



(WS) that had been prepared previously. Learners were required to work or conduct problem-solving activities.

During problem-solving activities, teachers sought to provide assistance process (scaffolding) at learners who needed it and conducted a review of the work produced by learners. Expected to be able to awaken constructivist shades, namely : (1) the process of scaffolding that giving an assistance to learners to achieve tasks that could not be controlled by him own, then gradually withdraw their support; (2) the coaching process, i.e. the process of motivating the students, analyzed their performance and provide feedback on their performance; (3) articulation, which encouraged learners to articulate ideas, thoughts, and their solutions; (4) reflection, which compared the obtained solution the students with the solution of the "experts" or othered learners at the same time thinking about how to solve the problem, the strategy used, and assess the effectiveness of the strategy; (5) collaboration, which emphasized the learners about how to learn from others; (6) exploratory and problem-solving activities, ie activities gained knowledge or information as possible from sources around him which was then used to solve the problem. Similarly during problem-solving activities, students were encouraged to do three metacognition learning strategies: (1) monitoring the activities of understanding metacognition, which included self-assessing, self-question, predict outcomes, and selecting a strategy; (2) the error control activities, including the use of lead or selective thinking and the use of discourse; and (3) revising activities, covering activity of critiquing and revising.

Phase 3: Evaluating

This phase contained an assessment of the activity of thinking and understanding related to knowledge / new topic. The scope of the evaluation activities undertaken was a self-assessment activities on a reasoning, thinking product and task progress, as well as an evaluation that led to the metacognition abilities improvement of learners. In this case, the learners are encouraged to do three activities: (1) present the results of activities and explain the entire problem-solving activities; (2) taking into account input from other groups or teachers, as well as analyze the strength or weakness of the arguments that have built; and (3) revising through cooperative and collaborative activities if found weaknesses in the argument. This was in line with the opinion of Muijs & Reynolds (2008) which stated that there were good teachers always remind learners of metacognitive processes applied in resolving the problem. Once the evaluation was completed, the teacher gave learners the opportunity to present the settlement of problems obtaining and asked other learners to criticize or give inputs. If possible the students were also asked to analyze and comment to the metacognitive activities, i.e. planning monitoring, and evaluating has done.

Stages III: Closing Activities

This stage was called the closing activity for the main purpose of this stage was to close the learning, knowing achievement of learning objectives, confirming the new topic that has been built, and making efforts to the next learning conditioning. Confirmation process aimed to provide confirmation of the knowledge / new topics that could be done in several ways, among others : providing feedback on what was produced learners, provide an appreciation of the strengths and weaknesses of learning outcomes, giving an appreciation of

the strengths and weaknesses of learning outcomes, providing additional information, encouraging students to use the knowledge further, asking students working on other tasks to create a concrete and contextual learning product. Through the closing activities were expected that learners gained a better understanding more about the new topic or material. While conditioning for subsequent learning was done through the provision of structured tasks related to the next topic. In this case the learners were given the task to carry out initial exploration on the next topic to be studied. With the provision of structured tasks, learners were expected to have the basis for the next step on the topic, so that the following learning activities could be run well and involved the interaction of students better.

2. The Social Systems of PME Learning Model

Social system with regard to interactions between teachers, learners, learning resources, and instructional media used. On the PME learning model, the learning process was recommended to use social constructivist teaching in small groups. This was in line with the opinion of Kramarski, Mevarech, & Arami (2002) which stated that metacognitive teaching strategies recommended to prepare the students in small groups to formulate and answer a series of questions metacognition.

Nuance of constructivist teaching could be found in the elements of PME learning model, as follows:

a. In the first stage (preparatory activities / introduction) did the coaching process, i.e. the process of motivating the students, analyzing their performance and providing feedback about their performance, which was part of the mental preparation activities of learners in the learning.

b. In the second stage (core activities) or planning phase carried out exploratory activities, i.e. activities which gained the knowledge or information as possible from sources around him; and collaboration, which emphasized the learners about how to learn from others.

c. In the second stage (core activities) or the monitoring phase of reflection, the students were encouraged to check the understanding that they get from the exploration and elaboration to look at their advantages and disadvantages.

d. In the second stage (core activities) the evaluating phase was done some process that showed shades of constructivist, namely: the scaffolding process that gave assistance to learners to achieve tasks that could not be controlled by their own, then gradually withdraw their support; articulation process that encouraged learners to articulate ideas, thoughts, and their solutions; reflection process to assess the effectiveness of the strategy; collaboration with others; explorative activities were used to support the resolution..

e. In the third phase (the closing) was given the structured task to encourage learners performed exploratory activities, i.e. activities gained knowledge or information as possible on the next material.

f. Teachers who have the flexibility and adaptability, the teacher could respond and react to the ideas of learners.

3. The Roles and Tasks of the Teachers



Roles / duties of the teachers with regard to the attitudes, duties, and functions of the teacher in the learning process, among others :Rusman (2013) argued, the teachers were the dominant factor in education because teachers played an important role in learning, and learning was an important element in the overall education. PME learning model was a model that put the roles and duties of teachers as counselors, classroom managers, motivators, facilitators, and evaluators. Roles as mentors could be seen in the activities of teachers: (1) Guiding learners found and recaptured the topic prerequisite or recalled the topic / previous material; (2) guiding learners to complete the task; (3) guiding the students understood and mastered the new topic / material; and (4) guiding learners made conclusions of the topic.

Role as a manager of a class could be seen in the activities of teachers conditioning learners in early learning activities and made the learning process in which little by little learners invited to reduce dependence on the teacher. PME instructional model which was developed from metacognition learning strategies, was essentially a learning model that aimed to improve the ability of metacognitive learners i.e. the ability to reflect on what had been thought and the ability to self-control of the action they had taken. Rusman (2013) stated that one of the good classroom management was to provide opportunities for learners to gradually reduce dependence on the teacher, so that learners were able to guide their own activities. Gradually learners should learn self-control and self-activity. A facilitating role could be seen in the activities of teachers made contact with learners to gain a higher understanding. This role appeared on the scaffolding process when teachers facilitated the learners by providing assistance to learners to achieve tasks that could not be controlled by their own, then gradually withdrew his support. By providing scaffolds, teachers have helped students gained the knowledge to a higher level in the zone of proximal development.

Role as a motivator could be seen in the activities of teachers asking students to work and asking questions that motivated learners to work. PME learning model emphasized on learning activities that encouraged learners had an understanding and a better awareness about the thinking process themselves. These activities were reflected in: (1) encouraged students to work; (2) encouraged students to do self-question and self-assessing; and (3) encouraged students to tell their thoughts with verbal language (think aloud). This activity was expected to encourage students to be a good learner. TEAL (2012) stated that teachers were required to encourage learners become better thinkers, helped them focus on their ways to process information through self-question, to reflect journal, and to discuss the thought process..

Role as evaluators could be found in the activities of teachers to give assignments or to conduct a discussion on the prerequisites topic / material or previous material; to conduct an evaluation to determine the success of learning; to evaluate for repair and reinforcement; and to provide structured assignment as a homework. Rusman (2013) stated that assessment was done to: (1) determine the level of achievement of the objectives that had been formulated; (2) determine the accuracy of the method used; (3) determine the position of the students in the class or group; and (4) feedback on the learning process to improve and enhance the learning process further.

4. The Support System in Learning Model PME

Support system with regard to the other components of learning that could be used to increase the effectiveness of the learning process. In this study the components of learning designed to support the effectiveness of the learning model PME included Lesson Plan (LP) and worksheets for students (WS). According to Duncan & Met (2010), LP or lesson planning helped to ensure that the learning process in line with the objectives of the curriculum and the learning objectives. Learning that had been well planned allowing students to become successful learners because learning was not only shaping how and what students learned, but also had an impact on students' attitudes about learning.

Jone (1998) stated that the lesson plan could be a characteristic of the professionalism of teachers. Lesson plan was the very core of the process of reflection that teachers did, the fundamental question as a reflection process with regard to what students should learn and how this would be achieved..

In general, the lesson plan contained the device and learning strategies that should always be considered when design it, namely: setting goals and learning agenda, the design of retreat, the context of the lessons learned in the unit, the strategy of research-based learning, assessment, feedback, learners-centered learning, teaching which according to students' abilities, differentiation or noticing the difference, motivation, mentoring from teachers or peers (scaffold), developing appropriate mental practices, Bloom's taxonomy, and the closing activity (Duncan & Met, 2010). Shape and charge of the LP developed adjusted to the rules of preparation of the LP and the curriculum set by the government or by the teacher. LP needed to be compiled in a complete and systematic manner that learning occurred in an interactive, inspiring, fun, challenging, efficient, motivating the students to actively participated and provided enough space for innovation, creativity, and independence in accordance with their talents, interests, psychological and physical development of learners. LP components usually consisted of identity, which was the name of the school or education units, allocation of time, learning objectives or the indicators of competencies achievement, learning materials, teaching methods, instructional media and learning resources, learning steps, and assessment of learning outcomes.

5. The Instructional and nurturant effects

Every model of learning was always expected to make instructional and nurturant effects. Instructional effects were the result of learning achieved immediately after the learners learned according to the model used, as a result of the achievement of the learning atmosphere was intentionally directed to achieve the goals that have been set or agreed upon. Nurnurant effects were another learning outcomes generated by the learning activities in accordance with the model used, as a result of the achievement of learning environment was experienced directly by the learners without guidance of teacher (Rochmad, 2009). Jacob and Paris (in Schraw&Moshman, 1995) stated, cognition arrangements regarding metacognitive activities helped to control thought or person learned, in this case there were three essential skills, namely planning, monitoring, and evaluation. Brown and Flavell (in Veenman et al, 2004) stated that the expertise in task analysis, planning, supervising, monitoring, and the recapitulation was a manifestation of metacognition skills. Me & Ho (2010)



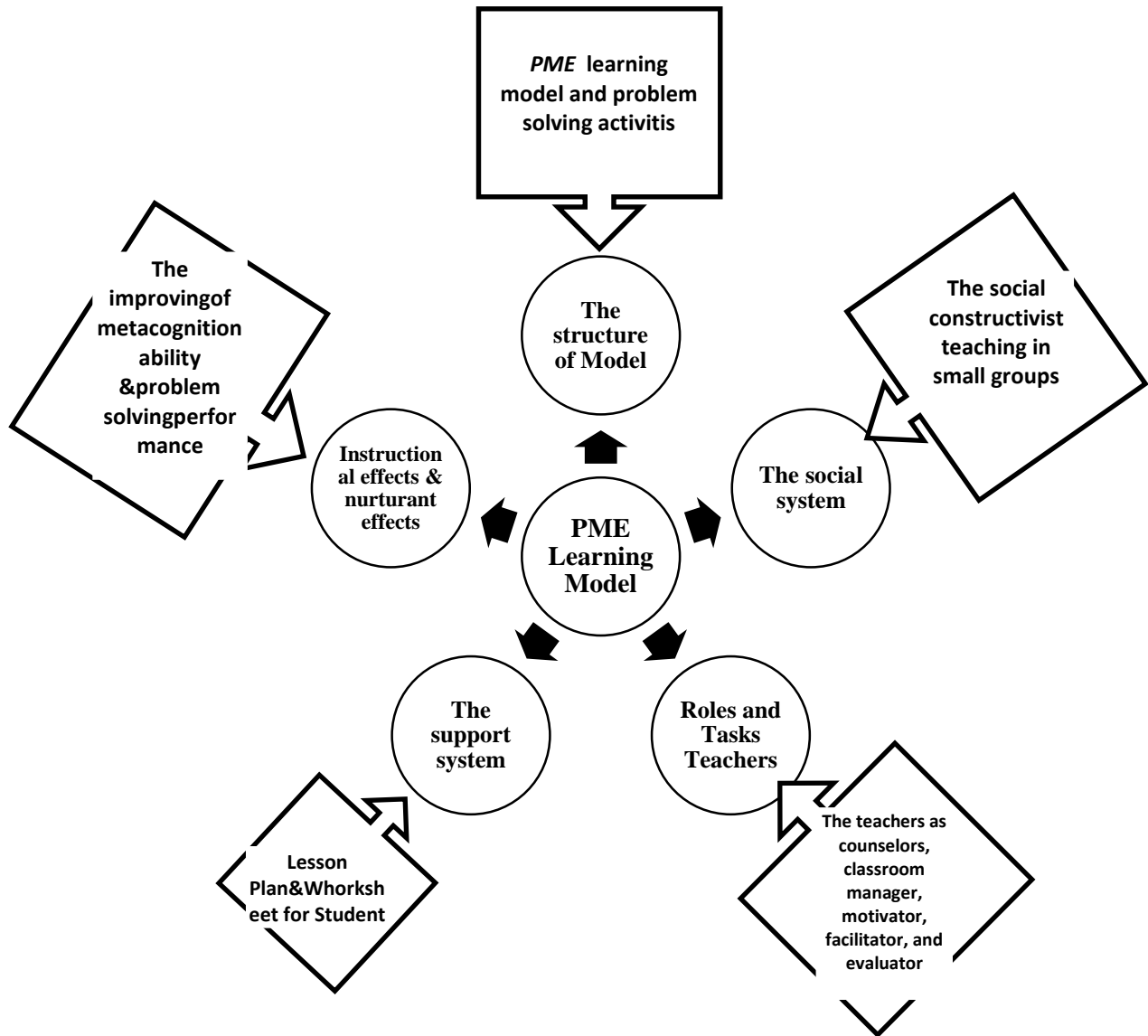
conducted a study and recommended the use of metacognitive strategies through metacognitive activities, i.e. planning, monitoring, and evaluating. So activities planning, monitoring, and evaluation were the most essential. So that metacognitive activities in this study the direct impact was expected from the application of PME model was the increasing ability of metacognition.

Several studies have concluded that the process of metacognition improved performance of problem solving. Darling-Hammond et.al (2003) argued that metacognitive strategies could make the learning process more streamlined and efficient for learners attempted to find information, assessment, and tried to apply different approaches to a problem. Davidson et al. noted that there were two metacognition processes that contributed important to the performance of solving the problem of planning how to proceed and evaluate what you know about your performance (Hoe, et al., 2001). Schoenfeld (in Toit, 2009) argued that the problem-solving activities meant to promote self-regulation. Problem-solving activities provided the ideal opportunity to improve metacognition strategies and metacognition abilities of learners with excel better in terms of problem solving. Of the overall the above opinions could be concluded that the metacognitive processes that were implemented in the activities of planning, monitoring, and evaluation could improve problem solving performance. Because of PME learning model was a learning model that specifically implemented planning, monitoring and evaluation activities, so that the PME learning model was believed to improve the problem solving performance. In this case the nurturant effects expected on the implementation of PME learning model was improving problem solving performance.

The Closing

From the whole descriptions above authors believed that the PME learning model could bring learning activities on the expected goals that made learners active, improved problem-solving abilities of learners, increased positive attitudes of learners, and improved the ability of learners' metacognition. This belief was based on several indicators, namely: (1) learning activities on PME learning model had been structured in sufficient detail, either in whole or activities at each of the stages and phases; (2) the process of giving material was started from reinforcement to the prerequisites or previous materials; (3) the core activities were designed and made possible the process of repetition (learning repetition activities and repetitive testing activities); (4) The PME learning model encouraged students to work and reflect on what they were doing; and (5) PME learning model nuanced constructivist teaching seeking the learners constructed enthusiastically their own knowledge. For further research, these conceptual ideas could be applied in the classroom and developed through research.

The Scheme of PME Learning Model Development



Disclosure statement

No potential conflict of interest was reported by the authors.

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