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Pre-service special education teachers' learning through recorded mini-lessons and peer review

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ABSTRACT

Despite the research regarding the importance of peer review and feedback in pre-service special education teachers, there exists a gap in teaching complex mathematical concepts such as fractional operations. This study sought to address this gap by investigating how pre-service teachers can effectively appraise and revise peer-generated teaching transcripts focusing on fraction operations and compare their feedback with those of experienced educators. The research sought to understand how this integrated approach can contribute to improving the instruction of pre-service special education teachers in the field of mathematics education. A modified version of Crespo's (2018) generating, appraising, and revising of representations was utilized to analyze the video content. Comparisons of the reviews showed that pre-service teachers may not have the content knowledge or experience to provide in-depth feedback to support learning as experienced educators. The article concludes with findings and recommendations for teacher educators who utilize anonymous peer review in teacher preparation for special educators.

Keywords: Crespo's framework, fractional operations, mathematics, pre-service teachers, reflection, video transcriptions

INTRODUCTION

Many teacher educators utilize a form of peer review with pre-service teachers. Although many studies demonstrate the positive characteristics of peer review, there may be situations in which it is not an effective instructional strategy (Howard et al., 2010). For example, peer review may not be advantageous when students rely on incomplete or incorrect knowledge as they critique others' work in a content area that may be challenging for them (Dijks et al., 2018). The purpose of this study was to enable special education pre-service teachers, who often find mathematics concepts difficult to grasp and explain, an opportunity to reflect and critique other's work while gaining a conceptual understanding of fraction operations. By using Crespo's (2018) generating, appraising, and revising model with strategic scaffolding to analyze instruction related to the operations of fractions in mathematics, the authors were able to study the benefits and potential difficulties pre-service teachers faced when learning and teaching fraction concepts . The following section will review the literature on teaching fractional operations with modeling and the difficulties pre-service teachers often encounter when teaching this topic. Peer review frameworks are also discussed, demonstrating the potential to develop professional reflective practices for pre-service teachers.

LITERATURE REVIEW

Teaching Fraction Operations with Modeling

Multiple models (i.e., set, area, length, and symbolic representations) are used when teaching fraction concepts and operations (Cramer & Henry, 2002). The set model involves a set of objects (i.e., two-color counters) representing a 'whole.' This whole comprises subsets of objects representing fractional parts (ex. ¼ of the whole or one red counter and three yellow counters) (Alqahtani et al., 2022). The area model is often utilized in schools and uses geometric shapes to convey fractions' part/whole meaning (Hodges et al., 2008). This model may be challenging for students because they need to partition the shape into equal parts (namely, thirds or fifths in circles and triangles). The length model (a linear model) assists students' understanding of fraction magnitudes and measurement. Fraction strips, number lines, and Cuisenaire rods are all examples of items used to represent the length model (Alajmi, 2012). Although various models are more beneficial than others, the National Council of Teachers of Mathematics (2014) advocates "making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem-solving."

Pre-Service Teachers' Difficulties With Fractions

Many studies attribute the lack of conceptual understanding of fractions found in children to be transferred from their classroom teachers' inability to consider alternative approaches that do not align with their teaching methods (Charalambous et al., 2020). The National Center for Education and Evaluation (Siegler et al., 2010) asserts, "far too many US teachers can apply standard computational algorithms to solve problems involving fractions but do not know why those algorithms work or how to evaluate and explain why alternative procedures that their students generate are correct or incorrect" (p. 8). This statement identifies the need to support pre-service teachers and addresses misconceptions of fraction concepts and operations before pre-service teachers enter the classroom (Crespo & Nicol, 2006; Hill et al., 2005). Thus, the preparation of future teachers must include as many opportunities as possible to investigate fractions using multiple representations using various models.

Among the misconceptions that pre-service teachers have regarding fractions are the misinterpretation and misrepresentation of concepts and operations (Novita et al., 2022). Regarding interpretation, researchers found that language could obscure the misunderstanding of fractions. For example, pre-service teachers needed help understanding what the referent whole was for a given fraction (Luo et al., 2011; Tobias, 2013). Regarding representation, pre-service teachers most commonly used the area representation of fractions instead of the length or set model. Furthermore, many pre-service teachers needed help to provide meaningful illustrations of improper fractions and operations using any model. Pre-service teachers' pictorial representations of fraction concepts given real-world applications were studied by Baek et al. (2016). Their research showed that pre-service teachers' experiences creating their drawings may help them deepen their understanding of fractions and operations and help them be more positive about providing their future students with similar learning opportunities. One way to utilize drawing while teaching mathematics is by incorporating the concrete, representation, and abstract strategy (CRA). The following section examines CRA strategy, commonly applied by pre-service teachers to teach fractions and demonstrate fraction operations (Kaya & Yildiz, 2023).

Concrete, Representation, & Abstract Strategy

CRA instructional sequence consists of three parts: concrete, representational, and abstract:

1. **Concrete:** At this stage, five modeled uses concrete materials (e.g., fraction circles, red and yellow chips, and squares). The teacher begins instruction by modeling each mathematical concept with concrete materials.
2. **Representational:** In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures, using circles, dots, and tallies; or using stamps to imprint pictures for counting.
3. **Abstract:** At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, −,×, and ÷) to indicate addition, multiplication, or division (Hauser, 2009).

CRA model can be utilized when instructing special education pre-service teachers on how to teach fractions. CRA is not a linear process; students should have all three strategies and understand the interaction among and between them (Crespo, 2018). CRA is not the only strategy that can be used to improve pre-service teachers' instruction regarding fractional operations. Teacher educators can also utilize peer review and feedback models to facilitate students' understanding of fractional operation models through reflection and revision of learning segments (Crespo, 2018).

Peer Review and Feedback Models

Positives & negatives of peer review

For this study, the authors defined peer feedback as interactions "aimed at redirecting and improving" the receiver's teaching (Griffth et al., 2020). A significant body of research documents the positive advantages of peer feedback. Benefits include pre-service teachers' increased ability to identify strengths and growth areas (Bas, 2021) and peer cooperation (Dijks et al., 2018; Howard et al., 2010). Peer feedback has also shaped professional practice among pre-service teachers (Dijks et al., 2018). Perhaps most importantly, utilizing peer feedback with pre-service teachers helps identify how teacher preparation programs can create and model learning communities in the classroom (Barrett & Frick, 2010).

Although there are many advantages to using peer review feedback in teacher education, there are also challenges. According to Topping (1998), the desire to avoid discouraging fellow students may also play a role in the types of feedback students give because they know that criticisms may damage relationships. The research found that students experienced embarrassment and anxiety when receiving reviews from face-to-face feedback (London, 1995; Lu & Bol, 2007). Following strategies have shown promise

(a) using computerized communication to avoid the possible embarrassment or discomfort faced by students in face-to-face feedback,

(b) using multiple evaluators to balance the uneven quality of peer feedback, and

(c) using anonymous peer feedback to minimize opportunities for students to reward friends or cheat during the peer feedback process (Machin & Jeffries, 2017).

This study collected written feedback to avoid face-to-face embarrassment for students, utilized multiple evaluators to balance uneven quality, and ensured anonymous participation to reduce bias.

Crespo's generate, appraise, & revise framework

It is challenging for pre-service teachers to transition from the role of a student to the teacher; therefore, teacher educators incorporate learning opportunities designed to support them in this shift. Typically, these types of activities, such as case studies and role play, position pre-service teachers as consumers of representations of teaching, leaving a gap in their ability to enact the instruction (Kennedy, 2006). For example, facilitating a whole-class discussion is a complex teaching practice that does not develop naturally and is an essential indicator of effective mathematics teaching (Danielson, 2013). The National Research Council (2001) recommends that pre-service teachers include conversation focused on the discussion instead of presenting polished mathematical ideas, which pre-service teachers may not be prepared to lead. Mathematics methods courses often give attention to productive and unproductive patterns of interactions (Herbel-Eisenmann & Breyfogle, 2005) and utilize Chapin et al.'s (2003) moves that promote discussion in a mathematics classroom. However, these professional learning activities follow a plan-teach-reflect model that limits learning opportunities for pre-service teachers.

Crespo (2018) proposed a teaching framework, "generating, appraising, and revising of representations," which differs from the traditional plan-teach-reflect structure. This approach invited pre-service teachers "to generate a representation of a mathematical discussion and to then sort and appraise the quality of their imaginary classroom dialogues" (Crespo 2018, p. 249). As a follow-up, pre-service teachers were also encouraged to revise and refine their mathematical representations and discussion to increase the quality of instruction. This process positions pre-service teachers as intellectual partners rather than technicians that reproduce the mathematics instruction they receive as students. In this way, pre-service teachers share the authority to generate, appraise, and revise representations of mathematics teaching.

Crespo (2018) investigated prospective teachers' thoughts on representations of mathematical practices. Initially, this took the place of creating a classroom dialogue between teachers and students and revising the exchanges. However, Crespo et al. (2004) supplemented their instruction by showing Carpenter and Romberg's video, Powerful Practices, which featured a teacher and students performing the same mathematical task. The students were then asked to revise their dialogues. Crespo's (2018) research demonstrated that consulting with others and revising based on what they learned through the Powerful Practices video appeared helpful to the pre-service teachers. Following the revision process, the pre-service teachers and teacher educators discussed the improved strategies and shared insights that emerged throughout the experience.

This study utilized a variation of the generate, appraise, and revise framework to examine the impact of using a peer feedback model in which teacher candidates had the opportunity to offer critical feedback to their peers with particular emphasis on analyzing the delivery of mathematics instruction. The study examined the extent to which students could provide constructive peer feedback. It also explored how teacher educators could optimize conditions and scaffolding throughout the peer review process to enhance the learning experience of pre-service teachers. The research question that guided the study was, "to what extent can pre-service teachers appraise and revise anonymous peer-generated teaching transcripts, and how does it compare to teacher educators' appraisal?"

MATERIALS & METHODS

This study was conducted in an undergraduate special education mathematics methods course. The pre-service teachers were explicitly taught to utilize multiple fractional models and CRA strategy to support understanding fractional concepts and operations. Pre-service teachers created instructional videos to teach fraction operations. Initially, these videos were viewed by four experienced educators (two mathematics professors, one education professor, and one K-12 intervention specialist) who provided feedback based on the content, vocabulary, and representations utilized in the video.

Pre-service teachers recorded an instructional video outside class time to ensure student anonymity. Each video was transcribed, and key images were added to demonstrate the mathematics (see **Appendix A**). Before students engaged in the peer review process, the course instructor modelled an approach to identify strengths and weaknesses in an instructional transcript and how to engage in the revisions process to increase the quality of instruction. After the scaffolded example, pre-service teachers peer-reviewed a sample of five mathematically correct instructional transcripts chosen by the course instructor. Pre-service teachers provided feedback based on content, vocabulary, and mathematical representations. Following the peer review, they engaged in the revision process. Pre-service teachers suggested ways to improve instruction, where they felt the content, vocabulary, or representations were unclear, incomplete, or ineffective. The thematic data analysis included comparing the feedback from four experienced educators and the pre-service peer reviewers. This comparison aimed to identify how pre-service teachers critically analyzed instructional methods and the extent to which their suggestions were appropriate, helpful, and research-based. Additionally, the researchers analyzed the pre-service teacher's transcript appraisal to understand how much pre-service teachers can strengthen instruction and learn from the peer-review process.

Thematic Analysis

This research aimed to understand how pre-service teachers appraised and revised anonymous peer-generated teaching transcripts, analyze how pre-service teachers' feedback compared to teacher educators' feedback, and explore how generating, appraising, and revising models can strengthen pre-service teacher preparation. To make sense of the appraisals and revisions, the researchers engaged in a thematic analysis process to bring "order, structure, and meaning to the mass of collected data" (Braun & Clarke, 2006, p. 111). Thematic analysis is defined as "a method for identifying, analyzing and reporting patterns (themes) within data" and "minimally organizes and describes the data set in (rich) detail" (Braun & Clarke, 2006, p. 79). This process was beneficial because it allowed the researchers to explore and compare the feedback of novice and experienced educators.

Table 1. Participant information

Participants	Gender	Class Rank
Student 1	Male	Junior
Student 2, 3, 4, & 5	Female	Juniors
Inclusion specialist	Female	7^{th} grade teacher
Three mathematics faculty	Female	College faculty

Note. Students responses were numbered, no pseudonyms or names were used in this study (student 1, 2, etc.)

In phase one of the thematic analysis, the researchers became familiar with the appraisals and revisions generated by pre-service teachers and teacher educators. They were immersed in the data to get a general overview of the depth and breadth of the written feedback. During this phase, the researchers took notes and jotted ideas for coding. In phase two, the initial codes were generated and served to "identify a feature of the data that appears interesting to the analyst and refers to 'the most basic segment,' or element, of the raw data or information that can be assessed in a meaningful way regarding the phenomenon" (Boyatzis, 1998, p. 63). Numerous codes emerged from the feedback using the principles of inductive content analysis. The analysis then shifted to broad themes in phase three. The researchers sorted the initial codes into potential themes. The themes represented distinctive patterns and captured the essence of the data. In phase four, the potential themes were consolidated, while others were separated based on the data within each theme. Finally, the themes were defined and refined in phase five. According to Braun and Clarke (2006), "define and refine" means "identifying the 'essence' of what each theme is about (as well as the themes overall) and determining what aspect of the data each theme captures" (p. 92). The themes were clearly defined by articulating what it was and were not formalized to guide the results and discussion.

Consistent with Crespo's (2018) analysis of pre-service teachers' explanations, the authors' analysis centered on examining the representations (pedagogical) and mathematical content of the transcripts (Crespo et al., 2011). The mathematical quality of the transcripts considered whether the operation was first correct and then if the symbolic and pictorial representations were consistent. The pedagogical qualities considered how the ideas of fraction operations were presented such, as the questions or explanations that pre-service teachers posed.

Participants

This study included five student participants, one middle school inclusion specialist, and three mathematics education faculty members. All students who were a part of this study were enrolled in a mathematics methods course for special education teachers. In order to be enrolled in the course, students must have an overall GPA of 3.0 in the education program. These students were required to also pass two courses in basic mathematics with at least a 2.0 GPA or above in those courses. The courses included K-8 mathematics content: whole number operations, fractions, decimals, percents, ratios, proportions, algebra, and geometry. The pre-service special education teachers also took several special education classes in which differentiation strategies were taught. After presented with the research study and a consent form from IRB approval, five students out of 22 in the one methods class volunteered as participants. The student work was reviewed and scored by a total of four faculty and educators. **Table 1** identifies further information about all participants (students and educators).

Course and Assignment

An early and middle school mathematics methods course (specifically designed for pre-service special education majors) met 180 minutes per week for 15 weeks within one semester. The study occurred during the eighth and ninth weeks of the semester when fractions were discussed. Pre-service teachers used fraction circles/squares, Cuisenaire rods, number lines, and fraction strips to represent fraction operations during the semester. Five of 10 students in the course consented to participate in the study. The instructions given prior to recording the video lesson included the following:

1. Submit one 2-4-minute video recording of your teaching of a particular type of fraction operation problem.
2. Use appropriate strategies and resources to adapt instruction to the needs of students with disabilities.
3. Use a wide variety of resources, including human and technological, to engage students in learning and to support student learning with high-quality media and technology.
4. Provide multiple models and representations of concepts and skills with opportunities (multiple routes to a solution, different forms of presentation) for students to demonstrate their knowledge in various ways.

Data Collection

The videos generated by five students who consented to participate in this study were analyzed by teacher educators and peers in a teacher education course. The professional educators included an inclusion specialist and three teacher education professors. Each professional provided written feedback on the instructional content, vocabulary, mathematical correctness, and usage of representations in each video.

Students enrolled in another section of the same course provided feedback and revised the student-generated work anonymously. The five videos were transcribed and included screen captures without student identification. The pre-service teachers' feedback and revisions were then analyzed for evidence of the ability to strengthen instruction and learn from the process.

RESULTS

A thematic analysis of the feedback provided by pre-service teachers and teacher educators revealed three major themes. Focus, depth, and clarity of the feedback emerged as critical components and are explained in detail in the subsequent sections.

Focus, Depth, & Clarity

Pre-service teachers

Pre-service teachers directed their attention toward minute intricacies and appeared to be diverted from deep, meaningful feedback by the potential misconceptions held by their peers. The feedback of pre-service teachers focused on surface-level aspects of instruction, such as tone and language that may confuse them. Regarding the teaching tone, pre-service teachers tended to address how the instruction would be perceived by students rather than the academic content of the transcript. When pre-service teachers discussed academic content, feedback was surface-level and often targeted vocabulary.

Pre-service teachers articulated segments of the teaching transcript that could confuse students and focus on how their peers gave directions. The pre-service teachers took the perspective of a student receiving the content instead of using a professional teacher lens. For example, one student reviewer asked, “are we supposed to know how to do this?” This comment illustrates that pre-service teachers still need to transition thinking deeply about instruction as a practitioner.

Although pre-service teachers could identify confusing segments within the teaching demonstration, they did not explain how their peers could teach or explain the section more effectively. For example, one peer reviewer stated, “this part is confusing; it is hard to understand what you are trying to say.” Sometimes, they identified a need for a more in-depth explanation of mathematical representations or operations. However, they offered only brief suggestions that could have been more meaningful to the content. For example, one student reviewer suggested changing the phrase “we are going” to “now we are going to try …” Feedback of this kind is insignificant when considering the big picture related to mathematical instruction.

Experienced educators

Meanwhile, the experienced teacher educators approached the task differently. They focused on the need for increased depth within the teaching segment and for pre-service teachers to explain fractional concepts using more detail. For example, one educator stated, “try to show the diagram first and then the procedure later. This visual will help explain why the numerator and the denominator are multiplied.” They focused on the instruction as a whole as opposed to separate segments and particular phrases.

Not only did the experienced educators identify weaknesses within the teaching segments, but unlike the pre-service teachers, they also suggested alternative ways to think about and explain the mathematical concepts. Their explanations were longer and more detailed, emphasizing mathematical concepts as their primary concern. For example, “the plate model and money were effective visuals of the subtraction process. The decimal discussion at the end may confuse students without seeing it written or by adding more explanation.” Feedback was primarily focused on academic content and strengthening instruction.

Comparison of feedback

When comparing the feedback from pre-service teachers and teacher educators, differences appeared in the length of feedback, the emphasis placed on content and vocabulary, depth of mathematical knowledge, and professional experience. Pre-service teachers prioritized surface-level aspects of instruction and displayed a student-centered perspective, focusing on how K-12 students would perceive teaching. Although it is essential to consider the learners’ needs, it is also important to understand and view the instruction from a prospective teacher’s lens. Experienced educators adopted a teacher’s viewpoint in addition to a student’s viewpoint, emphasizing the need for depth and detailed explanations of mathematical concepts. They offered alternative suggestions and strategies to enhance instruction and how it would impact a student’s understanding.

These divergent approaches highlight the disparity between pre-service teachers, still transitioning to a deeper understanding of instruction, and experienced educators who possess a broader perspective and prioritize content comprehension. Pre-service teachers primarily identified weaknesses in instruction and only offered brief suggestions for improvement. However, experienced educators provided comprehensive analyses and alternative strategies to enhance instruction and deepen students’ understanding of mathematical concepts.

CONCLUSIONS & DISCUSSION

This study involved multiple components in forming teachers’ content and pedagogical knowledge. Regarding content, modeling fraction operations proved difficult for pre-service teachers as they moved from their familiar procedural knowledge to helping children develop conceptual understanding. This study required pre-service teachers to create a video of their understanding of fraction operations after rational number and modeling instruction occurred. The findings of this study demonstrated a need to not only provide pre-service teachers with opportunities to reflect on their work but, more importantly, allow other students (peers) to revise and offer feedback anonymously.

Another component of pedagogical knowledge in this study was the importance of generating videos and appraising the instruction offered through peer review. It was apparent that peer, anonymous reviewers attended to superficial comments regarding their peer’s work while teacher educators were concerned with academic vocabulary, connections of representations, and the ability to convey the conceptual understanding of fraction operations. These differences point to the need for more scaffolding as pre-service special education majors negotiate the process of teaching complex content. Anonymous peer review was most helpful because peers offered non-biased feedback and served as another level of reflection and revision to impact their instruction positively. Revising is an untapped resource; more than identifying a weakness/point of confusion for students is needed. To improve instruction, students need to revise and strengthen their initial approach.

The study's findings both reinforce and deviate from the existing literature regarding teaching fractional operations with modelling and the effectiveness of peer review models. In alignment with the literature, the study confirms the multifaceted challenges encountered by pre-service teachers in understanding and conveying fraction concepts using various models. The results illustrate how pre-service teachers struggled with navigating between addressing surface-level concerns, often centered around students' immediate confusion, and delving into the profound understanding of mathematical concepts. This observation aligns with literature by (Crespo & Nicol, 2006; Hill et al., 2005) that portrayed the difficulties associated with transitioning from the role of a student to that of a mathematics teacher, emphasizing the need for support and training to bridge this knowledge gap effectively.

However, while the literature highlighted the potential benefits of peer feedback models in enhancing instructional strategies, the study's outcomes introduce a challenge to this approach. The findings echo the literature by Topping (1998), which acknowledge peer feedback's weaknesses and the lack of depth, or surface-level recommendations. Alternatively, experienced educators demonstrated a deeper grasp by providing comprehensive recommendations to enhance teaching strategies. This contrast signifies the importance of helping to cultivate pre-service teachers' capacity to provide specific and detailed feedback that can enhance their potential to contribute toward instructional improvement.

Limitations

For many pre-service teachers, this was the first time video generation and anonymous feedback exercises were given; therefore, students did not have prior experience or practice using this model. The number of pre-service teachers who participated was also limited due to enrollment numbers in the mathematics methods courses. Lastly, pre-service teachers and teacher educators used two different modes of analysis. Pre-service teachers used transcripts to reflect on their peer practice while teacher educators viewed the videos. These limitations lead to the following recommendations for future research.

Recommendations

This study revealed that peer feedback models could be beneficial for pre-service teachers and can be effective in improving mathematical instruction. However, peer review should be used cautiously, and pre-service teachers must be scaffolded throughout the process. It became evident that pre-service teachers may not yet be able to provide high-quality feedback to their peers. They are still in the early stages of developing their instructional toolbox; therefore, they often have difficulty envisioning alternatives and revisions. While this may seem discouraging, the researchers feel this is a promising model when carefully planned and implemented. The researchers recommend using peer feedback models with significant scaffolding, support, and practice. Teacher educators should consistently model and lead think-aloud discussions to help students appraise and revise instructional practices. Students should regularly work with partners or small collaborative groups to practice identifying strengths and weaknesses and should be encouraged to make revisions whenever possible.

Lastly, generating instructional videos, appraising instruction, and engaging in revision with their peers are meaningful throughout teacher preparation programs. This process should not be utilized merely as an assessment tool at the program's culmination. Instead, generating instructional videos followed by peer review and revision is another support we can provide to pre-service teachers to strengthen their instruction consistently in methods courses. Future research in this area could explore the impact of anonymous peer reviews on students' sense of freedom, reduced discouragement, and diminished embarrassment. Investigating the utilization of artificial intelligence technology for creating anonymous videos could be another avenue of study. Furthermore, it would be valuable to examine the application of these approaches beyond the mathematics classroom, such as in English or social studies methods, to assess their effectiveness and adaptability in different educational contexts.

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REFERENCES

Alajmi, A. H. (2012). How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics, 79*, 239-261. https://doi.org/10.1007/s10649-011-9342-1

Alqahtani, M., Powell, A., Webster, V., & Tirnovan, D. (2022). How a measuring perspective influences pre-service teachers' reasoning about fractions with discrete and continuous models. *International Electronic Journal of Elementary Education, 14*(3), 441-458. https://doi.org/10.26822/iejee.2022.255

Bas, G. (2021). Improving efficacy of pre-service teachers in educational research through peer review process. *The New Educational Review, 63*, 214-225. https://doi.org/10.15804/tner.21.63.1.17

Bley, N. S., & Thornton, C. A. (2001). *Teaching mathematics to students with learning disabilities*. PRO-ED.

Boyatzis, R. E. (1998). *Transforming qualitative information: Thematic analysis and code development.* SAGE.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

Buchanan, M., & Stern, J. (2012). Pre-service teachers' perceptions of the benefits of peer review. *Journal of Education for Teaching, 38*(1), 37-49. https://doi.org/10.1080/02607476.2012.643654

Carpenter, T. P., Romberg, T. A., & Carpenter, S. (2004). *Powerful practices in mathematics & scienc*e. National Center for Improving Student Learning and Achievement in Mathematics and Science.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2020). Classroom discussions using math talk in elementary classrooms. *Math Solutions*. https://mathsolutions.com/uncategorized/using-math-talk-in-elementary-classrooms/

Charalambous, C. Y., Hill, H. C., Chin, M. J., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: Exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*(6), 579-613. https://doi.org/10.1007/s10857-019-09443-2

Cramer, K., & Henry, A. (2002). Using manipulative models to build number sense for addition of fractions. In *Making sense of fractions, ratios, and proportions: Yearbook* (pp. 41-48). National Council of Teachers of Mathematics.

Crespo, S. (2018). Generating, appraising, and revising representations of mathematics teaching with prospective teachers. In R. Zazkis, & P. Herbst (Eds.), *Scripting approaches in mathematics education, advances in mathematics education* (pp. 249-264). Springer. https://doi.org/10.1007/978-3-319-62692-5_12

Crespo, S., & Nicol, C. (2006). Challenging pre-service teachers' mathematical understanding: The case of division by zero. *School Science and Mathematics, 106*(2), 84-97. https://doi.org/10.1111/j.1949-8594.2006.tb18138.x

Crespo, S., Oslund, J. A., & Parks, A. N. (2011). Imagining mathematics teaching practice: Prospective teachers generate representations of a class discussion. *ZDM, 43*, 119-131. https://doi.org/10.1007/s11858-010-0296-z

Danielson, C. (2013). *The framework for teaching: Evaluation instrument*. Danielson Group.

Dijks, M. A., Brummer, L., & Kostons, D. (2018). The anonymous reviewer: The relationship between perceived expertise and the perceptions of peer feedback in higher education. *Assessment & Evaluation in Higher Education, 43*(8), 1258-1271. https://doi.org/10.1080/02602938.2018.1447645

Griffith, A. N., Johnson, H. E., Larson, R. W., & Buttitta, E. K. (2020). A qualitative examination of critical feedback processes in project-based youth programs. *Contemporary Educational Psychology, 62*, 101892. ttps://doi.org/10.1016/j.cedpsych.2020.101892

Hauser, J. (2009). Concrete-representational-abstract instructional approach. *Access Center: Improving Outcomes for All Students K-8*. http://www.k8accesscenter.org/training_resources/CRA_Instructional_Approach.asp

Herbel-Eisenmann, B., & Breyfogle, L. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489. https://doi.org/10.5951/MTMS.10.9.0484

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hodges, T. E., Cady, J., & Collins, R. L. (2008). Fraction representation: The not-so-common denominator among textbooks. *Mathematics Teaching in the Middle School, 14*(2), 78-84. https://doi.org/10.5951/MTMS.14.2.0078

Howard, C. D., Barrett, A. F., & Frick, T. W. (2010). Anonymity to promote peer feedback: Pre-service teachers' comments in asynchronous computer-mediated communication. *Journal of Educational Computing Research, 43*(1), 89-112. https://doi.org/10.2190/EC.43.1.f

Kaya, S., & Yildiz, N. G. (2023). Using the concrete–representational–abstract sequence to teach math skills to a student with autism spectrum disorder in a general education classroom. *International Journal of Developmental Disabilities*. https://doi.org/10.1080/20473869.2023.2180539

Kennedy, M. (2006). Knowledge and vision in teaching. *Journal of Teacher Education, 57*(3), 205-211. https://doi.org/10.1177/0022487105285639

Lim, W., Son, J.-W., & Kang, S.-H. (2021). How reducing discomfort impacts peer assessments of pre-service teachers. *Sustainability, 13*(11), 6435. https://doi.org/10.3390/su13116435

London, M. (1995). Giving feedback: Source-centered antecedents and consequences of constructive and destructive feedback. *Human Resource Management Review, 5*(3), 159-188. https://doi.org/10.1016/1053-4822(95)90001-2

Lu, R., & Bol, L. (2007). A comparison of anonymous versus identifiable e-peer review on college student writing performance and the extent of critical feedback. *Journal of Interactive Online Learning, 6*(2), 100-115.

Luo, F., Lo, J., & Yuh, C. L. (2011). Fundamental fraction knowledge of pre-service elementary teachers: A cross-national study in the united states and taiwan. *School Science and Mathematics, 111*(4), 164-177. https://doi.org/10.1111/j.1949-8594.2011.00074.x

Machin, T. M., & Jeffries, C. H. (2017). Threat and opportunity: The impact of social inclusion and likeability on anonymous feedback, self-esteem, and belonging. *Personality and Individual Differences, 115*, 1-6. ttps://doi.org/10.1016/j.paid.2016.11.055

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Novita, R., Herman, T., Dasari, D., & Putra, M. (2022). Analyzing second-year university students' rational number understanding: A case on interpreting and representing fraction. *European Journal of Educational Research, 11*(3), 1747-1762. https://doi.org/10.12973/eu-jer.11.3.1747

Siegler, R., Carpenter, T., Fennell, F., Geary, D. C., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide*. National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, US Department of Education.

Tobias, J. M., Olanoff, D., & Lo, J. (2012). *A research synthesis of pre-service teachers' knowledge of multiplying and dividing fractions.* North American Chapter of the International Group for the Psychology of Mathematics Education.

Topping, K. (1998). Peer assessment between students in colleges and universities. *Review of Educational Research, 68*(3), 249-276. https://doi.org/10.3102/00346543068003249

APPENDIX A

Table A1.

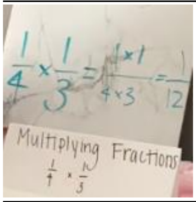
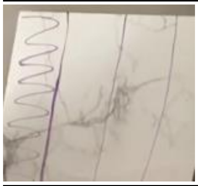
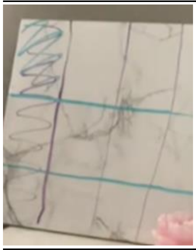
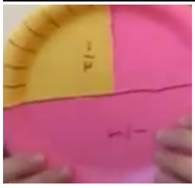
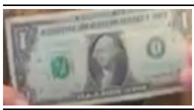
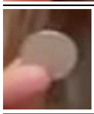
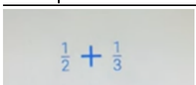
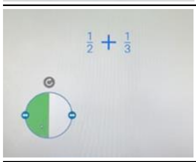
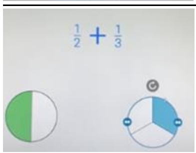
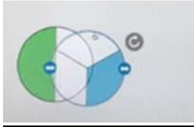
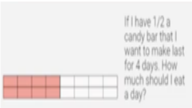
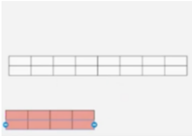
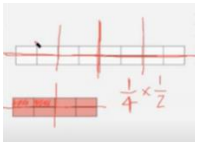
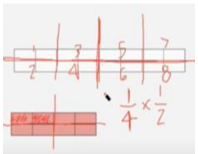
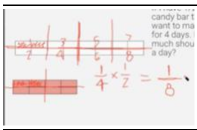
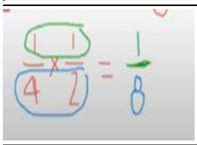
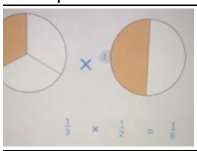
Examples	**Problems**
Example 1	Mathematics problem: ¼×⅓ video transcript
	Okay, so, we are going to learn how to multiply this fraction, which is ¼ times 1/ 3. So, if everyone could write it down, then first step to multiplying fractions is to multiply numerator.
	So, this is numerator, two ones, so, we are going to rewrite it over here so we are going to go one times one because we are going to multiply them together & then our second step would be to also whenever we do top we do to bottom when multiplying; so, we are going to multiply four times three & then one times one equals one four times three equals 12. So, we get answer of 1/12.
	I am going to show you guys another way on how we can figure this out. So, our first step is multiplying numerator. Second step multiplying denominator & then if the fraction is not in its simplest form you would have to reduce it. Second, way I am going to show you we are going to split our paper into fourths & then we are going to shade in 1/ 4. So that you guys can actually see 1/ 4 on paper. So, let's draw three lines to split it into fourths. So, now we have four equal parts: one, two, three, & four. We are going to shade one of them because our fraction is one-fourth. Okay & now we are going to demonstrate fraction one-third. So, now we are going to split our paper other way into three parts: one, two, & three. Try to make them as equal as possible, but it is not a big deal. Then since we are multiplying fractions, we want to show one-third of one-fourth; so, we are just going to color in we know that one-third of one-third would be this whole entire row, but since we want to show it of one-fourth we are just going to color in this one here & then that gives us 1/12.
Example 2	Mathematics problem: ½-¼ video transcript
	I am going to show how to solve problem one-half minus 1/4. A good model to use when looking at addition & subtraction of fractions is to use something like a plate model. So, here I have two plates put together & I have taken time to label a half of my plate one half & half of my plate & two, two fourths. So, as you can see, one half is equal to two-fourths.
	So, what would happen if we took away one of these fourths? So, now we will be able to see that a half minus one of those fourths is going to leave us with one-fourth; so, one half minus one fourth is one-fourth.
	Another good way to look at this would be like if we had a dollar. So, in terms of money this would be one whole.
	So, if we are looking at half a dot half of a dollar, we would be looking at two quarters or fifty cents.
	So, if we took away one of these quarters being the one-fourth we would be left with another quarter which is one-fourth. So, 0.5 minus 0.25 equals 0.25.
Example 3	Mathematics problem: ½+⅓ video transcript
	So, this week we have been talking about fractions. So, today we are going to be adding fractions we are going to add ½ plus 1/3.
	Now, how we are going to do this is we are going to take a circle down here and drag it over here. And now this circle is going to represent one half. So, how we are going to do that is we are going to divide it into two equal parts and then we are going to color in the one side green. And now this represents one half.
	Now, we are going to get another circle and we are going to drag it over here and this one is going to represent one third. So, we are going to divide this circle into three equal parts, and we are going to shade one section of it blue now this represents one-third.

Table A1 (Continued).

Examples	Problems
	Now to kind of visualize this we are going to turn this one third circle down like that and we are going to drag the one-third circle on top of the one half circle.
	So, now we can see since we put the one-third circle on top of the one-half that there is a lot shaded in a little bit that is not shaded in. So, from looking at this we can see that these spaces are bigger than these two spaces that is because this should be divided into six equal parts.
	So, we are going to take our line and we are going to draw all that out. Okay. So, this circle is now divided into six equal parts. Within these six equal parts, we can see what is shaded and what is not. So, we can count together and see that one, two, three, four, five. Five out of the six parts of this circle are shaded in which means that is five out of six. So, with this visual we now know that one-half plus one-third equals 5/6.
Example 4	Mathematics problem: $\frac{1}{4}+\frac{1}{2}$ video transcript
	Hey guys! Today we have multiplying fractions for bell work. I know we did not go over fractions a ton last week; so, I am not expecting anybody to be an expert. We are going to do slow & easy together. This will prepare you guys for what I have planned today. So, my question of the morning is. If I have half of the candy bar and I want to make it last for four days, how much should I eat a day? So, right over here, I have our candy bar. This is the entire candy bar, and this highlighted bit is the half we have left.
	I am going to drag this down for us to make life a little bit easier. So, here is my half of the candy bar and I want to eat equal parts of it over four days. Can anybody tell me what that is going to look like as a fraction? (wait time) Do we not know? Well that is okay.
	Um let is start by thinking about like this. I want to eat one equal part of a candy bar for four days; so, for four days if I want to eat one part I am going to divide it into four parts.
	Here, we go start to stamp our fraction over here. Here we have four because we divided the candy bar into four parts and how many parts do I eat a day? I ate one part a day; so, we are going to highlight this right here, one fourth. What is this top number? The numerator. And this bottom number? The denominator. Yeah, I thought I'd catch you guys with that one. Okay. So, one-fourth of one-half and another way to say that is one-fourth times one half. So, the question is really asking, what is one-fourth of one-half?
	To figure that out, I am going to have to make our pieces on our little half chocolate bar down here equal to what is going on up here. So, we are going to write 1/ 4 of one-half. This final fraction is going to show us uh what part we would get to eat all the original candy bar. So, to first do this I am going to divide our original candy bar in half and then I am going to divide it into four equal parts. Do you guys see how it is starting to look like the one we made down there?: They are half a candy bar. Um, okay now let is count the sections together: one, two, three, four, five, six, seven, eight. Okay. So, if we were to take one fourth of one half, how many sections would we have? It is going to find our denominator for our final fraction. Eight, yeah we just counted that good job guys and we would still just be eating one section a day. Remember this little highlighted section down here? If we bring it up here, I am just going to eat one part a day still; so, our final fraction would be 1/ 8.
	So, we are taking one-fourth out of our one-half we just drew that.
	What is another way we can do this guys? A little bit faster? Let me just rewrite down here for us. Nobody knows? Okay. Well another way we can approach this problem is simply multiply our numerators and our denominators. Now let me pick a new color. We are going to multiply our numerators. So, what is one time one times one everybody? One. I'd be worried if you didn't get that one. And then what is four times two? Eight. Yeah we see how these are the same and that is all guys. Okay. So, we are going to go into today is lesson I hope we are all ready.
Example 5	Mathematics problem: $\frac{1}{3}\times\frac{1}{2}$ video transcript
	Ok. So, we are going to be learning how to multiply fractions. So, we have $\frac{1}{3}$ times $\frac{1}{2}$. So, first we multiply the numerators. So, one times one equals one, and that is the one for our new fraction. And then we multiply three times two and then we get six because we also multiply the denominators. So, then we get six; so, 1/6 is our new fraction. And then here is a visual of one-third and then there is a visual of one half on a circle. And then that is what we get is one sixth.