reflect on instructional decisions, and gradually refine their pedagogical interpretations of mathematical content (Conceição et al., 2022). Similarly, research on teacher education demonstrates that structured opportunities to analyze teaching practice and pupils' responses can support the development of teachers' PCK in specific mathematical domains (Herreros-Torres et al., 2026). These studies highlight the importance of connecting collaborative lesson planning and reflection with explicit attention to how teachers interpret pupils' mathematical thinking.

RESEARCH METHODOLOGY

This research follows an interpretive and qualitative approach (Bogdan & Biklen, 2007) and considers as its data source the sessions of a lesson study undertaken during the initial teacher education of primary teachers at a higher education institution in Portugal. The lesson study was carried out in the 2023/2024 academic year and involved two prospective teachers (Júlia and Alda, pseudonyms) who were attending the second semester of the first year of their master's degree program in teaching. The lesson study sessions were facilitated by a higher education teacher, corresponding to the first author, who supervised the internship and took the role of researcher, preparing and conducting the lesson study sessions. This role included promoting a collaborative and reflective environment, by posing questions, for example, and also organizing the content of sessions, selecting resources such as tasks that could be explored by the prospective teachers.

The prospective teachers had already completed a degree in general education, with a total of 180 ECTS, including 26 ECTS in mathematics and 16 ECTS in internships. During the master's program (120 ECTS), the prospective teachers took several courses on mathematics (18 ECTS) and had longer internships (50 ECTS). Prior to the lesson study, they completed courses in subjects such as sociology and psychology of education, didactics of mathematics, and school organization. The prospective teachers observed several lessons taught by experienced teachers before the lesson study, so they were familiar with the pupils' dynamics in the classroom. The prospective teachers had already participated in a lesson study (Duarte et al., submitted) carried out during the first semester of the same academic year, in which Júlia conducted the research lesson, which learning aim was to understand the relationship between multiplication and successive addition by solving problems with rectangular arrangements, in a grade 2 class. Júlia led a productive whole-class discussion, following the exploratory approach, showing how to invite pupils to participate and to intervene according to the situation and the aim, and analyzing their work with a focus on mathematical ideas.

Table 1. Stages of the lesson study and corresponding sessions

Stages of the lesson study	Sessions (Sn)
Goal setting	S1, S2
Lesson planning	S3, S4, S5, S6
Research lesson	S7
Post-lesson discussion	S8
Reflection	S9

Table 2. Practices and actions to consider when preparing and leading the whole-class discussion and final synthesis (adapted from Duarte et al., 2024)

To prepare the whole-class discussion		To lead the whole-class discussion	To conclude the whole-class discussion with a final synthesis
Before class	**During class**		
Initial preparation - Identify the mathematical content - Define the learning aims considering the curriculum documents - Consult articles on pupils' common difficulties with the topic **Choose the task** - Select, adapt or create the task **Solve the task** - Anticipate pupils' solving strategies - Anticipate pupils' difficulties in interpreting and solving the task **Prepare to carry out the task in class** - Anticipate how to establish connections between the different strategies and the lesson aim - Prepare interventions	**Present the task** - Challenge the pupils and clarify aspects of the statement, without suggesting or indicating the answer **Monitor the pupils' work** - Observe their solving of the task and pose questions to understand their reasoning **Select pupils' answers** - To share in whole-class discussion **Sequence** - The selected answers	**Initiate the whole-class discussion and intervene according to the situation and the goal** - Invite pupils to present their solution according to the established sequence - Support/guide, including encouraging, redirecting and keeping the discussion on track - Challenge and urge pupils to clarify and justify their statements - Inform/suggest, including highlighting important mathematical ideas **Analyze pupils' work with a focus on mathematical ideas** - Make connections between ideas, solutions, representations and strategies used by pupils - Ask questions to support pupils when they express their reasoning - Emphasize important mathematical ideas - Promote situations in which pupils can reformulate erroneous or incomplete ideas **Promote pupils' participation** - Encourage pupils to participate in the discussion, involving the whole-class - Encourage pupils to listen, analyze and respond to their colleagues' reasoning - Ask another pupil to explain a colleague's answer - Promote situations of disagreement	**Conclude, with the collaboration of the pupils** - Summarize and highlight the main mathematical ideas or procedures explored - Show connections with other mathematical ideas already known to the pupils

"At the weekend I showed a colleague of mine some pentominoes. Do you know what pentominoes are? (after listening to the pupils' answers, the teacher explains what is meant by a pentomino if the pupils do not know). Well, I only managed to find five pentominoes, but my colleague told me there are more. So, I thought this would be a good challenge for my pupils! Can you help me find all the pentominoes there are?"

Figure 1. Task proposed to the pupils in the research lesson (Source: Field study, from Júlia's records)

Despite this, she did not always explore the opportunities that arose, and, above all, she found it difficult to get most of the pupils to participate. Taking part in this first lesson study allowed Júlia to develop her knowledge of how to lead a whole-class discussion, especially in terms of preparing for this moment in the lesson.

The sessions of the second lesson study in which Júlia took part are the object of analysis of this article and were organized as shown in **Table 1**. They took place weekly and lasted approximately 90 minutes each.

The lesson study was an integral part of the internship program, which was being carried out in a grade 3 class (pupils aged seven and eight). The prospective teachers were doing their teaching internship in this class and were supervised by a teacher (first author) from the higher education institution, who also took on the role of facilitator, preparing and leading the lesson study sessions. These sessions always focused on the contributions of the prospective teachers, in their interventions in the group discussions and in their analysis of the materials, resources, mathematical tasks and articles they selected and brought for discussion. The prospective teachers and the facilitator constituted the lesson study group.

After analyzing the timetable and planning provided by the school where the prospective teachers were doing their internship, the participants decided that the research lesson (which lasted 90 minutes) would focus on the theme of geometry and measurement and the introduction of the topic of area, with the learning aim of the lesson being to understand the concept of area and to recognize equivalent figures by solving exploratory tasks with pentomino shapes. All the work carried out for the research lesson was based on the participants' discussion of **Table 2** and also considered the exploratory approach, namely the selection of the task, shown in **Figure 1**.

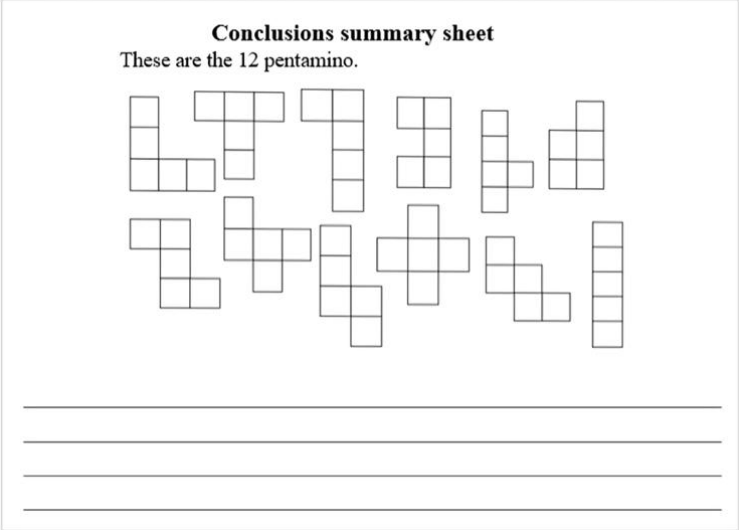

Figure 2. Conclusions summary sheet used in the final synthesis of the research lesson (Source: Field study, lesson study paticipants)

Each group was given five equal squares so that the pupils could manipulate them and build the various pentominoes, and a sheet to draw each construction they made. Larger squares were made available for the pupils to present their answers to their colleagues on the board during the whole-class discussion.

To promote the final synthesis, the prospective teachers decided to explore with the pupils the conclusions summary sheet shown in **Figure 2**, which shows the twelve possible pentomino shapes and includes a space to fill in with suggestions from the pupils about the main mathematical ideas explored in the lesson, particularly relating the concepts of area and equivalent figures to pentomino shapes.

Júlia conducted and Alda observed the research lesson, enabling us to access the results of all moments of preparation, enactment and reflection on the whole-class discussion, which is why we choose her as the focal case of this study. In this investigation we analyze how Júlia prepared, led and reflected on the whole-class discussion. The data was collected through observation of the lesson planning sessions, the carrying out of the research lesson, the discussion and reflection sessions, and, finally, through the collection of documents produced: the lesson plan; Júlia's written report (written before the research lesson) that presents, based on consultation of articles and other documents, the pupils' usual difficulties in learning the mathematical topic; the task solutions presented by Júlia; and her individual written reflection. The lesson study sessions were recorded and the relevant parts for this study were transcribed. The data was analyzed deductively (Amado, 2013), considering **Table 2**, which presents the teacher practices to be considered when preparing and leading the whole-class discussion and the final synthesis and the actions that make up these practices. We carried out a content analysis of the data, starting by making notes and initial analytical observations in order to have a global view of the data and what they suggest. Then, we selected transcribed episodes that highlight relevant situations regarding the presence of practices and actions shown in **Table 2**. Each extract was then coded using these practices and actions. To ensure trustworthiness in the interpretations, the three authors discussed the coding of data, in all cases reaching a consensus. This approach allowed us to identify substantial incidents from the preparation, leading and reflection stages of the research lesson led by Júlia. Through this analysis we sought to understand how she prepared, led and reflected on the whole-class discussion during her participation in the lesson study.

We followed the usual ethical procedures in qualitative research in education, informing the participants in a meeting prior to the beginning of the study of the aims and methods of the research, guaranteeing that they would remain anonymous and that the results would only be used for research and teacher education purposes. The participants signed an informed consent in the first lesson study session. The research was approved by the Ethical Committee of Instituto de Educação da Universidade de Lisboa by decision of 28.02.2022.

RESULTS

Preparing the Whole-Class Discussion

Initial preparation

After analyzing the timetable and the planning provided by the school, the prospective teachers tried to understand what mathematical content could be explored with the pupils by the time of the research lesson. They suggested that this lesson should focus on introducing the topic of area and, consulting the curriculum documents, they formulated the learning aim for this lesson:

> Alda: In *aprendizagens essenciais* [curriculum] it says, "interpret and model situations involving area and solve associated problems by comparing different solving strategies."

> Júlia: I think we need to change it a bit. It doesn't even mention pentominoes. Instead of problems, we could use exploratory tasks ...
>
> Facilitator: Why?
>
> Júlia: It depends on the task, I know. But I've seen somewhere they're supposed to explore the pentomino squares and find out how many there are. And that seems to me to be more of an exploration than a problem. What if it's just "Solving exploratory tasks with pentominoes that involve the concept of area," is that OK?
>
> Facilitator: And doesn't it make sense to include the concept of equivalent figures?
>
> Júlia: We can add it, yes ...
>
> Facilitator: Could it be "solving exploratory tasks with pentomino squares that mobilize understanding of the concept of area and recognition of equivalent figures?" (S2)

After defining the mathematical content and the learning aim of the research lesson, the prospective teachers consulted documents on the usual difficulties pupils have in learning the concept of area, stating that "the difficulties pupils experience in learning area stem from the fact that they sometimes confuse the concepts of length and area, since both involve measuring length" and "[t]his type of confusion has consequences for the use of units of measurement, i.e., selecting the most appropriate unit of measurement" (report). To help pupils overcome these difficulties, Júlia said that

> "the teacher should provide tasks that enable pupils to understand that the area of an object does not change by 'displacement' and that the measurement can be quantified by repeating a unit, with the resulting number depending on the size of the unit of measurement. The pupils develop this ability in contextualized situations, using materials and solving measurement problems" (report).

The analysis of the curriculum and of other materials provided the main foundation for the prospective teachers to identify important ideas to take into consideration in learning about area and what manipulative materials they could use. Analyzing these documents also guided them in clearly defining the learning aim, which in turn made them reflect on the type of task to select, opting for an exploratory task since they were going to introduce the concept of area in this lesson. The report they read allowed Júlia to analyze documents on the difficulties usually experienced by pupils in learning about the concept of area, which made anticipating their answers more consistent. Júlia's preparation demonstrated an integrated application of several practices from the whole-class discussion model. She began with initial preparation, consulting curriculum guidelines and relevant research to identify key mathematical concepts (area and equivalence) and pupils' common misconceptions. This study activities align with the necessity of connecting task design to documented learning difficulties. Júlia's documentation-based reasoning indicated an important form of pedagogical reasoning, moving beyond procedural planning toward conceptual alignment between aims, curriculum, and task.

Choosing the task

Júlia suggested giving the pupils an exploratory task in which they would build pentominoes with the purpose of finding out how many there are in total and, from there, understanding the concepts of area and equivalent figures:

> Júlia: I thought of a task with pentominoes, because I saw that there are 12 [different pentominoes] and the task I saw asked them to find them all. So, the pentominoes are all different ... They're always made up of 5 squares. [And] that the area is always the same! And if they're different figures in the same area, they're equivalent. And I thought I wouldn't give them a "normal" statement. As it's an exploratory task, we could use the "message of the day." (S3)

Based on Júlia's suggestion, the participants created the task to be proposed to the pupils in the research lesson, adapting the instructions that would be given to them through a statement shared orally by the prospective teacher (**Figure 1**). This task was challenging and required the pupils to work in a systematic way, and was meant as providing a motivating context, suitable to introduce the concept of area. Júlia designed an exploratory task using pentominoes to elicit pupils' exploration of area and equivalence, promoting the connection between task properties and learning goals—her reflection that "different figures with the same area can illustrate equivalence" reveals a conceptual connection between content and representation, which indicates the model's support for bridging task design and discussion aims.

Solving the task

After creating the task, the participants anticipated possible strategies that the pupils could use to solve it. They solved the task by creating the different pentominoes, being attentive to the correctness of each possibility:

> Júlia: As for the correct answers, they can only be these [points to the 12 pentominoes] ...
>
> Facilitator: What about the incorrect ones?
>
> Alda: Adding squares on top of the 5. Or using 4 or 3 squares instead of 5 ...

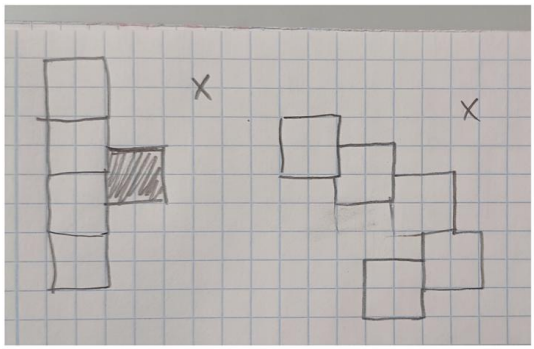

Figure 3. Júlia's anticipation of the pupils' possible wrong answers (Source: Field study, from Júlia's records)

> Júlia: Put a square in the middle, in other words, don't join sides and put a square against two sides of other squares ... Don't put side against side. Like this [draws **Figure 3**].
>
> Júlia: With regard to incomplete answers ... We consider groups that reach less than 12 pentominoes, don't we? (S4)

At the same time, the prospective teachers listed some difficulties that the pupils might have:

> Alda: I think they might have difficulties with the question of the directions of pentominoes. Realizing that two pentomino shapes, even in different directions, are the same.
>
> Facilitator: And how are we going to help them overcome this difficulty?
>
> Alda: Take the pentomino and rotate it, so they realize that the shape doesn't change, it's the same.
>
> Júlia: They may even have difficulty understanding what a pentomino is ... The concept itself. But I'll explain what it is in the introduction phase of the task and ask if there are any questions.
>
> Alda: But then they may also have difficulty understanding the concept of area and equivalent figures. They may have difficulty expressing themselves orally. (S4)

Júlia predicted both correct and incorrect pupil strategies, notably anticipating confusion between rotations and distinct shapes. This conversation reflected the whole-class discussion model's emphasis on predicting pupils' reasoning to inform later sequencing decisions. Her awareness of potential difficulties also led her to plan teacher interventions that would emphasize reasoning rather than correctness—a shift aligning with the main principles of productive whole-class discussions.

Preparing to carry out the task in class

In the next planning session for the research lesson, the participants prepared to carry out the task in class, considering how Júlia would select and sequence the pupils' answers and establish connections between them and the lesson's aim:

> Júlia: [Start with] the wrong ones and the incomplete ones. And finish with the correct ones. The wrong ones are, for example, the "bad use" or incorrect fitting of the squares to create pentominoes, or even the use of less than 5 squares to create a pentomino; and the incomplete [answers] are all the ones that don't include the 12 possible pentominoes, so 11, 10, 9 ...
>
> Facilitator: What about the sequencing of the correct answers?
>
> Júlia: So, I think it's best to always start with the group of pupils who have reached the fewest pentominoes. And also, from the simplest, most common [pentomino] and then on to the more complex ones.
>
> Facilitator: What if the class can't find all 12 pentominoes?
>
> Júlia: I could say "there are *x* more, find them." And, as a last resort, I introduce the missing ones. (S5)

Finally, they prepared some of Júlia's interventions, considering the degree of challenge of the questions she wanted to ask the pupils in order to promote their involvement in the discussion and to encourage them to explain their reasoning:

> Júlia: Not asking the whole-class so many open questions but asking specific groups to participate while a pupil is presenting their answer. It's important to ask a lot of challenging questions so that they can justify themselves.
>
> Facilitator: And what questions can we ask during the whole-class discussion?
>
> Alda: Imagine that one group came up with the T-shaped pentomino and another group came up with the upside-down T?

> Júlia: They both go to the board. And they have to conclude that it's the same pentomino. But questions to ask: the "why?" is obvious. "How did you think to create this pentomino?," "Group *x*, do you agree with your colleagues?" … to create disagreement. "Pupil *x*, explain what your colleague said," so that the same thing doesn't happen [lack of pupils' involvement in a previous whole-class discussion]. Don't ask the class closed questions [yes or no answers]. Ask open questions [in challenge] to a particular pupil or group. (S5)

Júlia outlined a structure for selecting and sequencing pupils' strategies, beginning with incomplete or incorrect answers and concluding with complete and correct solutions. This organization indicates a movement from surface procedural choices toward a clear didactical strategy, as she sought to use variation among responses to stimulate conceptual discussion.

Presenting the task

Júlia began the research lesson by presenting the task to the pupils. Given that they did not know the concept of a pentomino, as anticipated by the prospective teachers, she decided to explain this aspect without giving them too much information or leading them to an answer:

> Júlia: I was with a friend of mine this weekend and I was telling her that I knew five pentominoes. Do you know what pentominoes are?
>
> Pupils: No!
>
> Júlia: A pentomino is a polygon ... Do you know what a polygon is? Beatriz, what is a polygon?
>
> Beatriz: It's a geometric figure ...
>
> Júlia: ... Bounded by line segments. What about a pentomino? It's a polygon made up of 5 squares and that's the rule for pentominoes. After I told my friend that I found 5 pentominoes, she told me there were more. And I thought that a good challenge for my pupils would be to ask them to find out how many pentominoes there are in total. You already know that there are more than 5. I want you to find out how many there are. I'm going to give each group 5 squares to create pentominoes and a sheet where you draw and color in each pentomino you discover. Any questions? Has anyone not understood?
>
> Several pupils: No! (S7)

The presentation of the task was in line with the planning, relating the new notion of pentomino the pupils' known notion of polygon and providing the necessary information for the pupils to begin working.

Monitoring the pupils' work

During the pupils' autonomous work phase, Júlia circulated around the classroom, observing the work each group was doing and asking questions in order to understand the reasoning used to solve the task. Meanwhile, she also selected and sequenced the pupils' answers that she wanted them to share with their colleagues during the whole-class discussion:

> Júlia: What are you trying to do, Tomás?
>
> Tomás: Just moving a square.
>
> Júlia: Why?
>
> Tomás: It's easier, so I don't get mixed up. (S7)

As prepared in the previous phase, Júlia monitored the pupils' autonomous work, observing the strategies used and asking questions in order to understand the reasoning used by them, as was the case of Tomás, who was encouraged to explain his solution. As there were many superimpositions in the pupils' responses, Júlia designed a strategy to obtain all possible pentominoes and show them at the board without repetitions, choosing to mark with arrows the pentominoes that the pupils had to present to their colleagues.

Leading the Whole-Class Discussion

Episode 1

After the pupils selected by the prospective teacher had built their pentominoes on the board (**Figure 4**), Júlia began the whole-class discussion with an invitation to the group that she sequenced as being the first to present their answer: "Dinis, how did your group come up with these pentominoes?" (S7).

These inviting actions led the pupils to share their answers and explain them to their colleagues, as in the following episode:

> Dinis [group 1]: We experimented.
>
> Júlia: Why these pentominoes and not others?

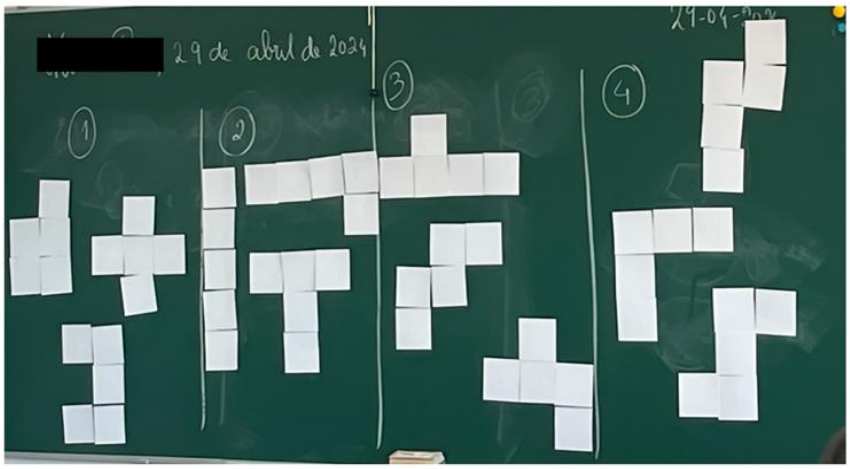

Figure 4. Answers from the different groups of pupils numbered 1 to 4 (Source: Field study, from board records)

Dinis: Because each one has 5 squares and no end is attached to another.

Júlia: OK, what about Bianca?

Bianca [group 2]: We always left 4 [fixed squares] and just moved one from side to side.

Júlia: But which pentomino did you start with?

Bianca: With this one. With all the squares next to each other.

Júlia: In a line, yes.

Bianca: And from this [pentomino], we took this [last] square over here and dragged it over here.

Júlia: They kept moving this square. Good strategy!

Dinis: They made a "T."

Júlia: Exactly. How did Tatiana's group do it?

Tatiana [group 3]: We just changed the squares and looked at which ones could be pentominoes. We tried a lot of them, but some turned out bad and we corrected them.

Júlia: Why was it wrong? How did your group realize it was wrong?

Tatiana: Because it was vertex to vertex.

Júlia: What do you mean, Lourenço?

Lourenço: We tried many and the ones that were wrong were because we joined vertices. Then we tried putting squares above and below them.

Júlia: What about Simão's group?

Simão [group 4]: We tried several. We saw if they followed the rules, if they were glued to the sides, if they weren't on top of each other and if they were 5. (S7)

At this point in the whole-class discussion, Júlia analyzed the pupils' work with a focus on their mathematical ideas and based on this analysis, asked various questions to support them in expressing their reasoning. This was the case when she asked Bianca which was the first pentomino her group built, supporting and guiding the pupil to explain her answer. Through her interventions, the prospective teacher encouraged the pupils to take part in the discussion and managed to involve most of the pupils, namely by asking a pupil, Lourenço, to explain the answer of his colleague Tatiana. Júlia intervened according to the situation and the goal at each specific moment of the discussion, so she varied the invitations, not only to begin the whole-class discussion, but also for the other selected pupils to present their answers. She also supported and guided the pupils by rephrasing some of their interventions and encouraging them to continue their explanations, ensuring that the discussion stayed on track. And through challenging questions, Júlia urged the pupils to clarify and justify their statements.

Episode 2

Although none of the groups managed to find all twelve pentominoes, Júlia selected the incomplete answers of four groups who together came up with all the possibilities, which is why she did not need to introduce new pentominoes. It should also be noted that the pupils did not come up with any wrong answers during the whole-class discussion, but these were corrected by the pupils themselves in the small groups during the autonomous work. The prospective teacher then continued the whole-class discussion, guiding the pupils through their interventions in order to establish connections among the various pentominoes and between them and the aim of the lesson:

Júlia: OK, so how many pentominoes are there in total?

Several pupils: 12!

Júlia: We have 12 on the board. Are they all the same? Or are they all different?

Several pupils: They're all different!

Júlia: But is there anything that's always the same ... Afonso?

Afonso: They're all 5 squares.

Júlia: And what else?

Rita: They all have the same corners.

Júlia: We have 12 pentominoes here and there are only 12. You've found them all. One of the characteristics of pentominoes is that they all have 5 squares. What other things can we say about these figures? Have you ever heard of equivalent figures?

Dinis: They're the same as each other! They're equal figures.

Júlia: So, are pentominoes equivalent figures?

Olga: Yes, because they always have 5 squares!

Júlia: Very good. They may not be the same, but they always have 5 squares. So, all pentominoes are equivalent. There's another very interesting thing about pentominoes, which has to do with the fact that they all have 5 squares... What could it be? I'll give you a hint: they occupy the same space.

Olga: Yes, they always occupy 5 squares.

Júlia: And what does that mean, Inês?

Inês: They use a measure of 5 squares.

Júlia: What can I call this measure?

Pedro: Square measure.

Carla: Centimeter!

Júlia: We once went outside to measure the perimeter of the part of the slide with our feet ... Is this measurement called a perimeter?

Several pupils: No ...

Júlia: What was the perimeter?

Several pupils: It's all around!

Júlia: So, the 5 squares are just around?

Several pupils: No, it's inside!

Júlia: What do we call this measurement? The space a figure occupies ...

Ana: Area! (S7)

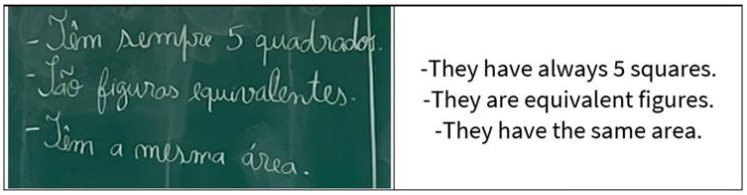

Figure 5. Júlia's written record on the board during the final synthesis (Source: Field study, from board records)

Again, based on analyzing the pupils' work with a focus on their mathematical ideas, Júlia made connections between the pupils' answers presented on the board and those she had selected as the ones to be discussed in class. She did this through questions designed to encourage the pupils to list similarities and differences between the twelve pentominoes. At other times, the prospective teacher used the analysis of the pupils' work to emphasize important mathematical ideas, such as the concept of equivalent figures, which she introduced and which Dinis and Olga developed. Once again, Júlia managed to promote the pupils' participation, encouraging them to listen, analyze and respond to their colleagues' reasoning, such as the intervention in which she asked Afonso to complement what his colleagues had said about the similarities between the twelve pentominoes. As in the previous episode, she asked a pupil, Inês, to explain her colleague Olga's answer, encouraging her participation. During this episode of the discussion, Júlia intervened according to the situation and the goal, supporting and guiding the pupils, making sure that the discussion stayed on track. At other times, she challenged the pupils, insisting that they clarify and justify their statements and informed or suggested with the intention of highlighting important mathematical ideas, such as the fact that all pentominoes are made up of the same number of squares, that they are equivalent figures and therefore have the same area–a concept introduced by Ana at the end of this episode, after Júlia used the example of another activity in which the pupils explored the concept of perimeter, thus also promoting connections with other mathematical ideas they already knew.

Leading the Final Synthesis

The whole-class discussion ended with a final synthesis promoted through the use of the previously prepared conclusions summary sheet, which also led to the whole-class discussion of some of the pupils' ideas:

> Júlia: Let's recap. What are these figures called?
>
> Rui: Pentominoes.
>
> Júlia: And why are they pentominoes?
>
> Ana: Because they have 5 squares and occupy 5 squares.
>
> Nádia: They're joined at the sides.
>
> Bruna: And they're not on top of each other.
>
> Tomás: The vertices aren't together.
>
> Júlia: What are these figures?
>
> Several pupils: Equivalent.
>
> Júlia: Very good, and why?
>
> Several pupils: Because they have the same area.
>
> Júlia: What is ...?
>
> Several pupils: 5 squares. (S7)

In this segment of the final synthesis, Júlia encouraged the pupils to get involved, and they spoke up when she asked them questions, reviewing the characteristics of the pentominoes, namely the fact that they all have the same area and are, therefore, equivalent figures. With their collaboration, she made a record on the board (**Figure 5**) summarizing and highlighting the main ideas explored in the lesson, such as "They have the same area" and "They are equivalent figures" and highlighting the relationship between pentominoes and the concepts of area and of equivalent figures.

This synthesis consolidated conceptual understanding and Júlia's written record on the board shows her transitioning from dialogue management to co-construction of knowledge, evidencing development in her discourse leading skills.

Reflecting on the Whole-Class Discussion

Reflecting on the lesson she led, Júlia began by saying that "overall, it went well, because they were able to understand the concepts of area and equivalent figures" (S8), stating that the learning aim set for the lesson had been achieved. She highlighted

monitoring the pupils' autonomous work as her main difficulty because, although she recognized that she felt secure and confident about the answers she wanted to select and their sequence, monitoring the pupils' work became complex, given that they "were always moving the pentomino squares. They recorded which ones they had create, but I couldn't keep up and memorize what each group had done. It was very confusing for me" (S8). So, the prospective teacher opted for

> Júlia: ... Carrying a sheet of paper to record the order of the groups I wanted for the whole-class discussion, and I also started to record which pentominoes each group created. And I left a little mark on the pupils' record sheet indicating which ones I wanted them to present on the board. I don't think this influenced the whole-class discussion or anything. It was more for me. So that I could manage the confusion better. (S8)

Júlia recognized that she had been surprised by the pupils several times during the lesson. Besides "[creating] the pentomino squares so easily, which for me were the hardest to find" (S8), many groups "moved one square around and fixed the others" (S8), a strategy she considered sophisticated and perceptive.

However, Júlia was somehow frustrated that, unlike the whole-class discussion she led in the previous lesson study, there were no situations of disagreement or wrong answers from the pupils, two aspects she considered important in a whole-class discussion: "I really felt sorry... It seemed like the whole-class discussion was less interesting. I didn't feel there was that challenge, that complexity. It was all too easy, so to speak. I expected something else" (S9). This reflection shows her concerns, but one should realize that such actions did not occur during the discussion because all disagreements among pupils were solved by them during the autonomous work.

Compared to the whole-class discussion she led in the previous lesson study, in which her biggest difficulty was getting most of the pupils to participate, the prospective teacher said that

> Júlia: I invited the pupils to present their solutions ... I asked other pupils to speak too. I challenged them a lot. I related the strategies they came up with. Unlike the other whole-class discussion, I got more pupils to participate and asked other pupils to explain a colleague's answer. (S9)

Júlia considered that "the whole-class discussion was interesting, with the pupils, under my guidance, concluding that all the figures were equivalent as they had five squares and, therefore, the same area" (written reflection). However, she said feeling "disappointed that there was no disagreement." Overall, in her reflection, she recognized that the weak points of her former discussion were overcome and that the main aims of the lesson were achieved by pupils.

During the post-lesson reflection, Júlia critically evaluated her practice, identifying two major learning insights: she recognized difficulty sustaining awareness of pupils' diverse strategies during monitoring, which affected sequencing decisions—an issue also reported in Stein et al. (2008); and she expressed disappointment at the absence of disagreements or errors in discussion, realizing that addressing errors prematurely during monitoring limited opportunities for in-lesson cognitive conflict. This awareness, linked to the use of the model in the reflection phase, suggests progression in her capacity for preparing whole-class discussions. She interpreted the lack of disagreement not just as pupil success but as a missed opportunity to prepare for richer whole-class discussion.

DISCUSSION

In this lesson study, Júlia and the other participants planned in detail the leading of the whole-class discussion, beginning with an initial preparation that considered the clear identification of the mathematical content and the learning aim, and which involved the study of curricular documents and literature on the most common difficulties of the pupils in learning the concept of the area (as suggested by Kooloos et al., 2023). Subsequently, they chose to create the statement of an exploratory task that would allow pupils to feel motivated and challenged to interpret and solve it (as suggested by Duarte et al., 2024) which, in this case, presupposed finding the twelve possible pentominoes. The fact that this task was open-ended in many respects suggested that it could lead to a productive whole-class discussion. The participants solved the task, anticipating possible responses from the pupils, correct and incorrect, which also allowed them to foresee any difficulties they might manifest, both in the interpretation of the statement and in the solution of the task, in line with what Kooloos et al. (2023) and Stein et al. (2008) say. When preparing to carry out the task in class, the participants considered the selection and sequencing of the pupils' answers to be shared in the discussion and prepared interventions to take in order to promote an active role of the pupils at that moment of the lesson, namely through the sharing of their answers and the explanation of their reasoning, as suggested by Stein et al. (2008). And, according to Duarte et al. (2024), these interventions were designed to have a certain degree of challenge that would encourage the constant involvement and participation of pupils during the whole-class discussion.

The professional development evidenced in Júlia's practice may be understood as the result of several complementary factors. First, the collaborative structure of lesson study provided opportunities for collective analysis of tasks, pupils' strategies, and instructional decisions, which has been widely recognized as a key mechanism for teacher learning. Second, the explicit model of whole-class discussion used in the lesson study offered a conceptual structure that supported her in identifying relevant practices and actions when preparing and leading the discussion. Finally, Júlia's participation in previous lesson study experiences during her initial teacher education may also have contributed to her capacity to assume a more active role in analyzing pupils' work and orchestrating classroom discourse, a process that has been associated with increased professional responsibility and growth in preservice teachers (Aitjanov et al., 2025).

During the lesson, Júlia began by presenting the task to the pupils, challenging them in their interpretation and clarifying aspects of the statement that raised questions. However, she did so without providing too much information and without inducing pupils to respond, as suggested by Duarte et al. (2024). Then, she monitored the pupils' autonomous work, identifying their responses and the reasoning involved in solving the task. Still at this stage, she selected and sequenced the pupils' answers that she considered important to be shared and discussed according to aspects defined *a priori* (in line with Fujii, 2018; Kooloos et al., 2023; Stein et al., 2008).

Júlia begun the whole-class discussion through an invitation addressed to the group she selected to first present their answer to their colleagues, thus promoting the initial involvement of these pupils. Through invitations to other pupils, she also promoted their participation and encouraged them to express their opinion on the answer presented by a colleague, as reported by Kooloos et al. (2023) and Ponte et al. (2013). At other times, the prospective teacher intervened according to the situation and the goal, using actions of: i) supporting and guiding the pupils in order to retell their explanations or encourage them to clarify their reasoning; ii) challenge pupils, insisting on them to advance their explanations or to validate the answers of their classmates, putting them mathematical questions and asking them for justifications and arguments; and iii) inform and suggest information and arguments that she introduced or validated, with the intention of highlighting important mathematical ideas (as reported by Duarte et al., 2024; Kooloos et al., 2023; Ponte et al., 2013).

At the same time, Júlia analyzed the pupils' work with a focus on the mathematical ideas to be explored in class, which led to the promotion of connections between ideas and answers presented by the pupils, as Kooloos et al. (2023) and Stein et al. (2008) say, particularly the different pentominoes, but also to highlighting of important mathematical ideas (as stated by Duarte et al., 2024) as well as the concepts of area and equivalent figures. She often asked her pupils questions to help them express their reasoning. However, there was no opportunity to promote moments of reformulation of pupils' wrong or incomplete ideas, an aspect to which Duarte et al. (2024) attribute relevance in leading whole-class discussions.

During the whole-class discussion, Júlia was able to promote the involvement of many pupils, particularly through invitations and questions that led pupils from different groups to participate and get involved regularly at this point in the lesson, as mentioned by Boerst et al. (2011) and Stein et al. (2008). The prospective teacher also encouraged pupils to listen, analyze and respond to their colleagues' reasoning, calling on pupils to speak up about a classmate's response (as suggested by Duarte et al. (2024). However, no situations of disagreement arose in this whole-class discussion, which, as Wood (1999) and Duarte et al. (2024) point out, could have promoted interesting opportunities for pupils to participate, justifying their answers and understanding their mistakes and difficulties. Situations of disagreement often arise from pupils' mistakes and incorrect solutions, but if the teacher solves these issues during the autonomous work they do not surface in the whole-class discussion. This appeared to be the case in this lesson.

To conclude the whole-class discussion, Júlia promoted the final synthesis that was built with the collaboration of the pupils, to summarize and highlight the main mathematical ideas explored in the lesson (as suggested by Duarte et al., 2024; Ponte, 2005), highlighting the concepts of pentomino, area, and equivalent figures. However, it was not found that Júlia promoted connections with other mathematical ideas previously known by the pupils, something that the teacher should consider during the final synthesis (Duarte et al., 2024).

In the reflection sessions of the lesson study, Júlia identified the monitoring of the pupils' autonomous work as her main difficulty, as she was not able to easily select and sequence the answers to be shared in the whole-class discussion (as reported by Kooloos et al., 2023). She was surprised that the pupils did not present wrong answers and was discouraged, since, for this reason, no situations of disagreement arose, which would make the discussion more interesting and challenging and with a stronger contribution to promoting pupil participation (Duarte et al., 2024). By analyzing the pupils' work, Júlia realized that they understood the concepts of area and equivalent figures, being able to relate them to the pentominoes they built during solving the task. And, finally, she highlighted that she was able to promote the participation of most pupils, involving the whole class through different interventions, as mentioned by Boerst et al. (2011).

These results also suggest that Júlia's participation in the lesson study contributed to the development of elements of PCK. In particular, her anticipation of pupils' strategies, identification of common difficulties, and orchestration of the discussion around key mathematical ideas indicate a growing ability to interpret and respond to pupils' mathematical thinking. Similar developments have been reported in studies examining preservice teachers' learning during lesson study, where collaborative planning and analysis of classroom interactions supported the construction of more sophisticated forms of PCK (Conceição et al., 2022; Herreros-Torres et al., 2026). In this sense, the lesson study context provided a setting in which Júlia could connect mathematical content, pupils' reasoning, and instructional decisions.

This practice, which constituted Júlia's main difficulty in leading the whole-class discussion carried out in the previous lesson study in which she participated (Duarte et al., submitted), seems to have been overcome through the preparation work done in this lesson study. Based on the analysis of the model presented in this article, simultaneously with the analysis and reflection on episodes of the previous whole-class discussion, this preparation consisted of giving greater attention to how Júlia could promote a broad and constant participation of the pupils, particularly through the anticipation of interventions that could encourage them to participate in the discussion, encourage them to listen, analyze and respond to their colleagues' reasoning, invite them to explain a colleague's response and promote situations of disagreement (as suggested by Duarte et al., 2024).

CONCLUSION

This study indicates that the whole-class discussion model functioned as both a design scaffold and a reflective framework supporting the prospective teacher's practice. While lesson study provided the collaborative and iterative structure for planning and reflection, the model supplied conceptual specificity, translating global goals ("lead a productive discussion") into tangible, actionable components (anticipate, select, analyze, and synthesize). The synergy between these two elements was critical: lesson study created a collective learning environment, and the model allowed Júlia to articulate and internalize discursive strategies.

Through engagement with the model, Júlia demonstrated growth in three interconnected domains of professional knowledge:

(1) from planning-focused to interpretive preparation, as she begun by focusing on "what to do" and evolved into analytical consideration of why certain tasks and sequences support conceptual learning,

(2) from procedural questioning to responsive facilitation, as she learned to use probing and connecting questions that fostered pupil's reasoning and participation, and

(3) from outcome evaluation to process reflection, as her reflections revealed the ability to analyze the relational impact of design, facilitation, and pupil learning.

While both the model and the lesson study contributed to Júlia's learning, their roles were distinct. The lesson study cycle—planning, implementation, reflection—provided a temporal and social structure enabling shared inquiry and feedback loops. In addition, the whole-class discussion model functioned as an analytic lens guiding Júlia's focus on specific teaching practices. Evidence of the role of the model lies in her capacity to name and analyze her own actions using its terminology ("inviting," "guiding," and "challenging"), showing conceptual appropriation beyond procedural reproduction. Thus, the model served as a cognitive framework enabling Júlia to translate collaborative experiences into articulated teaching knowledge.

These findings reinforce calls to integrate explicit discourse models into lesson study structures in initial teacher education (Kooloos et al., 2023; Ni Shuilleabhain & Bjuland, 2019) The combination promotes not only reflection "on what happened" but on how and why discussions unfold as they do. Moreover, Júlia's experience exemplifies how repeated engagement in structured lesson studies, supported by analytic models, can progressively bolster prospective teachers' capacity for leading mathematical discussions—a competence that is difficult to achieve during short-term practicum experiences.

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