International Electronic Journal of Mathematics Education
2025, 20(4), em0851
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Mathematics teachers' knowledge in the use of digital technologies for teaching: Insights from the TPCSK instrument

Iván Andrés Padilla-Escorcia [1,2], Martha Leticia García-Rodríguez [2], Álvaro Aguilar-González [3]*

[1] University of Atlantico, Barranquilla-Colombia, COLOMBIA
[2] Instituto Politécnico Nacional, MEXICO
[3] Univeristy of Oviedo, SPAIN
***Corresponding Author:** aguilaralvaro@uniovi.es

Citation: Padilla-Escorcia, I. A., García-Rodríguez, M. L., & Aguilar-González, A. (2025). Mathematics teachers' knowledge in the use of digital technologies for teaching: Insights from the TPCSK instrument. *International Electronic Journal of Mathematics Education, 20*(4), em0851. https://doi.org/10.29333/iejme/16836

ARTICLE INFO

Received: 24 Jun 2025

Accepted: 07 Aug 2025

ABSTRACT

This document analyzes the knowledge of teachers who use digital technologies for mathematics instruction through an instrument called Technological Pedagogical Content Specialized Knowledge (TPCSK) combining open-ended questions and a self-perception scale. The results are based on a case study involving ten teachers from different educational levels participated, and they produced a detailed characterization of teachers' knowledge, highlighting a strong alignment between self-perception scores and narrative evidence. The instrument's categories enabled a fine-grained, detailed, and robust analysis of teacher knowledge, distinguishing their understanding of technological tools, their grasp of the disciplinary potential of digital technologies in specific mathematical content, their ability to integrate them into classroom strategies, and their simultaneous articulation of mathematical, technological, and pedagogical knowledge when designing authentic tasks for teaching. In conclusion, this approach allows for the precise identification of the specific types of knowledge a teacher must master before representing mathematical content with digital technologies, as well as the pedagogical strategies that facilitate such representations and instruction in technology-mediated contexts.

Keywords: MTSK, TPACK, mathematics teachers, digital technologies

INTRODUCTION

The potential of digital technologies (DTs) for the teaching and learning of mathematics, and the roles that teachers´ knowledge and actions play in integrating them in the classroom are two topics that have become increasingly evident in the literature on mathematics education (Rocha, 2024), together with studies of the technological knowledge of mathematics teachers (Addico et al., 2022; Bretscher, 2023; Engin et al., 2023; Njiku, 2024).

Li et al. (2023) mention that the incursion of computers, the Internet, and mobile devices into the educational domain is reflected in their incorporation into the pedagogical practices of some educators. These technologies have progressively become part of daily teaching routines, influencing the design and delivery of mathematics instruction.

The COVID-19 pandemic in 2019-2022 demanded a reinvention of teaching practice that brought technology to the fore (Silva-Gumiero & Pazuch, 2024) as, in 2020, teachers were forced to develop classes in online formats, a reality that entailed numerous challenges and revealed a lack of training in areas like technological and pedagogical knowledge, specifically in the use of platforms like Moodle, various software programs, and other digital resources that facilitate interaction with students (Vàsconez-Paredes & Inga-Ortega, 2021).

This has brought to light the need for mathematics teachers to acquire knowledge, not only of their discipline (content knowledge) or of how they conceive teaching and learning in this field (didactic knowledge), but also of the knowhow required to efficiently integrate technology into the teaching of specific mathematics content.

According to the National Council of Mathematics Professors (National Council of Teachers of Mathematics, 2014) holds that using technology in classrooms is now imperative because, when utilized effectively, it helps students' reason and communicate mathematically, improves their academic performance, generates motivation and interest in learning this subject, and promotes development of advanced mathematical thinking. This includes abilities in problem-solving, reasoning, justifying, demonstrating complex mathematical concepts, communicating, and interacting. Moreover, it helps improve students' performance by integrating electronic classrooms and digital texts (Li et al., 2023; Pocan et al., 2023; Van Leendert et al., 2021).

Nevertheless, knowledge of technology is not directly proportional to its effective integration by mathematics teachers (Guzmán-González & Vesga-Bravo, 2024), for it cannot, by itself, produce a significant change in classroom practices. Barriers to the implementation of technology in classrooms have been identified, perhaps one of the most significant obstacles being deficient ongoing training in technology for teachers (Morales-López et al., 2021; Rodríguez-Muñiz et al., 2021). For authors like Rocha (2023) or Zambak and Tyminski (2020), the lack of knowledge of digital resources is the main factor that impedes teachers from using technology effectively when teaching mathematics.

In this regard, Tekeher et al. (2015) and Dilling et al. (2024) affirm that in-service teachers must receive continuous training in the use of DTs in class in order to apply that knowledge in their pedagogical practice in innovative ways that will lead them to inspire students' creativity and foment interest in interactive research and problem-solving (McLaughlin-Galanti et al., 2021).

Morales-López et al. (2021) argue that the inclusion of technology in education should be intelligent, relevant, appropriate, and of high quality. This perspective aligns with the concept of effective technology integration proposed by Koehler and Mishra (2005). These authors introduced the Technological Pedagogical Content Knowledge (TPACK) framework with the aim of promoting the effective use of technology in instructional settings. Their theoretical model is based on a triad of knowledge domains content, pedagogy, and technology and explores the various intersections that teachers may encounter in classroom practice. The TPACK framework has been widely adopted by the scientific community across diverse educational fields.

Despite its widespread use, Gamboa (2022) points out that the lack of specificity in the components of the model when applied to mathematics education has hindered the development of in-depth studies that analyze mathematics teachers' TPACK in relation to specific mathematical content.

A review of the state of the question for the years 2020-2025 shows that diverse studies have been conducted of mathematics teachers' knowledge in relation to digital technologies. Njiku (2024) and Smith and Zelkowski (2023) have analyzed this using self-reporting instruments, while Kurt and Çakıroğlu (2023) and Açıkgül and Aslaner (2020) oriented their work toward the use of specialized DTs for mathematics as tools for designing math tasks from the TPACK perspective.

A model that examines teachers' knowledge from both disciplinary and pedagogical perspectives was developed by Carrillo et al. (2018), known as the Mathematics Teacher's Specialized Knowledge (MTSK) framework. However, the analysis that this model and others allows in relation to mathematics teachers specialized technological knowledge appears to be limited. One possible reason is that within the MTSK framework, the subdomain Knowledge Mathematics for Teaching (KMT) includes a category labeled use of material or digital resources for teaching.

According to Gamboa (2022), this category is insufficient when considering the broader contributions that digital technologies are expected to make in mathematics learning. In line with this view, Badillo et al. (2023) argue that the aforementioned category does not adequately support the study of teachers' knowledge of technological tools for teaching specific mathematical content. They stress that it is not enough to know how to use a tool; it is also necessary to understand its properties, features, and pedagogical affordances that can facilitate the teaching and learning of mathematics.

In synthesis, few studies have examined the pedagogical-technological knowledge of teachers for a specific mathematical concept. In fact, there are no empirical records of instruments of analysis (categories or indicators) that would allow broad, detailed studies of this kind of teacher's knowledge.

In this context, the simultaneous use of the TPACK and MTSK frameworks both pioneering models in the study of teacher knowledge serves as a foundational basis for developing an alternative approach that explicitly defines the descriptors of the components related to content, pedagogy, and technology in teachers' professional knowledge. Building on these theoretical foundations, the aim of this study was to analyze the knowledge of mathematics teachers who use digital technologies in their instruction, through the application of a newly designed instrument: the Technological Pedagogical Content Specialized Knowledge (TPCSK) instrument.

THEORETICAL APPROACH

The TPCSK instrument was developed based on two main theoretical frameworks: the Technological Pedagogical Content Knowledge (TPACK) model (Koehler & Mishra, 2005) and the Mathematics Teacher's Specialized Knowledge (MTSK) model (Carrillo et al., 2018). The TPACK framework proposes the integration of three essential types of knowledge technological, pedagogical, and content knowledge for effective teaching with digital technologies, and it has been widely used to assess teachers' knowledge in technology-mediated educational contexts. In contrast, the MTSK model provides a more specialized structure for analyzing mathematics teachers' knowledge by distinguishing specific subdomains of mathematical knowledge (such as knowledge of topics, structures, and mathematical practices) and of pedagogical content knowledge (such as knowledge of learning characteristics, teaching approaches, and mathematical standards). Drawing on both frameworks, the categories of the TPCSK instrument were defined to precisely identify the knowledge required by teachers to effectively teach mathematics using digital technologies.

The TPCSK in Comparison with the MTSK and TPACK Models

Since there is no single theoretical framework that fully addresses the knowledge required for teaching mathematics with digital technologies, this study adopted the theoretical synthesis strategy proposed by Prediger et al. (2008) and Kidron and Bikner-Ahsbahs (2015). This strategy, grounded in the Networking of Theories approach, allows for the combination of complementary theoretical frameworks by recognizing their respective contributions without prioritizing one over the other (Rodríguez-Nieto et al., 2022).

Following this approach, the TPCSK instrument was designed by integrating elements from the TPACK model (Mishra & Koehler, 2006) and the MTSK model (Carrillo et al., 2018), with the aim of analyzing in detail the knowledge of mathematics teachers who incorporate digital technologies into their teaching practice.

Both the TPACK and MTSK frameworks present notable strengths. On the one hand, the TPACK model explicitly incorporates technology into the structure of teacher knowledge. However, it rarely addresses specialized digital technologies that fundamentally transform the teaching of mathematics. Additionally, its content knowledge (CK) and pedagogical knowledge (PK) components are defined in general terms and are not specific to mathematics education. The operationalization of this model has been carried out mostly through Likert-scale instruments focused on teachers' self-perception, without including evidence of their actual knowledge as demonstrated in pedagogical practice. On the other hand, the MTSK framework provides a specialized and detailed description of mathematical and pedagogical content knowledge, but it does not incorporate the technological domain as a central component of teachers' professional knowledge.

In this regard, the complementarity between the two models led to the development of an instrument that:

1. Integrates the disciplinary specificity of the MTSK framework, particularly in relation to Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), both of which are more elaborated than in the TPACK model.
2. Incorporates the TPACK perspective concerning the interaction among pedagogical, content, and technological knowledge, while overcoming its generality by adapting it specifically to the field of mathematics education.
3. Allows for the exploration not only of teachers' self-perceptions of their knowledge as is common in most TPACK-based instruments but also of their ability to design instructional tasks, represent specific mathematical concepts, and justify pedagogical decisions when teaching mathematics in technology-mediated environments.

Whereas the TPACK framework focuses on studying the changes that occur in pedagogical strategies when digital technologies are implemented, the TPCSK instrument aims to go further by posing deeper questions such as:

> What potential does the use of specialized digital technologies offer for the teaching of mathematics? And what are the benefits and strengths of implementing classroom strategies that incorporate digital technologies to support students' learning of specific mathematical content?

The TPCSK instrument is structured into four categories: GTMK (Generic Technology Mathematics Knowledge), DTMK (Digital Technology Mathematics Knowledge), DTPK (Digital Technology Pedagogical Knowledge), and DTPMK (Digital Technology Pedagogical Mathematics Knowledge). These categories represent a reconfiguration of components and subdomains from the TPACK and MTSK frameworks, respectively. This reconfiguration is intended to capture specific dimensions of the knowledge held by mathematics teachers who integrate digital technologies into their instruction. The instrument is designed to collect both qualitative and quantitative evidence of this specialized professional knowledge.

METHOD

This study adopts a mixed-methods approach with a qualitative emphasis (Creswell, 2007), as it aims to gain an in-depth understanding of the knowledge held by mathematics teachers who integrate digital technologies into their instruction. At the same time, it incorporates quantitative data to complement and triangulate the findings. The study involved ten in-service teachers selected through non-probability sampling. The participants had teaching experience at various educational levels, including elementary, secondary, and higher education. The sample consisted of eight men and two women; seven of them had completed postgraduate studies (specialization, master's, or doctoral programs), and three were master's degree candidates. To ensure confidentiality, each teacher was assigned a code: P1, P2, ..., P10.

Sample Selection Criteria

The sample was intentionally selected based on two key criteria:

1. The participants' experience in teaching mathematics supported by digital technologies (DTs), and
2. Their academic background, with preference given to teachers who had completed or were currently enrolled in postgraduate programs (specialization, master's, or doctoral degrees).

These criteria made it possible to gather professionals whose trajectories could provide rich and detailed information for analyzing their knowledge regarding the use of digital technologies in mathematics instruction. Additionally, since the instrument applied follows a mixed-methods approach, it was important to include participants with both the willingness and the professional experience to contribute relevant qualitative and quantitative data. Participation was voluntary, and all participants signed an informed consent form in accordance with ethical research principles (see **Table 1**).

Research Instrument

The TPCSK instrument is an adaptation of the instruments proposed by Handal et al. (2013) and Mailizar and Fan (2020), developed as a result of a document-based review of tools designed to assess mathematics teachers' knowledge. These instruments were selected due to the level of detail in their items related to Technological Knowledge (TK), Technological Pedagogical Knowledge (TPK), Technological Content Knowledge (TCK), and Technological Pedagogical Content Knowledge (TPACK), based on the framework proposed by Mishra and Koehler (2006). However, since the aim of the present study was to

Table 1. Professional profile of participating teachers

Participante	Academic Degree	Years of teaching experience	Years of experience teaching mathematics with technology
P1	PhD	9 years	7 years
P2	PhD	11 years	9 years
P3	PhD	17 years	10 years
P4	Master´s Degree	9 years	9 years
P5	Master´s Degree	24 years	15 years
P6	Master´s Degree	10 years	3 years
P7	Specialization	7 years	7 years
P8	Master´s Student	4 years	4 years
P9	Master´s Student	4 years	4 years
P10	Master´s Student	5 years	5 years

analyze mathematics teachers' knowledge of digital technologies for teaching in as much depth as possible, elements from the MTSK framework that were not considered in the aforementioned instruments were incorporated into the TPCSK.

Structure of the TPCSK Instrument

The TPCSK instrument consists of 55 items, divided into two sections Part A and Part B based on the nature of the questions. Both sections are designed to complement each other in capturing information about the knowledge of mathematics teachers who use digital technologies (DTs) in their instruction. Part A includes 30 items: 22 open-ended questions and 8 closed-ended questions. These items aim to gather qualitative data regarding teachers' experiences, reflections, and perceptions about their knowledge. Part B consists of 25 self-report items (Willermark, 2018) presented on a five-point Likert scale, where 5 represents the highest (expert) level of knowledge and 1 represents the most basic level. The goal of these items is to collect quantitative data on the teachers' perceived knowledge levels. Taking together, both parts form a mixed-methods instrument that integrates qualitative and quantitative data collection strategies, offering a richer, deeper, and more detailed view of the knowledge held by mathematics teachers who incorporate digital technologies into their teaching.

The items in the TPCSK instrument are designed to identify relationships between the components of the TPACK framework Technological Knowledge (TK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Technological Pedagogical Content Knowledge (TPACK) (Mishra & Koehler, 2006) and the subdomains of the MTSK framework Knowledge of Topics (KoT), Knowledge of Pedagogical Methods (KPM), Knowledge of Mathematical Structures (KMS), Knowledge of Features of Learning Mathematics (KFLM), Knowledge of Mathematics Teaching (KMT), and Knowledge of the Learning Standards (KMLS) (Carrillo et al., 2018). These relationships are organized into the four main categories that define the structure of the TPCSK instrument: Generic Technology Mathematics Knowledge (GTMK), Digital Technology Mathematics Knowledge (DTMK), Digital Technology Pedagogical Knowledge (DTPK), Digital Technology Pedagogical Mathematics Knowledge (DTPMK). A detailed description of each category is provided below:

Generic technology mathematics knowledge (GTMK)

This refers to teachers' knowledge of how technologies that are not specifically designed for mathematics education function. It includes two subcategories: Technological Knowledge of Hardware for Mathematics (MGTK-H) and Technological Knowledge of Software for Mathematics (MGTK-S). GTMK extends the TK component of the TPACK model, supported by the KoT and KMT subdomains of the MTSK framework. In other words, it details the disciplinary knowledge that teachers must possess in order to use digital technologies that, while not specific to mathematics, can still contribute meaningfully to mathematics instruction.

Digital technology mathematics knowledge (DTMK)

This refers to teachers' knowledge for understanding how specialized digital technologies in mathematics (such as GeoGebra, Statgraphics, GeoTic, etc.) influence the transformation of mathematics content instruction. It includes the subcategories: Specialized Mathematics Digital Technological Knowledge (SMDTK), Digital Technological Knowledge of Mathematical Practices (PMDTK), and Digital Technological Knowledge of Mathematical Structures (SrMDTK). DTMK integrates the TCK component of the TPACK framework with the KMS, KPM, and KoT subdomains of the MTSK model. This category captures how specialized digital technologies (SDTs) either complexify or simplify the teaching of specific mathematics content and support the development of mathematical practices such as modeling, proof, and the solving of challenging problems.

Digital technology pedagogical knowledge (DTPK)

This refers to teachers' knowledge for implementing SDTs within classroom strategies such as project-based learning, game-based learning, problem-based learning, and inquiry-based learning that enhances mathematics teaching. It also includes teachers' understanding of what motivates students to learn through SDTs and how these technologies help students grasp mathematical processes that may be difficult to understand through traditional methods. Moreover, it involves knowledge of how to prioritize mathematics content at a given academic level using digital tools. DTPK includes the subcategories: Digital Pedagogical Technology Knowledge for Classroom Strategies (Ce-PDTK), Technological Knowledge for Teaching (TKT), and Digital Technological Knowledge of Learning Standards (LSDTK). This category links the TPK component of TPACK with the KMT, KFLM, and KMLS subdomains of the MTSK framework.

Table 2. Categorization of TPCSK items (Parts A and B) by categories and subcategories

Category	Subcategory	Items in part A	Items in part B
Generic Technology Mathematics Knowledge (GTMK)	Technological Knowledge of Hardware for Mathematics (MGTK-H)	1, 2, 3, 4	1
	Technological Knowledge of Software for Mathematics (MGTK-S)	5, 6, 7, 8	2,3,4,
Digital Technology Mathematics Knowledge (DTMK)	Specialized Mathematics Digital Technological Knowledge (SMDTK)	10, 11, 12, 19, 20	5, 7, 9, 11
	Digital Technological Knowledge of Mathematical Practices (PMDTK)	15, 16,	15, 16, 17, 18
	Digital Technological Knowledge of Mathematical Structures (Sr-MDTK)	17, 18	19
Digital Technology Pedagogical Knowledge (DTPK)	Digital Pedagogical Technology Knowledge for Classroom Strategies (Ce-PDTK)	24, 25, 26, 27	12, 13, 23, 24, 25
	Technological Knowledge for Teaching (TKT)	9	6, 8, 10
	Digital Technological Knowledge of Learning Standards (LSDTK)	29, 30	20
Digital Technology Pedagogical Mathematics Knowledge (DTPMK)	Digital Technological Pedagogical Mathematics Knowledge for Developing Mathematical Competencies (MPDTK-C)	13, 14, 21, 22, 23	14
	Digital Technological Pedagogical Mathematics Knowledge for Solving Challenging Problems (MPTDK-PS)	28	21, 22

Table 3. Cronbach’s alpha for the TPCSK part B categories

Category	Cronbach´s Alpha	Interpretation
GTK	0,717	Acceptable
DTMK	0,920	Excellent
DTPK	0,930	Excellent
DTPMK	0,891	Very Good

Digital technology pedagogical mathematics knowledge (DTPMK)

This category refers to the integrated and simultaneous application of pedagogical, mathematical, and technological knowledge by the teacher. It reflects teachers’ ability to design instructional or pedagogical strategies that promote students’ understanding of specific mathematical content through the use of SDTs. It includes two subcategories: Digital Technological Pedagogical Mathematics Knowledge for Developing Mathematical Competencies (MPTDK-C) and Digital Technological Pedagogical Mathematics Knowledge for Solving Challenging Problems (MPTDK-PS). DTPMK simultaneously relates the TCK component of TPACK with the KoT, KMS, and KPM subdomains of MTSK, and the TPK component of TPACK with the KFLM, KMT, and KMLS subdomains of MTSK.

The categories are addressed in both sections of the TPCSK instrument, ensuring that the qualitative and quantitative questions align to cover each category from both perspectives. **Table 2** presents a detailed mapping that links the categories and subcategories with the corresponding items from Part A and Part B of the TPCSK. This mapping facilitates the triangulation and comparison of the collected data, ensuring coherence and complementarity between both sections of the instrument.

Instrument Validation

Given that the TPCSK is a mixed-methods instrument, its validation was conducted in two distinct phases. Part A, which is qualitative in nature, underwent a validation process through expert judgment, while Part B (**Table 3**), which consists of quantitative self-report items, was subjected to statistical analysis following standard psychometric procedures.

Part A: Qualitative validation through expert judgment

Five mathematics education experts with doctoral degrees and experience in digital technologies evaluated the 30 qualitative items based on four criteria: sufficiency, clarity, coherence, and relevance. Using a 4-point scale, they rated each item, and Content Validity Ratios (CVR) were calculated following Lawshe’s (1975) method, with a cutoff of 0.58. Items were revised or reworded based on expert feedback. For instance, items 2, 22, 23, and 29 were adjusted despite meeting the minimum threshold (CVR = 0.6), while item 18 below the cutoff in clarity was retained due to its conceptual relevance and strong ratings in the other dimensions.

Part B: Psychometric validation

The 25 Likert-scale items were assessed by five expert judges for clarity, relevance, sufficiency, and coherence. All items reached an Item-Level Content Validity Index (I-CVI) of 1.00, exceeding the recommended threshold of 0.99 (Polit & Beck, 2006).

An exploratory factor analysis (EFA) using principal component extraction and varimax rotation revealed a dominant factor aligned with the construction of Pedagogical Integration of Digital Technologies in Mathematics Teaching. Most items showed loadings above 0.50; items 18 and 19 were excluded due to low loadings. Four components with eigenvalues greater than 1 explained 72.9% of the total variance, supporting the instrument’s internal structure.

Discriminate validity was confirmed by calculating the Average Variance Extracted (AVE). The square root of the AVE (0.685) exceeded the inter-factor correlations, demonstrating conceptual independence between categories, in line with Fornell and Larcker’s (1981) criterion.

Reliability analysis yielded Cronbach’s alpha values above 0.70 for all subscales and 0.968 for the overall scale. Detailed alpha coefficients by category are provided in Appendix C. The 95% confidence interval, estimated with Bonett’s (2002) method (n = 131), was [0.96832, 0.96852], indicating excellent internal consistency.

Data Collection

Data were collected using the TPCSK questionnaire, administered in digital format through a Google Form. Completing the questionnaire took an average of two hours and was conducted virtually and asynchronously, allowing participants to respond independently. The data were exported to an Excel spreadsheet and systematized according to the four categories that structure the instrument.

Data Analysis

The analysis of the spreadsheet data was conducted using a mixed-methods approach, integrating qualitative and interpretative procedures for Part A of the instrument and descriptive statistical analysis for Part B. This strategy aimed to ensure complementarity and cross-validation between both sources of information.

The analysis of Part A was carried out in four phases:

1. Coding of participants' written responses;
2. Thematic grouping into emerging categories;
3. Triangulation with the categories established in the TPCSK instrument; and
4. Cross-validation with quantitative results to ensure the consistency of the information collected.

As for the closed-ended items in Part A, as well as all items in Part B (self-report on a 5-point Likert scale), a descriptive quantitative analysis was conducted, including the calculation of frequencies, means, and standard deviations for each set of items. Finally, qualitative and quantitative findings were contrasted through methodological triangulation, in order to examine the consistency between teachers' narrative responses and their self-perceived knowledge. This procedure ensured that the items corresponding to each category present in both parts of the instrument were effectively connected and mutually reinforcing, thereby enriching the overall interpretation of mathematics teachers' knowledge in using digital technologies for instruction.

To illustrate the triangulation method used in the analysis, the most substantial, detailed, and explicit responses from Part A of the instrument were selected. These responses contained ideas and evidence of teachers' knowledge aligned with the categories proposed in the TPCSK instrument. The selected responses correspond to items 7, 8, 10, 11, 12, 13, 14, 24, 25, 26, and 27. In addition, for each selected item, the corresponding quantitative results from the self-report section (Part B) were also considered.

Generic Technology Mathematics Knowledge (GTMK)

For this category, Items 7 and 8 from Part A of the TPCSK instrument were taken into account. These items are presented below.

Item 7: Which of the following general educational digital technologies have you used to teach any mathematics content? You may select more than one option: Inspiration, Macromedia Flash, Sketch up, Cmap Tools, Other

Item 8: What is the potential of using the general digital technologies mentioned in Item 7 for teaching mathematics? Please explain.

The data analyzed with the TPCSK instrument provides insights into teachers' knowledge regarding the topic of number sets. It was identified that the teacher is familiar with digital technologies such as Macromedia Flash and Inspiration, which allow for the representation of relationships between sets.

The analysis specifies the uses and advantages these tools offer in the teaching of mathematical content, particularly through the design of concept maps, mind maps, diagrams, and illustrations that depict membership relationships among number sets.

Although these tools do not specialize in mathematics, this type of information cannot be uncovered through a technological knowledge approach or a content-focused analysis alone. This highlights that the interaction between disciplinary and technological knowledge is essential for teachers to make the most of such digital technologies in their teaching practice, an aspect that falls under their Generic Technological Mathematical Knowledge (GTMK) (**Table 4**).

In the self-report section, P2 rated themselves at the expert level (five points) in their knowledge of hardware such as projectors, computers, smartphones, or tablets for mathematics instruction. This aligns with their responses in Part A of the instrument, where P2 explains how these tools support the visualization, dynamization, and representation of abstract mathematical content.

Similarly, P7 rated themselves at the expert level in using general digital technologies. This is consistent with their narrative in Part A of the instrument, where they described how they use tools such as Macromedia Flash and Inspiration to organize the hierarchical structure of number sets.

In contrast, P4 rated themselves at an intermediate level (three points) in their knowledge of such software. This aligns with P4's responses in Part A of the instrument, where they reported knowing that such tools can be used to insert images or design illustrative diagrams to represent mathematical content, but did not demonstrate evidence of being able to use them for designing concept maps that clearly reflect the hierarchical organization of mathematical content based on its properties.

General analysis of GTMK

It was found that general-purpose software for teaching mathematics such as Inspiration (for mind maps), SketchUp (for instrumental visualization), and CmapTools (for concept maps) were selected by four participants as digital technologies that support the presentation of mathematical content. Macromedia Flash (for animations) was selected by two participants.

Table 4. Analysis of GTMK

Category	Subcategory	Explanation	Evidence
Generic Technological Mathematics Knowledge (GTMK)	Technological Knowledge of Hardware for Mathematics (MGTK-H)	P2: Demonstrates knowledge of general hardware tools such as projectors, whiteboards, or smart boards to enhance the teaching of complex mathematical content through educational videos that visually represent graphs, diagrams, and three-dimensional models.	P2: "Using the projector allows me to present interactive educational videos as a way to facilitate the teaching of mathematics, especially for complex topics like graphing trigonometric functions, where visualization and simulation are key for understanding."
	Technological Knowledge of Software for Mathematics (MGTK-S)	P4: Demonstrates knowledge of general-purpose software for mathematics, such as Macromedia Flash, which enables the creation of illustrative diagrams to represent mathematical content. Also shows understanding of the commands needed to design such diagrams and how to install the software on a personal computer.	P4: "Using general software like Macromedia Flash allows me to insert images that illustrate relationships between mathematical concepts in a schematic way. For example, in teaching real numbers, it helps represent through interactive diagrams that all integers are rational, but not irrational, and that both irrational and rational numbers are real."
		P7: Demonstrates knowledge of general-purpose software used in mathematics for designing concept or mind maps that depict relationships between number sets.	P7: "With tools like Inspiration or Macromedia Flash, I can create an interactive mind map that visually structures the hierarchy of number sets and clearly shows the inclusion relationships among them."

Additionally, it was noted that participants most frequently use PowerPoint to prepare such presentations, as mentioned by P3. According to P4 and P7, Macromedia Flash is rarely used, as it is more commonly associated with graphic design. However, this digital technology holds high potential for animating mathematical problems, where a visual image can contribute significantly to the comprehension of a problem statement (Putri et al., 2022).

On the other hand, the self-report results made it possible to confirm or contrast the responses provided by participants in Part A of the instrument. It was found that half of the participants rated themselves at level four regarding their expertise in using hybrid digital technologies. Participants P4, P5, and P8 were the only ones who placed themselves at an intermediate level (three). It is worth noting that nine participants reported being able to use material digital technologies for teaching mathematics at an expert level (4 or 5), which provides evidence of a preference for this type of digital technology over hybrid digital technologies despite the significant rise in the use of the latter during the COVID-19 pandemic, when such tools were more frequently implemented.

Quantitative analysis of the self-report

The self-report results for the GTMK category showed a mean score of 4.40 and a standard deviation of 0.44, with values ranging from 3.75 to 5.00. This indicates a high and relatively homogeneous self-perception regarding the use of general digital technologies for teaching mathematics. The low dispersion suggests a broad consensus among teachers about their proficiency in using these types of tools.

This finding aligns with the statements from Part A of the instrument, where generic digital technologies such as PowerPoint, CmapTools, and Inspiration were frequently mentioned. This convergence between qualitative and quantitative findings not only supports the interpretive validity of the TPCSK instrument in this category but also demonstrates its internal consistency, thereby strengthening the instrument's structure and its capacity to differentiate mathematics teachers' generic technological knowledge.

Category: Digital Technology Mathematics Knowledge (DTMK)

For this category, Items 10, 11, and 12 from Part A of the TPCSK instrument were considered. These items are presented below.

Item 10: Which of the following specialized digital technologies for mathematics have you used to teach any content? You may select more than one option: Geogebra, Geometer´s, Blog de dibujo, Cabri, GeoTic, Autograph, Derive, Matemática, Maple, Máxima, Tinkerplots, R Studio, Winplot, SPSS, Statgraphics, Other

Item 11: Among the specialized digital technologies for mathematics listed in Item 10, which do you consider contribute to the teaching of arithmetic, geometry, statistics, algebra, and calculus, respectively? Please explain.

Item 12: What is the potential of using the specialized digital technologies mentioned in Item 10 for teaching mathematics? Please explain.

The data analyzed using the TPCSK instrument provides information about teachers' knowledge regarding the properties of functions and graphical representations in descriptive statistics. It was identified that teachers are familiar with specialized digital technologies in mathematics, such as GeoGebra, GeoTic, Excel, and Statgraphics, which enable them to represent these types of content.

TPCSK allows for the specification of functionalities, such as GeoGebra's ability to handle multiple representations and simultaneously compute and animate the properties of two or more functions. This kind of detailed evidence would be difficult to obtain using a general instrument that does not distinguish between disciplinary and technological components.

Likewise, it highlights how Excel or Statgraphics facilitate the selection of the appropriate type of frequency table or statistical graph based on the total number of data points in a statistical study. These findings show that the interaction between content knowledge and technological content knowledge helps reduce the level of abstraction in teaching mathematical concepts, thanks to the potential and tools offered by discipline-specific digital technologies (**Table 5**).

Table 5. Analysis of DTMK

Category	Subcategory	Explanation	Evidence
Digital Technologies in Mathematics Knowledge (DTMK)	Knowledge of Specialized Mathematics Digital Technologies (SMDTK)	P3 understands that the properties of functions and descriptive analysis of statistical data can be represented using specialized digital technologies for mathematics such as GeoGebra, GeoTic, SPSS, Excel, and Statgraphics.	P3: "Using specialized digital technologies like GeoGebra and GeoTic allows the simultaneous demonstration of function properties (domain, range, period, asymptotes, representation registers). Technologies such as Statgraphics or SPSS also help decide the appropriate frequency table (grouped or ungrouped) based on the data set and statistical variable, and facilitate the creation of bar graphs, pie charts, frequency polygons, histograms, ogives, etc."
		P6 recognizes the potential of specialized mathematics digital technologies like GeoGebra, Statgraphics, or GeoTic for simultaneously handling different representations of a function, such as domain, range, and asymptotes through animations. They can also interpret statistical graphs in Excel or Statgraphics, despite possible data approximations.	P6: "With GeoGebra, I can graph two similar quadratic functions (e.g., x^2+1 and x^2-3) in the same plane to determine their domain, type (even or odd), range, and period via animation and visualization, then analyze how they change when translated. Using Excel or Statgraphics, I can graphically represent a dataset as suggested by the software, considering the total number of data points."
	Digital Technological Knowledge of Mathematical Practices (PMDTK)	P5 understands that tools like GeoGebra allow the graphical demonstration of geometric theorems and conjectures without relying on axioms or definitions, thus supporting the understanding of abstract concepts from a concrete perspective. These tools also support modeling real-world situations related to mathematics.	P5: "Specialized software like GeoGebra enables the creation, manipulation, and visualization of mathematical models. For example, when teaching triangle properties, the teacher can create triangles, measure sides and angles, and observe changes in real-time by moving the vertices."
	Digital Technological Knowledge of Mathematical Structures (Sr-MDTK)	P2 understands that specialized mathematics digital technologies can simplify or enhance the teaching of mathematical content, providing dynamic visual tools that avoid traditional algorithms and allow for deeper analysis through complex graphs.	P2: "Mathematics-specific technologies like GeoGebra allow students to explore advanced analyses. For instance, simulating function graphs lets the teacher propose situations in which students must reason about function behavior: What changes occur in the domain? What happens to the period when the argument is modified? Where are the intercepts? All without relying on standard algorithms."

In the self-report section, P3 and P6 indicated that they are at an expert level (five points) in the use of specialized digital technologies for teaching mathematics. This is consistent with their responses in Part A of the instrument, where they reported using two pairs of specialized digital technologies (GeoGebra, GeoTic) and (Excel, Statgraphics) as tools to support the teaching of function properties and frequency tables, respectively.

P2 and P5 rated themselves at the advanced level (four points), which aligns with their responses in Part A. They demonstrated knowledge of how specialized mathematics digital technologies can support modeling, simulation, and demonstration of mathematical properties. They also noted that these tools allow for a more complex analysis and interpretation of graphs through dynamic visualizations.

General analysis of DTMK

Overall, the responses to item 10 revealed that the only specialized digital technology for teaching mathematics that all participants have used at some point is GeoGebra. This was followed by Statgraphics, used by five participants; then Drawing Blog and Derive, each used by three teachers; Maxima, GeoTic, Matlab, RStudio, and Winplot, each used by two participants; and finally, Geometer's Sketchpad, Cabri, Autograph, Mathematica, Maple, Tinkerplots, and SPSS, which were never used by the participants.

The participants stated that the main applications of specialized digital technologies for teaching mathematics (SMDTK) are found in analytic geometry, computational algebra, computational thinking, and dynamic and interactive representations of content particularly in the construction of geometric figures and graphs of functions.

On this point, P5 noted that SMDTs promote active learning and experimentation, fostering the development of a deeper understanding of mathematical topics through discovery and exploration.

Additionally, seven of the participants indicated that GeoGebra, among the technologies listed in item 10, is the most comprehensive tool for teaching mathematics including arithmetic, algebra, calculus, statistics, and geometry making it a versatile resource applicable at any educational level.

The results from the self-report (Part B) are consistent with the findings from Part A of the instrument. It was observed that seven teachers are able to use specialized digital technologies related to dynamic geometry, of whom five rated themselves at level 4 and two at level 5 of expertise.

This is noteworthy considering that all participants reported having used GeoGebra at some point in their professional experience, according to Part A of the instrument. The only exceptions were P7, P8, and P10, who rated themselves at intermediate or basic levels regarding the use of these digital technologies in teaching mathematics.

Table 6. Analysis of DTPK

Category	Subcategory	Explanation	Evidence
Pedagogical Knowledge with Digital Technologies (DTPK)	Pedagogical Digital Technological Knowledge of Classroom Strategies (Ce-PDTK)	P2 is knowledgeable about specialized digital technologies that support the implementation of strategies such as problem-based learning, project-based learning, among others. These approaches help bring students closer to mathematical content that aligns with their interests and meets their learning expectations.	P2: "Project-based learning with a focus on statistics requires the use of digital technologies such as Excel or Statgraphics to enable the detailed and accurate representation of data collected by students based on a problem they are interested in exploring."
		P4 is knowledgeable about didactic and pedagogical theories for teaching mathematics. They understand students' learning motivations and relate them to mathematical content.	P4: "In teaching grouped data frequency tables, I use project-based learning so that students themselves collect and analyze information on a topic of their interest."
		P6 is familiar with classroom strategies that facilitate the integration of digital technologies as support for teaching mathematical content.	P6: "Project-based learning, problem-based learning, and the design of authentic tasks are classroom strategies that require the use of digital technologies for their effective implementation in the classroom."

Three of the participants rated themselves at level 4 of expertise in their ability to use specialized digital technologies for teaching statistics. This aligns with the fact that, in Part A of the instrument, half of the participants reported having used Statgraphics at some point in their professional careers.

Despite this, five teachers placed themselves at an intermediate level (level 3), and two teachers reported being at the lowest levels (1-2) regarding the use of these technologies. This is also consistent with the fact that none of the participants have ever used the specialized digital technology SPSS in the teaching of statistics.

It is also noteworthy that only P2, P3, P5, and P6 consider themselves to be at an expert level (4-5) in the use of specialized digital technologies for algebra, such as Derive or Mathematica. These are also the only participants who rated themselves at expert levels in both geometry and statistics-related digital technologies.

Additionally, in Part A of the instrument, participants did not report having used specialized digital technologies for algebra. In the self-report, half of them indicated being at either an intermediate level (3) or a basic level (1-2) regarding the use of such technologies in algebra teaching.

This may be due to a lack of knowledge about these tools, or alternatively, to their limited application in educational practice (Ochkov & Bogomolova, 2015).

Quantitative analysis of the self-report

The items in Part B corresponding to the DTMK category show a mean score of 4.27, with a standard deviation of 0.47, and values ranging from 3.56 to 5.00. This result reflects a high and relatively homogeneous self-perception regarding the use of digital technologies for teaching mathematics. The low variability in responses suggests that the teachers feel confident using these tools, particularly GeoGebra, and to a lesser extent, Statgraphics. This consistency between the qualitative and quantitative findings reinforces the validity of the TPCSK instrument, demonstrating that the DTMK category effectively captures how disciplinary knowledge is integrated with digital technologies in real teaching contexts.

Category: Pedagogical Knowledge with Digital Technologies (DTPK)

This category considered items 24, 25, and 27 from Part A of the TPCSK instrument, which are cited below.

Item 24: Which of the following classroom strategies have you used as tools for mediating technology in your students' learning of specific mathematical content? (You may select more than one option): Inquiry-based learning, Project-based learning, Collaborative learning, Problem-based learning, Application of authentic and challenging tasks, Other

Item 25: How have you used technology as a tool to implement classroom strategies in your students' learning of mathematical content? Please provide examples based on your teaching experience.

Item 27: Based on your teaching experience, what are the strengths of using technology in the implementation of classroom strategies for teaching mathematical content?

The data analyzed using the TPCSK instrument provides information about teachers' knowledge of didactic and pedagogical theories for teaching mathematical content, as well as their ability to recognize students' motivations to learn. It was identified that teachers are familiar with classroom strategies, such as collaborative learning and inquiry-based learning, which enable the effective use of digital technologies to support mathematics instruction.

In addition, the analysis specifies the functionalities offered by various specialized digital technologies in mathematics, such as the ability to deepen and refine the development of a project based on statistical concepts, including frequency tables and statistical graphs. These tools help address contextualized problem situations that are of interest to students (**Table 6**).

In the self-report, P2 stated that they are at an expert level (five points) in the use of project-based learning (PBL) as a strategy for teaching mathematical content. Notably, this was the only classroom strategy for which P2 assigned themselves the highest score, which aligns with the findings from Part A of the instrument, where P2 provided an example of how a student can develop a project aligned with their interests, using Excel or Statgraphics as digital tools that enable precise descriptive statistical analysis.

However, it is noteworthy that P2 rated themselves at a basic level (2 points) regarding the use of collaborative learning to integrate technology into mathematics teaching. This is somewhat paradoxical, given that teamwork is a central component of any project-based activity in the classroom. In contrast, P4 and P6 rated themselves slightly higher (3 points) in their knowledge of how to promote collaborative work as a classroom strategy that supports the use of technology in teaching mathematics. However, they also assigned the same rating (3 points) to their knowledge of the PBL strategy. This is consistent with their responses in Part A, where both P4 and P6 mentioned being familiar with classroom strategies that support technology-mediated mathematics teaching, but did not elaborate in detail on any of them unlike P2, who provided a more in-depth explanation.

Analysis of DTPK

In Item 24 of Part A, it was found that eight participants have used problem-based learning (PBL), seven have used project-based learning (PjBL), five have applied collaborative learning (CL) and the implementation of authentic and challenging tasks, and three have used inquiry-based learning (IBL). Participants emphasized that PjBL supports the integration of digital technologies (DTs) in the classroom, as addressing a proposed problem often involving fieldwork and data collection requires systematizing information using DTs.

This is consistent with the self-report results, where seven participants rated themselves at expert levels (4-5) in their use of PjBL as a strategy for teaching mathematics with digital technologies, except for P4 and P6, who placed themselves at an intermediate level (3). A similar pattern was observed with collaborative learning, as four out of the five participants who reported having used this strategy also placed themselves at expert levels (4-5) in their ability to use digital technologies that promote collaborative learning in the classroom except for P2.

In Part B of the instrument, it was also confirmed that seven teachers rated themselves at level 4 or 5 in terms of their expertise in using problem-based learning (PBL) as a strategy for teaching mathematical content through digital technologies. Their placement at these levels is consistent with their knowledge of this strategy, as eight participants mentioned in Part A that PBL was the strategy they used most frequently to implement technology in a pedagogically meaningful way when teaching specific mathematical content.

Moreover, it was found that the self-assigned scores in the self-report regarding their ability to use digital technologies in PBL closely matched their scores related to the design of authentic and/or challenging tasks.

Quantitative analysis of the self-report

The results from Part B, associated with the DTPK category showed a mean score of 4.20, with a standard deviation of 0.53, and values ranging from 3.33 to 5.00. These results indicate a high self-perception, although slightly more dispersed than in other categories (GTK and DTMK). This variation may be related to differences in teachers' experience or familiarity with certain classroom strategies mediated by digital technologies.

In any case, the consistency between qualitative and quantitative data supports the validity of the DTPK category, as it effectively captures not only knowledge of specialized digital technologies in the field, but also their contextualized application within pedagogical classroom strategies.

Mathematical Pedagogical Knowledge with Digital Technologies – DTPMK

This category was based on Item 13 from Part A of the TPCSK instrument, which is cited below.

Item 13: What is the influence that the use of a specialized digital technology in the field has on a teaching strategy for a particular mathematical content? Explain and provide examples.

The data analyzed through the TPCSK instrument provide information about teachers' knowledge regarding changes in the properties and graphs of trigonometric functions resulting from modifications to their arguments. It was identified that teachers understand the functionality, usefulness, and versatility of GeoGebra for solving challenging mathematical problems whose solutions are not trivial.

Moreover, three essential components were recognized as interacting simultaneously: knowledge of classroom strategies (authentic tasks), knowledge of mathematical content (properties and graphs of trigonometric functions), and knowledge of specialized digital technologies (GeoGebra) (**Table 7**).

In the self-report, P3 stated they were at level four regarding their ability to design authentic tasks using specialized digital technologies in mathematics. This is consistent with the fact that the teacher designs activities related to properties and graphs of trigonometric functions, as they know that GeoGebra allows for the simultaneous graphing of multiple functions in a Cartesian plane. This, in turn, gives students greater opportunities to draw conclusions about how changes in the argument affect a function.

In Item 13, participants were asked about the influence of using a specialized digital technology on a teaching strategy for a particular mathematical content. One of the most significant responses was provided by P5.

P5: Technology has a significant influence on the teaching of mathematics because it enhances conceptual understanding, increases students' interest and motivation, develops problem-solving skills, and facilitates the visualization of abstract concepts.

P5's perspective reflects a belief that a proper integration of technological, pedagogical, and content knowledge is possible, if each of these components shares a common objective. An example that illustrates P5's DTPMK knowledge is their ability to simultaneously teach domain, range, and period of trigonometric functions using GeoGebra.

With this tool, function graphs can be dynamically visualized and represented; instant modifications to the function equations can be made and observed directly in the graphs; and collaborative work can be fostered so that students discover these

Table 7. Analysis of DTPMK

Category	Subcategory	Explanation	Evidence
Mathematical Pedagogical Knowledge with Digital Technologies (DTPMK)	Mathematical Pedagogical Digital Technological Knowledge in the Development of Mathematical Competencies (MPDTK-C)	P3 understands that the characteristics of trigonometric function graphs vary when the argument changes. Additionally, they know that GeoGebra enables the visualization and comparison of such changes.	P3: "Using GeoGebra, it is possible to observe how the period, domain, range, and type (even or odd) of a trigonometric function change when its argument is modified. These changes become instantly visible thanks to GeoGebra's versatility in displaying multiple functions on the same plane."
		P5 is familiar with specialized digital technologies that facilitate the teaching of mathematical content and can be applied within classroom strategies where traditional approaches are more complex.	P5: "GeoGebra's capabilities make it possible to design activities that are not trivial for students to solve nor for teachers to explain through traditional methods. The graphical behavior of a function as its argument changes can only be truly observed through simulation."
		P9 notes that implementing authentic tasks as classroom strategies requires the use of specialized digital technologies like GeoGebra, particularly for analyzing the graphical behavior of trigonometric functions when their argument changes.	P9: "The design of a non-trivial authentic task such as asking students to interpret what happens with functions like $\operatorname{sen} x$, $\operatorname{sen}(x+5)$, and $\operatorname{sen} x+5$ in terms of domain, range, period, type (even or odd), and asymptotes highlights the importance of using GeoGebra, as it allows multiple functions to be represented simultaneously."

relationships themselves. This example demonstrates how the interaction between pedagogical, technological, and mathematical knowledge is essential for the development of DTPMK in mathematics teachers. It also highlights the need for greater specificity within each of these components, contributing to more effective mathematics teaching with digital technologies.

Quantitative analysis of the self-report

The self-report results related to the DTPMK category showed a mean score of 4.33, a standard deviation of 0.50, and a range of values between 3.00 and 5.00. This distribution indicates that participants perceive themselves as competent in integrating mathematical content, classroom pedagogical strategies, and specialized digital technologies. The consistency between these data and the evidence gathered in Part A suggests that the TPCSK instrument effectively captures this category, which strengthens its interpretive validity. Furthermore, the homogeneity of the responses reinforces the relevance of including this category in the analysis of teachers' knowledge in the use of digital technologies for mathematics teaching.

DISCUSSION

The empirical data obtained through the TPCSK instrument showed that Part A effectively captures observable aspects of teachers' knowledge related to mathematical and pedagogical domains, while Part B complements these findings with data on their self-perceptions. This dual structure allows for an in-depth examination of how technological, pedagogical, and disciplinary domains intersect in actual teaching practice. The consistency between these two sources strengthens the internal validity of the instrument.

The use of the TPCSK instrument revealed nuances of teacher knowledge that are not commonly addressed in previous studies. For example, although previous research such as Morales-López et al. (2021) used the TPACK model to analyze the teaching of quadratic functions, they overlooked critical disciplinary components such as representation registers, mathematical properties, and didactic strategies that are essential for the effective integration of digital technologies. In contrast, the TPCSK instrument made it possible to identify how specific digital tools influence the representation of mathematical content and the strategies teachers use to mediate student learning, as evidenced in the case of participant P9, who used Excel to select appropriate graphical representations in a statistics project based on students' interests.

Similarly, Zambak and Tyminski (2020) focused on teachers' technological knowledge in geometry using Sketchpad but did not explore the disciplinary knowledge or pedagogical strategies necessary to use this tool effectively. In our study, these dimensions were made explicit through the DTMK and DTPK categories, which captured not only the teachers' knowledge of tools like GeoGebra and Statgraphics, but also their ability to align them with specific content, learning objectives, and student engagement strategies.

The main difference between previous instruments and the TPCSK lies in its ability to analyze technological knowledge as a component closely linked to content and pedagogy. For instance, the DTPMK category revealed how teachers such as P3 and P5 simultaneously mobilized multiple domains of knowledge to design authentic tasks involving trigonometric functions, enhancing students' conceptual understanding through dynamic visualizations.

Moreover, by comparing participants' narrative responses with their self-assessments, the study identified potential gaps between what teachers claim to know and what they can actually articulate or apply. The structure of the TPCSK instrument, designed to mitigate overestimation of an issue identified in studies such as Njiku (2024) enabled a more accurate characterization of teacher knowledge.

Finally, the Pearson correlation analysis revealed moderate to strong associations among the TPCSK categories. The correlation between DTMK and DTPMK was statistically significant ($r = 0.817$, 95% CI [0.386, 0.955]), while the correlation between DTPK and DTPMK ($r = 0.616$, 95% CI [–0.064, 0.906]) did not reach statistical significance at the 95% confidence level, likely due to the small sample size. However, both associations showed meaningful positive trends that support the theoretical coherence of the TPCSK framework. These findings suggest that pedagogical, technological, and mathematical knowledge are not developed in isolation, but rather function as an integrated system in classroom practice. This supports the theoretical validity of the instrument and highlights its potential as a diagnostic and professional development tool in mathematics education research. It is important to point out that the findings of this study are based on a purposive sample of ten participants. Therefore, the results are not intended to be generalizable, but rather to offer a contextualized view of the use of digital technologies in specific teaching practices. Future research might consider broader or comparative studies to further expand on these findings.

CONCLUSIONS

The literature review showed that the information regarding teachers' knowledge in using digital technologies for mathematics teaching, obtained through the MTSK and TPACK models, is complementary: the MTSK model focuses on disciplinary and pedagogical-didactic knowledge, while the TPACK model centers on technological knowledge. This complementarity was identified as an opportunity to design the TPCSK instrument, and the empirical findings confirmed that the information obtained through this instrument is more detailed in terms of its components or subdomains.

For example, the formulation of the DTMK category (Digital Technological Mathematical Knowledge) allows for a detailed analysis of the teacher’s knowledge regarding the influence that digital technology exerts on the way mathematical content is presented. The DTPK category (Digital Technological Pedagogical Knowledge) delves into the teacher’s understanding of how digital technologies enable the implementation of classroom strategies in mathematics teaching. The DTPMK category (Digital Technological Pedagogical Mathematical Knowledge) harmoniously and simultaneously integrates the teacher’s knowledge to design didactic or pedagogical strategies through the use of digital technologies, facilitating students’ understanding of a specific mathematical content.

In addition, relevant statistical relationships were found among the categories of the TPCSK. The Pearson correlation analysis revealed significant associations that demonstrate internal consistency within the instrument. For example, teachers’ self-perceived levels in DTPMK tend to increase when they also report high levels in DTPK and DTMK, which indicates that these types of knowledge are interrelated and mutually reinforce one another in practice.

From an educational perspective, the data reported in this study provide evidence to support the design of more comprehensive teacher training programs. The structure of the TPCSK instrument, organized into categories, makes it possible to precisely identify the areas in which a teacher may require further support or professional development whether in relation to specific digital technologies in the field, their didactic adaptation, or the integration of both within real classroom contexts. This makes the instrument a valuable tool not only for research, but also for the professional development of mathematics teachers.

One limitation of this study is that the data collected corresponds to in-service mathematics teachers. Therefore, it would be advisable to expand the sample to include pre-service teachers or teacher educators. A future line of work could involve the design of non-participant observation units, task design frameworks, or curriculum plan reviews, based on the categories DTMK, DTPK, and DTPMK, in order to carry out a more detailed analysis of mathematics teachers’ technological pedagogical knowledge related to specific mathematical content.

Author contributions: IAP-E, MLG-R, ÁA-G: conceptualization, methodology, writing – review & editing; **IAP-E:** investigation, resources, writing - original draft; **MLG-R:** supervision, writing - original draft; **ÁA-G:** supervision. All authors have agreed with the results and conclusions.

Funding: This research has been partially funded by Spanish Ministry of Science, Innovation and under Grants [MICIN-PID2024-155358NB-I00 and PID2024-155358NB-I00]; and the Asturian Agency for Science, Business Competitiveness and Innovation [GRUPIN-IDE/2024/000713] and Red Iberoamericana MTSK.

Ethical statement: The authors stated that the study was reviewed and approved by the Ethics Committee of the National Polytechnic Institute of Mexico on November 15, 2024. All participants provided informed consent prior to their inclusion in the research, ensuring their voluntary participation and full understanding of the study’s objectives, procedures, and possible implications. The authors further stated that to protect the privacy of the information, data were anonymized and stored in secure repositories with access restricted exclusively to the research team. Sensitive or identifiable data were handled in accordance with applicable regulations and used solely for research purposes.

AI Statement: The authors stated that generative AI tools (e.g., ChatGPT by OpenAI) were used to support language clarity and editing of the manuscript. All content was reviewed and validated by the authors.

Declaration of interest: Authors do not declare any competing interests.

Data sharing statement: Authors declare that no datasets were generated or analyzed during the current study.

REFERENCES

Açıkgül, K., & Aslaner, R. (2020). Effects of Geogebra supported micro teaching applications and technological pedagogical content knowledge (TPACK) game practices on the TPACK levels of prospective teachers. *Education and Information Technologies*, *25*, 2023-2047. https://doi.org/10.1007/s10639-019-10044-y

Addico, A., Ali, C., & Adzifome, N. (2022). Student-teachers’ technological pedagogical content knowledge preparedness for mathematics instruction. *Journal of Advanced Sciences and Mathematics Education*, *2*(2), 51-63. https://doi.org/10.58524/jasme.v2i2.116

Badillo, E., González, M., & Moreno, M. (2022). Una mirada crítica al MTSK. In J. Carrillo Yáñez, M. Á. Montes Navarro, & N. Climent Rodríguez (Eds.), *Investigación sobre conocimiento especializado del profesor de matemáticas (MTSK): 10 años de camino* [A critical look at MTSK. Research on mathematics teacher specialized knowledge (MTSK): 10 years of progress] (pp. 207-222). Dykinson.

Bonett, D. (2002). Sample size requirements for testing and estimating coefficient alpha. *Journal of Educational and Behavior Statistic, 27*(4), 335-340. https://doi.org/10.3102/10769986027004335

Bretscher, N. (2023). Conceptualising TPACK within mathematics education: Teachers' strategies for capitalising on transitions within and beyond dynamic geometry software. *Digital Experiences in Mathematics Education*, *9*, 232-254. https://doi.org/10.1007/s40751-022-00115-0

Carrillo, J., Climent, N., Montes. M.A., Contreras, L., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar, A., Ribeiro, M., & Muñoz, M. (2018). The mathematics teacher's specialized knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236-253. https://doi.org/10.1080/14794802.2018.1479981

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Sage Publications, Inc.

Dilling, F., Scheneider, R., Weigand, H., & Witzke, I. (2024). Describing the digital competencies of mathematics teachers: Theoretical and empirical considerations on the importance of experience and reflection. *ZDM-Mathematics Education, 56,* 639-650. https://doi.org/10.1007/s11858-024-01560-4

Engin, R., Karakus, D., & Niess, M. (2023). TPACK development model for pre-service mathematics teachers. *Education and Information Technologies*, *28*, 4769-4794. https://doi.org/10.1007/s10639-022-11381-1

Fornell, C., & Larcker, D. (1981). Evaluating structural equation models with unobservable variables and measurement error. *Journal of Marketing Research, 18*(1), 39-50. https://doi.org/10.2307/3151312

Gamboa, G. (2022). *Conocimientos tecnológicos pedagógicos de contenido matemático* [Technological pedagogical knowledge of mathematical content]. Universidad Internacional de Andalucia – Universidad de Huelva.

Guzmán-González, J., & Vesga-Bravo, G. (2024). TPACK in in-service secondary education teachers: A systematic review of the literature. *International Journal of Education in Mathematics, Science, and Technology (IJEMST)*, *12*(1), 282-296. https://doi.org/10.46328/ijemst.3198

Handal, B., Campbell, C., Cavanagh, M., Petocz, P., & Kelly, N. (2013). Technological pedagogical content knowledge of secondary mathematics teachers. *Contemporary Issues in Technology and Teacher Education*, *13*(1), 22-40.

Kidron, I., & Bikner-Ahsbahs, A. (2015). Advancing research by means of the networking of theories. In A. Bikner-Ahsbahs, C. Knipping, & N. C. Presmeg (Eds.), *Approaches to qualitative methods in mathematics education – Examples of methodology and methods* (pp. 221-232). Springer. https://doi.org/10.1007/978-94-017-9181-6_9

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, *32*(2), 131-152. https://doi.org/10.2190/0EW7-01WB-BKHL-QDYV

Kurt, G., & Çakıroğlu, E. (2023). Prospective mathematics teachers' TPACK development in graphical data displays: A microteaching lesson study. *International Journal of Mathematical Education in Science and Technology, 56*(3), 445-475. https://doi.org/10.1080/0020739X.2023.2246970

Lawshe, C. (1975). A quantitative approach to content validity. *Personnel Psychology, 28*(4), 563-575. https://doi.org/10.1111/j.1744-6570.1975.tb01393.x

Li, M., Qawi-Noori, A., & Li, Y. (2023). Development and validation of the secondary mathematics teachers' TPACK scale: A study in the Chinese context. *EURASIA Journal of Mathematics, Science and Technology Education*, *19*(11), Article em2350. http://doi.org/10.29333/ejmste/13671

Mailizar, M., & Fan, L. (2020). Indonesian teachers' knowledge of ICT and the use of ICT in secondary mathematics teaching. *EURASIA Journal of Mathematics, Science and Technology Education*, *16*(1), Article em1799. https://doi.org/10.29333/ejmste/110352

McLaughlin-Galanti, T., Baker, C., Morrow, K., & Kraft, T. (2021). Enriching TPACK in mathematics education: using digital interactive notebooks in synchronous online learning environments. *Interactive Technology and Smart Education, 18*(3), 345-361. https://doi.org/10.1108/ITSE-08-2020-0175

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Morales-López, Y., Chacón-Camacho, Y., & Vargas-Delgado, W. (2021). TPACK of prospective mathematics teachers at an early stage of training. *Mathematics*, *9*(15), Article 1741. https://doi.org/10.3390/math9151741

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Njiku, J. (2024). Assessing the development of mathematics teachers TPACK through an observation rubric. *Education and Information Technologies*, *29*, 2043-2066. https://doi.org/10.1007/s10639-023-11901-7

Ochkov, V. F., & Bogomolova, E. P. (2015). Teaching mathematics with mathematical software. *Journal of Humanistic Mathematics*, *5*(1), 265-285. https://doi.org/10.5642/jhummath.201501.15

Pocan, S., Altay, B., & Yasaroglu, C. (2023). The effects of mobile technology on learning performance and motivation in mathematics education. *Education and Information Technologies*, *28,* 683-712. https://doi.org/10.1007/s10639-022-11166-6

Polit, D., & Beck, C. (2006). The content validity index: Are you sure you know what's being reported? Critique and recommendations. *Research in Nursing & Health*, *29*(5), 489-497. https://doi.org/10.1002/nur.20147

Prediger, S., Bikner-Ahsbahs., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM Mathematics Education, 40*, 165-178. https://doi.org/10.1007/s11858-008-0086-z

Putri, S., Abdurahman, A., Andrian, D., Angraini, L., & Effendi, L. (2022). Development of interactive multimedia-based mathematics learning media macromedia flash 8. *International Journal of Trends in Mathematics Education Research*, *5*(2), 206-213. https://doi.org/10.33122/ijtmer.v5i2.133

Rocha, H. (2023). Models on teachers' knowledge to teach with digital technology: A systematic review. In *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*. Alfréd Rényi Institute of Mathematics.

Rodríguez-Muñiz, L. J., Burón, D., Aguilar-González, Á., & Muñiz-Rodríguez, L. (2021). Secondary mathematics teachers' perception of their readiness for emergency remote teaching during the COVID-19 pandemic: A case study. *Education Sciences, 11*(5), 228. https://doi.org/10.3390/educsci11050228

Rodríguez-Nieto, C., Font, V., & Rodríguez-Vasquez, F. (2022). Literature review on networking of theories developed in mathematics education context. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(11), Article em2179. https://doi.org/10.29333/ejmste/12513

Silva-Gumiero, B., & Pazuch, V. (2024). Digital technologies and mathematics teaching: An analysis of teacher professional knowledge. *Pedagogical Research, 9*(2), Article em0200. https://doi.org/10.29333/pr/14342

Smith, P., & Zelkowski, J. (2023). Validating a TPACK instrument for 7–12 mathematics in-service middle and high school teachers in the United States. *Journal of Research on Technology in Education*, *55*(5), 858-876. https://doi.org/10.1080/15391523.2022.2048145

Tekeher-Getenet, S., Beswick, K., & Callingham, R. (2015). Conceptualizing technology integrated mathematics teaching: The stamp knowledge framework. *Proceedings of the 2015 Annual Conference for the Psychology of Mathematics Education (PME 39), 2*, 321-328.

Van Leendert, A., Doorman, M., Drijvers, P., Pel, J., & Van Der Steen, J. (2021). Teachers' skills and knowledge in mathematics education for braille readers. *Technology, Knowledge and Learning*, 1171-1192. https://doi.org/10.1007/s10758-021-09525-2

Vàsconez-Paredes, C., & Inga-Ortega, E. (2021). El modelo de aprendizaje TPACK y su impacto en la innovación educativa desde un análisis bibliométrico [The TPACK learning model and its impact on educational innovation from a bibliometric analysis]. *Innova Research Journal, 6*(3), 79-97. https://doi.org/10.33890/innova.v6.n3.2021.1773

Willermark, S. (2018). Technological pedagogical and content knowledge: A review of empirical studies published from 2011 to 2016. *Journal of Educational Computing Research*, *56*(3), 315-343. https://doi.org/10.1177/0735633117713114

Zambak, V., & Tyminski, A. (2020). Examining mathematical technological knowledge of pre-service middle grades teachers with geometer´s sketchpad in a geometry course. *International Journal of Mathematical Education in Science and Technology*, *51*(2), 183-207. https://doi.org/10.1080/0020739X.2019.1650302