

Learning about Japanese verbal probability in elementary school mathematics

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ABSTRACT

This study aimed to identify a lesson design that could help elementary school students become aware of the ambiguity of Japanese verbal expressions of likelihood. A qualitative case study was conducted with 30 fifth-grade students. In a lottery-drawing lesson, students interpreted repeated simulation results, selected verbal expressions, represented the perceived likelihood on a number line, and compared their interpretations with those of their classmates. Video-recorded classroom interactions and worksheets were analyzed using observable indicators aligned with three lesson objectives. Ten students described characteristics of their own interpretations, 19 recognized that the same expression could represent different likelihoods for different people, and 11 referred to the trial frequency–likelihood relationship. Combining repeated simulations, visual representations, and peer comparisons appears to support reflection on verbal probability before the introduction of formal numerical probability. This study extends research on verbal probability to Japanese elementary mathematics and offers a design for early probability instruction.

Keywords: fifth-grade elementary school students, subjective probability, relative frequency, likelihood, Japanese language arts education

INTRODUCTION

Probability is encountered in many daily life situations, including weather forecasts, medical test results, and lotteries. Probabilistic information can be expressed numerically, as in “a 20% chance of rain,” or verbally, as in “it does not seem likely to rain.” In this paper, these are referred to as numerical and verbal probability, respectively. Although Japanese school mathematics mainly addresses numerical probability (Ishibashi, 2022; Ministry of Education, Culture, Sports, Science and Technology [MEXT], 2018b, 2019), verbal probability is common in everyday communication (Hirota, 2011; Honda & Yamagishi, 2006).

Ambiguity is an important characteristic of verbal probability, as individuals may diverge in the likelihood they attach to the same verbal expression. For example, Honda and Yamagishi (2006) showed that when participants interpreted an expression such as “It is conceivable that it will snow in Tokyo in November,” the numerical probabilities they associated with the expression varied considerably. Similarly, Fujii and Miyano (2000) found that Japanese verbal probability expressions differed in the breadth of the probability ranges associated with them. Some expressions, such as “almost certain” and “almost impossible,” were associated with relatively narrow ranges, whereas others, such as “conceivable,” “not certain,” and “unclear,” were associated with wider ranges. Thus, verbal probability does not map onto numerical probability in a fixed and uniform manner.

This ambiguity is educationally important. Gal (2005) argued that probability literacy should include awareness of the characteristics of verbal expressions of uncertainty. Such awareness is relevant because children often encounter and use these expressions before they formally study probability. Research has, therefore, increasingly examined young learners’ intuitive reasoning about probability and verbal expressions of uncertainty (Batanero & Álvarez-Arroyo, 2024; Groth et al., 2020; Kazak & Leavy, 2018). However, much of this research has focused on English expressions such as impossible, unlikely, equally likely, likely, almost certain, and certain. Japanese verbal probability has different linguistic and semantic characteristics, such that findings concerning English expressions cannot simply be transferred to Japanese. This creates the need to investigate how Japanese learners understand and reflect on verbal probability in their own language ¹.

¹ For example, concerning Japanese expressions of likelihood, Sugimura (2009) argued, from a linguistic perspective, that adverbs expressing likelihood were not arranged in a straight line on a single scale of likelihood and that the degree of likelihood should be attributed to sentence-

One relevant study in Japan was conducted by Ishibashi et al. (2024), who developed learning processes for second-year junior high school students. Their students compared numerical and verbal probability and, through this comparison, became aware of the characteristics and tendencies of their own interpretations of verbal expressions. However, this approach depends on students' prior learning of numerical probability and thus cannot be applied directly at the elementary school level, where formal numerical probability has not yet been introduced. At the same time, delaying attention to verbal probability until junior high school may be problematic because children already encounter these expressions in their daily lives, which may lead their informal interpretations to become established before formal instruction begins (Groth et al., 2020).

These considerations reveal a gap in existing research. Early probability education has emphasized probabilistic intuition development (Batanero, 2020; Batanero & Álvarez-Arroyo, 2024; Langrall, 2018), while Japanese studies have developed elementary-school learning activities concerning numerical probability (e.g., Matsuura, 2015; Okabe, 2007). Still, little is known about how elementary school students in Japan can be supported in becoming aware of the ambiguity of verbal probability before learning formal numerical probability². In particular, an alternative is needed to the numerical-verbal comparison used with older students.

The novelty of this study lies in the design of this alternative pathway. Although the details of the lesson design are described later, the lesson was designed to make the differences in students' interpretations of verbal probability visible without requiring prior knowledge of formal numerical probability. This study, thus, extends previous work on verbal probability to a younger age group and to the Japanese linguistic context.

This study contributes to mathematics education research in two ways. First, it examines whether awareness of the ambiguity of verbal probability can emerge before formal instruction in numerical probability. Second, it proposes a theoretically grounded lesson design that connects children's daily language about uncertainty with early probabilistic reasoning. By doing so, this study provides a possible approach to broadening elementary probability education beyond numerical representations. In so doing, this study addresses the following research question: *What kind of lesson would enable elementary school students to become aware of the ambiguity of verbal probability in Japanese?* This paper addresses this research question as follows: the lesson design is examined by dividing it into three components, namely objective setting, task setting, and lesson structure; the actual lesson is described and examined with reference to a classroom script; the outcomes of the students' learning are analyzed, and the significance of the lesson design is discussed.

METHODOLOGY

Lesson Design

Objectives

The target learners were fifth-grade students studying proportion, which forms the basis of probability in Japan (Science Council of Japan, 2020). Based on the study by Ishibashi et al. (2024), which focused on junior high school students, the lesson objectives were discussed and determined by the authors of this paper and an elementary school teacher. Three objectives were established.

Objective [1] (concerning the ambiguity of verbal probability): students can become aware of the characteristics and tendencies of their perceptions of verbal probability.

Objective [2] (concerning the ambiguity of verbal probability): students can notice that the degree of likelihood they associate with a particular verbal probability expression differs from one person to another.

Objective [3] (concerning probability instruction generally): students can notice the relationship between the frequency of simple experimental results and likelihood.

Regarding Objective [1], Ishibashi et al. (2024), drawing on findings from vocabulary instruction in Japanese language arts education, set the objective of enabling second-year junior high school students to become aware of the characteristics and tendencies of their perceptions concerning the range indicated by verbal probability. The aim was to encourage "the reorganization of each learner's words while reactivating from memory the learner's preexisting vocabulary network" (Tsukada, 2005, p. 6). Because fifth grade is a stage at which students can already objectively analyze their language use, this study adopted an objective for fifth-grade elementary school students that is similar to the one that Ishibashi et al. (2024) adopted for second-year junior high school students.

Since the current study focuses on elementary school children, particular attention was paid to Objective [2]. Ishibashi et al. (2024) asked junior high school students to estimate the number of winning lottery tickets in a box containing 100 tickets. In the problem setting, the shopkeeper, who knew the number of winning tickets, provided information on the likelihood of winning using verbal and numerical probability. This task was designed for "understanding the complexity and ambiguity of verbal probability through comparison with numerical probability." (p. 17) Such an objective was possible because the target students

final modality forms. However, doubts have been raised about this matter, such as the possibility that adverb usage differs depending on sentence-final modality forms, such as *kamo shirenai* ("may" or "might") (Moriyama, 2011). Thus, the debate remains unresolved, indicating that it is difficult to simply compare adverbs, a recurring practice in studies dealing with English, when examining verbal probability in Japanese.

² In implementing a classroom activity on verbal probability with Japanese elementary school students, Nakagoshi (2016) did not develop learning activities that addressed the ambiguity of verbal probability. Instead, students noticed that event likelihood may differ among individuals and that there were greater-than/less-than relationships among degrees of likelihood.

Table 1. The task addressed in the lesson in this paper

There is one box. This box contains 10 balls, and its composition is one of the following: 1 winning ball and 9 losing balls, 2 winning balls and 8 losing balls, or 3 winning balls and 7 losing balls. However, you cannot check what is inside the box. In fact, you may draw only one ball from the box at a time and check whether it is a winning ball or a losing ball, but the ball you draw must be returned to the box immediately. In this situation, infer whether the contents of the box are 2 winning balls and 8 losing balls, and choose from the options below the one that best matches your thinking. You may choose the same option as many times as you like.

- A. It does not seem likely (*'Nasa sou'*) that the box contains 2 winning balls and 8 losing balls.
- B. It is conceivable (*'Kangae rareru'*) that the box contains 2 winning balls and 8 losing balls.
- C. It is fifty-fifty chance (*'Gobu gobu da'*) that the box contains 2 winning balls and 8 losing balls.
- D. It is almost impossible (*'Hobo arienai'*) that the box contains 2 winning balls and 8 losing balls.
- E. It is unclear (*'Hakkiri shi nai'*) whether the box contains 2 winning balls and 8 losing balls.
- F. It is almost certain (*'Hobo kakujitsu da'*) that the box contains 2 winning balls and 8 losing balls.

had already learned, in the first year of junior high school, about “probability obtained through repeated trials” (MEXT, 2018b). However, the fifth-grade students in the present study had not yet received formal instruction on numerical probability. Thus, the purpose of the lesson was not to require them to convert verbal expressions into formal numerical probabilities, but to build on forms of reasoning already available to them. Previous research has shown that children can reason subjectively about uncertain events before receiving formal probability instruction (Kazak & Leavy, 2018). Therefore, it was considered that daily verbal expressions provided an accessible means for fifth-grade students to express their intuitive judgments. The number line used in the lesson served to make these judgments visible and comparable without requiring formal probability calculations.

In Objective [3], the “relationship between the frequency of simple experimental results and likelihood” refers to the fact that when the number of trials is small, the relative frequency varies greatly, whereas when the number of trials becomes large, variation becomes smaller. In the lesson discussed in this paper, students were asked to express the verbal probability for each of several simulation results of a “lottery drawing” (with numbers of trials of 10, 20, 40, 100, 500, and 1,000), and the lesson did not aim to formally teach relative frequency. Instead, the sequence of repeated simulations was designed to support students in noticing that the number of trials can affect how they interpret experimental results and judge the likelihood of an event.

Operationalization of the objectives and alignment with the lesson design

Each objective was addressed by a specific element of the lesson’s design. To support Objective [1], students compared their own sequence of verbal choices with the aggregated choices of the class and reflected on the characteristics of their own choices. To support Objective [2], students represented the likelihood associated with each verbal expression on a number line and then superimposed their representations on those of group members, making differences in interpretation visible. To support Objective [3], students repeatedly judged the same underlying situation after 10, 20, 40, 100, 500, and 1,000 simulated trials, allowing them to compare how their judgments changed as the amount of experimental evidence increased. Thus, the lesson design linked each objective to an observable activity through which the intended understanding could become visible in the students’ talk or written reflections.

Task

Setting the task situation

Based on the three objectives, the task used in the lesson was created with reference to the guessing game described by Brousseau et al. (2001) (see **Table 1**). Since children are able to reason subjectively about probability even if they have not yet studied probability in school mathematics (Kazak & Leavy, 2018), the lesson focused on subjective probability, that is, probability expressing a degree of belief concerning an event’s occurrence. Regarding the task context, among the 10 example contexts³ proposed by Gal (2005), involving chance and risk in the adult world, the context of a “lottery drawing” was selected because children experience it both inside and outside school. Regarding the task wording, the classroom teacher was asked in advance whether the wording would be understandable to the students, and the response was affirmative.

³ The 10 example domains presented by Gal (2005) are as follows:

1. The natural and physical world (e.g., weather and evolution)
2. Technological processes (e.g., quality assurance and manufacturing)
3. Human behavior (e.g., service encounters, sports, and driving)
4. Medicine and public health (e.g., genetic diseases and smoking-related risks)
5. Justice and crime (e.g., fingerprint and DNA matching)
6. Finance and business (e.g., investment markets and insurance)
7. Research and statistics (e.g., sampling and statistical inference)
8. Public policy and forecasting (e.g., vaccination)
9. Games of chance, gambling, and betting (e.g., dice, lotteries)
10. Personal decisions (e.g., wearing seat belts, getting into university)

The task designed in this paper corresponds to domain (9).



Figure 1. CODAP interface (100 trials) (Source: Field study, Teacher-created screenshot using CODAP software by The Concord Consortium (<https://codap.concord.org>))

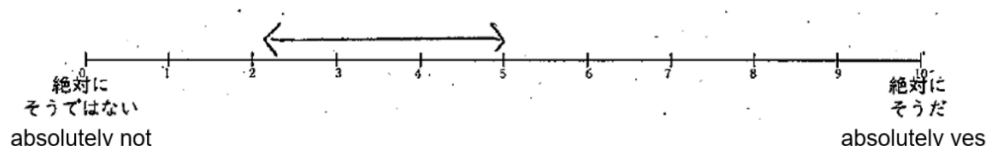


Figure 2. Examples illustrating how children may conceptualize verbal probability ⁴ (Source: Field study, Student worksheet collected during the study and reproduced in anonymized form)

Repeated trials of drawing one ball from a box were performed using the Common Online Data Analysis Platform (CODAP Version 2, <https://codap.concord.org/>) (Figure 1). Each time the number of trials reached 10, 20, 40, 100, 500, and 1,000, the students were asked to choose from options A to F (Table 1) the one that best matched their thinking. Additionally, they were asked to represent the likelihood they had in mind on a number line ranging from 0 to 10, where 0 indicated “absolutely not” and 10 indicated “absolutely yes.” Students made choices as the number of trials gradually increased to help them notice the relationship between the frequency of simple experimental results and likelihood. Moreover, verbal probability was presented in a multiple-choice format at each trial count to help students realize that the likelihood associated with each verbal probability expression could differ among individuals; shared options were considered preferable because they could serve as a common focus in the classroom. Notably, imagined likelihood can be represented as either a single point or a range. The students in the lesson represented their imagined likelihood as shown in Figure 2.

Setting the response options

In selecting the specific verbal probability expressions for options A to F, expressions that previous studies identified as representing various degrees of likelihood and having either narrow or wide interpretation ranges were included. Furthermore, as seen in the shared phrase, “The box contains 2 winning balls and 8 losing balls,” the wording was kept common across the options, while ensuring that the expressions sounded natural within the task context.

Notably, the expressions were chosen to be relatively close to spoken language. In Japanese, spoken language involves features such as particle omission and ellipsis more often than written language (Kurosawa et al., 2005). Since this study was conducted in the context of school education, especially Japanese language arts education, there was the possibility of using expressions based on normative grammar. However, using expressions based strictly on normative grammar would hinder learners’ natural and intuitive thinking. Therefore, in the original Japanese task, the study deliberately employed colloquial forms and omitted certain sentence-final particles (e.g., using *-nasasou* instead of the grammatically formal *-nasasou da*) to align with daily life expressions that may be more familiar to fifth graders. Importantly, the lesson did not investigate verbal expressions but sought instead students’ intuitive expressions of verbal probability. Since the lesson was intended to be implemented in

⁴ In the diagram representing the likelihood that students imagined for verbal probability expressions, 0 and 10 were given meanings: 0 indicated “absolutely not so” and 10 indicated “absolutely so.” In contrast, the meaning of the central point, 5, was not explicitly stated. In a mathematics textbook for Year 5 students in New Zealand (equivalent to Japanese Grade 4) that deals with verbal probability (Wilkinson, 2021), the diagram showing the probability range labels the central 5 on the number line as “even chance.” However, the meaning of the central 5 was not shown in the present lesson for two reasons. First, to the best of our knowledge, the Japanese translation of “even chance” is *gobu* (“fifty-fifty”), which overlaps with option C (“It is fifty-fifty whether the box contains 2 winning balls and 8 losing balls”). Although one might expect all students to interpret 5 as 50% for *gobu*, the findings of Fujii and Miyano (2000) showed that the imagined likelihood differed among individuals. Therefore, option C was considered one of the necessary options for achieving Objective [2]. Second, narrowing the range of the number line was not one of the goals of the present lesson, which was instead to help students notice that the degree of likelihood associated with the same expression may differ among individuals.

Table 2. Classification of the response options and ranges of evaluation values ⁵

	Wide range	Narrow range
The occurrence of the event is relatively likely	B (approx. 19–75)	F (approx. 75–100)
The occurrence of the event is relatively unlikely	E (approx. 18–61)	A (approx. 2–10) D (approx. 2–5)
Judgments about the occurrence of the event are neutral	C (approx. 42–58)	

mathematics education, expressions that were relatively close to the learners' spoken language were adopted. At the same time, the classroom teacher confirmed that these expressions would sound natural to the students.

Among the expressions shown in **Table 1**, option F, “It is *almost certain* (*Hobo kakujitsu da*) that ...,” and option B, “It is *conceivable* (*Kangae rareru*) that ...,” indicated that the event's occurrence was relatively likely. The former remained ambiguous because, although the expression used “certain,” it was modified by the adverb “almost,” which indicated an approximate quantity or degree. The latter remained ambiguous through the use of “can be thought,” which may be regarded as an expression of epistemic modality. Option D, “It is *almost impossible* (*Hobo arienai*) that ...,” option E, “It is *unclear* (*Hakkiri shi nai*) whether ...,” and option A, “It *does not seem likely* (*Nasa sou*) that ...,” indicated that the event's occurrence was relatively unlikely. Like options F and B, options D and E contained different adverbs that maintained ambiguity, whereas ambiguity arose in option A through the inclusion of a form of epistemic modality in Japanese. Additionally, option C, “fifty-fifty,” was constructed by irregularly arranging a group of expressions that may be regarded as neutral regarding the judgment of event occurrence (**Table 2**). The values shown in parentheses (percentages, ranging from 0 to 100) indicate the upper and lower bounds of the usage range, as evaluated by participants in the study by Fujii and Miyano (2000).

Including expressions previously shown to have both relatively narrow and wide interpretation ranges was intended to provide a common set of terms through which differences in students' interpretations could become visible. Therefore, this selection was directly related to Objectives [1] and [2], which focused on reflection on one's own interpretation and the recognition of differences between individuals, respectively.

Lesson Implementation

In the lesson introduction, a preliminary task was presented to help students understand the problem situation: “There is one box. This box contains 10 balls, and its composition is 10 winning balls and 0 losing balls. If you draw one ball from this box, is it a winning ball?” The teacher confirmed that this could be expressed as “It is *absolutely certain* (*Zettai ni*) to be a winning ball.” The teacher then presented another task: “There is one box. This box contains 10 balls, and its composition is 0 winning balls and 10 losing balls. If you draw one ball from this box, is it a winning ball?” The teacher confirmed that this could be expressed as “It is *absolutely certain* not to be a winning ball.” Students were then informed that in this lesson, they would think about words expressing possibility, such as “absolutely certain to be a winning ball” and “absolutely certain not to be a winning ball,” that is, verbal probability, using the provided number line (**Figure 2**; without arrows or circles drawn on it).

In the main part of the lesson, a worksheet was distributed and the task (**Table 1**) was presented along with an explanation of the problem situation. Students were instructed to think individually and not to consult others. The trial was carried out 10 times, and students chose a verbal probability from the response options and represented the likelihood they imagined on the number line. It was confirmed for students, through concrete examples placed on the blackboard, that the imagined likelihood could be represented either as a single point or as a range. Upon completing 20, 40, 100, 500, and 1,000 trials, the students chose a verbal probability and represented the likelihood on the number line. At this point, they were told that the range of any newly drawn arrow might overlap with the range of arrows drawn earlier. After 1,000 trials were completed, the students entered their choices into Google Forms, and the responses were aggregated.

While the teacher aggregated the responses, the students were placed in pairs or groups of four. They were instructed to explain to the other students the option they selected and the range of likelihood it represented for them. The teacher ensured that all students had the opportunity to express their thoughts. Once the students' choices were aggregated, the results were first shown in a table whose rows corresponded to options A to F and the columns corresponded to the numbers of trials. Using this table, students were encouraged to confirm that the words chosen to express likelihood may differ among individuals even for the same experimental results.

Students were then presented with the prompt “Compare your choices with those of the rest of the class. How would you describe the way you made your choices?” and asked to describe on their worksheets how their pattern of choices compared with that of the class. For each response option, students recorded the degree of likelihood they associated with each expression. They could leave options blank if they did not select them during problem-solving. Subsequently, students formed groups of four, superimposed the likelihoods imagined by the four members for each option, and represented the range from the minimum to the maximum with a double-headed arrow ($\leftrightarrow$) on a classroom whiteboard. Through these activities, students were encouraged to notice that even when the same word expressing likelihood was used, the degree of likelihood associated with that word may differ among individuals.

In the closing part of the lesson, the students wrote about their reflections, and several of them were invited to share their thoughts. It was expected that students would engage in activities aligned with the lesson objectives. However, the hypothesized outcome at this stage remained only a theoretical possibility. Therefore, this study demonstrates through a case study that

⁵ To make the wording sound natural within the Japanese task context, Fujii and Miyano's (2000) expression *-arizou mo nai* was changed to *-nasasou* (A), and *hobo fukanou da* was changed to *hobo arienai* (D).

phenomena predicted by this hypothesis can occur in practice. In other words, similar to much educational research, this study adopts an existence-proof approach, showing that phenomena consistent with a theoretically derived hypothesis can actually occur (Schoenfeld, 2008).

Data Collection and Analysis

The lesson was conducted as a mathematics lesson for 30 fifth-grade students at an elementary school attached to a national university when the Course of Study for Elementary Schools, announced in March 2017 (MEXT, 2018a), was being implemented. The teacher was an elementary school teacher who regularly taught this class. Before the lesson, the authors explained the background and purpose of the study to the teacher and designed the lesson while seeking guidance on whether the draft lesson design was feasible for the class. Apart from implementing the collaboratively developed lesson design, the teacher conducted the lesson in the usual manner so that the students could participate as they would normally do. The lesson was recorded using a video camera. Worksheets were distributed and collected after the lesson. The total lesson time was 62 minutes. The time shown in square brackets hereinafter indicates the elapsed time from lesson onset.

The lesson was video-recorded, and the students' worksheets were collected after the lesson. The classroom recordings were transcribed in Japanese, and the worksheet responses and classroom transcripts were analyzed in their original Japanese. English translations were prepared only for excerpts reported in the manuscript so that the analysis was based on the students' original Japanese expressions rather than on translated data.

For the analysis, evidence corresponding to each objective was identified through observable statements in the students' worksheets and classroom discourse. The evidence for Objective [1] comprised statements in which students explicitly described a characteristic or tendency in their own interpretation of verbal probability, particularly through comparison with classmates. The evidence for Objective [2] comprised statements indicating that different students associated the same verbal probability expression with different degrees of likelihood. The evidence for Objective [3] consisted of statements linking the number of trials to the variability, reliability, or stability of the experimental results or students' judgments of likelihood. These indicators were used to identify evidence corresponding to each lesson objective; they were not used as standardized measures of individual achievement.

The first author conducted the initial analysis and coding of all students' worksheet responses and relevant classroom discourse using these criteria. The coding decisions and the supporting excerpts from the data were then reviewed by the co-authors. Cases for which there were questions or different interpretations were discussed among the authors, with repeated reference to the original Japanese data and the coding criteria, until all authors reached consensus. The final coding used in the analysis therefore reflects the consensus of all authors.

Each student was coded as either showing or not showing evidence corresponding to each objective, and a student could show evidence corresponding to more than one objective. The numbers reported in the Discussion are based on these final consensus judgments. Classroom discourse was used both to identify relevant evidence and to contextualize and illustrate the students' written responses.

For reporting in the manuscript, excerpts from the worksheets and classroom discourse were translated from Japanese into English after the analysis. The translations were checked against the original Japanese data to ensure that the meanings relevant to the analysis were retained.

RESULTS

In the introduction, the teacher asked the following question: "There is one box. This box contains 10 winning balls. If you draw one ball from this box, how likely is it to be a winning ball?" The students responded, "You will win 100% of the time," "You will *definitely* win," and "You *are certain* to win." The teacher shared these responses with the class while showing on the board that they corresponded to "absolutely yes" on the diagram representing the imagined likelihood (**Figure 2**). Next, the teacher asked, "If all the balls were losing balls, how would you express it?" The students answered, "The likelihood of winning is 0%" and "You will *definitely* lose." The teacher shared these responses with the class while showing on the board that they corresponded to "absolutely not" on the number line representing imagined likelihood. The teacher subsequently distributed the worksheets [4 minutes].

In the main part, all students read the task to understand the problem situation. After they had read the task, the teacher asked, "Is there anything hard to understand?" The students asked whether they could mark a ball before returning it, whether the same ball could be drawn many times, and who would do the drawing. To these questions, the teacher answered, respectively, "You cannot mark the ball," "You may draw the same ball as many times as you want," and "A machine will do the drawing." Assuming that the box contained two winning balls and eight losing balls, the teacher used CODAP to conduct 10 trials. The 10 trials yielded three wins and seven losses. The teacher asked students to choose a response option. One student asked, "Teacher, do we also write the likelihood we imagine?" The teacher replied, "Can you do that? Some people may hesitate. What range did you choose?" Another student answered, "From 2 to 5." The teacher explained that the imagined likelihood could be written as shown in **Figure 2**. Additionally, when the teacher asked how it should be written if it was fixed at a single value, the students answered, "A circle," and so the teacher instructed them to use a circle in such a case. In each set of trials (20, 40, 100, 500, and 1,000), the students were asked to select a verbal probability and write down the likelihood they imagined (**Table 3**).

Table 3. The results of the trials

Number of trials	Wins	Losses
10	3	7
20	4	16
40	8	32
100	20	80
500	96	404
1,000	185	815

Table 4. The results of the students' selections ⁶

Number of trials	A	B	C	D	E	F
10	1	0	1	0	29	0
20	2	6	10	1	12	0
40	2	7	10	1	7	4
100	2	10	6	1	6	6
500	2	6	5	2	15	1
1,000	2	1	7	5	15	1

After all the trial sets had been completed, the students entered their choices into Google Forms. While the teacher organized the results, the students compared their ideas with those of the classmates seated next to them. After approximately one and a half minutes of discussion, the teacher asked, "Was anyone's answer the same?" but no student raised their hand [25 minutes].

The teacher showed the aggregated results of the students' choices (**Table 4**), and the students copied them onto their worksheets. The teacher then posed the question, "Compare your choices with those of everyone else in the class. How would you describe the way you made your choices?" and asked the students to write their responses on the worksheet [34 minutes]. After they had finished writing, the students compared their answers with those of the classmates seated next to them. After approximately one and a half minutes of discussion, several students were asked to present what they had written. The following is an excerpt from the interaction between the teacher and the students. Here, T refers to the teacher; Endo, Masuda, and Nomura are students' names; and S refers to multiple students. All student names are pseudonyms.

Endo : I think at first there was a comment that most people had chosen E.

T : Was that for the class? For everyone? Which was it?

Endo : The class.

T : Ah, so as a class.

Endo : As a class, E was the most common choice. Starting from that, when I was talking with Masuda, I noticed that for A, B, C, D, and F, there were responses like 0 or 1, but for E, the ranges mostly overlapped.

T : By the way, Endo, was your result the same as the class result? Or was it similar?

S : It was different.

T : It was different. Could you tell us how it was different?

Endo : While we were examining the results from 10 to 1,000 trials, I think someone mentioned that "it's being multiplied by some factor." Based on that, since it had not even reached half, I chose D most of the time. But since many people chose E, I thought that it must be unclear.

T : So, then does that mean that Endo was wrong?

S : No.

T : Then was the class wrong?

S : No.

⁶ The teacher entered responses in advance to check the system's operation: "E, E, C, C, E, E" (in the order of 10, 20, 40, 100, 500, and 1,000 trials). These were mistakenly not deleted at the time of aggregation. Therefore, from the 10-trial stage onward, the count for the indicated option at each stage was one higher than the actual number of students who selected it. The uncorrected counts are retained in Table 4 because they reproduce the aggregated results that were actually displayed to and discussed with the students during the lesson. Correcting the counts in the table would therefore make the reported table different from the information that constituted part of the students' actual learning environment. The test responses were not treated as student data in the analysis, which was based only on data from the 30 participating students.

S : There are different ways of seeing it.

T : You said “different,” but could you explain more concretely what you mean by “different,” Nomura?

Nomura : In my case, I chose C at 10 trials. The reason I chose C at 10 trials was that the number of results obtained was still small, so I thought it was hard to say whether the results really fit or not. But E says that it was unclear whether the contents of the box were winning or losing, so I think it depends on whether you are looking at it objectively or ... [inaudible].

T : So, when you say “different,” do you mean that the way of looking at it objectively is different?

Nomura : Yes.

T : Were Nomura’s and Endo’s choices the same, or different?

S : A little different.

The teacher then concluded, “Even with the same results, there are different ways of looking at them. The viewpoints and ways of thinking are different” [40 minutes].

After several students had presented their thoughts, the teacher instructed, “Please write down your final results. For example, if you chose A, show the range you selected for it. Can you write that on your worksheet? You do not need to write anything for options you did not choose. If there are two, you can combine them.” As shown in **Figure 3**, the students organized the likelihood that they associated with each of the response options on their individual worksheets. The teacher revealed the contents of the box that had been hidden on the left side of the CODAP (**Figure 1**), and the students confirmed that it had two winning balls and eight losing balls. Working in groups of four, the students superimposed the likelihoods imagined by the four group members for each response option and represented them on a whiteboard using a double-headed arrow ($\leftrightarrow$) extending from the minimum to the maximum. Any option that none of the four group members selected was left blank. During the activity, in one group, for example, when the range for option E became 2 to 8, one student commented, “That’s really wide” (**Figure 4**) [54 minutes].

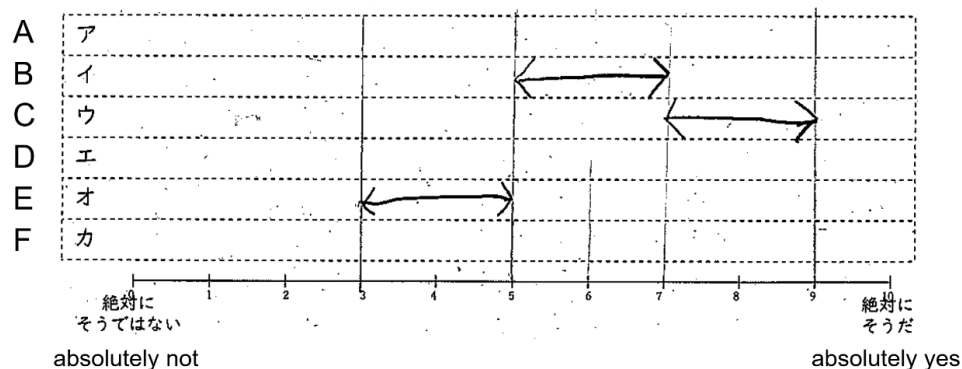


Figure 3. An example of organizing the possibilities a student envisioned based on the verbal probability this student selected (Source: Field study, Student worksheet collected during the study and reproduced in anonymized form)

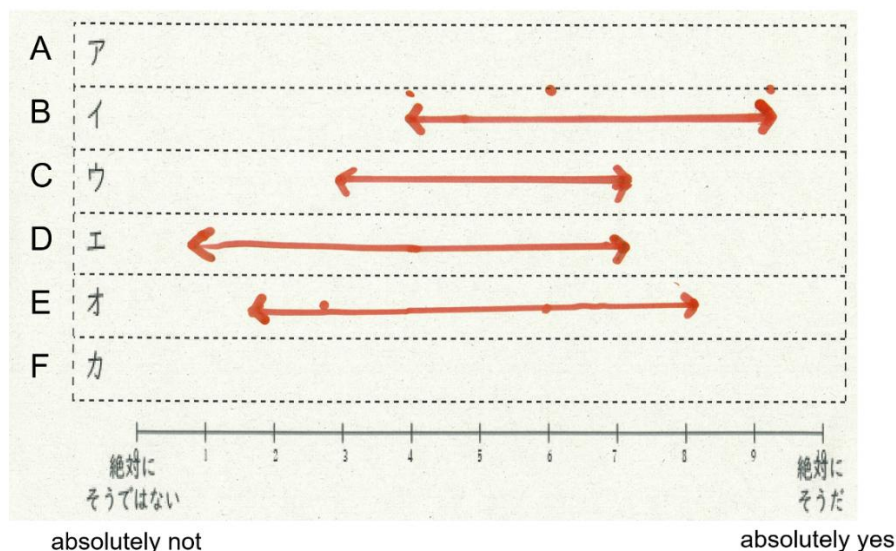


Figure 4. An example of organizing the images of the four team members (Source: Field study, Student worksheet collected during the study and reproduced in anonymized form)

After the groups had finished organizing their ideas, the teacher said, “Let’s write down what you understood from today’s lesson and what you would like to know more about.” After approximately three and a half minutes of reflection writing, several students were called on to present what they had written. The following is an excerpt from the interaction between the teacher and the students. Here, Tokoda and Kurohara are students’ names; the remaining labels are the same as in the previous excerpt.

Tokoda : Today, we drew up to 1,000 times, but even then, I did not reach the feeling expressed in option F, that it was almost certain. So I thought that if we did a larger number of trials, we might reach the feeling of certainty.

T : Is that how you feel, Tokoda?

Tokoda : Yes.

T : So that is your feeling. I see. You had not reached certainty. So you think the number of trials should be increased.

S : After class, let us keep drawing. Maybe up to about 10,000 times.

T : So you mean that if we draw more, we may come to know. All right, let us hear from one more person. Kurohara?

Kurohara: Until today, when doing this kind of thing, I thought everyone’s results would be almost the same. But when I looked at the table, that was certainly true at the beginning, yet from the middle, the results started to spread out. So I realized that if the point of view shifts even a little, the results of everyone’s thinking can change.

T : So, even with the same results, there were different ways of looking at them and different ways of thinking. And, Tokoda, you thought that perhaps the number of trials should be increased before reaching certainty, right? And you said that you had thought everyone’s results would be almost the same, “but” then came the word “but.” But in reality?

S : They were different.

T : The interpretations were different. Is that a bad thing?

S : No. Rather, it is a good thing.

The lesson ended after these exchanges [62 minutes].

DISCUSSION

This study aimed to examine, using the observable indicators specified in the Methodology section, whether students’ worksheets and classroom discourse contained evidence corresponding to each lesson objective.

Findings Related to Objectives [1] and [2]

For Objective [1], 10 students produced statements describing characteristics or tendencies in their own interpretations of verbal probability. Importantly, some of these statements explicitly referred to comparisons with their classmates’ interpretations. For example, Kurohara characterized his own judgments as relatively clear-cut, writing, “Compared with the people around me, I realized that I am someone who tends to give relatively clear-cut results rather than vague ones.” This suggests that the comparison with others provided at least some students with a reference point for reflecting on their use and interpretation of verbal probability. The significance of this finding lies not simply in students’ selection of different expressions but in some students’ ability to treat their own interpretations as objects of reflection.

This finding extends the work of Ishibashi et al. (2024). In their study, junior high school students reflected on their interpretations of verbal probability through comparison with numerical probability. The present findings show that a similar form of reflection was observed among fifth-grade students who had not yet learned formal numerical probability and that comparisons with classmates’ interpretations served as an alternative source of contrast. Thus, numerical probability may not be the only reference against which learners become aware of the characteristics of their interpretations of verbal probability.

Evidence for Objective [2] was found in the statements of 19 students. These students referred to differences in the degrees of likelihood that they and their classmates associated with the same verbal expression. The group activity was particularly relevant to this finding because the students did not merely compare the expressions they selected, but also superimposed the likelihood ranges they associated with the same expression. For example, Abe wrote, “I only chose B, C, and E, but I wrote $B \rightarrow 9$, $C \rightarrow 3$ to 5, and $E \rightarrow 2$ to 4. When we talked in the group, they were completely different, and sometimes there were no overlapping numbers at all, which was interesting. Some of the results were unexpected, and I was surprised.” Her reflection explicitly connected her awareness of differences to the group comparison, providing evidence that making individual interpretations visible and directly comparable can support the recognition of the ambiguity of verbal probability.

It is important to distinguish Objectives [1] and [2]. Objective [1] concerned reflection on one's own interpretation, whereas Objective [2] concerned the recognition of variation between individuals. The fact that evidence for Objective [2] was more frequent than that for Objective [1] suggests that recognizing interpersonal differences may be a more accessible form of reflection than articulating the characteristics of one's own interpretation. Still, this interpretation should be treated cautiously because the lesson was not designed to compare the relative difficulty of the objectives. Nevertheless, the difference indicates that noticing variations among people does not necessarily entail being able to characterize one's own interpretation pattern.

The students' use of particular Japanese expressions is an important contribution of this study to the literature. An interesting finding concerned the selection of "fifty-fifty (Gobu gobu da)." Watanabe wrote, "At first I wrote things like 'it is still unclear' because I did not know yet. But some people seemed to understand a little, so I was completely different from everyone. In the end, I still did not know, so I chose 'fifty-fifty'." Thus, Watanabe connected "fifty-fifty" with "not knowing." Three students, including Watanabe, made similar statements. Furthermore, Watanabe and Yoshida associated "fifty-fifty" with an imagined likelihood of "between 7 and 8." These results indicate that Japanese learners demonstrated interpretations similar to those previously reported in studies of English expressions, such as associating "50% probability" with "not knowing" (Konold, 1989), or using "50-50 chance" to represent outcomes that do not necessarily occur with equal probability (Tarr, 2002). However, the present study concerned specific Japanese expressions in one classroom context; therefore, future research is required before broader linguistic generalizations can be made.

Findings Related to Objective [3]

For Objective [3], 11 students produced statements linking the number of trials to the reliability or stability of their judgments. Some students regarded the results based on 1,000 trials as more trustworthy than those based on 10 trials, whereas others suggested that an even larger number of trials would be required before they could make a more confident judgment. These statements provide evidence of an emerging awareness that the amount of experimental evidence matters when judging likelihood. This finding is relevant to early probability education because the students in the sample had not yet learned relative frequency as a formal probability concept. Therefore, their statements should not be interpreted as demonstrating a formal understanding of the convergence of relative frequencies, but as an intuitive recognition that judgments based on a larger number of trials may differ from those based on a smaller number of trials. This finding corroborates the broader argument that probabilistic intuitions can begin to develop before formal probability instruction (Batanero, 2020; Batanero & Álvarez-Arroyo, 2024; Langrall, 2018).

Since the evidence for Objective [3] was limited to 11 students, this result can be understood in terms of the lesson structure. The central whole-class and group activities explicitly focused on differences in verbal choices and likelihood ranges associated with the same expressions. By contrast, students were not explicitly asked in the final reflection to explain how increasing the number of trials affected the reliability of the experimental results or their judgments. Thus, the lesson provided repeated-trial experiences relevant to Objective [3], but provided less explicit support for articulating this relationship than it did for recognizing the ambiguity of verbal probability. This difference in instructional emphasis provides a plausible explanation for the more limited evidence for Objective [3], although the present data do not establish a causal relationship.

Contribution to Research on Verbal Probability

The findings show one possible alternative lesson design through which elementary school students can become aware of the ambiguity of Japanese verbal probability. The key contribution of this study is that this awareness was observed among fifth-grade students before formal instruction in numerical probability. Previous research on Japanese verbal probability among junior high school students used a comparison between verbal and numerical probability as an important learning resource (Ishibashi et al., 2024). The present study extends this line of research by showing that differences in interpretation can be made visible through a combination of verbal judgments, visual representations of perceived likelihood, and comparisons with peers.

This study also extends research on young children's probabilistic reasoning. Children have been shown to be able to reason intuitively and subjectively about uncertainty before formal probability instruction (Kazak & Leavy, 2018), and some prior studies have examined the learning of probability vocabulary in English (Groth et al., 2020). The present study adds evidence from the Japanese linguistic context, where verbal probability does not necessarily share the same semantic structure as English probability vocabulary. These findings suggest that early probability education should attend not only to general probabilistic concepts but also to the linguistic resources through which uncertainty is expressed and interpreted.

The findings also support Gal's (2005) argument that awareness of verbal expressions of uncertainty forms a part of probability literacy. The present case indicates that such awareness can begin to be addressed in elementary mathematics, namely, before probability is formally introduced as a numerical concept. Verbal probability may serve as a bridge between children's everyday experiences of uncertainty and formal probability learning.

Implications for Lesson Design

The findings also suggest several refinements to lesson design. First, support for Objective [1] could be strengthened by confirming with the class how to respond to the question, "Compare your choices with those of everyone in the class. How would you describe the way you made your choices?" before students answered it individually. In Ishibashi et al. (2024), before asking junior high school students to explain the characteristics and tendencies of their interpretations of verbal probability, they engaged in an activity in which they examined the choices of a fictitious person and considered together the characteristics and tendencies of that person's interpretation of verbal probability. By including a similar activity, more children may be able to go beyond simply listing the verbal probability expressions they have selected and instead explain the characteristics and tendencies of their own interpretations.

Second, the activity addressing Objective [2] could include a whole-class discussion of the likelihood ranges produced by the groups. In the present lesson, students superimposed their ranges in groups, but these results were not subsequently discussed by the whole class before the final reflection. Making these differences explicit objects of whole-class discussion could reinforce the distinction between choosing different expressions for the same situation and assigning different meanings to the same expression.

The response options warrant further consideration. Option E, “it is unclear (Hakkiri shi nai),” was selected frequently and was associated with a wide range of likelihoods. For example, Nakazaki wrote, “I think everyone chose E at first because they hardly knew anything,” and Yabuta wrote, “Just like me, they did not really know, and in the end, they came back to E.” Hence, it may be inferred that some students used “it is unclear” in the sense of “I do not know.” This was an informative finding, but the frequent use of this option may have reduced the opportunities for some students to compare distinctive patterns in their own choices with those of others. Future implementations could examine whether replacing or supplementing this expression with an alternative, such as “not certain (Kakushin ga nai),” produces different interpretations.

Finally, Objective [3] requires explicit instructional attention. Students could be asked to compare their judgments at different numbers of trials and explain why their confidence changed. The actual results at 20, 40, 100, and 500 trials could be used to discuss why similar proportions based on different numbers of trials may be interpreted differently and why experimental proportions fluctuate even when the underlying situation remains unchanged. Such a discussion would preserve the intuitive character of the lesson while making the connection between repeated trials and judgments of likelihood more explicit.

CONCLUSIONS

This study developed and examined a fifth-grade mathematics lesson designed to help students become aware of the ambiguity of verbal probability in Japanese. These findings suggest that fifth-grade students, who have not yet received formal instruction in numerical probability, can begin to reflect on their own interpretations of verbal probability and recognize that the same expression may correspond to different degrees of likelihood for different people. In the present lesson, differences in interpretation were made visible through a combination of intuitive judgments of repeated experimental results, visual representations of perceived likelihood, and comparisons with classmates.

The original contribution of this study lies in the identification of a possible pathway for learning about the ambiguity of verbal probability before the introduction of formal numerical probability. Previous Japanese research on junior high school students compared numerical and verbal probability to encourage reflection on verbal probability (Ishibashi et al., 2024). The present study extends this line of research to the elementary school level by showing that comparison with peers’ interpretations can serve as an alternative source of contrast. It also extends previous research on young children’s probabilistic reasoning and probability vocabulary by providing evidence from the Japanese linguistic context.

These findings have important implications for elementary mathematics education. Verbal expressions of likelihood can be treated not merely as informal language used before students learn numerical probability, but as objects of mathematical reflection in their own right. Teachers can provide opportunities for students to express their intuitive judgments, represent the likelihoods they associate with verbal expressions, and compare their interpretations with those of others. Such activities may help students recognize that expressions of uncertainty do not necessarily have the same meaning for everyone. From a curriculum perspective, verbal probability may provide an early entry point for probability learning by connecting children’s everyday language of uncertainty with the later formal study of probability. The importance of distinguishing between students choosing different expressions for the same situation and those assigning different degrees of likelihood to the same expression is also emphasized and is something that should be addressed by teachers.

The findings concerning Objective [3] were more limited. Although some students produced statements indicating that the number of trials influenced judgment reliability or stability, the data did not show that students generally developed a formal understanding of the relationship between the number of trials and the variation in relative frequency. This suggests that repeated simulations alone may be insufficient to make this relationship explicit. If developing this aspect of probabilistic intuition is also an instructional goal, the lesson should include opportunities for students to compare the results obtained from different numbers of trials and explicitly discuss how and why their judgments change.

Several limitations of this study should be considered. It involved only one classroom in an elementary school attached to a national university in Japan. Such schools are not necessarily representative of ordinary public elementary schools because admission often involves a selection process and students may differ in their prior learning experiences and family backgrounds. University-affiliated schools also frequently serve as sites for teacher education and educational research; therefore, both teachers and students may be more accustomed to research-based lessons than those in ordinary school settings. The participating teacher also had previous research experience in lesson development in elementary mathematics and may have had greater expertise in probability and statistics education than a typical elementary school teacher. These contextual characteristics may have influenced both students’ responses and the instructional support provided. Therefore, the findings should be interpreted as evidence of what was possible in this particular classroom context, not as evidence that the same outcomes would necessarily occur in all elementary school classrooms. Future research should examine similar lesson designs across different schools and teachers. Longitudinal research is also required to investigate how early reflection on verbal probability contributes to students’ subsequent learning of formal probability, including classical and geometric probabilities.

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