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Mathematical connections in STEM-based modelling: From activity to reflection in pre-service teacher education

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ABSTRACT

Mathematical connections are a key component of meaningful mathematical learning, yet little is known about how pre-service teachers analyse them in STEM modelling contexts. This study examines how pre-service primary teachers establish, identify, and interpret mathematical connections in a population modelling task, and how this analysis informs their instructional decisions. Participants engaged in a modelling sequence including structured reflection activities. Data from written productions and instructional proposals were analysed using qualitative content analysis, complemented by exploratory quantitative analysis. Results show that participants mainly establish procedural and representational connections, while argumentative connections are less frequent. During reflection, connections are identified selectively, with representational ones overrepresented. A strong relationship is found between connections established and identified, but not with instructional strategies. These findings highlight the need to better support the recognition and didactical use of less visible mathematical connections in teacher education.

Keywords: mathematical connections, mathematical modelling, mathematics teacher education, pre-service teachers, reflective practice, STEM education

INTRODUCTION

The making of connections is widely recognized as a fundamental component of meaningful mathematical learning. Research in mathematics education has shown that mathematical knowledge develops through the coordination of concepts, representations, procedures, and meanings within coherent relational structures (Dubinsky & McDonald, 2001; Duval, 2006; Godino et al., 2019; Lakoff & Núñez, 2000). From this perspective, mathematical ideas acquire meaning not as isolated entities but through the network of relations established during mathematical activity.

This importance of connections is also reflected in curricular frameworks and educational standards. For instance, the NCTM Principles and Standards highlight connections as a central process of mathematics learning, emphasizing both connections among mathematical ideas and connections between mathematics and other contexts (NCTM, 2000). These connections may operate at different levels, from broad conceptual relationships to more specific relations such as coordinating multiple representations of the same concept.

Research has shown that establishing mathematical connections is associated with deeper conceptual understanding and more flexible use of knowledge (Askew et al., 1997; Barmby et al., 2009; Caviedes et al., 2021; Fyfe et al., 2017; León del Carmen et al., 2024). However, making connections is not straightforward. Learners may experience difficulties when coordinating multiple ideas, particularly when connections increase cognitive demands (Rittle-Johnson et al., 2009; Sweller et al., 1998). Moreover, teachers often face challenges in incorporating meaningful connections into classroom practice due to the lack of clear conceptualizations and instructional approaches (Frykholm & Glasson, 2005; Leikin & Levav-Waynberg, 2007).

Recent studies have advanced the analysis of mathematical connections in classroom activity by conceptualizing them as networks of relations among mathematical objects (De Gamboa et al., 2023; Rodríguez-Nieto, 2021). These approaches highlight both the structural complexity of connections and their functional role in mathematical reasoning.

In parallel, the integration of mathematics with other disciplines has become a central focus of STEM education. STEM learning environments often involve complex real-world situations requiring the coordinated use of ideas and practices from different disciplines (English, 2016). In this context, modelling activities play a key role, as they involve the construction and evaluation of representations linking mathematical structures with real-world phenomena (Blum & Niss, 1991; Gilbert & Justi, 2016; Wilensky & Reisman, 2006).

Despite these opportunities, research has shown that mathematics is often used in STEM contexts as an auxiliary tool rather than as a discipline with its own epistemic practices (Couso et al., 2021; Li & Schoenfeld, 2019). As a result, opportunities to develop rich connections between mathematical and scientific ideas may remain underexploited.

In STEM modelling, mathematics does not operate independently from the scientific domain. Rather, mathematical models are progressively refined through scientific assumptions, empirical constraints, and disciplinary knowledge. Consequently, mathematical connections emerge not only within mathematics itself but also through the interaction between mathematical structures and scientific explanations. Understanding how prospective teachers coordinate these disciplinary perspectives constitutes an important challenge for STEM teacher education.

While previous research has mainly focused on how learners establish connections during problem solving or modelling activities, less attention has been paid to how such connections are subsequently analysed, particularly in teacher education. Establishing a mathematical connection during activity resolution differs from identifying and interpreting it afterwards, which involves recognising the relationship, examining its structure, understanding its role, and considering its instructional implications.

This reflective dimension is especially relevant in teacher education. Pre-service teachers are expected not only to establish connections but also to evaluate them and use them to inform instructional decisions.

Accordingly, this study examines how pre-service primary teachers (PSTs) engage with mathematical connections in a STEM-based modelling activity on population growth. The study focuses on both the connections established during the activity and their subsequent analysis through structured reflection, as well as how this analysis informs instructional decision-making.

Specifically, the study addresses the following research questions:

1) What types of mathematical connections do pre-service primary teachers establish in a STEM-based modelling activity?
2) How do they identify these connections through structured reflection?
3) How does this analysis inform their instructional strategies?

These considerations highlight the relevance of further investigating how mathematical connections are not only established but also analysed within teacher education, particularly in STEM contexts where their role may remain underdeveloped. Despite the recognised importance of connections for meaningful mathematical learning, there is still limited understanding of how pre-service teachers interpret, evaluate, and use these connections to inform instructional decisions. Addressing this gap is essential to advance both theoretical and practical knowledge about the role of connections in supporting integrated STEM learning and the development of teachers' professional competence.

THEORETICAL FRAMEWORK

Research on mathematical connections has expanded significantly in mathematics education as scholars have sought to understand how mathematical knowledge is structured and mobilized in problem-solving and modelling activity. Mathematical understanding is no longer viewed as the possession of isolated concepts or procedures, but as the ability to relate different elements of mathematical knowledge in meaningful ways (Businskas, 2008; García-García & Dolores-Flores, 2018). From this perspective, mathematical activity can be analysed through the relations established between representations, concepts, procedures, and propositions when individuals engage with an activity.

This perspective is also closely connected to research on teachers' knowledge of mathematics. Models of mathematics teacher knowledge emphasize that effective teaching requires understanding the structure of mathematics as an interconnected system rather than as a collection of independent topics (Carrillo-Yañez et al., 2018; Climent et al., 2024). In particular, the Mathematics Teacher's Specialised Knowledge (MTSK) model highlights the importance of teachers' knowledge of the structure of mathematics, which includes the ability to recognize and mobilize connections between mathematical ideas. Such knowledge enables teachers not only to solve activities, but also to interpret solution strategies, anticipate students' reasoning, and design learning situations that foreground meaningful relations between mathematical concepts.

Mathematical connections occur when relations are established among mathematical objects belonging to the mathematical domain itself, such as representations, procedures, properties, or arguments (Duval, 2006). These connections allow us to analyse how mathematical ideas are articulated within mathematical activity, both during problem solving and in subsequent reflection.

Within intra-mathematical connections, further distinctions have been proposed in the literature. De Gamboa et al. (2023) differentiates between conceptual connections and process-related connections. Conceptual connections refer to relations among mathematical objects associated with a concept, representation, or property. Process-related connections, on the other hand, emphasize transversal aspects of mathematical activity, such as argumentation, justification, or strategic decision making. These distinctions are particularly relevant in teacher education contexts, where prospective teachers are expected not only to establish mathematical relations while solving activities, but also to analyse their structure, function, and potential instructional value.

In this study, mathematical connections are operationally identified when participants establish relations between mathematical objects, such as relating representations, linking concepts to procedures, or providing reasons that support a mathematical claim. This relational perspective allows us to analyse how mathematical ideas are articulated both during activity resolution and in subsequent reflection.

Several typological models have been proposed to characterise the relations established during mathematical activity (Businskas, 2008; De Gamboa et al., 2021; García-García & Dolores-Flores, 2018; Rodríguez-Nieto, 2021). A foundational framework was introduced by Businskas (2008), who identified different types of relations in teachers' discourse, including connections between representations, procedural relations, implications, and inclusion relations. Subsequent studies have refined and extended this classification (De Gamboa et al., 2021; García-García & Dolores-Flores, 2018; Rodríguez-Nieto, 2021).

In the present study, we adopt the reinterpretation of this extended model proposed by De Gamboa et al. (2023), as it provides a suitable analytical framework for examining connections both during activity resolution and during subsequent reflection. Within this perspective, mathematical connections are understood as elementary relations linking mathematical objects such as representations, procedures, and propositions.

One such relation corresponds to Equivalent Representation (ER), which occurs when two representations correspond to the same mathematical object within the same semiotic register. Such relations appear, for example, when different symbolic or graphical expressions are recognized as representing the same mathematical structure. A second type corresponds to Alternate Representation (AR), which links two representations of the same mathematical object expressed in different registers.

Another relevant type of relation is Procedure (P), which refers to the link between a mathematical concept and a procedure used to operate on it. This relation appears when a concept is associated with a rule, algorithm, or operational strategy applied to solve the activity. Closely related to this is Equivalent Procedure (EP), which occurs when two distinct procedures are recognized as valid approaches for solving the same problem. Identifying this type of relation reveals whether participants recognize the possibility of different mathematical strategies leading to the same outcome.

The framework also considers relations between propositions. Justification (J) refers to connections in which one proposition is supported by another through explicit mathematical reasons that establish its validity. Thus, the connected mathematical objects are propositions linked through justification. For example, justifying that the recursive rule of the sequence follows from the population-growth assumptions constitutes a justification connection, because one proposition provides the justification for the validity of another. Implication (IM) refers to connections between propositions expressed through conditional (if–then) reasoning, where one proposition logically follows from another.

Taken together, these categories provide an analytical framework for examining how participants coordinate representations, procedures, and arguments while engaging with the modelling activity. More importantly, they allow us to analyse not only the presence of connections, but also their structural role in mathematical reasoning and their potential relevance for instructional decision-making.

Although the original framework distinguishes a broader set of mathematical connections, including process-related and extramathematical connections (De Gamboa et al., 2023), the present study focuses exclusively on elementary intra-mathematical connections. This analytical delimitation was adopted because the research questions specifically address how PSTs establish, identify, and interpret relationships between mathematical objects during modelling and subsequent reflection. Accordingly, process-related and extramathematical connections were not included in the analytical framework.

This analytical framework is particularly relevant in STEM modelling contexts, where mathematical connections emerge through the coordination of mathematical and scientific ideas. In such contexts, modelling activities involve the construction, interpretation, and revision of representations that link mathematical structures with real-world phenomena (Blum & Niss, 1991; Gilbert & Justi, 2016; Wilensky & Reisman, 2006). However, previous research has shown that mathematics is often reduced to a computational tool in STEM activities, limiting opportunities to establish rich connections between mathematical and scientific ideas (Couso et al., 2021; Li & Schoenfeld, 2019).

Within STEM modelling, mathematical activity emerges through the continuous interaction between mathematical reasoning and scientific knowledge. In the population modelling task analysed in this study, biological assumptions such as rabbit maturity, reproduction rules, resource availability, and population dynamics do not merely provide a meaningful context, but also constrain the construction, validation, and progressive refinement of the mathematical model. Thus, although modelling involves interdisciplinary interactions, the present study focuses specifically on the intra-mathematical connections that participants establish among mathematical objects while developing and reflecting on the model. This delimitation is consistent with the research aims, which seek to understand how pre-service teachers establish, identify, and use mathematical connections within a STEM modelling environment.

From this perspective, the framework adopted in this study serves as the analytical lens to examine both the mathematical connections established by PSTs during the modelling activity and how these connections are subsequently analysed in a structured reflective phase. While the research analysis relies on a fine-grained typology of connections, participants engaged with a simplified framework during the instructional sequence, distinguishing between representational, procedural, and argumentative connections. This distinction is important for interpreting how PSTs move from mathematical activity to professional reflection.

MATERIALS AND METHODS

Context of the Research

The participants in this study were 20 pre-service primary teachers (PSTs) enrolled in the fourth year of a Primary Education degree. The study was conducted within an optional course in the teacher education program, which focused on mathematical connections as a key element in promoting meaningful mathematical learning. However, five participants were excluded from the analysis due to missing data, specifically the absence of written productions in one or more of the analysed activities. The final sample was therefore reduced to 15 PSTs. This reduction did not compromise the consistency of the analysis, as the remaining dataset provided sufficient and comparable information across all participants.

The course aimed to foster a relational understanding of mathematics beyond isolated procedures or fragmented content. To support this aim, PSTs worked with a simplified analytical framework that distinguished three types of mathematical connections: representational (relations between different representations of the same concept), procedural (relations between strategies, algorithms, or procedures), and argumentative (relations established through justification and reasoning).

Throughout the course, PSTs engaged in a sequence of activities in which they were required to establish mathematical connections, analyse them explicitly using the proposed framework, and reflect on how such connections could be intentionally promoted in classroom practice. These types of connections were addressed from a disciplinary perspective but also from a didactical standpoint. This way, PSTs examined how the intentional promotion of mathematical connections could strengthen conceptual coherence, support deeper understanding, and contribute to more integrated mathematical activity in primary classrooms.

The course comprised 46 hours in total and was organised over a semester. The population modelling activity analysed in this study was implemented in the final phase of the course, once PSTs had become familiar with the notion of mathematical connections and had had prior opportunities to work with and reflect on them. Within this framework, PSTs engaged in reflective and exploratory mathematical practices aligned with sense-making processes expected in primary education. The population modelling activity analysed in this study was implemented as part of the regular course activities, with all enrolled students participating. The instructional setting combined individual and collaborative work, whole-class discussion, and the explicit articulation of mathematical reasoning, thereby creating opportunities for the emergence and analysis of mathematical connections in a STEM-oriented context.

Teaching-Learning Sequence

The teaching–learning sequence (TLS) was designed to support the construction, evaluation, and revision of a model integrating mathematical and scientific ideas to interpret rabbit population growth. It constituted the empirical context for examining how mathematical connections emerge during modelling activity and how those connections are subsequently analysed from a professional perspective. The design was informed by modelling cycles involving iterative processes of model expression, evaluation, and revision, and built on a previous sequence implemented with primary pupils (De Gamboa et al., 2021) adapted here to a teacher education context through reflective activities.

The sequence was implemented across two sessions and involved a set of modelling and reflection activities centred on population growth, through which participants progressively refined a population-growth model by coordinating representations and analysing relationships between quantities.

More specifically, the sequence followed a modelling-based progression organised around the gradual construction and revision of a population growth model. It began with an exploratory activity (Activity 0), in which PSTs discussed the factors they considered relevant for the growth of a rabbit population, followed by an initial discussion on the plausibility of unlimited growth (Activity 1). PSTs then constructed an ideal mathematical model based on the Fibonacci sequence (Activity 2) and evaluated it using numerical and spreadsheet-based representations (Activity 3). The sequence subsequently introduced contextual constraints through a set of embodied and interactive tasks (Activities 4–6), including a role-play simulation to explore population changes as a function of resource availability, a card-based activity to incorporate competition and variability, and a synthesis task to consolidate emerging ideas. Activity 7 functioned as a prior synthesis of the model, before engaging in a transfer task (Activity 8), in which participants applied the constructed framework to a new real-world situation. Finally, Activity 9 focused on professional reflection, as PSTs identified the mathematical connections established throughout the sequence and proposed instructional strategies to foster their emergence in primary classrooms.

The main data sources for this study were the written productions generated in Activities 2 and 9 of the sequence. These activities were purposively selected due to their complementary analytical potential and because they provided the most relevant evidence for the research aims. While the remaining activities contributed to the overall development of the modelling process, the data they generated were less informative for the specific objective of this study. In contrast, Activities 2 and 9 offered access to two key and complementary moments: the emergence of connections during mathematical activity and their subsequent identification and didactical transformation through reflection.

In Activity 2, PSTs constructed a mathematical model of population growth based on the Fibonacci sequence. This activity required participants to coordinate representations, generate numerical structures, and articulate relationships between quantities. More specifically, PSTs worked on a contextualised version of the classical rabbit population problem (Gravett, 2010), in which they were asked to determine the number of rabbit pairs over different time spans and to justify the procedures used. Activity 2, therefore, provided evidence of the mathematical objects mobilised during model construction and of the connections established among them.

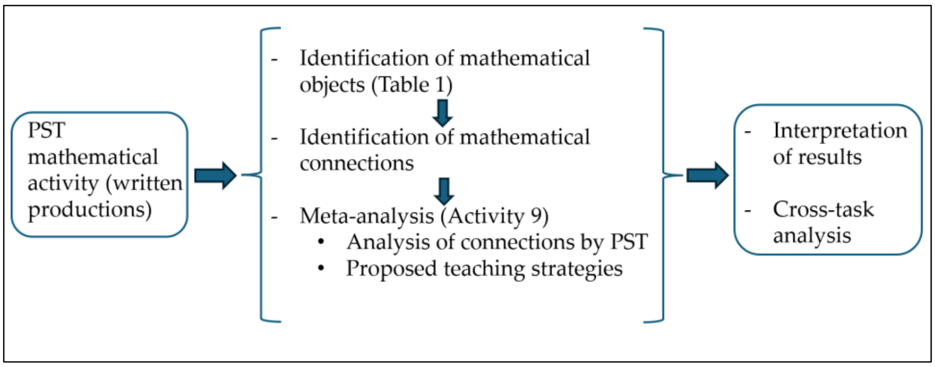


Figure 1. Analytical framework used in the study (Source: Authors' own elaboration)

Activity 9, a reflective activity implemented after a collective discussion of the modelling process was divided into two parts: in activity 9a PSTs analysed the mathematical connections that had emerged throughout the sequence; in activity 9b they proposed instructional strategies to foster such connections in primary classrooms. This activity invited participants to reflect not only on their own written productions but also on the connections that had emerged during whole-class interactions. Thus, whereas Activity 2 captured the emergence of mathematical connections during modelling activity, Activity 9 provided access to how those connections were subsequently identified, interpreted, and transformed into instructional decisions. Taken together, the written productions from both activities made it possible to analyse mathematical connections both as elements of mathematical activity and as objects of professional reflection.

Method of Analysis

The study employed a sequential explanatory mixed methods design (Tashakkori & Teddlie, 2010), in which qualitative content analysis was conducted first, followed by a complementary quantitative strand whose interpretation was informed by the qualitative findings. The qualitative strand followed the procedures of qualitative content analysis (Cohen et al., 2017), selected for its capacity to systematically identify and categorise conceptual elements within written mathematical productions. Its purpose was threefold: to identify the mathematical objects mobilised by PSTs, to characterise the connections established between those objects during the modelling task, and to examine how those connections were subsequently interpreted in the reflective phase. Coding was conducted through iterative readings and progressive category refinement, using an inductive-deductive approach in which initial codes were anchored in the theoretical framework while remaining open to categories emerging from the data (Tashakkori & Teddlie, 2010). Cases of ambiguity were resolved through discussion between the coders until consensus was reached, ensuring analytical rigour through negotiated agreement.

The quantitative strand examined numerical relationships between the three task components: connections established (A2), connections identified (A9a), and strategies proposed (A9b). All statistical analyses were performed using Python (v3.x), employing the scipy.stats library for statistical computation and matplotlib for data visualisation. Two complementary statistical procedures were applied. Pearson's product-moment correlation coefficient (r) was computed for each of the three variable pairs A2–A9a, A9a–A9b, and A2–A9b to assess the strength and direction of linear association, given the continuous and approximately linear nature of the variables (Bonett & Wright, 2000). Simple linear regression was additionally applied to the A2–A9a pair to model the rate of change between connections established and connections identified, yielding an explicit predictive equation. Whereas correlation quantifies the degree of co-variation symmetrically, regression specifies the directional relationship and magnitude of change, and both are reported here because they answer distinct but complementary questions about the data. Given the exploratory nature of the study and the small sample size ($n = 15$), all quantitative results are interpreted as complementary evidence rather than inferential conclusions (Creswell & Creswell, 2017).

Integration of the two strands was achieved through a joint display, in which scatter plots incorporating colour-coded qualitative indicators enabled simultaneous examination of quantitative trends and qualitatively derived patterns across participants.

Figure 1 summarises the analytical framework guiding the study. The analysis proceeded through three main stages: (1) identification of the mathematical objects mobilised by the PSTs in Activity 2, (2) identification and classification of the mathematical connections established between these objects, and (3) analysis of PSTs' reflections in Activity 9 with attention to both their interpretation of those connections and the instructional strategies they proposed.

This analytical process made it possible to address the research questions concerning (a) the types of mathematical connections established by PSTs during the modelling activity, (b) how these connections were interpreted in the reflective phase, and (c) how such interpretation informed the instructional strategies proposed for primary education.

Procedure of analysis of Activity 2

The analysis of Activity 2 was conducted in two phases. In the first phase, the mathematical objects mobilised by PSTs were identified in their written productions. These objects included representations, procedures, and justifications emerging in the modelling activity. The coding scheme was developed deductively from the theoretical framework and prior research on mathematical connections (De Gamboa et al., 2023) and was refined inductively during data analysis. The unit of analysis was a

Table 1. Category and coding system used to identify mathematical objects

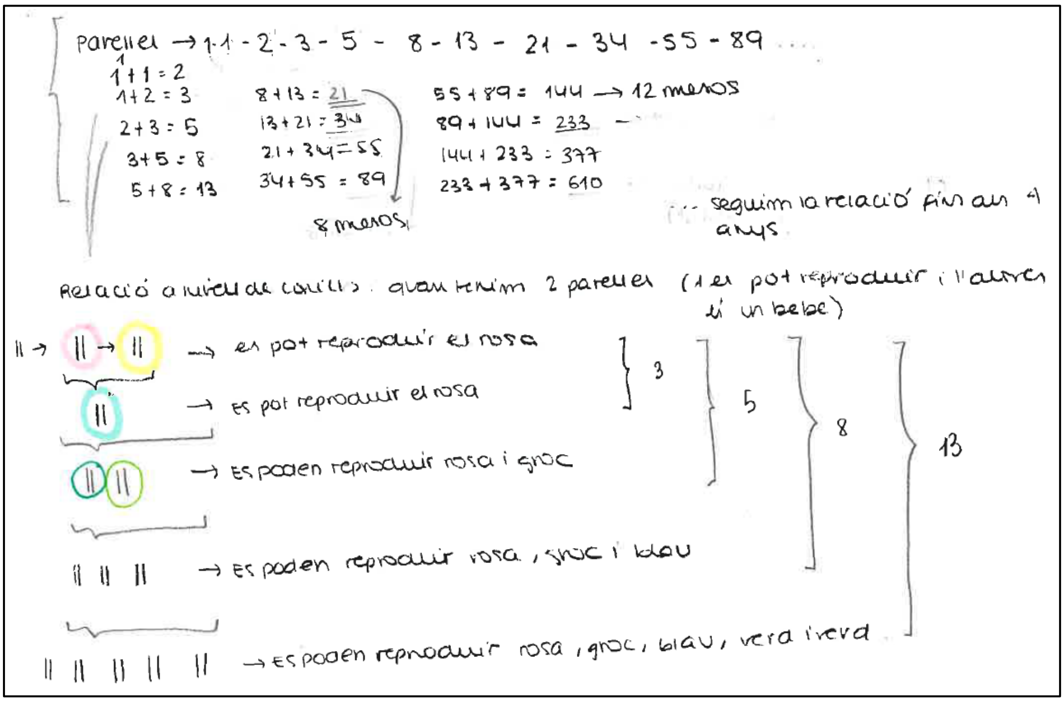
Code	Category	Definition	Description
R1	Representation	Manipulative Materials	Cards representing adult and young rabbit pairs
R2	Representation	Tree diagram	Population growth tree diagram
R3	Representation	Month-by month iconic representation	Month-by-month iconic non-tree representation of population
R4	Representation	Numerical sequence	1, 1, 2, 3, 5
R5	Representation	Month-by-month population table	Population count table: month – number of rabbit pairs
R6	Representation	Algebraic representation (generalization of the recursive formula)	$x_n = x_{n-1} + x_{n-2}$
P1	Procedure	Counting	Counting objects: I, I, II, III
P2	Procedure	Population calculation	Total population calculated as the sum of adult and young pairs month by month
P3	Procedure	Generalization of the growth pattern	1, 1, 1+1=2, 2+1=3, 3+2=5
J1	Justification	Verbal recursive rule	The number of pairs in each month equals the sum of the pairs in the two previous months
J2	Justification	Subpopulation decomposition	The number of pairs in a month is the sum of adult pairs and young pairs
J3	Justification	Maturity rule	A pair is young in the first month, becomes adult in the second month, and reproduces from the third month onward
J4	Justification	Reproduction rule	Each adult pair produces one new pair of offspring

meaningful segment of a written production in which a mathematical object could be identified. This phase resulted in the coding system presented in **Table 1**.

In the second phase, the intra-mathematical connections established in Activity 2 were identified and classified according to the framework proposed by De Gamboa et al. (2023) as described previously. A connection was coded when a PST established an explicit or inferable relation between two previously identified mathematical objects (**Table 1**). This phase made it possible to characterise not only which objects were mobilised, but also how PSTs related them during the modelling process. In the following, the two phases are carried out.

Figure 2 presents an excerpt from a PST production that illustrates the coordination of different mathematical objects during the modelling activity. This type of production was representative of a broader pattern observed across PSTs' solutions to this activity.

The PST uses rabbit cards as manipulative material to construct a population growth tree representing the temporal evolution of rabbit pairs (**Figure 2b**). Through this concrete representation, the student organises the emergence of new pairs month by



(a)

Figure 2. (a) PST written production from the modelling activity (A2). (b) use of manipulative materials to represent and explore the same situation. The original response is presented in Catalan. English translation of the handwritten text (following the order of appearance in the figure): Rabbit pairs; Initial rabbit population; The pink pair can reproduce; The pink pair can reproduce; The pink and yellow pairs can reproduce; The pink, yellow and blue pairs can reproduce; The pink, yellow, blue, green, and dark green pairs can reproduce; The pattern is extended up to Year 4 (Source: Field study)

(b)

Figure 2 (Continued). (a) PST written production from the modelling activity (A2). (b) use of manipulative materials to represent and explore the same situation. The original response is presented in Catalan. English translation of the handwritten text (following the order of appearance in the figure): Rabbit pairs; Initial rabbit population; The pink pair can reproduce; The pink pair can reproduce; The pink and yellow pairs can reproduce; The pink, yellow and blue pairs can reproduce; The pink, yellow, blue, green, and dark green pairs can reproduce; The pattern is extended up to Year 4 (Source: Field study)

month, making explicit the generational structure of the model. This tree is subsequently transformed into an iconic representation using tally marks, which allows the PST to progressively abstract from the concrete model while preserving the quantitative relationships. This transition illustrates a representational transformation that supports the coordination between material and iconic registers, providing evidence of an alternate representation connection (AR) grounded in the modelling process.

Building on these representations, as illustrated in **Figure 2a**, the PST constructs the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89…) through an iterative process based on the addition of consecutive terms. **Figure 2** also shows how the PST calculates the population of rabbits month by month by accumulating quantities, initially through counting strategies and progressively through additive procedures. This progression indicates an emerging understanding of the sequence as a process of accumulation over time, consolidating the transition from iconic to numerical representation. Moreover, it can be seen that the PST transitions from local counting procedures to the recognition of a recursive pattern, explicitly applying the rule that states that each term is obtained as the sum of the two previous ones (e.g., 1+1=2, 1+2=3, 2+3=5). This recursive strategy is extended to larger values (up to 377 and 610), suggesting a consolidation of the growth pattern and an emerging generalisation of the recursive structure of the sequence.

The PST solution also combines pictorial and numerical representations. At the initial stages, quantities are represented through tally-like marks grouped in pairs; these are later converted into numerical values. This coordination between representation systems provides evidence of an alternate representation connection (AR) between a concrete depiction of the rabbit population and its numerical expression.

In addition, the PST establishes relationships within the numerical sequence itself by linking consecutive terms through arithmetic operations (e.g., 13+21=34), thereby making explicit the internal organisation of the sequence and establishing an equivalent representation (ER) connection.

A further relevant feature is the explicit reference to the maturity rule, according to which a rabbit pair requires two months before reproducing. This validation links the numerical pattern to the modelling assumptions of the activity through a justification connection (J) providing a basis for interpreting the recursive relation in terms of the context. **Table 2** summarises the mathematical connections identified in this PST production.

Table 2. Mathematical connections identified in the analysed PST production

Type of connection	Category	Definition	Description
P	P1. Population calculation	Total population calculated as the sum of previous terms	The student calculates the population month by month by adding consecutive quantities, treating the sequence as an accumulative process.
EP	P1, P3. Transition from counting to recursive strategies	Initial counting procedures followed by the formulation of a recursive rule	The student shifts from procedural enumeration to recognising the recursive structure governing the sequence (1, 1, 1+1=2, 1+2=3, 2+3=5…) and extends it to larger values (up to 377 and 610), showing consolidation of the growth rule.
AR	R1, R3. Manipulative to iconic	Rabbit cards to rabbit diagrams as a tally mark	The student represents the rabbit population using manipulatives (rabbit cards) and later translates these quantities into iconic representations through tally marks, preserving the structure of the quantities while changing the semiotic register.
AR	R3, R4. Iconic to numerical sequence	Rabbit diagrams as a tally mark to numerical representation	The student represents the rabbit population month by month using tally marks grouped in pairs and later expresses the same quantities numerically.
ER	R4. Numerical sequence relations	Relations between consecutive terms within the same representation register	The student connects elements of the numerical sequence through arithmetic operations (e.g., 13+21=34), establishing relationships within the same symbolic register.
J	J3. Maturity rule	A new rabbit pair takes two months to reproduce	The student justifies the population growth based on the pictorial representation by referring to the maturity rule: a new pair takes two months before reproducing.

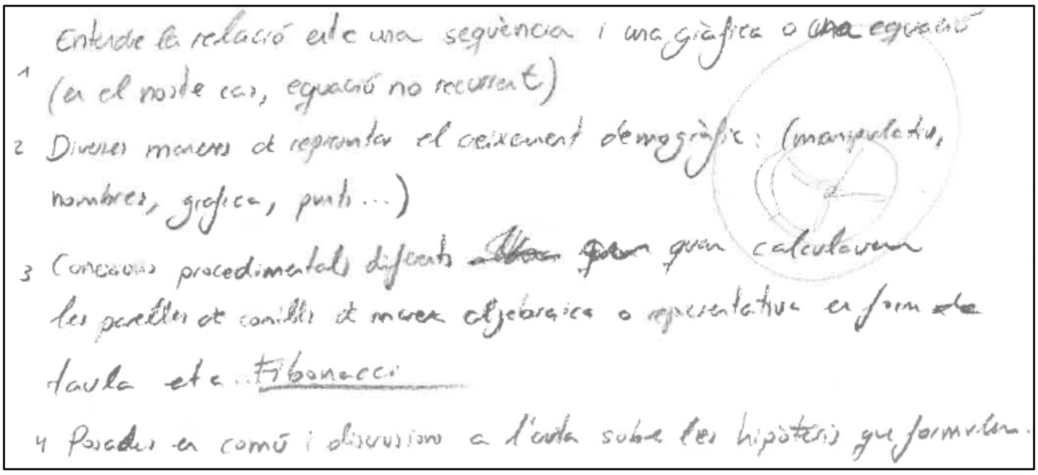

1 Entendre la relació entre una seqüència i una gràfica o ~~una~~ equació (en el nostre cas, equació no recurrent)
2 Diverses maneres de representar el creixement demogràfic: (manipulatius, nombres, gràfica, punts...)
3 Connexions procedimentals diferents ~~...~~ per quan calculaven les parelles de conills de manera algebraica o representativa en forma de taula etc... Fibonacci
4 Posada en comú i discussió a l'aula sobre les hipòtesis que formulen.

Figure 3. Excerpt of a PST written production from the reflection activity on connection identification (A9a). The original response is presented in Catalan. English translation of the handwritten text: Understanding the relationship between a sequence and a graph or an equation (in our case, a non-recurrent equation); Different ways of representing population growth (manipulatives, numbers, graphs, points, etc.); Different procedural connections for calculating the rabbit population using algebraic or tabular representations (Fibonacci); Whole-class sharing and discussion of the hypotheses formulated (Source: Field study)

Procedure of analysis of Activity 9

The sequence concluded with Activity 9, designed as a final reflective activity. This activity was separated in two main subactivities. In Activity A9a, the analysis focused on how PSTs identified, classified, and interpreted the mathematical connections present in their own productions and in the whole-class discussion.

To illustrate the application of the analytical framework, **Figure 3** presents a fragment of a PST's written production that was analysed following the criteria described above.

First, the PST states that "Understanding the relationship between a sequence and a graph or an equation". This statement was coded as an alternative representation connection (AR), as it establishes a relation between different semiotic registers (numerical sequence, graphical representation, and algebraic expression).

Second, the PST refers to "manipulatives, numbers, graphs, points…", indicating the use of multiple representations to describe the same phenomenon. This fragment was coded as a set of representational links (AR), where different registers are associated with the same mathematical object. Each representation was considered as an object within the analysis, and the relation between them was identified through their joint mention in the context of modelling population growth.

Third, the PST mentions the use of "using algebraic or tabular representations (Fibonacci)". This statement was coded as a procedural connection, linking the concept of population growth with procedures for generating the sequence (e.g., recursive or tabular approaches). The objects involved include the mathematical concept (sequence) and the procedures used to construct it, with the relation defined by their functional use in the modelling process.

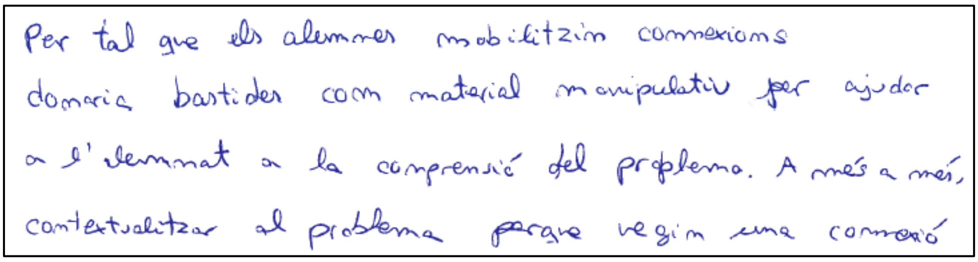

Per tal que els alumnes mobilitzin connexions
donaria bastides com material manipulatiu per ajudar
a l'alumnat a la comprensió del problema. A més a més,
contextualitzar el problema perquè vegin una connexió

Figure 4. Excerpt of a PST written production from the reflection activity on proposed teaching strategies (A9b). The original response is presented in Catalan. English translation of the handwritten text: To encourage students to establish mathematical connections, I would provide scaffolding, such as manipulative materials, to help them understand the problem. In addition, I would contextualise the problem so that they could recognise the connections (Source: Field study)

Additionally, the PST refers to the use of manipulatives to construct a population growth structure that is later represented through diagrams or tally marks. This fragment was coded as a representational conversion (AR), involving a transition from a manipulative register to an iconic register. The relation between objects was identified through the sequential use of different representations to express the same quantities.

Overall, this example illustrates how the analytical framework allows the identification of different types of connections (representational, procedural, and argumentative) within a single written production, based on explicit relations between mathematical objects as expressed in the student's discourse.

In Activity A9b, the analysis focused on the instructional strategies proposed to foster mathematical connections in primary classrooms. These responses were coded according to the type of strategy or didactical resource proposed.

To analyse PSTs' proposals for fostering the emergence of mathematical connections, a coding scheme was developed focusing on the instructional strategies described in their written productions. The analysis followed a qualitative, interpretative approach grounded in prior research on classroom practices and connections, taking as unit of analysis each excerpt in which a strategy to promote connections was explicitly articulated. The initial coding framework was informed by the literature on mathematical connections and mathematics teacher education presented in the theoretical framework of this study (De Gamboa et al., 2021). During the analysis, this preliminary framework was iteratively refined through constant comparison with the participants' written productions, allowing categories to be adjusted and consolidated in response to patterns emerging from the data. Through an inductive–deductive process, six categories were identified: contextualization strategies (CS), which situate mathematics in meaningful contexts; manipulative-based strategies (MS), based on the use of tangible artefacts; teacher questioning strategies (TQS), aimed at eliciting and deepening relational reasoning; digital and simulation strategies (DS), which use technological tools to explore connections; role-play and embodied strategies (RS), where students act out mathematical situations through roles, gestures, or physical interaction to make abstract relationships more tangible; and other strategies (OS), capturing proposals not fitting previous categories.

The coding process was iterative and based on constant comparison to ensure internal coherence and conceptual distinction between categories. The resulting coding scheme was progressively refined through collaborative review and discussion, ensuring the consistency and credibility of the analysis.

To illustrate the analytical procedure used to identify strategies for promoting connections, we present an example of a PST's written production (see **Figure 4**). The excerpt states: "To help students make connections, I would provide scaffolding through manipulatives. Moreover, I would contextualize the problem so that they can see a connection."

Following our analytical framework, the unit of analysis was segmented into meaningful statements referring to instructional actions. Each segment was coded according to the type of strategy proposed and the kind of connections it may potentially promote.

First, the reference to "provide scaffolding, such as manipulative materials" was coded as a manipulative-based strategies (MS). This code captures instructional actions aimed at supporting the coordination between different registers of representation. In this case, the use of manipulatives was interpreted as a resource to facilitate transitions between concrete, iconic, and symbolic representations, thus enabling the emergence of representational connections.

Second, the statement "contextualise the problem" was coded as a contextualization strategy (CS). This category includes instructional actions that establish links between mathematical content and extra-mathematical situations. The purpose of this strategy is to promote the recognition of connections between mathematical ideas and real-world contexts, thereby supporting meaning-making processes.

Finally, a cross-analysis was then conducted to identify broader patterns in the PSTs' responses, and to examine the relationship between their established mathematical connections, their interpretation of mathematical connections and the instructional decisions they proposed.

Table 3. Distribution of mathematical connections identified in the modelling activity

Type of connection	Description	n
P	Procedural construction of the sequence (tables, iterative counting, numerical generation)	17
AR	Coordination of multiple representations (iconic, tabular, symbolic)	15
ER	Same representational register with different objects	3
J	Explicit recursive justification	5
EP	Transition from counting procedures to recursive formulation	3

Table 4. Distribution of representations mobilized in the modelling activity

Representations connected	Connection type	Description	n
R2/R3 to R5 (Iconic to Tabular)	AR	Rabbit diagrams or tree structures to Months-population table	5
R2/R3 to R4 (Iconic to Numerical)	AR	Rabbit diagrams or tree structures to Numerical growth sequence	5
R1 to R2/R3 (Manipulative to Iconic)	AR	Rabbit cards to rabbit diagrams or tree structures	3
R4 (Numerical to Numerical)	ER	Number of rabbits per month and monthly increase in the rabbit population	3
R4 to R6 (Numerical to algebraic)	AR	Numerical growth sequence to recursive formula	2

RESULTS

Mathematical Connections Established in the Modelling Activity

The first stage of the analysis focused on the mathematical connections established by PSTs while solving the population modelling task (Activity 2). The analysis began with the identification of the mathematical objects mobilised in students' written productions, including representations, procedures, and justifications. These objects constituted the basis for identifying the mathematical connections established between them.

Students approached the modelling task using a variety of representations and procedures. Most productions involved the construction of a month–population table, which enabled students to track the evolution of the number of rabbit pairs over time. This representation was frequently combined with iterative calculations, analysis of increments between consecutive terms, and, in some cases, attempts to formulate general rules describing population growth. Taken together, these elements provide insight into how PSTs progressively move from local calculations based on counting towards a more general understanding of the recursive structure of the sequence.

Table 3 summarises the distribution of the main types of connections identified in the full corpus of data. The results show a clear predominance of procedural connections (n=17), followed by representational connections across registers (AR, n = 15). In contrast, justification-based connections (J, n=5), connections within the same representation system (ER, n=3), and transitions from counting procedures to recursive formulation (EP, n=3) appear less frequently.

Three main features emerge from these results. First, the modelling process is predominantly driven by procedural strategies. Most students generate values through iterative processes such as constructing tables or extending numerical sequences. However, these strategies do not always involve explicit recognition of the recursive structure governing population growth. In this sense, procedural connections reflect an operational engagement with the task, centred on calculation rather than on structural understanding.

Second, a substantial proportion of students establish connections between multiple representations. Representational connections across registers (AR) frequently involve transitions between iconic, tabular, and numerical representations. In particular, students often move from rabbit diagrams or tree structures to tabular representations (n = 5) and numerical sequences (n = 5), as well as from manipulative materials to iconic representations (n = 3). These results suggest that students are not only calculating values but also coordinating different ways of representing the situation, with iconic representations often playing a mediating role between the contextual interpretation of the problem and its mathematical organisation.

Third, only a smaller group of students articulates justificative connections. These cases reflect a qualitatively different form of reasoning, in which students explicitly identify and justify the recursive relationship underlying the sequence. Similarly, transitions from counting procedures to recursive formulation (EP) are scarce (n = 3), indicating that only a limited number of PSTs move from iterative strategies towards a more general and structural understanding of the model.

A more fine-grained analysis of representational connections further clarifies how PSTs coordinate different representation systems (**Table 4**). Within this category, most connections correspond to changes of register (AR), while fewer involve relations within the same register (ER). The latter are mainly observed in numerical contexts (n = 3), where students relate the total number of rabbits to the monthly increase in the population. In addition, a small number of connections link numerical and algebraic representations (n = 2), reflecting initial attempts to express the recursive structure symbolically. Although limited, these cases are particularly relevant as they indicate emerging processes of generalisation and formalisation.

Taken together, these results suggest that Activity 2 primarily gives rise to procedural and representational connections. While some productions show evidence of PSTs' structural recognition of the recursive model, such cases are less frequent than those centred on calculation and representational coordination. This distribution points to a partial development of deeper forms of mathematical reasoning, highlighting the need to further support the transition from procedural activity to explicit structural understanding within modelling contexts.

Table 5. Distribution of mathematical connections identified by PSTs in A9a

Type of connection	Description	n (%)
Representations	Connections between different representations of the same mathematical object, either equivalent (within the same register) or alternative (across registers).	29 (60%)
Procedures	Connections related to the use, comparison or coordination of procedures and solution strategies for addressing a task.	15 (32%)
Arguments	Connections based on reasoning processes, including justification, generalisation or validation of relationships between mathematical objects.	4 (8%)

PST Analysis of Mathematical Connections and Proposed Teaching Strategies

Identification of mathematical connections

In Activity A9a, PSTs were asked to identify in writing the mathematical connections that had emerged during the modelling sequence. This second stage of the analysis focused on PSTs' ability to identify mathematical connections in previously produced modelling solutions. In this phase, the emphasis shifts from the construction of connections to their recognition and explicit articulation, providing insight into PSTs' interpretative and professional noticing skills.

Table 5 presents the distribution of the types of connections identified by PSTs. A total of 48 connections were reported, with a clear predominance of representational connections (n = 29; 60%), followed by procedural connections (n = 15; 32%), and a much smaller proportion of argumentative connections (n = 4; 8%).

These results reveal a strong imbalance in the types of connections that PSTs are able to identify. Representational connections are not only the most frequent but also significantly overrepresented compared to other types. In particular, the most identified connections correspond to alternative representational (AR) connections involving iconic representations, such as diagrams or visual structures of the rabbit population. These representations appear to play a central role in PSTs' recognition processes, likely due to their visual and accessible nature, which makes relationships more immediately observable.

In contrast, procedural and especially argumentative connections are identified less frequently. Procedural connections, although present, require recognising underlying processes or strategies (e.g., iterative construction or recursive reasoning), which are less directly visible than representations. Argumentative connections, which involve justification, explanation, or validation of relationships, are particularly scarce. This suggests that PSTs experience greater difficulty in identifying connections that rely on higher levels of mathematical reasoning.

A key finding emerging from this analysis is that, although PSTs are generally able to correctly identify the mathematical objects involved (e.g., representations, quantities, procedures), they often struggle to establish explicit connections between them. In many cases, their analyses point to the presence of related elements without fully articulating the nature of the relationship that links them.

Overall, PSTs' productions show a predominance of descriptive and representational connections, with a relatively limited explicit articulation of the underlying relationships that sustain them. While multiple potential links are suggested, only a few are fully developed in terms of explicit relations, justification, or coordination. This indicates that PSTs can recognise the existence of connections but encounter difficulties in operationalising them mathematically and didactically, particularly when moving from identification to explicit relational reasoning.

This pattern can be partially explained by the nature of the different types of connections. Representational connections, especially those involving iconic representations, are more perceptually salient and cognitively accessible, which facilitates their identification. By contrast, procedural and especially argumentative connections require participants not only to recognise mathematical objects but also to establish explicit relationships between them through reasoning and justification. Consequently, these connections involve higher cognitive demands and are less immediately visible during reflective analysis, which may explain why they are identified less frequently by PSTs.

Taken together, these results suggest that PSTs' ability to identify mathematical connections is strongly influenced by the level of explicitness and visibility of the relationships involved. While they show competence in recognising connections at a representational level, further support appears necessary to foster the identification of more complex connections, particularly those involving reasoning, justification, and structural understanding.

Proposed teaching strategies

In the final part of the activity (A9b), PSTs were asked to propose teaching strategies that could help primary school students identify and construct mathematical connections during similar modelling activities. Across the cohort, 20 strategies were identified. Similar responses expressed with slightly different wording (for example "teacher asks" and "good questions", or "use of manipulative material" and "rabbit cards") were grouped under the same analytical category.

As shown in **Table 6**, the strategies proposed by PSTs mainly focused on three dimensions: contextualisation, the use of materials, and teacher questioning. Contextualisation was one of the most frequently mentioned strategies. Several PSTs suggested maintaining the modelling context throughout the activity so that students could continuously relate the mathematical work to the real-world situation being analysed. For example, PSTs proposed returning to the context of rabbit population growth when interpreting results or discussing the meaning of the mathematical model.

Table 6. Teaching strategies proposed by PSTs to promote mathematical connections

Strategy	Description	n
CS	Introduces real-world or meaningful contexts to situate mathematical ideas and promote connections beyond the purely formal domain.	5
MS	Uses concrete or physical materials to support the construction and transformation of representations, facilitating the emergence of connections.	5
TQS	Relies on purposeful teacher questioning to elicit, guide, and make explicit relationships between mathematical objects or ideas.	4
DS	Incorporates digital tools or simulations to explore, visualize, or dynamically represent mathematical relationships.	3
RS	Engages students in role-playing or embodied actions to model situations and support the understanding of relationships.	2
OS	Includes additional or hybrid strategies not captured in previous categories but aimed at fostering connections.	2

Another frequently proposed strategy was the use of manipulative materials. PSTs suggested employing concrete resources that allow students to physically represent the evolution of the rabbit population. These materials were seen to help students visualise relationships, explore patterns, and build links between concrete representations and more formal mathematical representations.

Teacher questioning also appeared repeatedly among the proposed strategies. PSTs emphasised the importance of posing meaningful questions and encouraging students to explain their reasoning to guide reflection and discussion. These responses indicate that PSTs recognised questioning as a key instructional tool for making students' reasoning explicit and supporting the construction of connections between mathematical ideas.

Other strategies involved the use of technological and informational resources. Some PSTs suggested using simulators or digital tools to explore the evolution of the rabbit population dynamically, while others proposed incorporating informational documents that could provide additional background about the phenomenon being modelled. These resources were described as ways of enriching the learning environment and helping students connect the mathematical model with the underlying scientific context.

Finally, a small number of PSTs proposed participatory strategies such as role play in which students could adopt different perspectives or verbally describe the situation being modelled. These proposals suggest that some PSTs considered discussion and storytelling as potential ways of helping students articulate and interpret the mathematical relationships involved in the activity.

Taken together, these results show that PSTs mainly emphasised strategies combining contextualisation, concrete materials, and guided discussion to promote mathematical connections. At the same time, the relatively limited diversity of strategies suggests that PSTs relied on a small set of familiar didactical resources when considering how to support the development of mathematical connections in classroom practice.

Cross-Task Analysis

Figure 5 presents a bubble plot showing the relationship between the number of mathematical connections established in A2 and the number subsequently identified in A9a, while incorporating the number of teaching strategies proposed in A9b as a third variable encoded through dot size and colour intensity, where larger and brighter (yellow-green) dots indicate a higher number of strategies proposed and smaller darker (purple) dots indicate fewer. The plot reveals a strong positive linear association between connections established and connections identified ($r = 0.82$, $p < .001$, $R^2 = 0.67$), indicating that PSTs who evidenced greater capacity to establish mathematical connections during the modelling task also tended to identify a higher number of connections in the reflective phase. Within the analysed sample, the regression line ($y = 0.92x + 1.11$) suggests a positive association between the number of connections established and those subsequently identified. Although the estimated slope is close to one, this pattern should be interpreted as an exploratory trend rather than as a generalisable relationship. Notably, the majority of data points lie above the identity line ($y = x$), indicating that most PSTs identified more connections in A9a than they had explicitly established in A2. This pattern may reflect the scaffolding effect of A9a, in which PST were presented with a set of connections, supporting recognition beyond spontaneous production. PST3 and PST1 are particularly illustrative of this tendency, since both are positioned at the upper right of the figure and lie clearly above the identity line, having established 5 and 4 connections respectively while identifying 7 and 6.

By contrast, the colour and size of the dots, encoding A9b, do not follow a similarly ordered pattern along the regression line. If strategies proposed were strongly associated with connection-making performance, dot colour and size would shift gradually and consistently from small and dark on the left of the figure to large and bright on the right. This gradient is not observed. Instead, colour and size are distributed irregularly across the figure regardless of position on the axes. For example, PST11, positioned centrally at (3, 5), displays the largest and brightest dot in the entire scatter plot (A9b = 4), while PST5, positioned at the high end of both axes at (5, 5), is represented by one of the smallest and darkest dots (A9b = 0). Similarly, PST13, among the lowest scorers on both A2 and A9a, proposed three strategies, represented by a large green dot at the lower left, while PST2 occupies virtually the same position yet proposed only one strategy, appearing as a small dark dot beside it. Pearson correlation coefficients computed separately for each variable pair confirmed this absence of significant association involving A9b (A9a–A9b: $r = 0.36$, $p = 0.185$; A2–A9b: $r = 0.20$, $p = 0.466$). This visual irregularity and the absence of statistically significant correlations suggest that, within the analysed sample, the capacity to translate reflective analysis of mathematical connections into concrete didactical proposals does not appear to be directly associated with the number of connections established or identified. However, given the exploratory nature of the study and the limited sample size, this pattern should be interpreted with caution and warrants further investigation in studies involving larger samples.

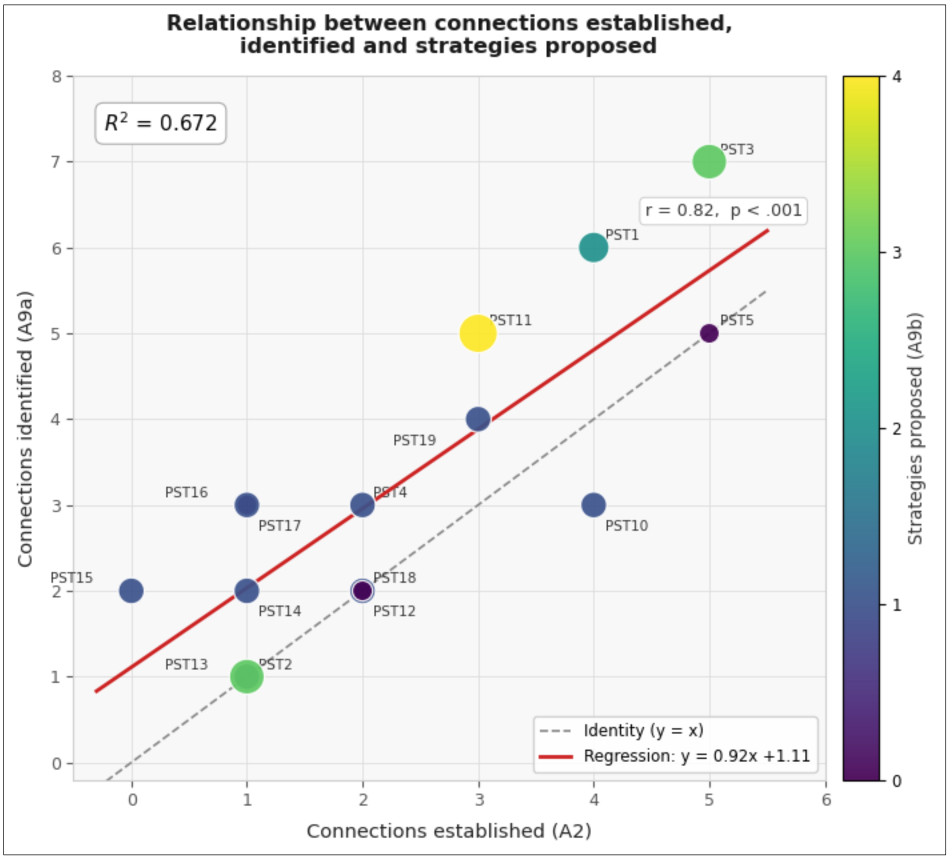


Figure 5. Correspondence between the connections established (x-axis) in the modelling activity and those identified (y-axis) during the professional reflection activity. The dots represent the strategies proposed. The red line shows the linear regression, and the dotted line represents the identity line (y = x) (Source: Authors' own elaboration)

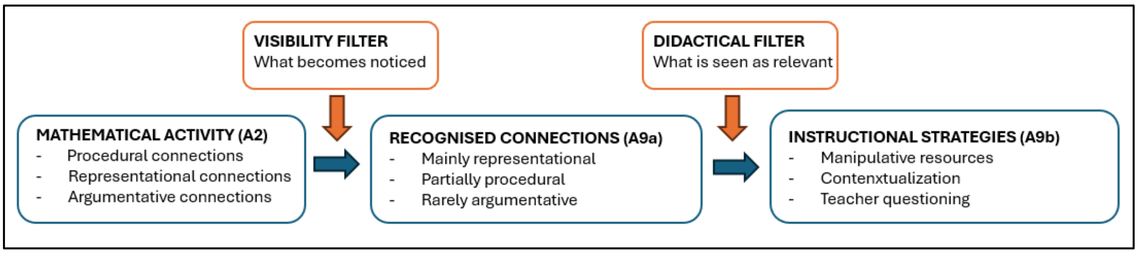


Figure 6. Interpretative synthesis of the transition from mathematical activity to instructional decisions-making (Source: Authors' own elaboration)

Taken together, these findings support the tentative hypothesis that the transition from mathematical activity to instructional decision-making may be mediated by processes of selection and interpretation, influenced by the visibility of mathematical connections and their perceived didactical relevance. Given the exploratory nature of the study and the limited sample size, this interpretative framework should be considered provisional and examined in future research involving larger cohorts (**Figure 6**).

DISCUSSION

This study shows that the transition from establishing mathematical connections during modelling to identifying, interpreting, and using them to inform instructional decisions is selective rather than direct. The results provide insight into how PSTs engage with mathematical connections in a STEM-based modelling context, both during activity resolution and through subsequent reflection. Overall, the findings reveal a clear distinction between the connections enacted during mathematical activity and those later identified and interpreted from a professional perspective.

During the modelling activity, PSTs predominantly relied on procedural strategies, such as iterative calculations and the construction of tables, to generate the sequence of values. These processes were often accompanied by the coordination of multiple representations, particularly iconic and tabular forms, which supported the interpretation of the modelling situation. However, only a limited number of participants articulated the recursive structure underlying the population model. This result is consistent with previous research showing that, although modelling activities can promote rich mathematical activity, the transition from procedural work to structural understanding remains cognitively demanding (Blum & Niss, 1991; Lehrer & Schauble, 2007).

In contrast, the reflective analysis carried out in Activity 9a shows a shift in the types of connections identified by PST. Representational connections were the most frequently recognised, whereas procedural and especially argumentative connections were less prominent. This asymmetry suggests that the visibility of mathematical objects plays a key role in shaping reflection. Connections externalised through representations, such as diagrams or tables, are more easily identified, whereas those embedded in procedures or underlying structures tend to remain implicit. This finding aligns with studies emphasising the role of semiotic representations in making mathematical relationships accessible (Duval, 2006).

A further relevant result concerns the relationship between mathematical activity and reflection. The analysis reveals a strong correspondence between the connections enacted during the modelling activity and those later identified by PSTs. This finding indicates that reflection tends to be bounded by prior activity, with PSTs predominantly identifying connections grounded in their prior modelling activity, although the reflective phase enabled the recognition of additional connections beyond those explicitly established. This finding highlights a limitation of reflection when less visible connections are not explicitly scaffolded and suggests the need for instructional interventions that explicitly support the identification of less visible or more complex connections.

At the same time, the results show that this correspondence does not extend to the instructional dimension. No clear relationship was observed between the number of connections enacted or identified and the diversity of teaching strategies proposed by PST. This suggests that the ability to analyse mathematical connections and the ability to translate them into didactical actions are not directly linked. Instead, the generation of instructional strategies appears to depend on additional dimensions of professional knowledge, including didactical repertoire and beliefs about teaching and learning.

Taken together, these findings point to a selective and mediated transition from mathematical activity to professional reflection. A key contribution of the study is to conceptualise mathematical connections not only as elements of mathematical activity but also as objects of professional reflection, whose recognition is shaped by their visibility and perceived didactical relevance. This transition appears to be influenced by at least two factors: the visibility of mathematical connections and their perceived didactical relevance. As a result, certain types of connections, particularly those related to representations, are prioritised both in reflection and in instructional proposals, whereas structural connections tend to remain underexplored.

From a teacher education perspective, these results highlight the importance of designing learning environments that explicitly address the analysis of mathematical connections as an object of reflection. It is necessary to support PSTs in recognising less visible connections, such as those related to underlying mathematical structures, and in translating these connections into meaningful instructional strategies. This suggests that modelling activities, when combined with structured analytical activities, can play a key role in bridging the gap between mathematical activity and professional knowledge in STEM teacher education.

CONCLUSIONS

This study examined how PSTs establish and analyse mathematical connections in a STEM-based population modelling activity, and how this analysis informs their instructional decision-making. The results show that PSTs engage in rich mathematical activity during modelling, primarily through procedural and representational connections, while only a limited number explicitly recognise the underlying recursive structure.

The findings indicate that the subsequent analysis of connections is selective and influenced by the visibility of mathematical objects. Representational connections are more frequently identified during reflection, whereas procedural and especially argumentative connections tend to remain less explicit, likely because they require a deeper analysis of mathematical structure and explicit justification. In addition, the number of connections identified by PSTs is closely related to those previously enacted, suggesting that reflection is largely bounded by prior activity.

No clear relationship was found between the connections established or identified and the diversity of teaching strategies proposed. This points to a critical gap between mathematical activity and didactical action, suggesting that the knowledge of mathematical connections does not automatically translate into the ability to design instructional strategies that promote them.

This result is consistent with previous research on teacher knowledge and professional learning, which shows that recognising mathematically relevant aspects of students' activity does not necessarily entail the ability to interpret them didactically or to make appropriate instructional decisions (Fernández et al., 2023). Recent studies highlight that one of the main difficulties in the professional development of prospective teachers lies precisely in interpreting students' mathematical thinking and linking this interpretation to classroom decision-making (Aoki, 2023; Creswell & Creswell, 2017; Moreno et al., 2025). From this perspective, the ability to analyse mathematical connections can be understood as a necessary but insufficient condition for effective teaching, unless it is explicitly connected to instructional action. More broadly, this finding reflects the lack of automatic coordination between different domains of teacher knowledge and the well-documented gap between theoretical constructs and their enactment in classroom practice (Dreher et al., 2016; Leikin & Levav-Waynberg, 2007).

Overall, the study contributes to research on mathematical connections by conceptualising them as both elements of mathematical activity and objects of professional reflection in teacher education. From a practical perspective, the results underscore the need to design teacher education experiences that explicitly support not only the identification of less visible connections, but also their interpretation and transformation into meaningful instructional strategies.

Given the relatively small sample and the exploratory nature of the quantitative analyses, these findings should be interpreted with appropriate caution. Future research involving larger and more diverse samples could examine the robustness and transferability of the relationships identified in this study, as well as explore how different forms of scaffolding support the development of this integrated professional competence and how the relationship between analysing mathematical activity and making instructional decisions evolves over time in teacher education programs.

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REFERENCES

Aoki, M. (2023). Mathematics in a Japanese overseas school: Connecting classroom study and curriculum. *Asian Journal for Mathematics Education*, *2*(2), 204-219. https://doi.org/10.1177/27527263231187804

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy*. King's College.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2009). The array representation and primary children's understanding in multiplication. *Educational Studies in Mathematics*, *70*, 217-241. https://doi.org/10.1007/s10649-008-9145-1

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects. *Educational Studies in Mathematics*, *22*, 37-68. https://doi.org/10.1007/BF00302716

Bonett, D. G., & Wright, T. A. (2000). Sample size requirements for estimating Pearson, Kendall and Spearman correlations. *Psychometrika*, *65*, 23-28. https://doi.org/10.1007/BF02294183

Businskas, A. M. (2008). *Conversations about connections: How secondary mathematics teachers conceptualize and contend with mathematical connections* [Doctoral dissertation, Simon Fraser University].

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, *20*(3), 236-253. https://doi.org/10.1080/14794802.2018.1479981

Caviedes, S., de Gamboa, G., & Badillo, E. (2021). Mathematical objects that configure partial area meanings. *International Journal of Mathematical Education in Science and Technology, 54*(6), 1092-1111. https://doi.org/10.1080/0020739X.2021.1991019

Climent, N., Contreras, L. C., Montes, M., & Ribeiro, M. (2024). The MTSK model as a tool for designing tasks for teacher education. *ZDM–Mathematics Education*, *56*, 1123-1135. https://doi.org/10.1007/s11858-024-01605-8

Cohen, L., Manion, L., & Morrison, K. (2017). *Research methods in education* (8th ed.). Routledge.

Couso, D., Mora, L., & Simarro, C. (2021). De las mates como instrumento a las mates como práctica [From math as an instrument to math as a practice]. *Uno: Revista de Didáctica de las Matemáticas, 93*, 8-14.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). SAGE.

De Gamboa, G., Badillo, E., Couso, D., & Márquez, C. (2021). Connecting mathematics and science in primary school STEM education: Modeling the population growth of species. *Mathematics*, 9, Article 2496. https://doi.org/10.3390/math9192496

De Gamboa, G., Badillo, E., & Font, V. (2023). Meaning and structure of mathematical connections in the classroom. *Canadian Journal of Science, Mathematics and Technology Education*, 23, 241-261. https://doi.org/10.1007/s42330-023-00281-2

Dreher, A., Kuntze, S., & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *International Journal of Science and Mathematics Education*, *14*, 363-382. https://doi.org/10.1007/s10763-015-9633-6

Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton (Ed.), *The teaching and learning of mathematics at university level* (pp. 273-280). Springer. https://doi.org/10.1007/0-306-47231-7_25

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*, 103-131. https://doi.org/10.1007/s10649-006-0400-z

English, L. D. (2016). STEM education K–12: Perspectives on integration. *International Journal of STEM Education*, *3*, Article 3. https://doi.org/10.1186/s40594-016-0036-1

Fernández, C., Ivars, P., & Llinares, S. (2023). El desarrollo de la competencia mirar profesionalmente el pensamiento matemático de los estudiantes durante los períodos de práctica. *Revista Interuniversitaria de Formación del Profesorado*, *98*(38.2), 127-146. https://doi.org/10.47553/rifop.v98i37.2.99296

Frykholm, J. A., & Glasson, G. E. (2005). Connecting science and mathematics instruction. *School Science and Mathematics*, *105*, 127-141. https://doi.org/10.1111/j.1949-8594.2005.tb18047.x

Fyfe, E. R., Alibali, M. W., & Nathan, M. J. (2017). The promise and pitfalls of making connections in mathematics. In E. Galindo & J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 717-724). Hoosier Association of Mathematics Teacher Educators.

García-García, J., & Dolores-Flores, C. (2018). Intra-mathematical connections made by high school students in performing calculus tasks. *International Journal of Mathematical Education in Science and Technology*, *49*(2), 227-252. https://doi.org/10.1080/0020739X.2017.1355994

Gilbert, J. K., & Justi, R. (2016). *Modelling-based teaching in science education*. Springer. https://doi.org/10.1007/978-3-319-29039-3

Godino, J. D., Batanero, C., & Font, V. (2019). The onto-semiotic approach: Implications for the prescriptive character of didactics. *For the Learning of Mathematics*, *39*, 37-42. https://www.jstor.org/stable/26742011

Gravett, E. (2010). *The rabbit problem*. Macmillan Children's Books.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Lehrer, R., & Schauble, L. (2007). Scientific thinking and science literacy. In W. Damon, R. M. Lerner, K. A. Renninger, & I. E. Siegel (Eds.), *Handbook of child psychology*. Wiley. https://doi.org/10.1002/9780470147658.chpsy0405

Leikin, R., & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. *Educational Studies in Mathematics*, *66*, 349-371. https://doi.org/10.1007/s10649-006-9071-z

León del Carmen, A., León del Carmen, W., García-García, J., & Salgado-Beltrán, G. (2024). Mathematical connections made by preservice mathematics teachers when solving tasks about systems of linear equations. *International Electronic Journal of Mathematics Education*, *19*(4), Article em0799. https://doi.org/10.29333/iejme/15590

Li, Y., & Schoenfeld, A. H. (2019). Problematizing teaching and learning mathematics as "given" in STEM education. *International Journal of STEM Education*, *6*, Article 44. https://doi.org/10.1186/s40594-019-0197-9

Moreno, M., Valls, J., Sánchez-Matamoros, G., & Aráuz, D. F. (2025). Desarrollo profesional de futuros docentes de matemáticas de educación secundaria. *Enseñanza de las Ciencias*, *43*(2), 61-80. https://doi.org/10.5565/rev/ensciencias.6320

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples. *Journal of Educational Psychology*, *101*, 836-852. https://doi.org/10.1037/a0016026

Rodríguez-Nieto, C. A. (2021). *Analysis of mathematical connections in the teaching and learning of the derivative* [Doctoral dissertation, Universidad Autónoma de Guerrero].

Sweller, J., van Merriënboer, J. J. G., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, *10*, 251-296. https://doi.org/10.1023/A:1022193728205

Tashakkori, A., & Teddlie, C. (2010). *SAGE handbook of mixed methods in social & behavioral research* (2nd ed.). SAGE. https://doi.org/10.4135/9781506335193

Wilensky, U., & Reisman, K. (2006). Thinking like a wolf, a sheep, or a firefly: Learning biology through constructing and testing computational theories—An embodied modeling approach. *Cognition and Instruction*, *24*, 171-209. https://doi.org/10.1207/s1532690xci2402_1