Mathematical Communication and Its Relation to the Frequency of Manipulative Use

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Many studies on manipulatives describe communication in mathematics as a component for properly implementing manipulatives in the classroom. However, no empirical research is available to support this relationship. Secondary analysis of data collected by the National Center for Educational Statistics from the Early Childhood Longitudinal Study was used to examine whether a relationship between students' manipulative use and communication in mathematics learning exists. Correlational analyses found a significant relationship between students' verbal and written communication and manipulative use.

Keywords: manipulatives, mathematical writing, mathematical discussion, mathematics communication

"Communication is an essential part of mathematics and mathematics education" (National Council of Teachers of Mathematics [NCTM], 2000, p. 60). Both writing and discussion are seen as integral parts of communication that promote deeper understanding of concepts (Cramer & Karnowski, 1995; NCTM, 2000). Writing is seen as a way for individuals to reflect on or explain in detail certain mathematical ideas (Silver, Kilpatrick, & Schlesinger, 1990; Whitin, 2004). It helps students to articulate strategies, therefore increasing their procedural knowledge and producing cognitive benefits in general (Jordak & Abu Zein, 1998; Kroll & Halaby, 1997). Discussion between students is another avenue in deepening understanding of concepts through social interaction. It enables students to reflect upon concepts through interactions with others engaged in the same activity as well as allow students to become familiar with certain ways of describing mathematics while they are doing mathematics—therefore providing students opportunities to become more knowledgeable (Lee, 2006).

Manipulative use is also seen as a way of increasing mathematical understanding. Manipulatives are typically concrete objects used to represent mathematical concepts (Bruner, 1973; Uttal, Scudder, & DeLouche, 1997). Many of the benefits associated with manipulative use can be found in Bruner's (1973) investigations with young children learning about the distributive and commutative properties. Bruner (1973) found through a series of detailed observations of children that concrete materials can be used to develop deep understandings of certain mathematical concepts. The process described involves transitioning from manipulating concrete materials to creating images from the student's perception of the concept, and finally to the development or adoption of some form of symbolic notation representing the concept. Throughout his descriptions of using manipulatives to move students from concrete to abstract understanding, Bruner (1973) describes language as an integral part of manipulative use. "Where does the language begin and the manipulation of materials stop? The interplay is continuous" (Bruner, 1973, p. 431-432).

Since Bruner's (1973) descriptions of the interconnectedness of language and manipulative use, much of the literature focusing on manipulatives appears to take the relationship of manipulative and language use as an assumed relationship (e.g., Cramer & Karnowski, 1995; Moch, 2001; Moyer, 2001). However, studies focusing specifically on the relationship could not be found by the authors. Practical implementation of Bruner's work on development of abstract mathematical concepts through use of concrete materials is evident in publications such as the *Principles and Standards for School Mathematics* (NCTM, 2000). Additionally, there is empirical evidence supporting the cognitive benefits of manipulative use (e.g. Moch, 2001; Warrington & Kamii, 1998), mathematical writing (e.g. Jurdak & Abu Zein, 1998; Kenney, 2005), and mathematical discussion (e.g. Mercer & Sams, 2006; Hiebert & Wearne, 1993). Since literature on manipulative use (e.g. Moch, 2001; Moyer, 2001; Stein & Bovalino, 2001) illustrates written and verbal communication as part of manipulative use, and the converse has also been discussed (Kroll & Halaby, 1997; Whitin, 2004), an interactive relationship between these cognitively benefitting practices would appear to exist. Thus, an examination of such a relationship between each of these aspects of pedagogy would provide empirically documented support for the implementation of such practices. On the contrary, results showing either a negative or no relationship between these behaviors would suggest that many of the assumptions used as the basis for mathematical pedagogy are ill-conceived. It is therefore the primary purpose of this study to investigate the relationship between students' mathematical communication and manipulative use. More specifically, we consider the following research question: Is there a relationship between the frequency that students use manipulatives and the frequency that they write about and discuss mathematics?

Manipulatives

The purpose of using manipulatives in mathematics is to help the learner understand abstract concepts. Successful use of manipulatives occurs when they are used as symbols as opposed to literal representations of what they are (e.g. pattern blocks representing their shapes with no use beyond such representation). For children to gain an understanding using manipulatives, they must identify the mathematical concept being learned with the manipulatives used (Bruner, 1967; Bruner, 1973; Uttal et al., 1997).

Warrington and Kamii (1998) found that students using manipulatives can learn multiplication of fractions before the introduction of an algorithm. Using student-created representations and engaging in class discussion, students were able to find solutions to problems such as "1/3 of 1/3" which is written symbolically as "1/3 x1/3." By learning to think of the problems in this way, students were able to transition to using the symbolic representation more easily than if it had been introduced first as a normal multiplication problem.

When describing different forms of representation, Cramer and Karnowski (1995) identify manipulatives as concrete representations that should be followed by pictorial representations and then verbal and written representations. These last forms of representation are identified as critical for linking informal mathematical knowledge to abstract representations and understandings. Isolating a student's informal understandings formed through manipulative use from talking and writing about the concepts with formal abstract symbols decreases the likelihood of deeper understanding for a student. In this context, language use in both the written and spoken form is what takes the informal understanding gained from manipulative use to a solid formal understanding of the concept (Bruner, 1973; Cramer & Karnowski, 1995).

Communication in Mathematics

Student Discussion

When students talk about mathematics, they use informal language that makes it easier for them to understand the concepts. Typical language in textbooks, or used by teachers, can sometimes act as a barrier for student understanding. Though the language of mathematics, mathematics register, needs to be taught to students, they do not necessarily come to the classroom understanding certain mathematical concepts in that language (Lee, 2006). Even when students do understand mathematics in the formal language, they discuss it informally with each other (Pimm, 1987). By encouraging student discussion about mathematics, students are able to communicate in a language with which they are comfortable, rather than the foreign language of mathematics.

Students asked to communicate their ideas about mathematics to other students are encouraged to find a way to explain and justify their reasoning (Silver et al., 1990). The attempt to put thought into words helps students to structure and clarify their reasoning. Talking about mathematics communicates the concepts to others but also helps communicate the concept(s) to the individual speaking (Pimm, 1987; Silver et al., 1990). Yet students do not have to communicate verbally in order to gain individual benefits of communication. Writing about mathematics also generates benefits to understanding (Jurdak & Abu Zein, 1998; Kenney, 2005).

Writing about Mathematics

Students who write to explain or describe solution strategies experience an improvement in their problem solving skills (Borasi & Rose, 1989). A student's mathematical writing can illustrate their reasoning of a problem or concept (Kenney, 2005). This illustration can be used by the teacher to identify methods for improving understanding, but it can also be used by students in small groups to compare their solution strategies and explanations with each other. Research discussing the use of writing to improve understanding often involves some sort of verbal communication between either the student and teacher or the student and peers. Therefore, writing can be incorporated into discussion to deepen understanding (Kenney, 2005; Whitin, 2004).

Connecting Mathematical Communication and Manipulative Use

During an initial review of the literature, research empirically documenting a link between manipulative use and written or spoken communication was not found—though an article (Cramer & Karnowski, 1995) discussing a connection was found. Cramer and Karnowski (1995) identified manipulatives as a form of representation that interacts with other forms of representation such as real-life contexts, pictures, verbal symbols, and written symbols. Yet Cramer and Karnowski's (1995) descriptions did not give evidence of a connection; only descriptions of one. In reviewing other studies, however, certain trends in the methods of research conducted on manipulative use were identified that linked manipulatives and communication.

In a study conducted by Moyer (2001) on teachers' use of manipulatives in teaching mathematics, aspects of discussion were identified as part of manipulative use. Moyer sought to investigate how and why middle school teachers used manipulatives. The treatment applied in the study included gaining a familiarity with certain manipulatives. However, there was also an emphasis placed on classroom dialogue, which was not identified as a main aspect of the research. Student interaction leading to discussion of the concept was clearly outlined as being an important part of the treatment and the effectiveness of the manipulatives. The results found by Moyer (2001) did briefly mention teacher directed manipulative use and discussion, but did not seem to place as much emphasis on discussion as was placed on manipulative use.

Moch (2001) had a similar focus in her study "Manipulatives Work!" Though it is mentioned in the methods section that several teaching strategies thought to be effective were used, the main drive of the study was the effect of manipulative use on improving proficiency in the different mathematics strands. Moch (2001) stressed the point that manipulatives were used only 18 hours in seven weeks and improved test scores by an average of ten percentage points. What is not stressed is the amount of group and whole class discussion that went along with using the manipulatives. Students were also given instructions to summarize their experiences with the lessons in writing. Even when students did assignments individually, there seemed to be a point where the topic was discussed or written about.

Moyer (2001) and Moch (2001) seem in step with Stein and Bovalino (2001) who included discussion between students as part of an activity using manipulatives. The article by Stein and Bovalino (2001) is a practitioner piece exemplifying effective ways to use manipulatives—thus showing that articles geared to both teachers and researchers see discussion and writing as linked to manipulative use.

Another observation linking manipulative use and communication is the many examples displayed in texts such as *Mathematics Assessment: A Practical Handbook for Grades 6-8* (NCTM, 2000) and *Literacy Strategies for Improving Mathematics Instruction* (Kenney, 2005). Both texts show many examples of students' written descriptions of different solution methods and concepts in mathematics. Most of these examples include pictorial representations of the situation with their description. Pictorial representations are in and of themselves a way of abstracting concrete situations (Kamii, Kirkland, & Lewis, 2001). As it has been shown that instruction with manipulatives prompts students to draw pictorial representations when they are unable to grasp the concept symbolically (Butler, Miller, Crehan, Babbit, & Pierce, 2003), it is possible to see a connection between manipulative use in a student's ability to write about mathematics.

In *Making Sense*, the authors identify ways in which to help students understand mathematics. In one section of the text, students use base-ten blocks to compute addition problems of two digit numbers (Hiebert et al., 1997). When one student described adding 18+23+37, she used symbolic representations to explain what she had done with the base-ten blocks. This type of connection is exemplary of Cramer and Karnowski's (1995) argument for keeping informal meanings of concepts connected to the formal meanings of concepts.

The literature above illustrates how communication is used frequently and described as important in implementing the use of manipulatives in mathematics instruction. This relationship, though, is not one-sided. Whitin (2004) used manipulatives prior to having students write and discuss problems involving perimeter and area of different figures, therefore showing that the relationship between manipulatives and communication may not be causal but interactive.

Kroll and Halaby (1997) identify manipulatives as being important in developing writing skills in mathematics. They identify writing in mathematics as a long process of development. First, it is suggested that manipulatives are used concretely to describe a solution strategy, then symbols as manipulatives, and finally algorithms communicated by students. Writing solutions and solution methods with pictures, words, and numbers is said to help students voice their strategies (Kroll & Halaby, 1997). This description is remarkably similar to the process described for learning new concepts with manipulatives themselves. A manipulative is used to represent an actual object and over time the concept the manipulative is used to teach is discussed in terms of symbolic representation—or abstract thinking (Bruner, 1967; Bruner, 1973; Clements, 1999; Kamii et al., 2001; Moyer, 2001). Students must find patterns and relationships among concepts in order to make a transition from initial to intermediate writing (Shepard, 1993). So manipulatives are used to introduce students to the notion of writing and discussion in mathematics (Kroll & Halaby, 1997) and both writing and discussion are used to move from different levels of thinking when using manipulatives (Bruner, 1973; Moyer, 2001).

The different stages to grasping abstract concepts through use of manipulatives described by Bruner (1973) and others (Cramer & Karnowski, 1995; Kamii et al., 2001; Moyer, 2001) uses manipulatives to introduce concepts but eventually phases them out in place of symbolic notation. That last stage in effective use of manipulatives involves language extensively. Language is viewed as the key to making the leap to abstract understanding of a concept and it is still seen as a part of manipulative use by evidence of the way manipulatives are implemented in research (Moch, 2001; Moyer, 2001). Yet the converse has also been suggested (Kroll & Halaby, 1997; Whitin, 2004). Manipulatives have been used in facilitating discussion and writing about mathematics. Therefore, the relationship between concrete representations (manipulatives) and verbal representations (discussion and writing) should not be viewed as unidirectional, but interactive. In order to provide a basis for this conjecture, the present study sought to answer the aforesaid research question.

Method

Data

In this study we conducted secondary analysis of data from the Early Childhood Longitudinal Study (ECLS)-Fifth Grade Year, a nationally representative sample of U.S. fifth grade students. The data included 11,820 fifth grade students in different geographical locations around the United States. Of the school settings, 34% were urban schools, 36% suburban, and 22% were rural, with 8% unidentified. Data from a subsample of these students (N=5,381) was collected by the students' mathematics teachers who completed a questionnaire related to classroom practice and information on each individual student in their classroom that participated in the study. In order to maintain generalizability of the subsample, sampling weights were created by the National Center for Educational Statistics (NCES) and included in the ECLS database. The specific sampling weight from the ECLS database used in the present study is C6CPTM01. Sampling weights, as used in the analysis of the present study, adjust for oversampling of portions of the population so that the data of such subjects is proportionally representative of the population. For example, in the current sample, students of Asian and Hawaiian/Pacific Islander ethnicity were oversampled, and this oversampling was controlled for with the use of weights. Weights allow for a sample to be generalizable to the full population. As a result of using the sampling weights, the working sample size was reduced (N=4,922).

Table 1

Student demographic information with and without weights applied

	Without Weights Applied	With Weights Applied
Caucasian	60.1% *n=2959*	57.1%
African American	9.9% *n=488*	16.1%
Hispanic	17.8% *n=876*	19.1%
Asian	6.3% *n=312*	2.5%
Hawaiian / Pacific Islander	1.3% *n=65*	.7%
American Indian / Alaskan Native	2.1% *n=105*	1.7%
More Than One Race, Non-Hispanic	2.3% *n=112*	2.3%
Not Ascertained	.1% *n=5*	.5%
Total	100% *n=4922*	100%

Note: Percentages in the "Without Weights" column do not add to 100% due to rounding.

[1] Due to the large sample size resulting from the use of the C6CPTM0 sampling weights in the ECLS dataset, sampling weights were divided by 100 in order for statistical software to calculate the nonparametric statistics used in this study. This maintained the intent of using the sampling weights so that a sample proportionate to and representative of the general population was created. This simultaneously provided a more conservative estimate of the statistics.

The data were collected by the NCES as part of the ECLS beginning in 1998. Students in the study were followed from kindergarten to eighth grade with information collected in kindergarten and the first, third, fifth, and eighth grade years, although only data from the fifth grade year was used in the current study. The fifth grade data was used because measures for discussion, writing, and manipulative use assessed at the student level were collected only in this year. Data were collected from students, their parents, teachers, and schools in each phase of collection. As stated before, sampling weights were present in the analysis to adjust for oversampling and attrition, as well as missing data from the math and science questionnaires. The distribution of student demographic data calculated with and without the use of sampling weights is presented in Table 1.

Measures

In the Spring of students' fifth grade year, teachers filled out one survey for each student they taught participating in the ECLS. Teachers answered questions regarding their observation of individual students' actions. On the questionnaire were three items of particular interest for this study. Teachers were asked how often the student was observed to:

(a) Write a few sentences about how to solve a mathematics problem,

(b) Discuss solutions to mathematics problems with other children,

(c) Work with manipulatives (ECLS, 2004, pp. 6–7).

Teachers rated these questions on a four point Likert-like scale (*1 = Almost Everyday, 2 = Once or Twice a Week, 3 = Once or Twice a Month, 4 = Never or Hardly Ever*).

Some teachers had several students from the study in their classes, while others had as few as one student. In cases where teachers had only one student, no variance would be expected in their observations. However, for teachers who observed several students, some variance between their observations should be expected. This was found not to be the case for all teachers. There were many teachers who had multiple students and zero variance in their responses. This may indicate that instead of recording individual student actions in their classrooms, they may have instead recorded classroom actions, thereby inadvertently changing the unit of analysis in the study. However, this can neither be confirmed nor dismissed as a possibility. Some teachers with multiple students had variance in their observations and some did not. Additionally, there is a lack of information on how teachers made their observations related to manipulative use, discussion, or mathematical writing. Responses from the surveys were the only information in this regard. Therefore, the potential differences in the way that teachers rated student actions in this study may be a limitation.

Analysis and Results

Frequency distributions for each variable were calculated using appropriate sampling weights and are presented in Table 2. As described in the previous section, lower scores represent more frequent student engagement in each teacher-observed action (*1 = Almost Everyday, 2 = Once or Twice a Week, 3 = Once or Twice a Month, 4 = Never or Hardly Ever*). The largest proportions of students were reported to use manipulatives once or twice a week (39.8%) and once or twice a month (39.2%). Regarding the frequency students were reported to "write a few sentences about how to solve a mathematics problem," the largest proportion of students (40.8%) was reported to do so once or twice a week. Additionally, the largest proportion of students (41.3%) was reported to "discuss solutions to mathematics problems with other children" almost every day, with a slightly smaller proportion of students reported to do so once or twice a week (38.6%).

Table 2
Frequency distributions of study variables

	Almost Everyday	Once or Twice a Week	Once or Twice a Month	Never / Hardly Ever
Manipulative Use	9.2%	39.8%	39.2%	11.8%
Writing	15.1%	40.8%	29.5%	14.5%
Discussion	41.3%	38.6%	13.6%	6.5%

Note: The C6CPTM0 sampling weight was used in determining these statistics. Percentages in the Writing row do not add to 100% due to rounding.

In order to examine differences in the frequency of discussion, writing, and manipulative use, a Wilcoxon signed-rank test was used. Analyses indicate that fifth grade students in the sample discuss mathematics with their peers statistically more often than they use manipulatives (z=109.50, p<.01) and write about mathematics (z=101.83, p<.01). Fifth grade students were also found to write about mathematics statistically more often than they use manipulatives (z=20.55, p<.01). Implications of these results are discussed in the conclusions section of this article.

In order to investigate the relationship among the frequency of students' manipulative use, writing about mathematics, and discussing mathematics problems with other students, we used Spearman's rho correlation coefficient (see Table 3). The correlation between manipulative use and writing was found to be positive and statistically significant ($\rho=.32, p<.01$). The correlation between manipulative use and discussion was also positive and statistically significant ($\rho=.32, p<.01$). Additionally, the correlation between writing and discussion was found to be positive and statistically significant ($\rho=.43, p<.01$). The magnitude of each of these correlations is considered moderate (Cohen, 1988). These results indicate a positive direct relationship between students' observed manipulative use and their observed mathematical writing and discussion. That is, the more frequently fifth grade mathematics students are observed using manipulatives, the more frequently they are observed writing about and discussing mathematics, and vice versa.

Table 3
Spearman Rho correlations for study variables

	Manipulative Use	Writing	Discussion
Manipulative Use	-		
Writing	.32*	-	
Discussion	.32*	.43*	-

Note: *N=4,922; *p<.001*

Conclusion and Implications

The current study sought to determine if there was a relationship between students' mathematics communication and manipulative use in mathematics. The common assumption in the research suggests a relationship between manipulative use and communication in mathematics (Bruner, 1973; Kroll & Halaby, 1997; Moch, 2001; Moyer, 2001; Stein & Bovalino, 2001; Whitin, 2004). As stated previously, such a link—to the knowledge of the authors—has not recently been empirically investigated. The results of the current study do suggest that such a positive direct relationship exists. The correlations between observed manipulative use and both writing ($\rho=.32$) and discussion ($\rho=.32$) were found to be statistically significant. The sizes of the correlations are moderate and at first glance may not seem impressive. However, it must be remembered that the current analysis looked at manipulative use, discussion, and writing as they are generally implemented. In other words, there were no *specific* criteria for effective pedagogical practices in regards to manipulative use, discussion, and writing referenced to teachers who made their observations in the study. It is altogether likely that many teachers of students in the sample may not have used best practices in implementing manipulatives, discussion, or writing in their normal classroom practices. Rather, a range of more and less effective practices in regards to *each* of the observed variables (manipulative use, discussion, and writing) is what is most likely to have been measured and analyzed in the current study.

Taking into account that correlations represent a range of effective and ineffective use of manipulatives and mathematics communication by students, the size of the correlations are impressive and the relationship they indicate should be considered important. The correlation results indicate a relationship between the frequency U.S. fifth-grade students are observed using manipulatives with how frequently they are observed to discuss and write about mathematics. This relationship appears to hold for what is likely to be a range of best and less-than-best implementations of manipulative use, discussion, and writing in the various classrooms. It is the fact that in the face of such a variance in pedagogical implementation that the relationship posited in the current study still holds.

Since the literature (Moch, 2001; Moyer, 2001; Stein & Bovalino, 2001) illustrates written and verbal communication as part of manipulative use, the results of the current study were not unexpected. However, the current study intended to establish an empirical basis for a relationship that has to this point been only an assumed one. The nature of this relationship is still very much in question. The results presented here show support for a relationship, but do not provide us with a description of the nature of this relationship. The correlations found could represent statistical evidence of students' cognitive processes related to mathematical representation and language use (e.g., manipulative use and discussion). However, the correlations could also represent the relationship of specific teacher practices that may tend to happen in conjunction. A third possible explanation of the relationship found in the current study is that this is both a cognitive and environmental relationship. In other words, while certain teacher pedagogy may happen in conjunction, certain cognitive processes the student engages in may also happen in conjunction, and in a reflexive nature with teacher pedagogical actions.

The review of literature shows evidence of different uses of communication in manipulative use and at different stages (e.g., Moch, 2001; Moyer, 2001; Stein & Bovalino, 2001). Because many of these studies focused only on the effectiveness of the manipulatives and did not take into account the effects of communication used, further research is needed. Based on this study, we know now that students who are using manipulatives to learn mathematics are more likely to be engaged in mathematical communication and vice versa. Gaining a greater understanding of why this happens must now be investigated.

Additional results from the present study suggest that fifth grade students discuss mathematics significantly more often than they write about mathematics or use manipulatives. Also, fifth grade students were reported to write about mathematics significantly more often than they used manipulatives. While the findings from this study do not provide us with specific evidence as to why these results were present, certain conjectures can be made. Some helpful information is provided by Broderick (2009), who compared two groups of mathematics students in which one group engaged in face-to-face mathematical discussions and another in written online exchanges. Broderick found that students in the face-to-face setting asked more mathematical questions than those students in the online setting. The reason suggested for this is the static nature of writing. Whereas discussion is a fluid environment where exchanges occur more frequently, when students write there is less frequent exchange between others (Broderick, 2009). For the present study, Broderick's results imply that discussion may occur more frequently because it is, simply, much easier to initiate and maintain. Likewise, a similar argument might be made with why mathematical writing was found to be more frequent than manipulative use. Since proper use of manipulatives requires certain structures to be in place, it may be that having students write about their mathematics was an easier task to initiate and maintain. Such conclusions are logical and may give a plausible explanation for the present results.

While many questions concerning the nature of the relationship between mathematical communication and manipulative use still remain, the results of the current study do have some practical implications for mathematics teaching. The present study found a statistically positive relationship between frequency of manipulative use and mathematical communication. Whether this relationship is cognitive, environmental, or both, there is empirical evidence that manipulative use (e.g. Moch, 2001; Warrington & Kamii, 1998), mathematical writing (e.g., Jordak & Abu Zein, 1998; Kenney, 2005), and mathematical discussion (e.g. Mercer & Sams, 2006; Hiebert & Wearne, 1993) each provide opportunities for a deeper understanding of mathematics. Combining these results of previous investigations with those of the present study suggests that teachers should implement these practices within their classrooms and at similar levels of frequency. However, specific aspects of how and why this relationship functions, as well as aspects of practical implementation, need further investigation. The very fact that the relationship between manipulative use and mathematical communication exists in the context of varied student uses or teacher implementations suggests that the connection between manipulatives and communication is an important one.

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