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Pre-service mathematics teachers investigating the attributes of inscribed circles by technological and theoretical scaffolding

Ruti Segal [1,2*], Moshe Stupel [2,3]

[1] Oranim Academic College of Education, ISRAEL
[2] Shannan Academic College of Education, ISRAEL
[3] Gordon Academic College of Education, ISRAEL
*Corresponding Author: rutisegal@oranim.ac.il

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ABSTRACT

The benefits of technological and theoretical scaffolding were observed when pre-service teachers aiming to teach upper elementary grades were given three learning-based geometrical inquiry tasks involving inscribed circles. They were asked to collaboratively examine the accompanying geometrical illustration and data for some new or interesting feature and then propose a hypothesis resulting from their observations and prove them.

Due to the difficulty generally involved in proposing and proving geometrical hypotheses, two forms of scaffolding were provided: theoretical scaffolding based on revising previous learning or specific attributes of the given data and technological scaffolding in the form of specifically designed GeoGebra applets that allowed dynamic observation of the attributes of the geometrical shapes and the changes they underwent during modification.

We found that the two forms of scaffolding led to relatively pre-service teachers' high levels of success. They exhibited high levels of interest and participation, were engaged in the tasks, and underwent high-quality learning processes. In follow-up interviews, they confirmed that the exercise improved their inquiry skills, and developed their pedagogical and technological knowledge.

Keywords: dynamic geometry environment, GeoGebra, inscribed circles, pre-service mathematics teachers, task design

INTRODUCTION

Euclidean geometry has been studied in depth throughout history, from ancient times to the present. Despite its ancient origins, mathematicians continue to discover new features. Any discovery of a "new" or unfamiliar property requires proposing a hypothesis and then offering a precise mathematical proof.

The proof process is at the heart of mathematics, and knowledge of how to write and present a proof is vital for pre- service and in-service mathematics teachers so that they may impart these skills to their future students (Haj-Yahya, 2022; Hanna, 2000). Unfortunately, the school curriculum does not always foster the development of these skills. As a result, knowing how to properly write a proof in geometry often poses a challenge for both pre- service and in-service mathematics teachers (Noto et al., 2019; Oflaz et al., 2016) either due to their lack of sufficient mathematical knowledge or insufficient experience in the teaching process.

To improve the teaching and learning processes, it is important to expose teachers how to submit, to examine and to manage geometrical situations, analyze specific attributes, for propose a hypothesis, and write a proof (Oxman et al., 2016, 2018; Segal et al., 2015, 2017). Overcoming the difficulties inherent in such a task often requires the instructor to offer a meaningful and supportive environment such as scaffolding to help and guide the students regarding what attributes are important and to refresh their memories with respect to knowledge that they may have (Cirillo & May, 2021; Dove & Hollenbrands, 2013; Tall, 1995).

Teachers are interested in integrating technology in their teaching, but at the same time they encounter obstacles including lack of confidence, lack of ability and skills and lack of access to appropriate sources. All of these are significant and main barriers that prevent teachers from integrating technology into teaching. Accessibility to appropriate technological sources includes familiarity with technological learning environments, software, and more. Hence, pre-service teacher should receive support during being experience in integrating technology, and gradually deal with these obstacles in their training and during their teaching (Bingimlas, 2009; Kihoza et al., 2016; Triutami et al., 2020).

There are myriad of technological tools available today to assist the development of mathematical theoretical knowledge and enhance the process of determining attributes and exposing teachers to the variety of technological resources and options available will advance their teaching skills (Krause et al., 2017; Sutiarso et al., 2017). Imparting them positive attitudes toward the use of digital media in the classroom will encourage them to adapt new technologies and integrate them into the learning and teaching processes. Gaining knowledge and experience regarding the use of technology may give teachers the motivation to actually incorporate such technologies into their instruction (Bingimlas, 2009; Lim et al., 2013).

In this paper, we offer three tasks involving features of circles inscribed in triangles or other polygonal shapes that we believe it can be used by teacher educators to expand their students pedagogical and technological geometric knowledge, aiming to teach upper elementary grades. Alongside the static illustration that accompanies each task, a dynamic representation (GeoGebra applet) to augment the variety of examples suitable for the data related to the static task, was prepared for each. The students (in groups) were asked to research the data with the aid of this technological tool and attempt to discover an interesting or novel (to them) feature regarding the inscribed circles that had to do with conservation of a property. They were then asked to propose a hypothesis, articulate it, and present a complete proof.

The aims of these activities were to allow students to experience inquiry tasks while working in a dynamic environment, which have the potential to develop their pedagogical and technological geometric knowledge. This may raise their awareness of geometry as a rich and diverse field and encourage them to offer fertile ground for inquiry activities for their students in school.

Because the students' often demonstrated difficulty in conjecturing and pinpointing the aimed-for hypothesis or proving it, we submitted students tasks, and in addition, provided two types of scaffolding (Cirillo & May, 2021) to assist students in their learning process: One of the scaffolding was technological scaffolding, via GeoGebra software, includes dynamic representations of the objects evolved in the given task to guide students' attention to attributes that remained constant when the vertices and/or sides of the triangle were dragged. The second was theoretical scaffolding included data about previously learned concepts that related to the tasks such as, pointing out geometric relationships, imparting advice regarding working with the dynamic software, and/or reminding them of the use of auxiliary constructions.

LITERATURE REVIEW

Current literature offers a number of models to describe knowledge related to integrating technology into teaching and the professional development of pre-service and in-service teachers in this context.

One model is the TPACK (technological, pedagogical, and content knowledge) model (Koehler & Mishra, 2009), which is based on terms coined by Shulman (1986) and which is, as the name implies, an amalgamation of technological, pedagogical, and content knowledge (TK, PK, and CK, respectively). Specifically, TK is knowledge about the various technologies available for teaching in the classroom environment, PK is knowledge about methods and instruction processes, and CK is knowledge of the topics relevant to learning or teaching (i.e., the suggested curriculum for each subject). TPACK implies that all these bodies of knowledge intersect at various levels of complexity as reflected in the common components of basic knowledge. For example, TCK (technological content knowledge) concerns how technology can create different representations for a specific concept. TCK demands that the teacher recognizes how using specific technologies can affect learners' skills and understanding of the relevant concepts and content.

Another model regarding the integration of technology into the design of learning processes is the SAMR (substitution, augmentation, modification, redefinition) model (Puentedura, 2013, 2014), which suggests that there are four levels of technology application in teaching: substitution, augmentation, modification, and redefinition. "Substitution" means that the technology replaces previous tools with new, technological ones, without any functional change. "Augmentation" implies the use of technology to replace previous tools while at the same time incorporating additional performance that was not there originally. "Modification" means that the technology is integrated into familiar tasks and contributes to the achievement of learning goals. At the highest level, "redefinition," technology leads to output and products that could not have been created otherwise.

These two model TPACK and SAMR serve as a theoretical framework for studies dealing with the challenges of teachers in integrating technologies, as well as in characterizing the knowledge and the competences required for integration of technology in teaching (Falloon, 2020; Kihoza et al., 2016),

Integrating Dynamic Inquiring as Technological Scaffolding

The use of technology in teaching and learning processes, enables dynamic presentation and exploration of various geometric attributes. It provides a quick and much more accurate replacement for traditional geometry drawing tools: pencil, straightedge, and compass.

Implementing technology as a scaffolding such as media scafolding, during geometry teaching and learning processes can assist students to complete a geometrical problem, help shorten learning time, and give a strong impetus to understanding concepts in geometry (Sutiarso et al., 2017). Submission of an assignment to learners by using scaffoldings can promote opportunities for explaining, reviewing, restructuring, and developing conceptual thinking (Dove & Hollenbrands, 2013). Teachers can provide scaffolding to support their students, especially those who have difficulty understanding a concept or solving a problem. Scaffolding can be provided when a task is beyond the ability of the students and can be adapted to the proximal developmental area of the students to enable acquisition of a new skill or new knowledge (Baxter & Williams, 2010). It is important to give teachers skills in providing scaffolding so as to be able to provide students with adequate conceptual and strategic roadmaps that help them understand the process of inquiry. Knowing how to properly plan for scaffolding reduces the amount of spontaneous, unstructured scaffolding they may have to deal with (Saye & Brush, 2002).

Research Question

What are the benefits of learning geometry-based inquiry with technological and theoretical scaffolding to pre-service teachers' professional development?

METHODOLOGY

Participants

The 17 participants were enrolled in college program leading to a B.Ed. in teaching mathematics in upper elementary grades and participating in a course devoted to implementing technology into geometry instruction. They were divided into five groups of three-four students each. Each group was assigned a task that focused on the attributes of inscribed circles. The tasks were presented to students both as hard copy (worksheet) and via applets prepared with GeoGebra dynamic software (The three tasks are described below presented with links to GeoGebra applets, which provided as part of the task descriptions.)

The Process

In accordance with the framework for incorporating tasks with technology proposed by Trocki and Hollebrands (2018), students were given prompts that included questions or directions for inquiry that required written proof. The tasks were displayed alongside the applets prepared for the task. This allowed the students to explore the concepts involved in the tasks, identify properties, propose hypotheses, and test their hypotheses before proving them (Cirillo & May, 2021).

The inquiry process of the given tasks took up two 90-minute lessons. In the first stage, the students were asked to explore and examine their assigned task, propose a hypothesis based on their work with the GeoGebra applets (by technological scaffolding), and prove it. If they felt it necessary, they could request additional information/hints (i.e., theoretical scaffolding) to assist them in the proof process.

In the second stage of the lesson, each group of students presented their hypothesis and its proof to their peers. This was followed by a plenary discussion about the process itself, the value of the technological tool, and the contribution this activity made to expanding their knowledge as future teachers.

Task 1-Group 1: Circles Inscribed in a Triangle and the Triangles Formed by a Cevian

Given: Triangle ΔABC with inscribed circle (O, r). Cevian AD divide the triangle into two triangles ΔABD and ΔACD (**Figure 1**). (S_1, r_1) and (S_2, r_2) are the areas of and the radii of the circles inscribed in triangles ΔABD and ΔACD, respectively.

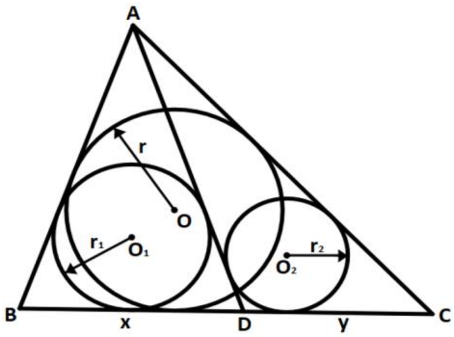

Figure 1. Diagram for task 1 (Source: Authors' own elaboration)

Let $AD=z$, $DC=y$, and $BD=x$.

Propose and prove a hypothesis about the relationship between the radii of the circles.

Process: The students began the inquiry process by examining the objects in the exercise and tried to refresh their knowledge about theorems and properties of inscribed circles in a triangle, and then turn to work in technological environment.

Technological scaffolding: A GeoGebra applet had been prepared where students could drag each of the vertices as well as vertex D (https://www.geogebra.org/m/nprbvfvc). The following dimensions appeared on the screen: r, r_1+r_2, ∡ABC, ∡ACB.

The students accessed the attached applet. They began by observing individual cases obtained by dragging vertices on the screen and looking at the different triangles such as those shown in **Figure 2** and **Figure 3**, respectively.

Following observation of the individual cases and by dragging the triangles' vertices and sides and tracking the lengths of the sides and radii of the circles, the students noticed that the radius of the circle inscribed in triangle ΔABC was always smaller than the sum of the radii of the other two circles. To verify this supposition, they added a text box that allowed corresponding data to be recorded on the screen while dragging the vertices and sides:

As a result, they proposed the following:

Hypothesis: $r_1+r_2>r$. However, the students in the group found it difficult to prove the hypothesis.

During the discussion that took place, the students helped each other refresh existing knowledge about the center of a circle inscribed in a triangle and that it is the intersection point of the triangle's bisectors. Nevertheless, they still were not able to use

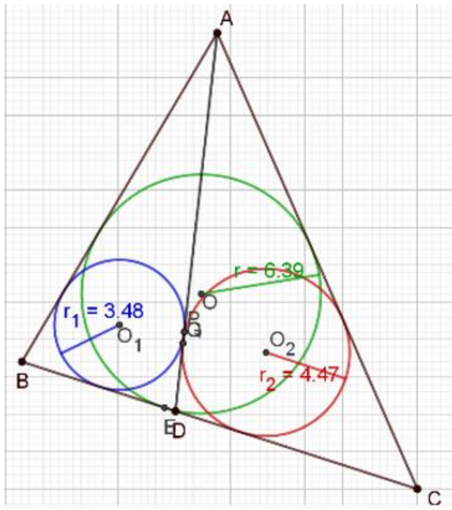

Figure 2. Examples of changes to the triangle observed using the Applet (Source: Authors' own elaboration)

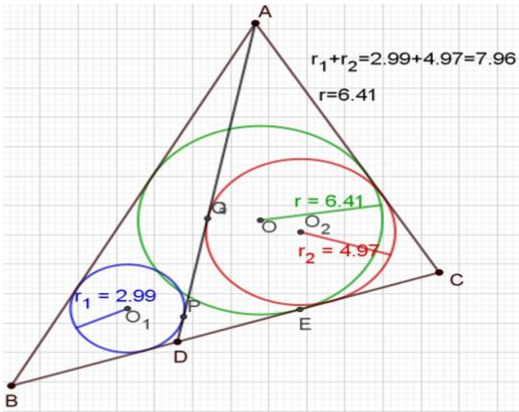

Figure 3. Examples similar to **Figure 2** with the addition of text boxes (Source: Authors' own elaboration)

this information to provide a conjecture on how to calculate the lengths of the radii and express what the relationship between them might be.

Theoretical scaffolding (hint): The formula $r = \frac{S_\Delta}{p}$ (already learned in a course on Euclidean geometry that preceded the present course). This hint enabled students to define each of the radii that appear in the illustration by using the appropriate formula and, from there, to successfully prove the hypothesis.

Proof:

$$r_1 = \frac{2S_1}{AB + x + z}, r_2 = \frac{2S_2}{AC + y + z}, r = \frac{2(S_1 + S_2)}{AB + BC + AC}$$

From ΔACD we obtain $z < y + AC$, then

$AB + x + z < AB + x + y + AC = AB + BC + AC$

And so, $r_1 > \frac{2S_1}{AB+BC+AC}$.

Similarly, $r_2 > \frac{2S_2}{AB+BC+AC}$. Thus $r_1 + r_2 > \frac{2S_1+2S_2}{AB+BC+AC} = r$.

Task 2. Circles Tangential to Each Other at a Point on the Cevian

Given: Triangle ΔABC. Inscribed circle (O, r) is tangent to the sides of the triangle at points E, F, and G. Cevian AD forms two triangles ABC and ACD in which circles (O_1, r_1) and (O_2, r_2) are inscribed and which are tangential to the Cevian at points M_1 and M_2 (**Figure 4**).

Propose and prove a hypothesis about the relationship between the location of tangent points M_1, M_2, E, and point D.

Process: The students start to think about the task, and very quick start to work with the applet because they did not find a suitable theorem that may help them as a starting point. They were curious about how the locations of points changed when changing the triangle size by dragging its vertex.

Technological scaffolding: A GeoGebra applet (**https://www.geogebra.org/m/wqzcmqm4**) had been prepared where students could drag points A, B, C, and D, thereby changing the side lengths and angles for each triangle. They could thus observe that when points D and E coincide, circles (O_1, r_1) and (O_2, r_2) were tangential to Cevian AD at the same point (**Figure 4**). The

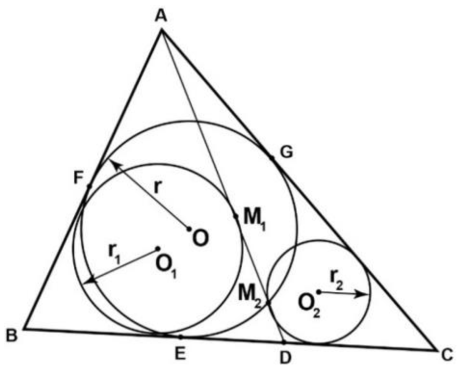

Figure 4. Diagram for task 2 (Source: Authors' own elaboration)

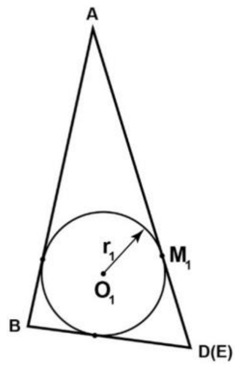

Figure 5. D and E coincide in the diagram for task 2-1 (Source: Authors' own elaboration)

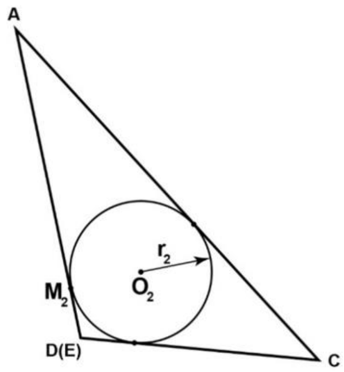

Figure 6. D and E coincide in the diagram for task 2-2 (Source: Authors' own elaboration)

students succeeded to formulate hypothesis, but as they identified difficulties in presenting proof, they asked for a hint that would allow them to prove the hypothesis, namely a theoretical scaffolding.

Hypothesis: When point D of the Cevian coincides with tangent point E, points M_1, M_2will also coincide.

Theoretical scaffolding (hint): The students were reminded of some geometric relationships.

Note the following:

$AB = c, BC = a, AC = b$

$P_{\Delta ABC} = \frac{a+b+c}{2}$,

which means that,

$AG = AF = p_{\Delta ABC} - a$

$BF = BE = p_{\Delta ABC} - b$

$CE = CG = p_{\Delta ABC} - c$

Proof: When point D has coincided with point E (**Figure 5** and **Figure 6**), then:

When point E coincides point D (**Figure 4**), we can look separately at the two triangles (**Figure 5** and **Figure 6**). According to **Figure 4**, **Figure 5**, and **Figure 6**,

$BD = BF = p_{\Delta ABC} - b$

$DC = CG = p_{\Delta ABC} - c$

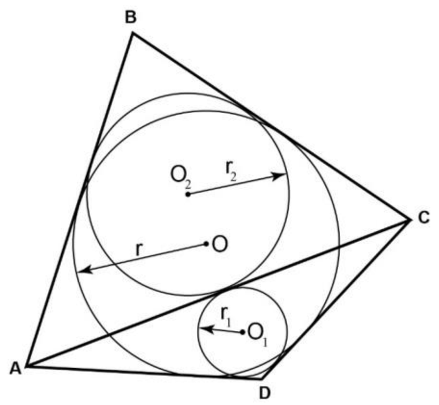

Figure 7. Diagram for task 3 (Source: Authors' own elaboration)

According to **Figure 5**,

$AM_1 = p_{\Delta ABD} - BD = \frac{AB+AD+BD}{2} - BD = \frac{AB+AD-BD}{2}$.

According to Fig. 5b,

$AM_2 = p_{\Delta ACD} - CD = \frac{AC+AD+CD}{2} - CD = \frac{AC+AD-CD}{2}$.

According to **Figure 3**,

$BD = p_{\Delta ABC} - b, CD = p_{\Delta ABC} - c$.

By substituting the expression for BD and DC into the expressions for AM_1 *and* AM_2, we obtain

$$AM_1 = \frac{AC + AD + BC - p_{\Delta ABC}}{2}, AM_2 = \frac{AC + AD + BC - p_{\Delta ABC}}{2}$$

From this we see that $AM_1 = AM_2$ and therefore the tangent points of each circle to Cevian AD will coincide.

Task 3: Circles Inscribed in a Quadrilateral and Triangles

Given: Circle (*r*, *O*) is inscribed in quadrilateral ABCD. Diagonal AC is drawn in the quadrilateral to obtain two triangles in which (r_1, O_1) and (r_2, O_2) are inscribed circles (**Figure 7**).

Propose and prove a hypothesis about the relationship(s) between the radii of the circles.

Process:

Technological scaffolding: A GeoGebra applet had been prepared to investigate the case of any quadrilateral in which a circle with a given radius is inscribed https://www.geogebra.org/classic/kkwr2nue and the radius of the circle can be changed by dragging on a slider. Similarly, two of the quadrilateral's vertices can be dragged, thereby changing the side lengths and angles of the two triangles that compose the quadrilateral and changing the radii of the inscribed circles. For each quadrilateral, the values of the radii of the inscribed circles were displayed, alongside the sum of the radii of the circles inscribed in the triangles.

The students observed the changes that came about as a result of dragging the triangle's vertices and sides and traced the lengths of the sides and the circles' radii. They proposed the following hypothesis:

Hypothesis: $r_1+r_2>r$. However, they could not prove it.

Theoretical scaffolding (hint): The lecturer reminded the students of the formula, $r = \frac{S_\Delta}{P}$ (previously learned). This hint enabled the students to define each of the radii that appear in the illustration using the appropriate formula and, from there, to successfully prove the hypothesis.

Proof:

$$r_1 = \frac{S_{\Delta ACD}}{p_{\Delta ACD}}, r_2 = \frac{S_{\Delta ABC}}{p_{\Delta ABC}}, r = \frac{S_{ABCD}}{p_{ABCD}}$$

$$r_1 + r_2 = \frac{S_{\Delta ACD}}{p_{\Delta ACD}} + \frac{S_{\Delta ABC}}{p_{\Delta ABC}}, r = \frac{S_{\Delta ACD}}{p_{\Delta ACD}} + \frac{S_{\Delta ABC}}{p_{\Delta ABC}}$$

$since\ p_{\Delta ACD} < p_{ABCD}$ and $p_{\Delta ABC} < p_{ABCD}\ then, r_1 + r_2 > r$.

The above attribute is true for any polygon with *n* sides in which a circle is inscribed. Passing diagonals from any vertex, (n - 2) will form triangles in which the radii of the inscribed circles follow:

$r_1 + r_2 + r_3 + \cdots + r_{n-2} > r$.

CONCLUSION

The activities presented above gave the students a challenge unlike anything they had faced before; it included proposing hypotheses based on static data alongside dynamic data provided by the GeoGebra software. The addition of technological and theoretical scaffolding, i.e., supplying applets to enable them to observe the given task from a dynamic point of view. It is offering hints to allow them to refresh existing knowledge of already-studied theories and formulae or discover an interesting connection between the concepts incorporated in the task. In addition, working with GeoGebra applets allowed students to recognize which attributes remained constant and thereby formulate suitable hypotheses and prove them.

The activity improved the students' mathematical, pedagogical, and technological knowledge, as explained in the following.

Mathematical-Technological (MT) Knowledge:

Having to cope with the task in a dynamic environment encouraged the students to **revise** their knowledge of Euclidean geometry such as the formula for finding the value of the radius of an inscribed circle based on the area of the triangle and half its circumference ($r = \frac{S_\Delta}{p}$), what conditions the sides of a quadrilateral must meet so that a circle may be inscribed in it, how to find the center of an inscribed circle in such a quadrilateral, and the like. In addition, during the support (scaffolding) stages, the students successfully **expanded** their mathematical knowledge with respect to concepts heretofore unknown to them, such as those regarding the conservation of properties. This was the result of both the support bestowed by the technological environment (which allowed them to explore, measure, and calculate values; propose hypotheses; and prove them) and the support bestowed by the theoretical scaffolding.

Following are some relevant quotes made by some of the students during the concluding discussion: "The technology helped me discover mathematical attributes about inscribed circles that were unfamiliar to me. I want my future students to undergo such an experience"; "Working in the technological environment contributed to my confidence in understanding the mathematical concepts involved in this task. It helped me with the inquiry process."; "During these activities I underwent a novel learning experience in which I discovered new, unfamiliar properties, just like real mathematicians."

It is important to select tasks that are related to each other, such as in this case, where all the tasks involved inscribed circles and the students were already familiar with the relevant theorems from their high school curriculum. However, the tasks presented in this paper allowed them to explore features beyond what were taught in the curriculum and observe the mathematics–in particular the geometry–as an all-encompassing, rich subject that they would be able to explore with their students later similarly at school.

Technological Pedagogical Knowledge (TPK)

The students expanded their TPK as a result of their exposure to a task that was also presented in a technological framework, thus allowing the learner to discover interesting phenomena. The students experienced the benefits of technological and theoretical scaffolding when solving geometry tasks, an approach that can be applied in the classroom environment in accordance with the subject being taught.

Some relevant comments: "This activity taught me that pupils can be given exercises with data and then be directed to identify an unfamiliar geometric feature using technology. I imagine my pupils, too, will find this a fun and productive exercise".

Technological, Pedagogical, and Content Knowledge (TPACK)

Students expanded their mathematical TPACK as a result of working in a technological environment and discovering the contribution this environment can make to the processes of research, proposing hypotheses, identifying multiple cases from which to speculate about conservation, and more. They appreciated how the technological environment exposes learners to dynamic representations of geometric concepts and the different approaches it offers (scaffolding) to support the process of building and designing exploratory tasks.

Some remarks: "Carrying out the inquiry, discovery, and proof processes alongside peers in the technological environment helped us–as future teachers–better understand the critical importance of integrating diverse types of knowledge"; "While working together on our task, we discovered that with an effective and necessary combination of geometric knowledge, geometry instruction, and technological knowledge, we will be able to construct appropriate GeoGebra applets to support learning processes."

DISCUSSION

"Teachers should be able to prepare various scaffolding with attention [sic] level of ability" (Sutiarso et al., 2017, p. 100). This citation emphasizes the critical role that mathematics teachers should have in integrating technology to support their students' learning processes. The adjustment process that students undergo as a result of scaffolding is of great importance in their ability to solve the tasks, so it is imperative that the scaffolding matches students' prior knowledge and abilities.

The technological scaffolding that included a geometric-dynamic view of the task allowed them to identify the geometric objects integrated in the task, to understand them better, to see the interrelationships and the preserved properties and from that to make hypotheses. The theoretical framework that included refreshing or expanding students' knowledge was a bridge that helped overcome the first hurdle in the proof process. Similar to the research of Triutami et al., (2020). the scaffolding helped the students to identify, making connection, explain, reason, and justify generalizations and thereby develop their geometric knowledge.

With respect to the SAMR model (Puentedura, 2013, 2014), having to cope with the task led students to reach new insights of how technology can enhance teaching. Being exposed to a task in a technological environment led them along a unique route toward discovering a conservation feature and later proving it. The technology enabled them to carry out an authentic exploration process by dragging vertices, sides, and angles; measuring geometric concepts; accordingly, observing a wide array of individual cases; and eventually arriving and then proving a hypothesis.

Whether due to a lack of appropriate knowledge or insufficient experience (Oflaz et al., 2016), mathematics students often have difficulty presenting proofs in geometry. However, the research process proposed herein, that is, the combination of technology and supporting scaffoldings, allowed them to explore, discover, and offer proofs for new and unfamiliar features in geometry in a comfortable and non-threatening environment.

Another important aspect to consider is the challenge presented to teachers in general, and mathematics teachers in particular, in the wake of the COVID-19 over the previous two years and the resultant need to introduce distance learning. It is important to note that the activities presented above are suitable for online synchronous and asynchronous distance teaching. They thus provide a solution for teachers to provide research activities for their students even without a face-to-face meeting in the classroom.

The motivation to give student-teacher familiarity with technology-integrated inquiry assignments was to introduce the students to the opportunities for integrating scaffolding in their future teaching, they would be able to "*identify, assess, and select digital resources for teaching and learning. To consider the specific learning objective, context, pedagogical approach, and learner group, when selecting digital resources and planning their use*" (Punie & Redecker, 2017, p. 20).

We have described an example of an activity that clearly demonstrates how technology can be integrated into educational processes in general and mathematical education in particular. It supports teaching in the following aspects:

a) Organization: of the teacher's work (producing tasks with GeoGebra applets);
b) Representation: new ways of doing and representing mathematics;
c) Collaboration: communicating and sharing materials during research; and
d) Independence: students can work more independently and focus on practicing and assessing previously taught mathematical knowledge and skills.

Pre-service mathematics teachers who undergo the experience of such technological scaffolding while collaborating in solving a geometrical task based on inquiry will develop competence in wisely "selecting digital resources" based on the learning environment and develop appropriate skills in teaching and learning (Clark-Wilson et al., 2020).

The tasks presented in this paper can be further explored using a variety of exploration strategies such as "what if not?" (Brown & Walter, 1993), and "what if instead" (Segal et al., 2018). Through these exploration strategies the tasks can be linked. For example, around **Figure 2** and **Figure 3** we can ask "what would have happened if we had inscribed another circle tangent to the circle inscribed in ABD triangle and the two sides of the triangle ABD as shown later in **Figure 7**."

In this paper we presented high-level geometry tasks that were scaffolded with both dynamic geometry software and theoretical information relevant to the task solution. The combination led students to deepen their existing mathematical knowledge whether it involved facts, tenets, formulae, or definitions. The supporting scaffolding allowed the students to explain the mathematical concepts, processes, or relationships illustrated and led them to go beyond what they first understood from the original diagram and be able to generalize the concepts and improve the quality of their geometrical knowledge.

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Ethical statement: Authors stated that the research was conducted sensitively and strictly following the rules of ethics. The students who participated in the course filled out an informed consent form to confirm their participation in the study. Authors further stated that the data was analyzed for research purposes only after the end of the course, after the students received the final grade of the course. After that, only the data of the students who expressed consent to participate in the study were analyzed. Also, in the process of analyzing and presenting the findings, the anonymity of the participants was preserved.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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