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Mathematical resilience through a culture-based geometry e-module integrating Van Hiele theory and problem-based learning

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ABSTRACT

Geometry learning often presents both cognitive and affective challenges, highlighting the need for instructional approaches that foster not only conceptual learning but also mathematical resilience. This study aimed to develop a culture-based geometry e-module grounded in Nias culture that integrates Van Hiele theory through Problem-Based Learning (PBL), and to examine its validity, practicality, and effectiveness in enhancing students' mathematical resilience. The study employed a Research and Development approach using the 4D model (define, design, develop, and disseminate). Participants were 47 eighth-grade students from SMP Negeri 4 Maniamolo, comprising an experimental group (n = 24) and a control group (n = 23). Data were collected using expert validation sheets, student response questionnaires, and a mathematical resilience questionnaire. Data analysis included descriptive statistics, normalized gain (N-Gain), tests of normality and homogeneity, and an independent-samples t-test. The developed e-module achieved a very high level of validity (83.12) and practicality (84.10%). The experimental group demonstrated moderate normalized improvement in mathematical resilience, whereas the control group showed low improvement. An independent-samples t-test revealed a statistically significant difference in N-Gain between the experimental and control groups, with the experimental group demonstrating greater improvement than the control group ($t(45) = 8.59$, $p < 0.001$). The effect size was large (Cohen's $d = 2.51$), indicating a substantial difference in normalized improvement between the two groups. These findings suggest that the integrated learning environment provided through the culture-based geometry e-module, combining cultural contexts, Van Hiele learning progression, Problem-Based Learning, and digital scaffolding, was associated with higher mathematical resilience than conventional geometry instruction.

Keywords: geometry e-module, mathematical resilience, Nias culture, Problem-Based Learning, Van Hiele theory

INTRODUCTION

The success of mathematics learning is increasingly understood as a combination of cognitive competence and affective capacity, particularly students' ability to persist when facing difficulties (Boaler, 2016). In geometry learning, students are often confronted with unfamiliar representations, non-routine problems, and abstract reasoning processes, which may lead to frustration and disengagement (Evans et al., 2021; Riegel, 2021). These affective challenges highlight the importance of mathematical resilience, which refers to a positive stance toward mathematics that enables learners to persist, engage, and respond adaptively when encountering mathematical difficulties (Goodall & Johnston-Wilder, 2015; Johnston-Wilder & Lee, 2010). Rather than being viewed as a fixed characteristic, mathematical resilience has been conceptualized as a disposition that can be developed through supportive learning experiences and intentional pedagogical practices (Goodall & Johnston-Wilder, 2015; Johnston-Wilder & Lee, 2010).

To support empirical research on mathematical resilience, Kooken et al. (2016) developed and validated the Mathematical Resilience Scale (MRS), conceptualizing resilience through valuing mathematics, persistence in facing challenges, and adaptive responses to difficulty. Their findings indicate that learning environments can influence students' resilience, consistent with systematic reviews showing that mathematical resilience is shaped by psychological as well as social and learning-environment factors (Xenofontos & Mouroutsou, 2023). Thus, mathematical resilience can be considered an intentional and measurable outcome of mathematics education, highlighting the need for pedagogical frameworks that connect cognitive development with affective development, particularly in geometry learning.

In geometry education, Van Hiele theory explains how students' geometric thinking develops through qualitatively different levels, from visualization to rigor, through appropriately structured learning experiences rather than age or maturation alone (Van

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Hiele, 1986). Difficulties may arise when instructional demands are not aligned with students' current thinking levels. Research indicates that Van Hiele-based instruction supports conceptual understanding and coherent geometry teaching (Burger & Shaughnessy, 1986; Fujita & Jones, 2007). Its progressive sequencing may also support affective engagement by providing logically connected and attainable learning experiences, which can strengthen self-efficacy, perceived control, and persistence when facing challenges (Hiebert et al., 1996; Schoenfeld, 1989; Xenofontos & Mouroutsou, 2023; Zimmerman, 2000). Thus, Van Hiele-based instruction provides a theoretical basis for connecting structured geometric reasoning with mathematical resilience.

To complement this cognitive progression, Problem-Based Learning (PBL) provides opportunities for students to actively engage with mathematical challenges through meaningful problem situations (Barrows & Tamblyn, 1980; Halat et al., 2008). By presenting complex problems whose solutions are not immediately apparent, PBL encourages exploration, collaboration, reflection, and productive engagement with difficulty (Barrows, 1996; Hmelo-Silver, 2004; Savery, 2006). Such experiences can help students view struggle as a normal part of learning and thereby support persistence and confidence, which are important aspects of mathematical resilience (Boaler, 2016).

To further support the affective potential of PBL, culture-based mathematics learning can provide meaningful contexts for mathematical problem situations. Culturally contextualized tasks have been reported to enhance engagement, motivation, and the meaningfulness of abstract concepts (Luzano, 2025; Solihin et al., 2025; Yanti, 2025). From an ethnomathematical perspective, mathematics is embedded in cultural practices rather than existing solely as an abstract system (D'Ambrosio, 2006). Local cultural contexts can therefore increase students' interest and sense of relevance in mathematics learning (Bishop, 1988a; Rosa & Orey, 2016). In geometry, cultural artifacts and spatial practices provide meaningful contexts for exploring geometric concepts and may support students' engagement and willingness to persist when facing difficulties, which are important aspects of mathematical resilience.

To translate culturally grounded learning principles into systematic classroom practices, learning resources are needed that can organize cultural contexts within coherent instructional sequences. The use of contextual and historically grounded resources in mathematics education, however, predates digital learning. For example, Toeplitz's genetic approach to mathematics teaching emphasized reconstructing the development of mathematical ideas through historical insights and problems so that mathematical concepts could be understood as products of intellectual development rather than as static bodies of knowledge (Fried & Jahnke, 2015). Although the genetic approach differs conceptually from ethnomathematics and culture-based mathematics education, both perspectives highlight the value of situating mathematical ideas within meaningful contexts rather than presenting them as context-free abstractions. In this broader tradition, culture-based mathematics learning can similarly connect formal mathematical concepts with learners' cultural experiences and practices (D'Ambrosio, 2006; Rosa & Orey, 2016).

Digital modules do not create this contextual approach, but they provide particular affordances for organizing and integrating such learning experiences within a coherent instructional environment. Their capacity to combine structured sequences, visual representations, interactive tasks, multimedia resources, immediate feedback, and reflective prompts can facilitate the presentation of culture-based geometry problems while maintaining a systematic progression of learning activities (Mayer, 2009; Schindler et al., 2017; Sun et al., 2008). In this study, these affordances are used to integrate Nias cultural contexts with Van Hiele-based progression and Problem-Based Learning within a single learning environment. Thus, the contribution of digitalization lies not in introducing cultural contextualization itself, but in providing a flexible medium through which multiple pedagogical components can be organized, presented, and experienced coherently.

The e-module was designed as a student-oriented digital learning resource for use within regular classroom instruction rather than as a replacement for teacher facilitation. Its interactive features allow students to access explanations, visual representations, cultural contexts, problem situations, and guided activities independently and at their own pace. However, the learning sequence is implemented within a teacher-facilitated environment in which students engage in problem exploration, discussion, solution development, and reflection. Thus, the e-module serves both as a resource for individual interaction and as a structured medium supporting collaborative classroom learning.

Despite these developments, a clear research gap remains. Existing studies tend to examine Van Hiele-based instruction primarily in relation to geometric thinking outcomes (Burger & Shaughnessy, 1986; Teppo, 2020; Vieira & Cyrino, 2022), PBL in terms of problem-solving or achievement (Albanese & Mitchell, 1993; Strobel & van Barneveld, 2009), and culture-based mathematics learning as a means of contextualization and engagement (Hunter et al., 2022; Samo et al., 2018; Suh & Calabrese, 2025). Very few studies explicitly position mathematical resilience as the primary outcome while simultaneously integrating culture-based geometry contexts, digital e-module formats, Van Hiele theory as the instructional framework, and Problem-Based Learning as the learning model. As noted by Goodall and Johnston-Wilder, (2015), resilience does not emerge automatically from innovative pedagogy; it requires intentional design that normalizes difficulty, provides structured progression, and supports reflection on learning challenges.

This perspective provides a basis for understanding the complementary roles of the instructional components within the integrated learning environment. PBL primarily addresses students' encounters with difficulty by engaging them in challenging problems, investigation, strategy revision, and solution development (Hmelo-Silver, 2004; Savery, 2006). Van Hiele theory provides a structured progression of geometric reasoning by organizing tasks according to increasingly sophisticated levels of thinking (Burger & Shaughnessy, 1986; Van Hiele, 1986). The cultural approach provides relevance and meaning by connecting geometric concepts with familiar cultural artifacts and practices (D'Ambrosio, 2006; Rosa & Orey, 2016), while digital technology provides scaffolding, visual representations, feedback, and opportunities for self-monitoring and reflection (Mayer, 2009; Schindler et al., 2017).

These functions were not intended to represent independent effects but complementary pedagogical functions within a single integrated intervention. PBL provided the context for engaging with difficulty, Van Hiele structured the progression of geometric

reasoning, cultural contexts provided relevance and value, and digital technology coordinated scaffolding, feedback, and reflection. Together, these components were intended to create learning conditions in which students could engage meaningfully with challenging tasks, progressively develop geometric reasoning, and reflect on their learning. Accordingly, the theoretical contribution lies in coordinating these functions rather than attributing mathematical resilience outcomes to any single component.

Although previous studies have reported positive outcomes from culture-based mathematics learning, PBL, digital modules, and Van Hiele-oriented instruction, limited empirical evidence has examined their integration in relation to mathematical resilience compared with conventional geometry instruction. The present study therefore evaluates these approaches as an integrated instructional environment rather than as independently manipulated components. Consequently, the specific and interactive contributions of individual components cannot be determined from the present design and remain an important direction for future research.

Therefore, this study addresses the identified gap by developing and implementing a culture-based geometry e-module integrating Van Hiele theory through PBL, with mathematical resilience as the primary outcome. The study evaluates the validity, practicality, and effectiveness of the developed e-module and compares the magnitude of improvement in mathematical resilience with conventional geometry instruction. The novelty lies in: (1) positioning mathematical resilience as the primary outcome of geometry learning; (2) aligning Van Hiele learning progression with the PBL cycle within a digital module; (3) embedding geometry learning in culturally grounded Nias contexts to enhance relevance and engagement; and (4) evaluating the integrated e-module through comparison with conventional geometry instruction. This integrated approach contributes to research on affective outcomes in mathematics education by providing a theoretically grounded and design-oriented approach to supporting students' mathematical resilience.

THEORETICAL FRAMEWORK

Culture-Based Geometry E-Module Integrating Van Hiele Theory Through Problem-Based Learning

The development of the e-module was based on the assumption that well-designed learning experiences can support students' cognitive and affective development by integrating geometric thinking, problem-based pedagogy, cultural contexts, and digital support (Hiebert et al., 1996; Mayer, 2009; Schoenfeld, 1989; Zimmerman, 2000). Accordingly, the e-module was designed not merely to deliver geometry content but to provide a coherent learning environment connecting students' geometric development with their engagement in mathematics learning (Attard & Holmes, 2019; Borba et al., 2017).

From a cognitive perspective, the e-module was grounded in Van Hiele's theory of geometric thinking, which describes the development of geometric understanding through hierarchical levels from visualization to rigor (Van Hiele, 1986). Because geometric thinking develops through structured learning experiences rather than age alone, the activities were organized progressively according to increasing levels of geometric reasoning (Clements & Battista, 1992; Usiskin, 1982). Students initially observed culture-based objects and identified geometric forms, then examined properties such as faces, edges, vertices, and congruent components, before establishing relationships, formulating generalizations, and providing justifications. Thus, the progression increased the level of geometric reasoning rather than merely numerical complexity, helping students move from visual recognition toward analysis, relational reasoning, and justification (Burger & Shaughnessy, 1986; Fujita & Jones, 2007; Hiebert et al., 1996).

For example, in a surface-area activity using a culture-based prism, students first identified the three-dimensional form and its faces, then examined their properties and congruence, related the areas of individual faces to the total surface area, and finally explained or justified the resulting relationship and applied it to a related problem. The same geometric concept was therefore revisited through progressively more demanding forms of reasoning rather than introduced directly as a formula.

This progression was complemented by Problem-Based Learning (PBL), which used contextual problems as the starting point for investigation and solution development. Students were encouraged to explore strategies, discuss and revise their solutions, and reflect on their reasoning (Barrows, 1996; Hmelo-Silver, 2004; Savery, 2006). Thus, Van Hiele determined the progression of geometric reasoning, whereas PBL structured students' engagement with the tasks through problem orientation, investigation, solution development, presentation, and reflection.

The e-module also incorporated selected Nias cultural artifacts and practices as contextual resources for three-dimensional geometry (**Table 1**). Omo Hada, the traditional Nias house, was used to introduce three-dimensional flat-faced solids through the identification of familiar architectural forms and their components. Bawu, a traditional Nias kitchen, was used to explore nets by examining how three-dimensional structures can be represented through interconnected two-dimensional faces. Baa Fakhe, the traditional Nias rice barn, provided a context for surface area and volume of prisms by connecting its geometric structure and measurable components with corresponding geometric quantities. Finally, Lompat Batu (stone-jumping) provided a culturally meaningful context for exploring the surface area and volume of pyramids through geometric dimensions and quantities.

Table 1. Integration of Nias cultural contexts into geometry learning

Geometry topic	Nias cultural context	How the cultural context is used	Main geometry focus
Introduction to three-dimensional flat-faced solids	Omo Hada (traditional Nias house)	Students observe the structure of Omo Hada and identify geometric forms and basic components represented in the architectural structure.	Recognition of three-dimensional shapes, faces, edges, vertices, and spatial structure
Nets of three-dimensional flat-faced solids	Bawu (traditional Nias kitchen)	Students examine the structural components of Bawu and explore how three-dimensional forms can be represented through connected two-dimensional faces.	Nets, faces, relationships among faces, and spatial visualization
Surface area and volume of prisms	Baa Fakhe (traditional Nias rice barn)	Students investigate the geometric structure of Baa Fakhe, identify relevant faces and dimensions, and use these relationships to determine surface area and volume.	Prism, surface area, volume, dimensions, and relationships among faces
Surface area and volume of pyramids	Lompat Batu / Hombo Batu (stone-jumping tradition)	Students use the culturally meaningful stone-jumping context to investigate dimensions and geometric quantities associated with a pyramid model and apply the corresponding surface-area and volume concepts.	Pyramid, surface area, volume, dimensions, and spatial relationships

These cultural contexts are not included merely as decorative illustrations. Rather, they function as starting points for observation, problem formulation, exploration, mathematical reasoning, and reflection. The contexts therefore provide concrete and culturally familiar situations through which abstract geometric concepts can be introduced and progressively developed. By connecting geometry with cultural objects and practices that are part of the students' local environment, the learning activities are intended to enhance the relevance and meaningfulness of mathematics and to connect formal geometric concepts with students' lived experiences.

The digital dimension in the e-module functions as an integrator of these three components. Pedagogically designed digital modules enable the presentation of structured materials, interactive visualizations, and systematic feedback and reflection. Effective digital learning environments not only deliver content but also support active engagement and students' self-regulation (Mayer, 2009; Schindler et al., 2017; Sun et al., 2008; Zimmerman, 2000). In mathematics education, digital technology also allows the integration of various learning strategies within a coherent and interactive learning environment (Attard & Holmes, 2019; Borba et al., 2017). Thus, the e-module functions as a learning ecosystem that combines cognitive structure, problem exploration, and cultural relevance within a unified design. Conceptually, the integration of Van Hiele theory, PBL, cultural context, and digital support forms a complementary learning framework. The gradual structure supports the development of understanding, PBL encourages exploration and reflection, cultural contexts strengthen meaning, and digital media provide flexible scaffolding. Overall, this design is positioned as a pedagogical intervention deliberately developed to support not only cognitive achievement but also the development of students' affective dispositions in geometry learning.

Mathematical Resilience

Mathematical resilience in this study is understood as students' disposition to remain engaged, persistent, and adaptive when facing difficulties in mathematics learning. Resilience is not merely the ability to endure challenges but also the capacity to interpret difficulties as part of the learning process while maintaining productive engagement (Goodall & Johnston-Wilder, 2015; Kooken et al., 2016). Empirical research indicates that mathematical resilience is influenced by both psychological factors and learning environments, and therefore it can be developed through appropriate instructional design (Xenofontos & Mouroutsou, 2023).

Mathematical resilience can be understood through four interrelated dimensions identified by Lee and Ward-Penny (2022): growth mindset, value of mathematics, knowing how to work at learning mathematics, and knowing how to find appropriate support. Growth mindset refers to the belief that mathematical capability can develop through effort and persistence; value of mathematics concerns recognizing mathematics as meaningful and valuable; knowing how to work at learning mathematics involves engaging productively with mathematical difficulty and learning from challenges; and knowing how to find appropriate support refers to recognizing and using appropriate assistance when difficulties arise (Lee & Ward-Penny, 2022). These dimensions are interconnected, as beliefs about developing mathematical capability, recognizing the value of mathematics, productive engagement with difficulty, and appropriate help-seeking can collectively support persistence in learning.

The integrated learning design was intended to provide complementary opportunities to exercise these dimensions rather than strengthen them independently. Growth mindset was supported through the progressive task demands of the Van Hiele sequence and PBL activities that normalized difficulty, strategy revision, and persistence. The value of mathematics was supported through Nias cultural contexts, including Omo Hada, Bawu, Baa Fakhe, and Hombo Batu, which connected geometric concepts with familiar cultural objects and practices. Knowing how to work at learning mathematics was addressed through PBL activities involving investigation, strategy comparison, error revision, solution development, and reflection, supported by the Van Hiele progression. Finally, knowing how to find appropriate support was supported through digital scaffolding, including visual representations, guiding questions, hints, feedback, peer interaction, and teacher facilitation. Thus, the four dimensions were addressed through an integrated set of learning experiences rather than through independently tested components.

Research Questions

Based on the literature review and identified research gap, geometry learning requires approaches that support both students' cognitive development and their affective dispositions when facing mathematical challenges. The integration of Van Hiele theory, Problem-Based Learning, and Nias cultural contexts through a digital e-module provides a potentially effective framework for

addressing these aspects. However, empirical evaluation is needed to establish the quality of the developed e-module and examine its effectiveness in supporting students' mathematical resilience.

In line with the educational design research approach, the developed e-module was evaluated in terms of validity, practicality, and effectiveness. Because mathematical resilience was positioned as the primary outcome, the study also examined changes in students' mathematical resilience and compared the magnitude of improvement between students who used the integrated e-module and those who received conventional geometry instruction. Accordingly, this study addressed the following research questions:

RQ1 How can a culture-based geometry e-module integrating Van Hiele theory through Problem-Based Learning be developed to meet the criteria of validity and practicality?

RQ2 To what extent does the developed e-module improve students' mathematical resilience?

RQ3 Is there a significant difference in mathematical resilience between students who learn through the developed integrated e-module and those who receive conventional geometry instruction?

METHOD

Research Design

This study employed a Research and Development (R&D) approach to develop a culture-based geometry e-module grounded in Nias culture by integrating Van Hiele theory through Problem-Based Learning to enhance students' mathematical resilience. The development process followed the 4D model (Define, Design, Develop, and Disseminate) proposed by Thiagarajan et al. (1974). The quality of the developed product was evaluated based on the criteria of validity, practicality, and effectiveness, as recommended by Nieveen (1999).

To evaluate the effectiveness of the developed e-module, a quasi-experimental design involving an experimental group and a control group was employed during the field-testing stage. Students in the experimental group learned through the developed culture-based geometry e-module, which integrated Nias cultural contexts, Van Hiele learning progression, Problem-Based Learning, and digital learning support. Students in the control group received conventional geometry instruction. These components were intentionally implemented as a single integrated instructional environment rather than as independently manipulated variables. Therefore, the between-group comparison was designed to examine the overall effectiveness of the integrated learning environment relative to conventional instruction, rather than to estimate the separate effects of cultural context, Van Hiele Theory, Problem-Based Learning, or the digital medium.

Participants and Context

The study involved 47 eighth-grade students at SMP Negeri 4 Maniamolo from two pre-existing Grade VIII classes: 24 students in the experimental group received geometry instruction using the culture-based geometry e-module integrating Van Hiele theory and Problem-Based Learning, while 23 students in the control group received conventional geometry instruction. The classes were selected purposively in consultation with the school based on instructional feasibility and curriculum alignment. Both groups were taught by the same mathematics teacher and received instruction over the same six instructional meetings. Because the classes were pre-existing, students were not individually randomized. Baseline mathematical resilience was assessed using the pretest administered to both groups before the intervention, and the pretest results were examined to evaluate the initial comparability of the groups.

The experimental learning activities incorporated selected Nias cultural artifacts and practices based on their cultural relevance and geometric affordances for the targeted concepts. Omo Hada was selected for the introduction to three-dimensional flat-faced solids, Bawu for nets of three-dimensional solids, Baa Fakhe for the surface area and volume of prisms, and Lompat Batu/Hombo Batu for the surface area and volume of pyramids. These cultural contexts were selected based on their observable forms, structures, dimensions, and spatial characteristics and their relevance to students' local cultural environment. Their rationale and pedagogical use are described further in the Theoretical Framework section.

The sample was relatively small and limited to two intact classes within the participating school; therefore, the findings may be sensitive to sampling variation and should be generalized cautiously beyond the participating context. The assumptions of normality and homogeneity of variance were examined before the between-group comparison, and Cohen's d was reported to complement statistical significance by indicating the magnitude of the observed difference.

Development Procedures

The development procedures used in this study followed the 4D model, which consists of the stages of define, design, develop, and dissemination (Thiagarajan et al., 1974). To address the needs of this study, the procedures were slightly modified so that each stage explicitly included the activities conducted by the researchers, starting from the define stage until the dissemination stage (**Figure 1**).

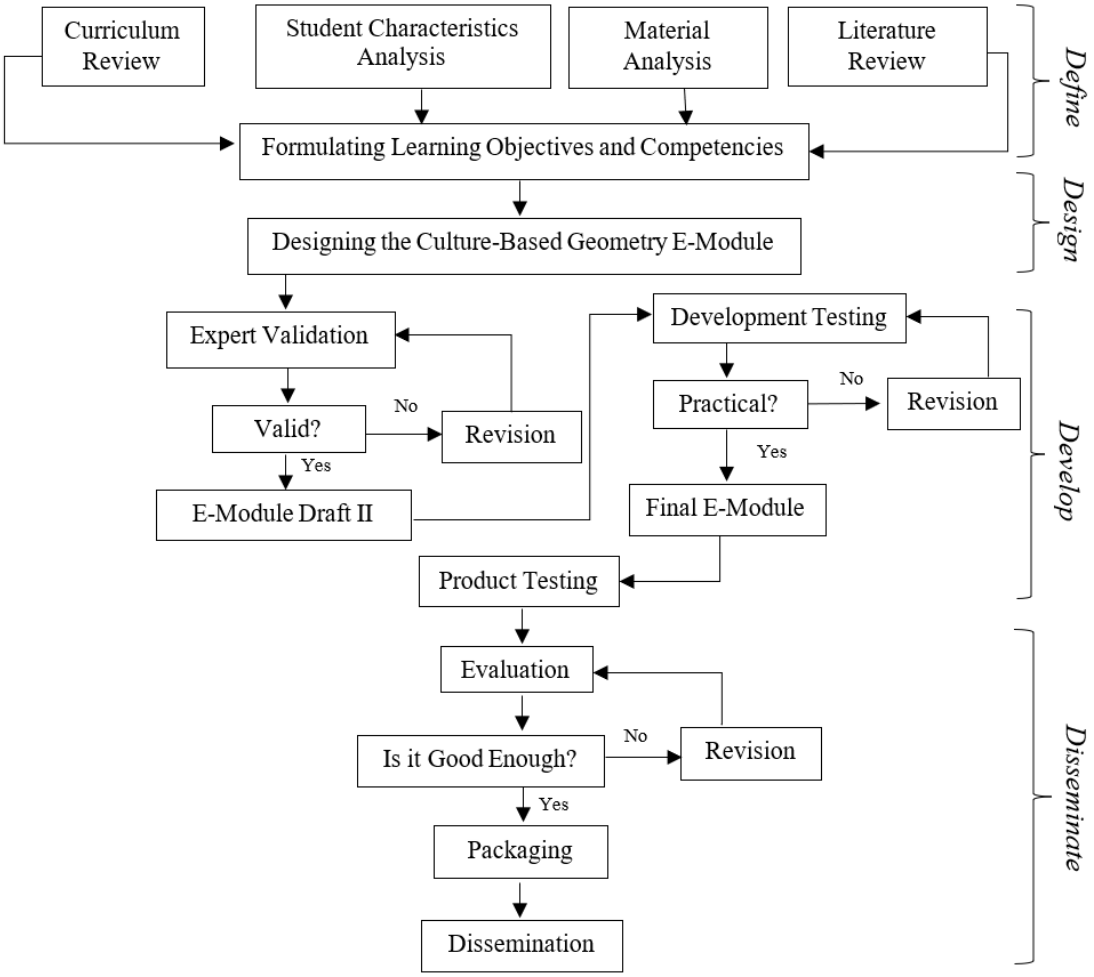


Figure 1. Modified 4D development procedure (Adopted from Thiagarajan et al., 1974)

Instruments and Data Collection

To address the research questions concerning the validity, practicality, and effectiveness of the developed e-module, data were collected using expert validation sheets, a student response questionnaire, and a mathematical resilience questionnaire. The e-module was evaluated through five domain-specific validation questionnaires administered to five experts, with one expert assigned to each domain: material/content, media, pedagogy, culture, and language. Each questionnaire used a 1–10 rating scale, with the number of statements varying according to the scope of each domain: 30 statements for material/content, 34 for media, 33 for pedagogy, 18 for culture, and 33 for language, comprising 148 statements in total. The items assessed domain-specific characteristics, and experts were also invited to provide comments and recommendations for revision. The validation procedure followed the principles of content validity suggested by Aiken, (1985), while the product quality criteria were aligned with the validity, practicality, and effectiveness framework proposed by Nieveen (1999).

The second instrument was the student response questionnaire, which was used to examine the practicality and engagement of the e-module. Practicality was considered in terms of ease of use and feasibility of implementation in real classroom settings (Plomp & Nieveen, 2013). The questionnaire covered seven indicators: ease of navigation, clarity of instructions, conciseness and time efficiency, accessibility flexibility, relevance to classroom conditions, adaptability to users, teacher and student satisfaction, and attractiveness (engagement). These indicators addressed both the functional usability and user experience of the e-module, including navigation, instructional clarity, accessibility, classroom suitability, adaptability, satisfaction, and student engagement. The questionnaire design was also informed by usability principles (Nielsen, 2004) and digital learning engagement in technology-supported mathematics education (Crompton, 2017; NCTM, 2019). The questionnaire was reviewed by two mathematics education experts and one educational technology expert, who evaluated the relevance, clarity, language, and suitability of the items. Their ratings and qualitative comments were used to revise the questionnaire before its administration to students.

The final instrument was the mathematical resilience questionnaire, developed based on the dimensions and indicators of mathematical resilience proposed by Lee & Ward-Penny, (2022). It consisted of 30 statements covering four dimensions: growth mindset, value of mathematics, knowing how to work at learning mathematics, and knowing how to find appropriate support. The questionnaire was reviewed by two mathematics education experts for content and construct relevance, item correspondence with the intended dimensions and indicators, and clarity and appropriateness of wording. The questionnaire was revised based on their feedback and subsequently pilot-tested with 30 respondents who were not included in the main study. The

Table 2. Reliability statistics

Cronbach's Alpha	N of Items
0.982	30

Table 3. Interpretation categories of validity

Score Range	Interpretation
$80 < x \leq 100$	Very Valid
$60 < x \leq 80$	Valid
$40 < x \leq 60$	Fairly Valid
$20 < x \leq 40$	Less Valid
$0 \leq x \leq 20$	Very Less Valid

Table 4. Practicality level

Criteria (%)	Practicality Level
$80 < P \leq 100$	Very Practical
$60 < P \leq 80$	Practical
$40 < P \leq 60$	Fairly Practical
$20 < P \leq 40$	Not Practical
$0 \leq P \leq 20$	Very impractical

pilot data were used to examine the internal consistency of the 30-item questionnaire using Cronbach's Alpha, as presented in **Table 2**.

All questionnaire-based instruments used in the study employed a 1–10 response scale. These included the five domain-specific e-module validation questionnaires, the student practicality questionnaire, and the mathematical resilience questionnaire. The specific number of items and indicators differed according to the purpose of each instrument.

The mathematical resilience questionnaire was administered to students before and after the instructional intervention to examine changes in their mathematical resilience. The pretest was administered prior to the intervention to establish students' baseline levels of mathematical resilience, while the posttest was administered after the completion of the intervention to assess their levels following the learning experience. The same questionnaire was used at both measurement points to ensure consistency in assessing the four dimensions of mathematical resilience: growth mindset, value of mathematics, knowing how to work at learning mathematics, and knowing how to find appropriate support. The pretest and posttest scores were subsequently compared to examine changes in students' mathematical resilience following the intervention. In addition, normalized gain scores were calculated to describe the magnitude of improvement in students' mathematical resilience from pretest to posttest.

Data Analysis

Data analysis was conducted to process the data obtained from the research instruments in order to determine the validity, practicality, and effectiveness of the developed e-module.

Validity

The validity of the e-module was determined based on the evaluation scores provided by the five experts for their respective domains. Because each domain was assessed by one expert, the scores were analyzed descriptively within each domain. The obtained scores were converted from the original 1–10 rating scale to a 0–100 scale and then interpreted according to the predetermined validity criteria. The average score from all validation results was then calculated and compared with the interpretation categories of validity (Samsu, 2017), as presented in **Table 3**.

Practicality

Practicality data were obtained from a student response questionnaire administered after students used the e-module during the classroom geometry intervention. The questionnaire examined students' experiences as primary users of the digital learning resource, including navigation, clarity, accessibility, adaptability, classroom relevance, attractiveness, and time efficiency. Although the e-module supported individual interaction with learning materials, it was implemented within regular classroom instruction, with the teacher acting as a facilitator while students explored problems, interacted with representations, developed solutions, and reflected on their learning. The responses were analyzed descriptively to obtain percentage scores and compared with the practicality criteria in **Table 4** to determine the usability and feasibility of implementation, following product quality criteria that define practicality as the extent to which a product can be used easily and consistently in real classroom environments (Samsu, 2017).

Effectiveness

The effectiveness of the developed e-module was evaluated through a quasi-experimental comparison involving an experimental group and a control group to examine changes in students' mathematical resilience and differences in the magnitude of improvement between the two groups.

First, descriptive statistics were used to summarize the pretest and posttest scores for both groups, including the number of students, mean, standard deviation, minimum, and maximum scores.



(a) (b) (c)

Figure 2. (a) Cover of the geometry e-module; (b) E-module navigation; (c) Animated video feature in the e-module (Source: Authors' own elaboration)

Second, normalized gain (N-Gain) analysis was conducted using the pretest and posttest scores of both groups to describe the magnitude of improvement in mathematical resilience (Hake, 1998). N-Gain was calculated as the ratio of the actual score increase to the maximum possible increase and was interpreted as high ($g \geq 0.70$), moderate ($0.30 \leq g < 0.70$), or low ($g < 0.30$). N-Gain scores were calculated for individual students and then summarized for each group.

Third, because the two groups were pre-existing classes and showed some difference in their descriptive pretest means, ANCOVA was initially considered to compare posttest scores while accounting for pretest scores. However, the assumptions required for ANCOVA were not adequately satisfied. Therefore, individual N-Gain scores were used as an alternative gain-score approach to examine differences in the magnitude of improvement between the two independent groups. The N-Gain scores were examined for normality and homogeneity of variance. Because these assumptions were satisfied, an independent-samples t-test was conducted to determine whether the mean N-Gain differed significantly between the experimental and control groups. This approach is consistent with gain-score comparisons in pretest–posttest designs and previous educational research comparing normalized gains between independent groups (Bao, 2006; Hake, 1998; Meltzer, 2002).

Fourth, the statistical decision criterion was specified a priori at $\alpha = 0.05$. The null hypothesis was rejected when the two-tailed p-value was less than 0.05; otherwise, the null hypothesis was not rejected. The expected direction was that students in the experimental group would demonstrate greater normalized improvement in mathematical resilience than those in the control group.

Finally, Cohen's d was calculated to estimate the standardized magnitude of the difference in N-Gain between the two groups (Cohen, 1988). Effect sizes were interpreted as small (0.20), medium (0.50), and large (0.80). Given the relatively small sample and the use of pre-existing classes, the inferential findings were interpreted cautiously and within the context of the participating school.

RESULTS

Characteristics of the Developed E-Module

The developed geometry e-module is an interactive digital learning material designed to support meaningful geometry learning through the integration of cultural contexts, Van Hiele theory, and Problem-Based Learning (**Figure 2**). The e-module is designed to be used both independently and in facilitated learning settings, with a systematic learning sequence beginning with problem orientation, followed by exploration activities, concept reinforcement, reflection, and formative evaluation. These characteristics enable the e-module not only to function as a source for delivering content but also as an active learning medium that encourages students' cognitive and affective engagement. In terms of design, the e-module is adapted to the characteristics of secondary school students through the use of communicative language, visualizations that support conceptual understanding, and digital navigation that is easy to access.

One of the main characteristics of this e-module is the utilization of cultural contexts as the foundation for geometry learning (**Figure 3**). Local cultural elements are presented as sources of problems and objects of mathematical exploration so that geometric concepts are not introduced in an abstract manner from the outset. Through culture-based problems, students are directed to observe, identify, and explore geometric forms and patterns present in their surrounding environment. This approach allows students to connect familiar cultural experiences with the geometric concepts being studied while simultaneously increasing the meaning and relevance of mathematics learning.

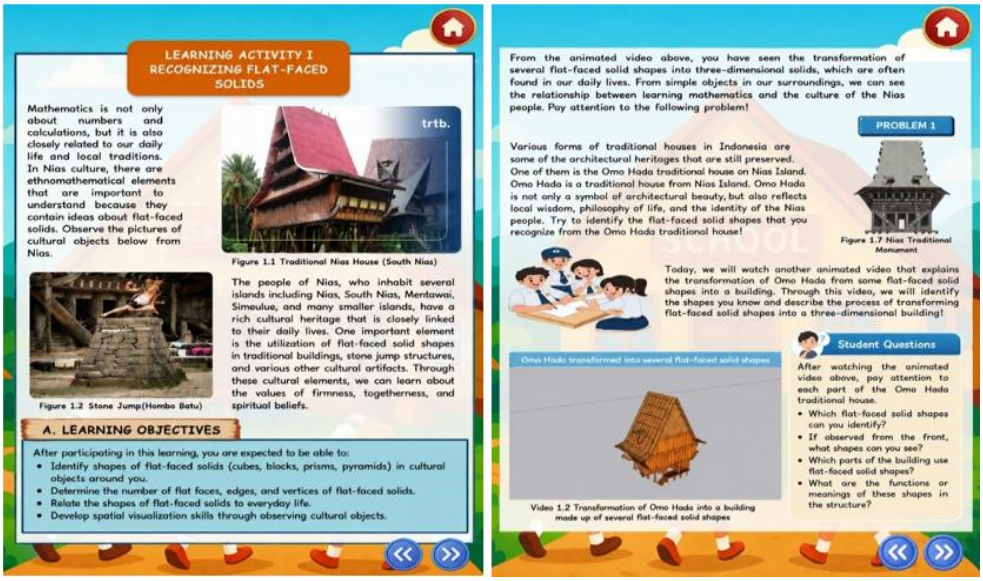


Figure 3. Integration of local cultural contexts as objects of mathematical exploration in the e-module (Source: Authors' own elaboration)

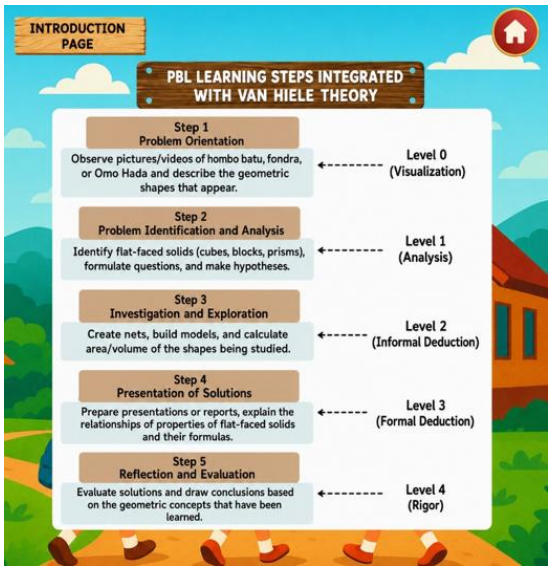


Figure 4. PBL learning process integrated with Van Hiele Theory in the e-module (Source: Authors' own elaboration)

The structure of the e-module is organized based on the Van Hiele stages of geometric thinking, enabling students' conceptual understanding to develop gradually and systematically (**Figure 4**). Learning begins with visualization activities through the observation of culture-based objects, followed by the analysis stage, where students examine geometric properties through exploratory tasks and guiding questions. At the stages of informal abstraction and rigor, students are guided to relate various properties of three-dimensional shapes, formulate definitions, and develop simple arguments and justifications. The gradual arrangement of learning materials was designed to support the progressive development of geometric thinking in accordance with the Van Hiele sequence. Rather than assuming that this progression directly eliminates misconceptions, the sequence provided opportunities for students to revisit geometric properties, compare representations, refine their reasoning, and construct more formal relationships as the level of task complexity increased.

Furthermore, the e-module is designed according to the syntax of Problem-Based Learning, where contextual problems serve as the starting point of learning. Students are encouraged to explore various solution strategies based on their prior knowledge, while the e-module provides adaptive scaffolding through hints, partial examples, and reflective questions. These supports are intended to guide students' exploration without directly providing complete solution procedures. At the final stage, students are given the opportunity to reflect on both the learning process and the outcomes. These characteristics provide space for students to face challenges, manage learning difficulties, and interpret errors as part of the learning process, thereby contributing to the strengthening of students' mathematical resilience in geometry learning.

Table 5. Expert validation result of geometry e-module

Aspect	Score	Category
Content	69.39	Valid
Media	83.53	Very Valid
Pedagogical	84.24	Very Valid
Cultural	93.89	Very Valid
Language	84.55	Very Valid
Total Score	415.6	Very Valid
Overal Mean	83.12	

Table 6. Student response questionnaire results

Indicator	Mean Percentage	Category
Ease of navigation	84.72	Very Practical
Clarity of instructions	83.96	Very Practical
Concise and time-efficient	78.33	Practical
Accessibility flexibility	88.75	Very Practical
Relevance to classroom conditions	86.04	Very Practical
Adaptability to users	79.72	Practical
Teacher and student satisfaction	80.83	Very Practical
Attractiveness (engagement)	90.42	Very Practical
Overall Mean	**84.10**	**Very Practical**

Expert Validation

To establish the validity of the developed geometry e-module, expert validation was conducted across five domains: content, pedagogy, media design, Nias cultural relevance, and language. Five experts evaluated the corresponding domains using domain-specific questionnaires and provided comments and recommendations for product revision. The results of the expert validation are presented in **Table 5**.

Table 5 shows the validation scores obtained from experts for each aspect of the developed e-module. When compared with the validity interpretation criteria presented in **Table 3**, the results indicate that overall, the aspects fall within the very valid category, except for the content aspect, which was categorized as valid due to several elements that required revision and improvement. The revisions identified through expert validation were incorporated into the e-module before it was used in the subsequent development testing and classroom implementation. Thus, the version evaluated for practicality and effectiveness reflected the revisions made based on expert recommendations.

Practicality

The practicality of the developed geometry e-module was evaluated to determine the level of ease of use and feasibility of implementation in the learning process. The results of the practicality analysis are presented in **Table 6**.

The results indicate that all practicality indicators were categorized as very practical (**Table 4**), except for conciseness and time efficiency (78.33%) and adaptability to users (79.72%), which were categorized as practical. Although these two indicators obtained relatively lower scores, they remained within the practical category. For example, comments and suggestions provided by students in the student response questionnaire indicated that some activities required more time than expected, while some students reported that they still needed teacher assistance when navigating or completing particular activities in the e-module. These comments suggest that some learning activities may require additional time and that certain features may need further adjustment to accommodate differences in students' familiarity with digital learning resources. Nevertheless, the overall mean of 84.10% indicates that the e-module was perceived as very practical for geometry learning.

Effectiveness of the E-Module on Mathematical Resilience

The effectiveness of the developed e-module was evaluated using data from both the experimental and control groups. During the intervention, the experimental group used the culture-based geometry e-module across six geometry lessons covering three-dimensional flat-faced solids, nets, surface area of prisms, surface area of pyramids, volume of prisms, and volume of pyramids. The learning activities integrated Nias cultural contexts, Van Hiele levels, and Problem-Based Learning throughout the intervention. The control group received conventional geometry instruction covering the same topics and instructional period but without the developed e-module. Thus, the comparison evaluated the integrated e-module as a whole instructional resource, rather than the effects of individual components or selected sections.

The effectiveness analysis consisted of two stages: (1) examining the improvement in mathematical resilience among students in the experimental group through N-gain analysis, and (2) comparing the mathematical resilience outcomes of the experimental and control groups through an independent samples t-test.

Descriptive statistics

Descriptive statistical analysis was first conducted to provide an overview of students' mathematical resilience before and after the intervention in the experimental and control groups. The descriptive results are presented in **Table 7**.

Table 7. Descriptive statistic of experimental and control class

Statistic	Experimental			Control		
	Pretest	Posttest	*N-Gain*	Pretest	Posttest	*N-Gain*
N	24	24	24	23	23	23
Mean	57.51	72.46	0.35	53.22	64.25	0.23
Standard deviation	10.89	7.66	0.05	12.13	9.26	0.04
Minimum	39.33	59.00	0.24	36.33	51.67	0.15
Maximum	73.00	83.33	0.45	75.33	81.00	0.31

Table 8. Tests of normality

Class		Kolmogorov-Smirnova			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	Df	Sig.
NGain_Score	Experimental	0.09	24	0.200*	0.98	24	0.93
	Control	0.15	23	0.200*	0.96	23	0.55

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

As shown in **Table 7**, the experimental group consisted of 24 students, while the control group consisted of 23 students. At the pretest stage, the experimental group obtained a mean mathematical resilience score of 57.51 (SD = 10.89), with scores ranging from 39.33 to 73.00. The control group obtained a lower mean pretest score of 53.22 (SD = 12.13), with scores ranging from 36.33 to 75.33. Thus, the experimental group had a 4.29-point higher mean pretest score than the control group. The control group also showed slightly greater variability in the initial scores, as indicated by its higher standard deviation (12.13 compared with 10.89).

Following the intervention, the mean mathematical resilience score of the experimental group increased to 72.46 (SD = 7.66), with scores ranging from 59.00 to 83.33. In the control group, the mean posttest score increased to 64.25 (SD = 9.26), with scores ranging from 51.67 to 81.00. Thus, the experimental group obtained a higher mean posttest score than the control group, with a difference of 8.21 points. Compared with the respective pretest scores, the experimental group showed an increase of 14.95 points, whereas the control group increased by 11.03 points. The standard deviation also decreased in both groups, from 10.89 to 7.66 in the experimental group and from 12.13 to 9.26 in the control group, indicating that the posttest scores were less dispersed around their respective group means. Descriptively, therefore, the experimental group showed a larger increase in mean mathematical resilience scores than the control group.

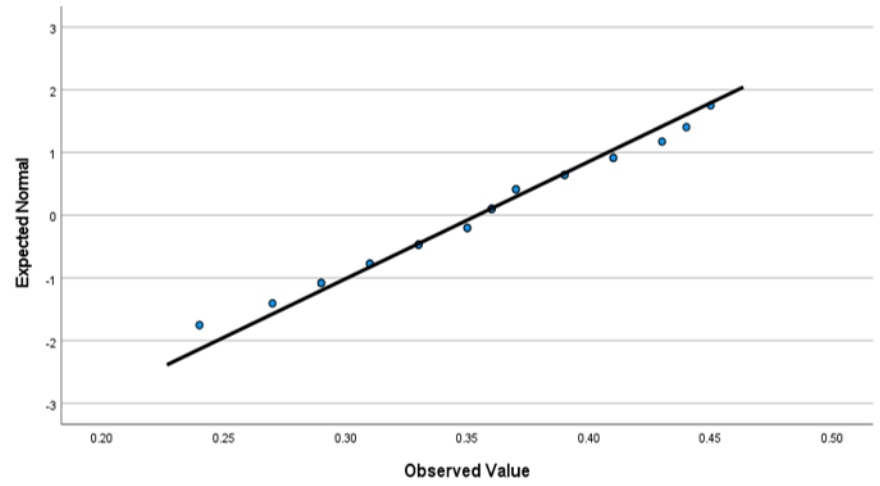
To further describe the magnitude of improvement from pretest to posttest, normalized gain (N-Gain) was calculated for both groups. The experimental group obtained a mean N-Gain of 0.35 (SD = 0.05), with values ranging from 0.24 to 0.45. Based on the predetermined N-Gain criteria, the mean value of 0.35 falls within the moderate improvement category. The control group obtained a mean N-Gain of 0.23 (SD = 0.04), with values ranging from 0.15 to 0.31, corresponding to the low improvement category. Thus, both groups demonstrated improvement in mathematical resilience, but the experimental group showed a higher mean normalized gain than the control group (0.35 vs. 0.23). These descriptive results indicate a greater magnitude of improvement in the experimental group.

To determine whether this observed difference in normalized improvement was statistically significant, the individual N-Gain scores of the two groups were subsequently examined for the assumptions required for an independent-samples t-test. Specifically, normality and homogeneity of variance were assessed before conducting the inferential comparison.

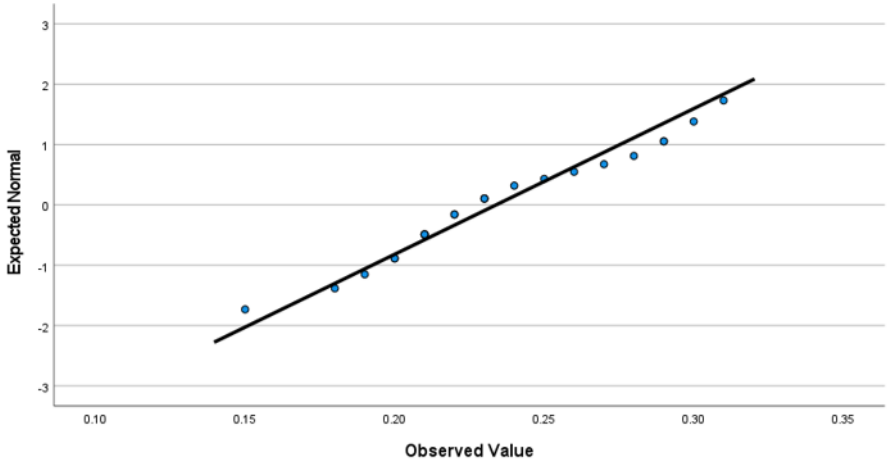
Normality test

The results of the normality test used in this study are presented in **Table 8**.

Considering that the sample sizes were relatively small (n = 24 for the experimental group and n = 23 for the control group), the normality of the N-Gain scores was primarily examined using the Shapiro–Wilk test. As presented in **Table 8**, the Shapiro–Wilk test yielded a significance value of 0.926 for the experimental group and 0.551 for the control group. Both values exceeded the significance level of 0.05, indicating that there was no statistically significant evidence of deviation from normality in either group. Therefore, the N-Gain scores in both the experimental and control groups were considered to satisfy the normality assumption required for the subsequent independent-samples t-test. Furthermore, the Normal Q–Q Plots show that the observed data points closely follow the reference line, providing additional evidence that the normality assumption was satisfied (**Figure 5**).



(a)



(b)

Figure 5. (a) Normal Q-Q Plot of NGain_Score for Experimental Class; (b) Normal Q-Q Plot of NGain_Score for Control Class (Source: Authors' own elaboration)

Table 9. Tests of homogeneity of variances

		Levene Statistic	df1	df2	Sig.
NGain_Score	Based on Mean	0.887	1	45	0.351
	Based on Median	0.889	1	45	0.351
	Based on Median and with adjusted df	0.889	1	41.29	0.351
	Based on trimmed mean	0.856	1	45	0.360

Homogeneity test

The homogeneity test was conducted to determine whether the variances of the datasets being compared were equal. The results are presented in **Table 9**.

Based on the homogeneity of variance test presented in **Table 9**, Levene's test based on the mean yielded a significance value of 0.351, which is greater than the predetermined significance level of 0.05. Therefore, the null hypothesis of equal variances was not rejected, indicating that the N-Gain scores of the experimental and control groups had homogeneous variances.

Combined with the normality results presented in **Table 8**, which showed that the N-Gain scores in both groups satisfied the normality assumption, the results indicated that the assumptions for the independent-samples t-test were met. Therefore, the independent-samples t-test was subsequently conducted at a significance level of $\alpha = 0.05$ to examine whether the mean N-Gain differed significantly between the experimental and control groups.

Table 10 presents the group statistics for the N-Gain scores of the experimental and control groups. The experimental group consisted of 24 students and obtained a mean N-Gain score of 0.35 (SD = 0.05; SE = 0.01). Meanwhile, the control group consisted of 23 students and obtained a mean N-Gain score of 0.23 (SD = 0.04; SE = 0.01). Thus, the experimental group showed a higher mean normalized gain than the control group, with a difference of 0.12 points. The standard deviation indicates that the N-Gain scores were relatively less dispersed in the control group than in the experimental group.

Table 10. Group statistics

	Class	N	Mean	Std. Deviation	Std. Error Mean
NGain_Score	Experimental	24	0.35	0.05	0.01
	Control	23	0.23	0.04	0.01

Table 11. Independent samples test

		Levene's Test for Equality of Variances		t-test for Equality of Means							
		F	Sig.	t	df	Significance		Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
						One-Sided p	Two-Sided p			Lower	Upper
NGain_Score	Equal variances assumed	0.887	0.351	8.59	45	<0.001	<0.001	0.12	0.014	0.092	0.148
	Equal variances not assumed			8.64	43.14	<0.001	<0.001	0.12	0.014	0.092	0.148

Descriptively, these results show that the experimental group experienced a greater magnitude of normalized improvement in mathematical resilience than the control group. However, the difference in mean N-Gain scores alone does not establish whether the observed difference is statistically significant. Therefore, the results of the independent-samples t-test presented in **Table 11** were examined to determine whether the difference in N-Gain between the two groups was statistically significant.

The independent-samples t-test was subsequently conducted to determine whether the observed difference in normalized improvement between the experimental and control groups was statistically significant. As shown in **Table 11**, Levene's test indicated that the assumption of homogeneity of variance was satisfied, $F(1, 45) = 0.887$, $p = 0.351$. Therefore, the equal-variances-assumed results were used for interpreting the t-test. The analysis revealed a statistically significant difference in N-Gain scores between the experimental and control groups, $t(45) = 8.59$, $p < 0.001$. The mean difference in N-Gain was 0.12, with a standard error of 0.014 and a 95% confidence interval ranging from 0.092 to 0.148. The statistically significant result indicates that the magnitude of normalized improvement in students' mathematical resilience differed between the two groups, with the experimental group demonstrating greater improvement than the control group. Thus, the null hypothesis of no difference in mean N-Gain between the two groups was rejected at the $\alpha = 0.05$ significance level.

To complement the significance test, an effect size was calculated to estimate the magnitude of the difference in N-Gain between the experimental and control groups. Cohen's d for the between-group difference in N-Gain scores was approximately 2.51, with a 95% confidence interval ranging from 1.73 to 3.27. Following Cohen (1988), this value represents a large effect because it exceeds the conventional threshold of 0.80. The confidence interval further indicates that the estimated standardized difference remained within the large-effect range.

Overall, the effectiveness findings indicate that the developed e-module was associated with greater improvement in students' mathematical resilience. The N-Gain analysis showed a moderate level of normalized improvement in the experimental group, compared with a low level of improvement in the control group. The independent-samples t-test further demonstrated a statistically significant difference in normalized improvement between the two groups, with students in the experimental group showing greater improvement than those in the control group. The effect-size analysis also indicated that the observed difference was substantial in magnitude. Overall, these findings provide empirical evidence that the learning intervention using the developed e-module was more effective in fostering students' mathematical resilience than the comparison instruction.

DISCUSSION

The results indicate that the Nias culture-based geometry e-module integrating Van Hiele theory through Problem-Based Learning met the criteria of validity, practicality, and effectiveness in supporting students' mathematical resilience. The improvement observed from pretest to posttest, together with the greater normalized improvement in the experimental group than in the control group, suggests that the integrated learning environment was associated with greater development of mathematical resilience. This finding supports the view that mathematical resilience can be developed through intentional learning experiences rather than being solely an inherent student characteristic (Goodall & Johnston-Wilder, 2015; Kooken et al., 2016; Xenofontos & Mouroutsou, 2023). Learning experiences that normalize difficulty, encourage reflection, and foster beliefs in the development of mathematical ability may contribute to more adaptive dispositions toward mathematics (Boaler, 2016; Dweck, 2015; Johnston-Wilder & Lee, 2010).

The comparative results further indicate that students who used the developed e-module demonstrated greater normalized improvement in mathematical resilience than those who received conventional geometry instruction. This finding should be interpreted as evidence concerning the integrated instructional environment, rather than the independent effect of any single component, because the Nias cultural context, Van Hiele progression, PBL, and digital scaffolding were implemented simultaneously. The substantial effect size also indicates that the observed difference was meaningful in magnitude beyond statistical significance. These findings are consistent with previous research suggesting that learning environments that promote

active engagement, reflection, and productive engagement with difficulty can support positive affective dispositions toward mathematics (Boaler, 2016; Hmelo-Silver, 2004; Xenofontos & Mouroutsou, 2023).

The learning structure was designed according to the Van Hiele stages of geometric thinking to provide a gradual progression of geometric tasks and reasoning demands. The sequence of activities, from visualization toward increasingly formal forms of reasoning, was intended to provide students with opportunities to engage with geometric challenges progressively and to strengthen the relationships among geometric concepts. In this framework, difficulties encountered during geometry learning can be viewed as part of the progression of geometric thinking rather than as evidence of fixed mathematical ability (Fujita & Jones, 2007; Teppo, 2020; Van Hiele, 1986). The structured sequencing of tasks may also provide opportunities for mastery experiences, which constitute an important source of self-efficacy development (Bandura, 1977). Repeated opportunities to engage with increasingly challenging tasks, revise strategies, and experience progress may support students' beliefs that their mathematical capabilities can develop through effort and learning, consistent with the concept of growth mindset (Dweck, 2015). In turn, such experiences are theoretically aligned with the dimension of mathematical resilience that emphasizes persistence and constructive responses when facing mathematical challenges (Burger & Shaughnessy, 1986; Schoenfeld, 1989).

The implementation of Problem-Based Learning (PBL) within the e-module further strengthens these learning experiences by placing students in problem situations that require exploration, decision-making, and reflection. Within the PBL framework, the focus of learning lies not only on the final answer but also on the reasoning process, strategy selection, and monitoring and evaluation of solution steps (Barrows, 1996; Hiebert et al., 1996). This process encourages students to develop self-regulation and the ability to adjust strategies when encountering obstacles, which are essential aspects of effective mathematics learning (Schoenfeld, 1989; Zimmerman, 2000). A learning environment that normalizes exploration and errors as part of the thinking process helps students perceive difficulty as a natural component of learning mathematics (Boaler, 2016). Therefore, the implementation of PBL in the e-module supports students' understanding of how to work at learning mathematics, which aligns with the dimension of mathematical resilience emphasizing persistence, reflection, and adaptive strategies (Lee & Ward-Penny, 2022; Strobel & van Barneveld, 2009).

The superiority of the developed e-module compared with conventional instruction may also be explained by the integration of multiple pedagogical supports within a single learning environment. In the present study, the control group received geometry instruction using the school's regular learning materials and the same general mathematical content as the experimental group, but without the developed culture-based e-module. Although problem-based activities were included in the control condition, they were not supported by the integrated digital scaffolding, Nias cultural contexts, and Van Hiele-based progression embedded in the developed e-module. Therefore, the difference between the two conditions was not simply the presence or absence of problem-based learning, but the specific integration of the cultural, cognitive, pedagogical, and digital features within the developed learning resource. In contrast, the developed e-module combines structured cognitive progression, contextual problem-solving activities, reflective prompts, and digital scaffolding. This integrated design enables students to experience challenge in a supportive environment, thereby strengthening their confidence, persistence, and willingness to engage with mathematical difficulties. Such characteristics are consistent with contemporary perspectives that view resilience as an outcome of supportive learning environments rather than solely as an individual trait (Goodall & Johnston-Wilder, 2015; Kooken et al., 2016).

It is important to note that the present study was designed to evaluate the integrated instructional environment rather than to isolate the contribution of each component. The culture-based context, Van Hiele learning progression, Problem-Based Learning, and digital scaffolding were implemented simultaneously within the experimental condition. Therefore, the observed difference in mathematical resilience cannot be attributed independently to any single component. Rather, the findings indicate the effectiveness of the integrated learning environment as implemented in this study. Future research using factorial or multi-condition experimental designs could examine the relative contribution and interaction of these components more specifically.

The integration of Nias cultural contexts within the e-module was theoretically aligned with the value of mathematics dimension of mathematical resilience by connecting geometric concepts with familiar cultural objects, practices, and experiences. By presenting cultural artifacts and practices as sources of problems and objects of geometric exploration, the learning activities provided opportunities for students to recognize mathematics as relevant to their everyday cultural environment rather than merely as a collection of abstract formulas. Such contextualization is consistent with culturally responsive approaches, which emphasize the importance of connecting learning to students' cultural experiences, identity, and sense of belonging (Gay, 2018; Ladson-Billings, 1995). From the perspective of expectancy-value theory, perceiving mathematics as meaningful and relevant may strengthen students' willingness to engage with mathematical tasks and persist when difficulties arise (Wigfield & Eccles, 2000). This interpretation is also consistent with the ethnomathematics perspective, which views mathematical ideas and practices as embedded within social and cultural contexts (Bishop, 1988b; D'Ambrosio, 2006; Rosa & Orey, 2016), as well as with previous studies suggesting that culturally meaningful contexts can support students' engagement and persistence in mathematics learning (Hunter et al., 2022; Samo et al., 2018; Suh & Calabrese, 2025). However, because the present study did not analyze changes in individual dimensions of mathematical resilience separately, this explanation should be understood as a theoretically informed interpretation of the learning design rather than as empirical evidence of a specific improvement in the value-of-mathematics dimension.

The large effect size observed in this study further strengthens the practical significance of these findings. While statistical significance indicates that the difference between groups was unlikely to occur by chance, the effect size demonstrates that the integrated learning design produced a substantial impact on students' mathematical resilience. This result suggests that the combination of cultural relevance, structured geometric reasoning, problem-based learning experiences, and digital support created learning conditions that meaningfully influenced students' affective engagement with mathematics. Consequently, the

developed e-module represents not merely a statistically effective intervention but also a pedagogically meaningful approach for supporting students' resilience in geometry learning.

The availability of scaffolding in the e-module is conceptually related to the "finding the right support" dimension of mathematical resilience, which emphasizes recognizing and using appropriate learning support constructively. In the present study, this dimension was assessed as part of the mathematical resilience questionnaire administered to the students, and the findings provide evidence regarding students' mathematical resilience in this dimension. However, the study did not separately examine whether students' use of the e-module's hints or guiding questions directly contributed to this dimension. Similarly, although the flexible and structured digital environment provided opportunities for students to work through tasks, check their understanding, and reflect on their solutions, self-regulated learning was not measured as an independent construct. Therefore, these features are discussed as affordances of the learning design rather than as empirically established changes in students' self-regulated learning processes.

Overall, the moderate N-gain improvement observed among students in the experimental group, together with the significant difference between the experimental and control groups, indicates that the developed e-module functions as an effective pedagogical intervention for fostering positive dispositions toward geometry learning. Although the improvement is categorized as moderate, this result is understandable considering that resilience is an affective construct that develops gradually and requires repeated and sustained learning experiences (Kooken et al., 2016; Xenofontos & Mouroutsou, 2023). The integration of Van Hiele cognitive structure, Problem-Based Learning, cultural contexts, and digital support within a coherent instructional design suggests that mathematical resilience may be supported through an intentionally designed integrated learning environment.

The contribution of this study lies in the integration of cognitive, pedagogical, and cultural frameworks within digital learning media, with mathematical resilience positioned as the primary outcome of geometry learning. Nevertheless, the study is limited by the relatively small number of participants and the relatively short intervention period, which consisted of six instructional meetings covering the targeted three-dimensional geometry topics. This duration may restrict the extent to which the observed changes in mathematical resilience can be interpreted beyond the immediate post-intervention period. Longer implementation periods would provide students with repeated opportunities to encounter mathematical challenges, explore and revise strategies, seek appropriate support, and reflect on their learning, thereby providing a stronger basis for examining whether changes in mathematical resilience are sustained over time. Future research could therefore involve larger samples and extend the intervention across a complete instructional unit or, where feasible, a semester, accompanied by a delayed follow-up assessment to examine the persistence of changes in students' mathematical resilience.

CONCLUSION

This study developed and evaluated a culture-based geometry e-module integrating Van Hiele theory and Problem-Based Learning to support students' mathematical resilience in learning three-dimensional geometry. The developed e-module demonstrated a very high level of validity and practicality, indicating that its content, pedagogical design, media features, cultural relevance, and language were appropriate and that the e-module was feasible for use in geometry learning. The findings also indicate that the integrated learning environment was associated with greater improvement in students' mathematical resilience compared with the control condition. The e-module provided a structured sequence of geometric activities that combined progressive reasoning demands, problem-based exploration, Nias cultural contexts, and digital learning support. These elements were designed to provide students with opportunities to engage with challenging geometric problems, explore and revise solution strategies, seek appropriate support, and reflect on their learning.

The findings should be interpreted at the level of the integrated instructional design rather than attributed to any single component of the e-module. Although the learning sequence and digital scaffolding were theoretically aligned with progressive geometric thinking, adaptive help-seeking, and self-regulated learning, these processes were not independently measured in the present study. Likewise, the study did not directly assess students' geometric conceptual understanding or misconception reduction. The primary empirical outcome was mathematical resilience, assessed through pretest and posttest administration.

The study was conducted with a relatively small sample of 47 eighth-grade students in two pre-existing classes and over six instructional meetings. Therefore, the findings should be interpreted within the context of the participating school and should not be generalized broadly without further evidence. Future research involving larger and more diverse samples, longer implementation periods, and delayed follow-up assessments is needed to examine the persistence of changes in mathematical resilience over time. Further studies may also investigate the specific contribution of cultural contexts, Van Hiele-based progression, PBL, and digital scaffolding, as well as students' self-regulated learning processes, through designs that allow these components to be examined more directly.

Author contributions: EG: study conception, and design, development of the theoretical framework, instructional design, data collection, data analysis, interpretation of results, and preparation of the initial manuscript draft. The second, third, and fourth authors served as academic supervisors, contributing through methodological guidance, theoretical and conceptual review, and critical feedback on the analysis, discussion, and overall coherence of the manuscript. All authors have agreed with the results and conclusions.

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Ethical statement: The authors stated that the study was conducted in accordance with ethical standards in educational research. Participation in this study was voluntary, and all students were informed about the purpose of the research before the data collection process

began. Permission to conduct the research was obtained from the school authorities, and students participated with the understanding that their responses would be used only for research purposes. All data collected from participants were treated confidentially and used solely for academic research. No personal identifying information was recorded, and the results were reported in aggregate form to ensure participants' anonymity. The authors declare that this research did not involve any form of physical or psychological risk to the participants.

AI statement: authors stated that AI-based tools were used exclusively for translation and language editing purposes. The authors confirm that the use of AI did not affect the originality, scientific content, data interpretation, or conclusions of the study, and that full responsibility for the manuscript rests with the authors.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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