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Pre-service teachers' professional noticing when viewing standard and holographic recordings of children's mathematics

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ABSTRACT

Professional teacher noticing is a key skill in preparing future elementary mathematics teachers, with the goal of increasing the specificity and sophistication of noticing children's mathematical reasoning. One common effort scholars have engaged to improve teachers' professional noticing is to improve the quality of representation. Recently, various scholars have found that extended reality based representations, such as 360 video and virtual reality, positively affect teachers' noticing by increasing the realism (via perceptual capacity). The present study explored the use of holographic representations to determine whether the increased perceptual capacity afforded by an ability to "lean in" and observe children's mathematics affected professional noticing. Findings and results provide preliminary support for this medium.

Keywords: professional noticing, extended reality, holograms, representations of practice, pedagogical content knowledge

INTRODUCTION

Professional teacher noticing is a fundamental skill for educators that involves attending to students' actions and strategies, interpreting their reasoning, and deciding how to respond to such reasoning (Jacobs et al., 2010; van Es & Sherin, 2021). Like most knowledge-based skills, noticing must be learned over time (Barnhart & van Es, 2015; Stockero & Rupnow, 2017).

Initially, novice teachers may focus on generic aspects such as classroom management, student engagement or behavior, how the teacher is managing the classroom, etc. (Barnhart & van Es, 2015; Huang & Li, 2012). Teachers then begin to attend to students' procedural reasoning before eventually attending to and interpreting students' conceptual reasoning (Barnhart & van Es, 2015; Tyminski et al., 2021; van Es et al., 2017). Although experience in the classroom can and does facilitate growth in noticing, developing robust and sophisticated professional noticing typically requires professional development (Jacobs et al., 2010; Simpson & Haltiwanger, 2017; Stockero et al., 2017).

A common feature in facilitating professional noticing is the incorporation of representations of practice. Such representations typically involve standard video recordings of classrooms or student interviews (Christ et al., 2017; Gaudin & Chaliès, 2015; van Es & Sherin, 2002). In recent years, various forms of extended reality (XR) have begun to be used to facilitate teachers' noticing including 360 video (Buchbinder et al., 2021; Kosko et al., 2021b) and animated simulations in virtual reality (Huang et al., 2021; Luke et al., 2021). Within this emerging body of literature, scholars argue that such representations better approximate the sense of being in the classroom (Ferdig & Kosko, 2020; Gold & Windscheid, 2020) and that teachers' engagement with such media is more embodied (Kosko et al., 2021b; Walshe & Driver, 2019; Weston & Amador, 2021).

Affordances of such advantages have been characterized by XR-based representations of practice having a higher degree of *perceptual capacity*, or "a medium's capacity for aspects of the scenario to be perceivable" (Kosko et al., 2021b, p. 286). The present study builds upon this emergent scholarship by focusing on a novel form of XR-based representation of practice: holograms. Holographic representations convey a sense of depth and volume such that the viewer can move around and closer/further from the recorded hologram. The purpose of this exploratory study is to examine whether use of holographic representations of practice, and the perceptual capacity they afford, affect pre-service teachers' (PSTs) professional noticing of children's mathematical reasoning.

THEORETICAL FRAMEWORK

Professional Noticing

Professional teacher noticing involves attending to pedagogical events of significance, interpreting those events, and deciding how to respond (Jacobs et al., 2010; Yang et al., 2021b). The pedagogical events that teachers attend may include aspects of classroom management, how particular concepts are scaffolded by the teacher, or students' mathematical procedures and/or reasoning. There is strong evidence that novices entering their teacher education programs initially focus on attending to aspects of classroom management, with a heavy focus on what the teacher is doing (Huang & Li, 2012; Jacobs et al., 2010; Yang et al., 2021b). Following professional development and experience in the classroom, teachers begin to focus less on the teacher in recorded scenarios and more on students. This focus is initially on aspects of classroom management and participation, but eventually transitions into observing students' engagement in procedures related to the content (Barnhart & van Es, 2015; Kosko et al., 2022; Stockero et al., 2017). The most sophisticated form of professional noticing is when teachers attend to students' conceptual understanding of the content and interpret this reasoning (van Es et al., 2017).

Although there is ample research that suggests more experienced teachers demonstrate more sophisticated professional noticing (Huang & Li, 2012; Jacobs et al., 2010; Yang et al., 2021b), Yang et al. (2021b) note that "teaching experience indeed acts as a main–though not sufficient–factor in the development of noticing" (p. 37). Rather, it provides initial and necessary support for development of noticing, but targeted professional development experiences are needed for further growth (Jacobs et al., 2010; Tyminski et al., 2021). One approach to this is to engage teachers (novice or experienced) in various forms of decomposing practice. As described by Grossman et al. (2009), *decomposing practice* involves "breaking down complex practice into its constituent parts" (p. 2069). For example, some scholars have engaged PSTs in using StudioCode to mark specific moments in standard videos of classroom instruction, and discuss these moments with peers or their course instructor (Stockero et al., 2017; Teuscher et al., 2017). Others have used specific resources such as frameworks for posing questions (Tyminski et al., 2021), scaffolds for specific types of noticing (Amador et al., 2016; van Es & Sherin, 2002), descriptions of forms of students' mathematical reasoning (Jacobs et al., 2010; Schack et al., 2013), and so forth. In most cases, the increased specificity in resources and prompts described above facilitated an increased specificity in how teachers describe students' reasoning of mathematics.

A primary goal in decomposing practice is to make more explicit the forms of professional knowledge involved in professional noticing. Some scholars have found a positive relationship between professional noticing and professional knowledge (Dick, 2017; Simpson & Haltiwanger, 2017; Yang et al., 2021a), while others have found more mixed relationships between the two constructs (Cross Francis et al., in press; Jong et al., 2021). For example, both Jong et al. (2021) and Yang et al. (2021a) used variations of the TEDS-M pedagogical content knowledge (PCK) assessment. Yang et al. (2021a) found that Chinese mathematics teachers with higher PCK scores also demonstrated more sophisticated mathematical noticing. By contrast, Jong et al. (2021) conducted a quasi-experimental design and compared a group who engaged in professional noticing activities with one that did not. They found that the experimental group did improve on their noticing but their PCK remained unchanged. Cross Francis et al. (in press) provide one possible explanation for such mixed results in their comparison of two educators with different levels of professional knowledge. Namely, the teacher with higher assessed professional knowledge engaged in less sophisticated noticing due to their disposition towards students and pedagogy. The teacher with lower (but not low) professional knowledge had a higher degree of anxiety related to their decisions and, therefore, attended to students' reasoning more diligently. This notion of pedagogical dispositions moderating the effects of knowledge on noticing has been discussed by others (Fisher et al., 2018; Jong et al., 2021). However, another issue that is seldom discussed is the conceptualization of professional knowledge itself (Copur-Gencturk et al., 2019; Hill et al., 2008; Zolfaghari et al., 2021).

Pedagogical Content Knowledge for Fractions

PCK was initially conceptualized by Shulman (1986) as an interconnection between pedagogical and content-based knowledge "which goes beyond knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching" (p. 9). Hill et al. (2008) further defined a subconstruct of PCK referred to as knowledge of content and students (KCS). KCS is knowledge of how students reason and engage with content. Given the focus of professional noticing literature on attending to students' mathematical thinking (i.e., Jacobs et al., 2010; Tyminski et al., 2021), KCS is of particular concern in the current paper. Interestingly, most scholarship examining connections between professional knowledge and noticing do not have a central focus on KCS but in various other domains and constructs that compose mathematical knowledge for teaching (Cross Francis et al., in press; Dick, 2017; Fisher et al., 2018; Jong et al., 2021; Simpson & Haltiwanger, 2017; Yang et al., 2021a). Interestingly, defining KCS or PCK as distinct from content knowledge is an ongoing issue (Copur-Gencturk et al., 2019; Hill et al., 2008; Zolfaghari et al., 2021).

Recently, Zolfaghari et al. (2021, in press) began the work of theoretically conceptualizing KCS for elementary fractions through construction of a validity argument for their PCK-fractions measure. Following the efforts Ball et al. (2008), Hill et al. (2008), and Zolfaghari et al. (2021) constructed an initial construct map, or a framework conceptualizing lower to higher levels of PCK for teachers' understanding children's fraction reasoning. After an initial pilot of the PCK-fractions measure and examination of cognitive interview data, Zolfaghari et al. (2021) revised their construct map. Then, Zolfaghari et al. (in press) verified the new construct map with a separate sample. Zolfaghari et al. (in press) observed that PSTs with field experience in upper elementary grades (where fractions are initially taught) demonstrated higher PCK scores than those without field experience in those grades. Such a relationship is similar to that of sophistication of professional noticing being related, but not completely informed, by experience (Yang et al., 2021b). Thus, I conjecture that the PCK-fractions construct map serves as a useful reference in understanding how PSTs may engage in their professional noticing.

Table 1. Construct map for PCK-fractions

Level	Description
Level 1	Assess children's creation and/or use of fractional parts.
Level 2	Assess children's coordination of parts and of the whole.
Level 3	Assess children's creation and use of non-unit fractions and comparison of fractions.
Level 4	Assess children's coordination of non-unit fractions with the whole and comparison of fractions and wholes.

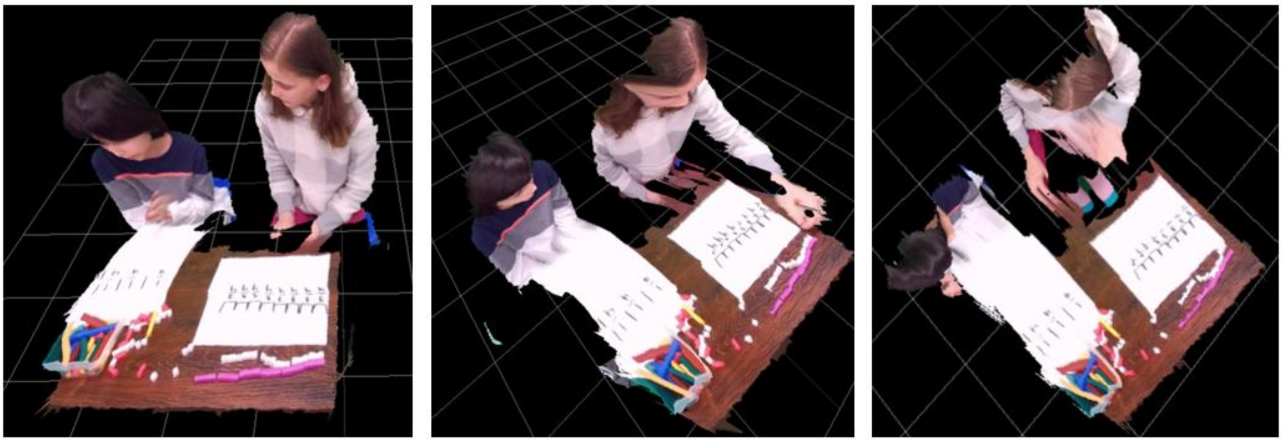

Figure 1. Illustration of single-perspective recorded hologram from three different angles

The construct map posed and verified by Zolfaghari et al. (2021, in press) is illustrated in **Table 1**. Initially, teachers at level 1 can learn to assess children's creation and/or use of fractional parts. This essentially is a focus on whether and how students partition a whole into smaller parts or engages in fair sharing. At level 2, teachers begin to assess how children coordinate these parts to the whole (i.e., part-whole reasoning). This also includes the creation and coordination of unit fractions ($\frac{1}{3}$, $\frac{1}{4}$, etc.). At level 3, teachers demonstrate ability to assess children's creation and coordination of composite, or non-unit, fractions ($\frac{3}{4}$, $\frac{2}{5}$, etc.). This also includes comparison of such fractions and basic arithmetic (addition & subtraction). Finally, level 4 includes an ability for teachers to assess fraction multiplication and division, which often involves assessing children's coordination of fractions of a fraction. Hackenberg (2010) describes children's reasoning at this stage as evident of coordination of units of units of units. In this manner, the construct map vetted by Zolfaghari et al. (in press) is aligned with assessing unit coordination of children's fractional reasoning.

Representing Practice

Beyond decomposing practice and facilitating growth in teachers' professional knowledge, another approach to improving teachers' professional noticing involves improving the representations of practice. For example, Seidel et al. (2011) compared teachers' noticing when viewing videos of others' teaching versus video of their own teaching. Although those who watched their own teaching reported a higher degree of perceived immersion, there was no clear distinctions between groups regarding the quality of what was noticed. In a similar manner, scholars investigating the potential for animated representations of practice have found it has elicited both higher (Friesen & Kuntze, 2018) and lower (Herbst et al., 2013) degrees of immersion, but, similar to Seidel et al. (2011), found no differences regarding specificity of noticing. By contrast, Kosko et al. (2021b) compared standard video with 360 video and found that 360 video elicited a higher degree of perceived immersion and elicited higher specificity of professional noticing. 360 video is a form of XR that records omnidirectionally such that the viewer can choose where to look in the recorded scenario. Kosko et al. (2021b) suggested that by allowing PSTs to decide where to look, 360 video provided a higher degree of perceptual capacity or "the medium's capacity for aspects of the scenario to be perceivable" (p. 286).

Various scholars have observed benefits of incorporating 360 video in studying teachers' noticing. For example, Walshe and Driver (2019) found that participants expanded upon descriptions of practice more when viewing 360 video of themselves versus when asked to reflect on their own recollection of teaching. Theelen et al. (2022) found that using 360 video reduced professional anxiety and increased self-efficacy of PSTs ahead of their field placements. Using 360 video of PSTs' own teaching, both Buchbinder et al. (2021) and Weston and Amador (2021) found that the ability to turn the camera perspective improved teachers' ability to attend to different aspects of students' reasoning. In each of these various analyses, the ability for teachers to look in any direction allowed them to see more examples of students' thinking and reasoning. This, in turn, led to a higher sense of immersion and more sophisticated professional noticing.

The present study examines a novel form of XR to represent practice: holograms. As previously defined, holograms record "3D images onto a space" (Yoo et al., 2022, p. 2). **Figure 1** provides an illustration to convey how such three-dimensional information can be conveyed "onto a space." In this particular illustration, only a partial hologram is provided from a single camera recording, as multiple cameras are needed for a whole object or person to be volumetrically recorded. To date, there has been little research on the use of holograms in education, and none identified for teacher education (Yoo et al., 2022). However, Kosko et al. (2021a) conjectured that holographic representations of practice would have key advantages in teacher education due to the perceptual capacity they afford. Although Kosko et al. (2021a) described the potential of holograms of a whole class, the present study

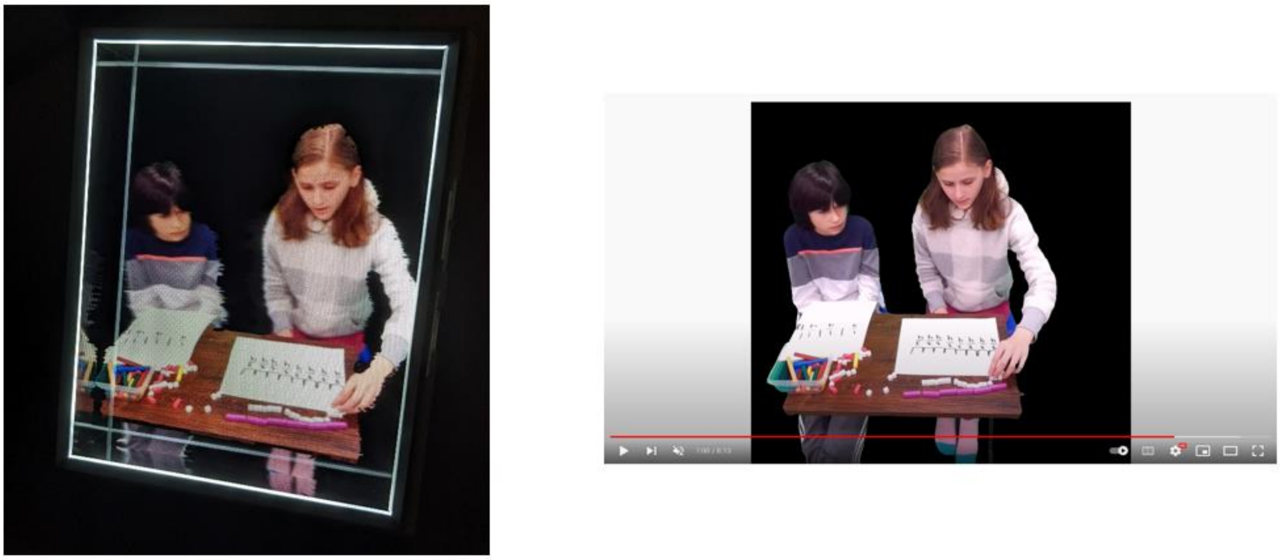

Figure 2. Video of Ben and Katherine on a LookingGlass holographic display (left) and as a standard video on YouTube (right)

examines holograms of a single student or a pair of students at the same table (see **Figure 1**). In such cases, a hologram's perceptual capacity lay in the ability to lean into and look to the sides of students' work as they engage in a task. This property is commonly referred to in XR-based media as six degrees of freedom (or 6DoF) versus three degrees of freedom (3DoF). In 3DoF representations, like 360 videos, one can adjust their view along three axes: one can look in any direction in a recorded classroom but cannot lean into or walk around. To date, all 6DoF representations of practice are animated in VR (Atsikpasi & Fokides, 2022). Given the limited scholarship on more realistic 6DoF representations and their potential perceptual capacity, it is worth considering whether and how such a feature affects PSTs' professional noticing.

Increases in perceptual capacity have been found to be beneficial to teachers' noticing when increasing the physiological realism of spatiality of a representation (Gold & Windscheid, 2020; Kosko et al., 2021b), spatial audio (Ferdig et al., 2020), and embedding abstracted information (via augmented reality) with embodied experiences (Prestridge et al., 2021). However, as noted by Kosko et al. (2021a), more research is needed to better understand the role of perceptual capacity in facilitating teachers' professional noticing. The present study focuses on a form of spatial sense promoted by the perceptual capacity afforded by holographic representations of practice. To better understand how this facet of perceptual capacity may influence professional noticing, the following research question was asked:

Do holographic recordings affect PSTs' professional noticing of children's mathematics?

METHODS

Participants

Participants included a convenience sample of 15 PSTs majoring in either a primary (n=10) or middle grades (n=5) licensure program at a Midwestern US university. Primary certification in this US state included grades preK-5 (ages 3-11 years) and middle grades certification included grades 4-8 (ages 9-14), with the latter requiring participants to focus on as many as two content areas. While all middle grades participants focused on mathematics, their additional focus areas included science (n=3), social studies (n=1) and English (n=1). PSTs majoring in primary grades were enrolled in their first of two mathematics pedagogy courses taken before their final year of the program (i.e., second semester of junior year). PSTs majoring in middle grades were enrolled in their second of two mathematics pedagogy courses taken the semester prior to full-time student teaching (i.e., first semester of senior year). All participants self-identified as white, with 14 identifying as female and one as male.

Materials and Procedure

Participants were asked to watch a set of recordings of two students, Ben and Katherine, solving two fraction division problems using a length-based mathematics manipulative (Cuisenaire rods). Both recorded children were actors playing the part of students with specific mathematical understandings and strategies. Thus, their actions and dialogue were scripted–a feature Herbst (2017) notes as useful in engaging teachers in pedagogical discourse and actions. Both actors were recorded on a green screen using an Azure Kinect depth-sensing camera and DepthKit software. The actors were recorded playing their characters working independently and then as a pair discussing their mathematical strategies. The recordings were edited to produce a holographic recording and a standard video version of the same recording (see **Figure 1** and **Figure 2**). The standard video version was placed on YouTube and viewed on a tablet by participants (XRi, 2022a). The holographic recording was placed on a LookingGlass display–a hardware device that conveys the spatiality of holograms with the naked eye (viewable at XRi, 2022b). Thus, the front of the desk in **Figure 2** appears to be physically closer than the back of the desk in the left-hand image, and so does Katherine's right hand as it points at white Cuisenaire rods near the front of the desk.

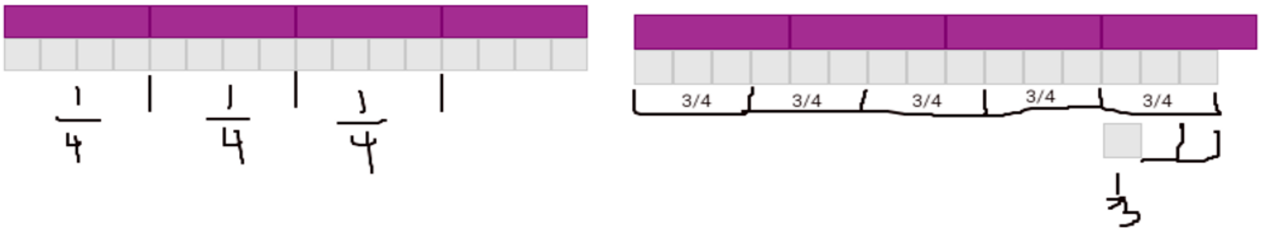

Figure 3. Ben (left) and Katherine's (right) individual work for solving 4÷¾

Within the recording, the two depicted students solve two fraction division tasks: $6 \div \frac{3}{4}$ and $4 \div \frac{3}{4}$. The character Ben mistakenly solves the fraction division tasks as if they are fraction multiplication. For example, in **Figure 3** (left side), Ben solves $4 \div \frac{3}{4}$ by partitioning the whole number 4 into four parts and counting white rods to find a solution of 12. Katherine also represents a whole as a purple rod but identifies three white cubes as one $\frac{3}{4}$ unit before considering how many $\frac{3}{4}$ units fit into four wholes. In discussing their reasoning, the character Ben is unable to explain mathematically why he did what he did before Katherine tells him "you're wrong" and explains her own strategy.

Each participant was provided a 4-page resource packet that included Battista's (2012) stages, which children develop fraction reasoning and example descriptions with children's work. Participants watched the recorded students twice: via the holographic recording or as a standard video. Specifically, participants were randomly placed into one of two conditions. *Hologram-first* participants (n=8) viewed the holographic recording and then described evidence of each child's mathematical reasoning with fractions. Then, these participants viewed the standard video version of the same recording and, with the aid of their initial written description, re-assessed the children's reasoning with fractions. Participants in the *video-first* condition (n=7) followed the same procedure as their peers but viewed the standard video before viewing the holographic recording.

Analysis

This study follows a convergent mixed methods design in which qualitative and quantitative data were collected within the same phase of the study. Specifically, a data transformation variant of the convergent design was used (Creswell & Plano Clark, 2018). In such studies, the qualitative data is collected first and analyzed before transforming it into quantitative data for statistical analysis via a process Creswell and Plano Clark (2018) term *quantitization*. This quantitization allows for statistical analyses to be interpreted more robustly as they are informed through findings from the qualitative analysis. Such interpretations are at the heart of this mixed methods approach and serve as the manner in which findings and interpretations are merged. Customary in such analyses, the findings and results are presented sequentially, and this paper follows this custom.

In the qualitative phase, participants' written noticings of children's fraction reasoning were collected and analyzed using systemic functional linguistics (SFL) (Halliday & Matthiessen, 2004). SFL is a form of discourse analysis that examines how grammar functions to convey meaning across various discursive systems. The present study is concerned with the information system, which is conveyed through information units. As noted by Halliday and Matthiessen (2004) these units exist in "the tension between what is already known or predictable and what is new or unpredictable" (p. 89). To examine information units, I examined how participants used the system of reference with attention to transitive and nominal groupings. Nominal groupings represent objects or entities that may reveal the actor or goal in a clause, while transitivity, often conveyed through verbs, transfer (or transition) actions onto another goal or referent (Halliday & Matthiessen, 2004). Analysis of information units focused on information regarding a child's actions and reasonings with fractions. To aid in explaining the analytic process, consider the excerpt below. Transitive processes are in bold, with hedging italicized as an aid to understand positionality conveyed by the participant. Referents are underlined and include nominal groupings of terms that convey particular meanings. For example, when the participant references "1/3 of her 3/4," it is a reference to a particular quantity which is later referenced as "a remainder of 1/4." The referent is consistently paired with material transitive processes "**figure** out…**had** 1/3 of her 3/4…**had** a remainder of 1/4" conveying the participant's observations of Katherine's procedures. Rather, there is a prevalent use of material processes and references that point to a focus on answers and procedures. Thus, when the isolated reference "computational reasoning" is used, it conveys a meaning focused on procedures and not on the underlying concept and meaning of fractions. The reference to "reasoning" is, thus, constructed through the information units and reference chains provided throughout the participant's text. It is in this manner that written noticing was analyzed in the present study.

> She [Katherine] was *able to* **figure** out // that when she **had** 1/3 of her 3/4 left, // that **meant** // she **had** a remainder of 1/4. // She *was also able to* **explain** to Ben why he was wrong and **explain** computational reasoning for solving the problem.

The construct map presented in **Table 1** served as a reference, but not a determinant, in establishing themes related to the detail provided by participants in their written noticing. Specifically, reference chains and information units conveyed through written text conveyed attending to specific actions and reasonings of children. The different levels in the **Table 1** construct map provided an initial framing of trends in how the information units that emerged. Although participants wrote what they noticed twice (once after viewing one medium and again after viewing another), analysis of these written noticings were treated collectively. Rather, teachers tend to modify their original written noticing instead of creating a new version of the written account initially provided. Given this, it was more pragmatic to analyze both written accounts as if they were two parts of a whole narrative.

Qualitative findings were next quantitized as ordinal data and analyzed quantitatively through calculation of the Somers' D statistic. Somers' D is a nonparametric statistic for measuring the asymmetrical association of ordinal variables (Siegel & Castellan, 1988; Somers, 1962). A key advantage of the statistic is that it allows for one ordinal variable to serve as the dependent variable and the other as independent variable. Somers' D reports the percent difference between one category of the independent variable and the other as a statistical effect. In the present paper, the quantitized variable of PSTs' noticing was the dependent variable and PSTs' assigned condition was the independent variable (0=*video-first;* 1=*hologram-first*). Quantitization of qualitative themes served as the mechanism for merging data in the present study (Creswell & Plano Clark, 2018). Given the nature of data in this exploratory study, both qualitative and quantitative data were given equal prioritization in interpreting findings and results.

Ethical Statement

The Kent State Institutional Review Board approved data collection for this study (IRB #179) on March 28, 2022.

FINDINGS AND RESULTS

Qualitative Findings

Findings revealed key distinctions in the type of information linguistically conveyed by participants. As noted previously, analysis was informed by the construct map in **Table 1** such that it was used as a reference to better understand patterns in the data and create themes for the type of information about children's reasoning participants conveyed. Five themes were observed that appear to align with the initial constructs described in **Table 1**, but also expand such descriptions.

The first theme, what may be referred to as level 0, involved PSTs *assessing children's engagement with fractions at a basic level*. This is exemplified in the below excerpt, where the PST references Ben's use of "manipulatives to assemble fractions" and "explain his thinking." Within the displayed excerpt, references are underlined, transitive processes are in bold, hedging is italicized, and clauses are separated by // to illustrate how information units were constructed and modified via reference chains. In terms of information about Ben's fraction reasoning, there is relatively little information provided beyond his engagement with manipulatives to solve fraction problems. There are no references, tacit or explicit, provided to describe the type of mathematics he engaged or how he reasoned about that engagement. Three participants demonstrated this level of detail in their noticing, and did so in assessing both Ben and Katherine's reasoning.

> He *was able to* **use** manipulatives to assemble fractions. // He *was able to* **explain** his thinking. // [He] **process** only part way // and then realized he **did not do** it correctly–**solved** the problem incorrectly.

The most common theme emerging from the data was *assessing children's creation and/or use of fractional parts* (level 1 in **Table 1**). This was typically exemplified by PSTs' references to students' partitioning and an emphasis on these parts being equal. The emphasis on partitioning in this theme was also characterized by a lack of reference to the child coordinating parts and wholes. For example, in the below excerpt, the PST describes Ben dividing "the whole number into parts" and counting "the parts." Although the term "fourths" is referenced and used in noting Ben added "3 partitions," the explicit use of the term does not occur with a description of coordination between a partition and any other type of unit. Rather, use of the term "fourths" is often replaced with "partitions" or "parts" and use of "adding" is conveyed as identical to counting. Thus, despite the use of terms that may appear, at face value, to convey assessment of deeper reasoning, the structure of grammar presented in this excerpt suggests information about the child's reasoning is focused on their ability to partition and count partitions. Of the 30 written noticings about children's fraction reasoning (15 of Ben and 15 of Katherine), 9 were classified as being evident of this theme.

> Ben **used** manipulatives for both problems, // **divided** the whole number into parts (pink blocks for whole numbers, white blocks for parts). // Then [he] **counted** the parts after partitioning into fourths // and **adding** 3 partitions.

Contrasting the lack of attention to part-whole reasoning described above, one participant did attend to such reasoning in their noticing of both Ben and Katherine. This was evident in one such description where they described Ben connecting "that 4 white blocks **made** 1 purple block," and emphasized this relationship as why each partition was ¼. This sole participant appeared to demonstrate reasoning that corresponded to level 2 in **Table 1**.

The next most common theme to emerge was PSTs' *assessing children's coordination of non-unit fractions*, which appeared to align with level 3 in **Table 1**. PSTs demonstrating this theme referenced children's coordination of non-unit fractions–specifically $\frac{3}{4}$. One PST noted "[Ben] **splitting** the 24 into 4 groups // and **adding** total of 3 groups" to find $\frac{3}{4}$ of 24. Though this brief excerpt appears to emphasize partitioning and coordinating the number of these partitions, much the same way an above example does, this participant also referenced coordination of the four groups to the whole (of 24 cubes). Additionally, their use of the term "adding" coincided with reference to ¼ of the whole being equivalent to 4 white cubes out of 24 (i.e., an implicit reference to equivalent fractions). Such references construct a different meaning of "adding" and "fourths" than some other PSTs' written noticing, which is characterized in this brief excerpt.

The last theme to emerge was in many ways similar to assessing children's coordination of non-unit fractions, but conveyed more nuance in the references to children's coordination of fractions. This theme, *assessing children's nuanced coordination of fractions*, appears to align with level 4 in **Table 1**. In the current data, common features included participants referencing children "fitting fractions" into other units (whole numbers or other fractions) and/or considering a unit as having more than one numeric identity (i.e., a white cube being both $\frac{1}{4}$ and $\frac{1}{3}$ depending on context). The below excerpt provides examples of many such common

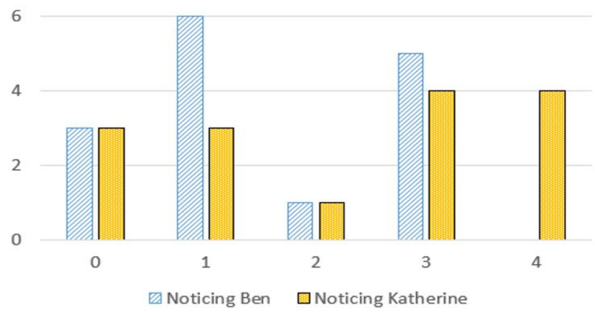

Figure 4. Distribution of noticing categories across the sample

features of this theme. Namely, this PST referenced Katherine's use of one white block as $\frac{1}{4}$ of a whole (pink block) and that three make a $\frac{3}{4}$, with $\frac{3}{4}$ being referenced repeatedly as a singular entity. This linguistic feature is referred to in SFL as *nominalization* in which an action or set of actions are synthesized into a single referent. What truly characterizes this written noticing as evident of the theme, though, is referencing "the leftover [white block] is $\frac{1}{3}$ and not $\frac{1}{4}$," despite previously referencing the white block as ¼ in their assessment of Katherine. As such, the PST conveys Katherine's renegotiation of the value of a white block in the fraction division task such that she was able to arrive at the correct answer of 5 and $\frac{1}{3}$.

> For 4 ÷ ¾, she **sets up** 4 pink blocks with 16 white blocks underneath (each [white block] **representing** ¼). She knows to **make** groups of 3 (**representing** ¾). She **gets** 5 total [groups of ¾] // but with 1 [white block] left. She knows that the leftover **is** 1/3 and not ¼. She then is able to explain to Ben how she got her answer.

Quantitative Results

Given the different ways participants appeared to assess and interpret Ben and Katherine's reasoning, PSTs' noticing of their reasoning was quantitized into two separate ordinal variables, but with the same ordinal classifications:

0. Assessing children's engagement with fractions at a basic level;
1. Assesses children's creation/use of fraction parts;
2. Assesses children's coordination of parts and of the whole;
3. Assesses children's coordination of non-unit fractions;
4. Assessing children's nuanced coordination of fractions.

The median level participants assessed Ben's fraction reasoning was at level 1 and the median for Katherine was at level 3 (both student characters were actually at level 4). The variation in the level of language used by participating teachers is illustrated in **Figure 4**. Although it was expected that Ben's reasoning might be described at a lower level than Katherine's, since his script was written to demonstrate a slightly lower level of reasoning, it is interesting how low a level some participants described his reasoning. Given such patterns, a Somers' D statistic was calculated for each noticing (one for noticing of Ben and one for Katherine).

Results indicated a positive and statistically significant effect of participants being assigned to the hologram-first condition and their level of noticing Ben's fraction reasoning (D=.625, p=.026). Additionally, a positive and statistically significant effect at the .10 level was observed for participants being assigned to the hologram-first condition and noticing of Katherine's reasoning (D=.446, p=.087). Thus, the sophistication of participants' noticing of Ben's fraction reasoning was found to increase by 62.5% when assigned to the hologram-first condition instead of the video-first condition. Similarly, sophistication of participants' noticing of Katherine's fraction reasoning increased by 44.6% when assigned to hologram-first rather than video-first condition.

DISCUSSION

Improving the specificity that PSTs attend to students' mathematical reasoning is a central goal of teacher education (Jacobs et al., 2010; Kosko et al., 2021a; van Es & Sherin, 2021). Various XR-based representations of practice have been found to beneficial in this endeavor due to their increased perceptual capacity (Buchbinder et al., 2021; Kosko et al., 2020, 2021b; Weston & Amador, 2021). Yet, because such scholarship is emergent, it has also been limited on the forms of perceptual capacity explored and the ways such perceptual cues may be conveyed through practice-based representations (Kosko et al., 2021a). The present study expands this literature by providing preliminary evidence that 6DoF conveyed through holographic representations of practice may support more sophisticated noticing when viewed before standard video of the same event. Although results and findings should be considered both preliminary and exploratory, they are also quite significant. Specifically, creating more realistic representations of practice has been a goal of many teacher educators for decades (Sherin et al., 2008; van Es et al., 2015). Perceptual capacity provides a theoretical lens for examining how such realism is conveyed through one's embodied experience, and the degree to which certain perceptual experiences are worth representing. For example, taste is a perceptual experience but one that is less likely to be worth representing in recording pedagogical practice. By contrast, the current study provides evidence that spatial sense in terms of depth and movement is a perceptual sense that is worth representing for teacher education.

Kosko et al. (2021a) conjectured that holographic representations could provide a high-level embodied experience for teachers to learn to notice pedagogical events. Results presented here suggest this may be the case and expands the literature on application of holograms beyond educating students to preparing future teachers. However, the different statistical effects associated with each represented student suggests that what, and perhaps who, is recorded matters significantly. Specifically, viewing Ben as a hologram first increased the level of detail in PSTs' writing by 62.5%. Ben solved the fraction division problem as if it were fraction multiplication which may have resulted in lower ratings across the sample. By viewing Ben as a hologram, it is possible that the ability to "lean in" and look more closely at Ben's actions allowed for more attention to how he coordinated rods and blocks, and conceptualized the relationship between various units. To be clear, others have discussed how placing cameras proximally closer to students allows for more detailed noticing of their mathematical reasoning (Kosko et al., 2021b; van Es et al., 2015), and the ability to "zoom in" on recorded videos has existed as a feature for quite some time. YouTube, the viewing platform used in the present study, allows users to magnify portions of a playing video. It appears that holographic recordings provide a unique advantage over magnification. However, future study is needed to better understand why such affordances exist for "leaning in" over "zooming in" when teachers view representations of practice.

Although not the primary focus of the present study, results provide support for Zolfaghari et al.'s (2021, in press) conceptualization for how teachers develop PCK for fractions. Additionally, findings support those of prior scholars who have observed how teachers' professional knowledge manifests in their noticing of students' reasoning (Dick, 2017; Simpson & Haltiwanger, 2017; Yang et al., 2021a). It should be noted that the analysis used here is limited to what PSTs chose to write about students' mathematical reasoning. However, as Halliday and Matthiessen (2004) note, language is choice and what one chooses, consciously or not, to say, write or draw confers a choice of what and how one communicates. I conjecture that such choices are informed by one's prior experiences and, therefore, partly through their professional knowledge. However, as others have observed (Cross Francis et al., 2018, in press; Jong et al., 2021), the influence of professional knowledge on one's linguistic choice when engaged in professional noticing is moderated by other factors. This is an area of scholarship in need of further study, and one that can benefit from more rigorous conceptualizations of PCK. Stated differently, much of the scholarship on PCK has often confounded it with the domain of content knowledge (Copur-Gencturk et al., 2019; Hill et al., 2008), or otherwise limited it to a "you either have it or you don't" conceptualization of the domain. Scant work has been done on what evidence of PCK looks like at varying degrees (high, middle, low). Findings from the present study contribute to such efforts and deserve further attention.

Limitations

The present study was exploratory in nature. As such, there are several limitations that should be considered both in terms of referencing the work presented here and in conducting future study. First, the sample size was limited in size with 15 participants. Although nonparametric statistics allowed for a statistical comparison, a large sample is needed to better understand and verify the observed results. A larger sample size would also be useful in understanding the role of other factors not explored here including PSTs' PCK, beliefs, and professional experiences. Such relationships could not be explored here due to the sample size and the exploratory nature of the study. Another limitation is in the representation itself. The holograms recorded were partial and did not include full bodies of participants. The displays they appeared (LookingGlass) allowed for viewing the spatiality of holograms with the naked eye. However, the models used were smaller with a 7.9 inch display size and 58 degree viewing cone (i.e., the degree to which one can move side-to-side to view the hologram) and was actually smaller than the display for the standard videos (a 10 inch tablet). Such factors may have limited how much of the recorded children's actions was viewable by participants when watching the holograms. Alternatively, participants can view holograms using dedicated headset displays, which have been shown to be more effective in comparing other forms of XR (Kosko et al., 2021b). These various limitations should be taken into account when citing or considering replications or extensions of the current study

CONCLUSION

The purpose of this exploratory study was to examine whether, and how, holographic representations of practice affected PSTs' professional noticing of children's fraction reasoning. Findings and results suggest that viewing holograms prior to standard videos is more beneficial than viewing standard videos first. Thus, this paper adds to scholarship on XR-based representations of practice highlighting the affordances of the perceptual capacity such representations provide (Buchbinder et al., 2021; Kosko et al., 2021b; Walshe & Driver, 2019; Weston & Amador, 2021). However, future study is needed to better understand not simply whether certain perceptual factors are beneficial but underlying reasons for why (i.e., "lean in" vs "zoom in"). Additionally, given the benefits sighted in the growing literature base on XR-based representations, more mathematics teacher educators should consider using them–as only a small percentage do so currently (Austin & Kosko, 2022). Holographic representations of practice appear to have significant potential, but the current study is exploratory. As scholars begin to create and explore use of such representations of practice, the present paper may serve as a resource not only regarding the use but the creation of such representations of practice.

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