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Language demands in undergraduate mathematics courses

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ABSTRACT

This study examines language demands in undergraduate university mathematics classes with a view to better preparing and supporting international and other culturally and linguistically diverse (CLD) students in the United States universities. We observed and analyzed the language demands of 13 entry-level mathematics classes across two large universities. Findings showed that mathematics instruction required extensive and varied listening skills requiring students to listen to long and uninterrupted stretches of sometimes fast paced speech. Technical vocabulary contributed to the language demands, but instructors used both academic and colloquial varieties of language in instruction, interweaving mathematical nouns with phrasal verbs to explain mathematical processes. We suggest a greater focus on development of academic listening skills in English language classes as well as potential collaboration with mathematics campus resources and faculty for additional student support for preparing CLD students for potential future discourse practices.

Keywords: language demands, international students, culturally & linguistically diverse students, mathematics

INTRODUCTION

United States colleges and universities serve a highly diverse student body. In addition to the 1.1 million international students in higher education comprising nearly 6% of all students (Israel & Batalova, 2021), approximately 10% of graduates from United States high schools were once classified English learners (National Center for Education Statistics [NCES], 2020a, 2020b), which suggests that many domestic students have learned or are learning English as an additional language. To serve these culturally and linguistically diverse (CLD) students, many universities are seeking ways to provide appropriate support to ensure these students succeed. Although these students meet minimum English proficiency requirements, they may have limited experience learning academic content in English. As Clegg (2007) warns, learning new curricular content through the medium of the new language increases the cognitive demands on those learners, adding, “if you ask a learner to do this without support, you reduce their capacity to learn” (p. 114). Without adequate support, CLD students may face higher rates of course failures or retakes, delayed graduation, or dropout, and university faculty unfamiliar with language learning may struggle to teach CLD students in their courses.

To provide the support necessary for success, some universities, including the two focal universities in this study, offer both academic English and pathway/bridge programs in which graduate and undergraduate students entering the university take English for academic purposes (EAP) courses alongside courses in their academic program of study. These programs offer resources such as tutoring, workshops, and language support students need to access, transition to, and find success in higher education. For these support resources to be effective, they must accurately reflect current classroom realities and prepare students for the classroom language they will encounter–what we refer to as language demands. In other words, CLD student success depends on EAP courses, support resources, and programs preparing students for they language they need to participate in today’s classrooms. To that end, this study seeks to examine and better understand the language demands of university classrooms with a view to intentionally informing EAP and support curricula and aligning the language demands required of students with the skills taught.

We define language demands broadly as the language needed in a given context or for a specific purpose. Within education, this “language of schooling” (Schleppegrell, 2004) requires that students speak, listen, read, and write in specific and complex, yet often implicit ways. Meeting these demands requires that English learners, international students, and other CLD students in United States schools go beyond “speaking English” to the use of specialized and discipline-specific language skills (Moschkovich, 2007). Understanding the language demands of classroom tasks is a necessary prerequisite to supporting CLD students and scaffolding language (Lucas et al., 2008), so more detailed understanding of classroom language demands might allow for more targeted and specific language support for CLD students.

This study focuses specifically on undergraduate mathematics instruction because of its centrality to general education requirements and the high concentration of international and CLD students in disciplines requiring advanced mathematics, such as engineering, business, and computer science (International Institute of Education, 2020). We focus on entry-level classes that students are likely to take in their first semesters that are prerequisites for more advanced courses in students' majors.

This study reports the first of two related studies examining language in university mathematics. The first focuses on the classes themselves and the type of language used in mathematics instruction while the second study focuses on students and their experience with the language used in mathematics instruction. This paper reports findings for the first phase of the study exploring the language used in university mathematics classes, guided simply by asking what language demands are present in undergraduate, entry-level university mathematics classes.

MATHEMATICAL LANGUAGE

Immediately identifiable aspects of mathematics language include the numbers, symbols, and images that comprise a complex semiotic system of language used "in precise and specific ways unique to mathematics" (Hughes & Powell, 2020, p. 81). However, doing mathematics and completing a mathematics course require far more language than this complex notational system. The common perception of mathematics as a "universal language" belies the uniqueness of mathematical language as well as the dependence on listening, speaking, reading, and writing in mathematics teaching and learning.

Research focused on mathematics education and teacher preparation for primary and secondary students has reiterated that mathematical language differs from both everyday, informal language as well as generic 'academic' language (e.g., Halliday, 1978; Schleppegrell, 2007). Some of these differences are reflected in current academic standards (e.g., common core state standards & WIDA English language development standards) that call for language-based skills such as reasoning and arguing to be used specifically within the context of mathematics. To foster these skills among English learner students, primary and secondary mathematics teachers have been encouraged to provide frequent and multiple opportunities for students to speak, listen, read, and write using the language of mathematics (Moschkovich, 2013).

Importantly, literature focused on lower grades has emphasized that developing proficiency in the language of mathematics requires instruction beyond that of mathematical vocabulary (Kersaint et al., 2014; Moschkovich, 2013; Schleppegrell, 2007; Uccelli et al., 2015). Mathematics-specific terminology such as parabola or circumference is certainly a prominent feature of mathematical language and essential for instruction (Schleppegrell, 2007), but this vocabulary can overshadow common words such as place or function that have additional technical meanings within mathematics, which create additional semantic complexity. Simply translating these words into other languages can be problematic and misleading, as Durand-Guerrier et al. (2015) note, "the English word *field* could be translated into French as *domaine*, *champs*, or *terrain*, but none of these translations are valid when considering the use of the term field in algebra, where the correct French translation is *corps*" (p. 86).

At the discourse level, Schleppegrell (2007) identified the length and density of noun phrases, implicit logical relationships, and conjunctions with technical meanings as important features of mathematical language. These features and patterns provide further insights into the complex construction of mathematical meaning through language, seen for example in phrases such as "the volume of a rectangular prism with sides 8, 10, and 12 cm" (p. 143). More recently, mathematical discourse has been conceptualized as one component of academic literacy in mathematics, alongside mathematical proficiency and mathematical practice (Moschkovich, 2015), echoing the skills outlined in United States K-12 academic standards for mathematics.

Corresponding academic standards for mathematics in higher education do not exist and no extant studies found detail the language demands placed on university mathematics students. Although the skills needed to demonstrate college readiness in K-12 standards are presumably similar to the skills that are needed, used, and valued in college study, this is a presumption; this study is needed to identify aspects of mathematical language specifically in higher education so that support offered to CLD students in universities is based on the demands of universities themselves rather than on those of K-12.

MATERIALS & METHODS

This study focuses on identifying and describing language demands in university mathematics courses. This study is part of a broader study that also includes student perceptions and experiences related to language and mathematics, but the present study specifically focuses on the language that international and CLD students are likely to encounter in their mathematics classes. The intent of this study is not to provide exhaustive or generalizable results to all of mathematics education; instead, this qualitative study seeks to explore language demands in a few contexts relevant to students at our specific universities and inform directions for later phases of the broader study as well as subsequent studies on tertiary language demands.

Data Collection

Data were collected at two large public universities in United States, one in the southeast, and one in the northwest. These universities had robust international student populations with nearly 7,000 enrolled international students, including over 3,000 undergraduates, across both institutions. Not included in these numbers are those domestic students whose home language is not English and recent immigrants not on student visas.

To narrow the study scope and provide consistency across research sites, we focused on introductory-level courses typically taken by first-year students, including college algebra, pre-calculus, and calculus I. Emails were sent to mathematics faculty to

Table 1. Participant & course descriptions

Participant*	Site	Position	Course	Class type
Ms. Ahn	B	Graduate teaching assistant	Introductory algebra	Online synchronous & small
Ms. Allen	B	Graduate teaching assistant	College algebra	Online synchronous & small
Ms. Campbell	A	Graduate teaching assistant	College algebra	Face-to-face & small
Ms. Porter	B	Graduate teaching assistant	Introductory algebra	Face-to-face & small
Mr. Reed	B	Graduate teaching assistant	Calculus I	Online synchronous & medium
Ms. Ives	A	Adjunct instructor	College algebra	Face-to-face & small
Mr. Richfelt	B	Adjunct instructor	Calculus 1	Online synchronous & large
Mr. Bergman	A	Visiting instructor	Pre-calculus	Face-to-face & large
Mr. Ramsey	A	Visiting instructor	Pre-calculus	Online asynchronous & large
Ms. Dai	B	Senior instructor	Mathematics for business & cconomics	Face-to-face & large
Mr. Elliott	B	Teaching assistant professor	Calculus for business & economics	Face-to-face & large
Ms. Jones	B	Teaching assistant professor	Calculus I	Face-to-face & large
Dr. Getz	A	Tenured professor	Intro to computer science	Online asynchronous & small

Note. *Pseudonyms were assigned

recruit instructors, with participation open to all professors and instructors of an introductory course. Participants agreed to allow a member of the research team to observe a "typical" class period and participate in a semi-structured interview about their experiences teaching CLD students. In total, five instructors at site A and eight instructors at site B participated. 12 of the 13 participants held a temporary, part-time teaching appointment.

The research team developed a four-column observation protocol to be used in all classroom and online observations. This observation protocol was adapted from existing STEM classroom observation protocols by Hora et al. (2013) and Smith et al. (2013) that classified different pedagogical techniques and student actions in two-minute increments. We then added a column focusing on the type of language the instructor used, noting any specialized vocabulary, colloquial language, or linguistic complexity, as well as a column noting any instance in which an instructor explicitly attended to any language, such as defining a word or term that was used. Before data were collected, members of the research team across the sites participated in two norming activities focused on identifying language demands within mathematics classes. One site was also able to audio record class sessions. These sessions, along with all interviews, were transcribed and de-identified. Data were collected during the Spring and Summer 2020 semesters and included classes of approximately 15 students to more than 100 students. Observed classes included both face-to-face format and synchronous and asynchronous online instruction, subsequent to the COVID-19 pandemic restrictions. In total, 13 classes were observed, documented in **Table 1**.

Data Analysis

After data were transcribed, digitalized, and de-identified, research team members independently open coded (Saldaña, 2016) their site's data, specifically attending to the language demands present in the teaching. This included identifying which language skills were required of students, technical and non-technical vocabulary used, language functions, and any slang, idiomatic language or references to culture.

Research team members at each site then discussed their open coding and engaged in an iterative process of examining and re-examining data to identify similar patterns observed across both sites. After open coding, research team members then returned to the data for more fine-grained analysis that included process coding (Saldaña, 2016). This coding focused specifically on instructional actions and processes (e.g., asking questions and diagramming) as well as the mathematical sequences represented in the instruction (e.g., solving and simplifying).

Field notes showed that members of the research team perceived some instructors to speak quickly; consequently, we also calculated the approximate rate of speech for instructors in words per second for those with audio recordings. We identified at least two continuous sections of teacher speech of more than 50 words for each instructor, measured the time it took to say each, and simply calculated an average. These sections of speech included pauses of less than two seconds that occurred between phrases or sentences as part of each instructor's natural speech patterns, but did not include extended pauses for changing topic, writing on the board, or any student questions.

FINDINGS

Language demands in the observed classes primarily required receptive language skills. Classes were lecture-based and students in both face-to-face and online settings seemed largely expected to listen to the professor speak while taking notes and visually following projected slides, graphs, depictions, or representations on the board or on the instructor's interactive whiteboard. Although lectures were common, it is important to note that no instructors preferred this format, citing lectures as "ineffective ... whether or not English is [a student's] first language" (Dr. Getz, interview, April 3). Mr. Ramsey lamented, "it was not my desire to be a traditional lecturist, it's just that in the cramped timeframes of the college schedule, you try to hit the topics you need to in the most efficient timeframe" (Mr. Ramsey, interview, April 13), and unfortunately, Mr. Bergman felt he could not "spend five minutes in these lectures waiting" for students to complete an interactive task (interview, March 11). These sentiments about instructors–especially those of larger classes–having to resort to lectures echo previous research on the negative impact of class size on active learning pedagogy (Monks & Schmidt, 2011).

Pace

The desire to cover topics quickly also seemed to influence some instructors' pace of speech. Recorded videos for online courses could be closed captioned, paused, rewound, or viewed at a slower or faster speed, making them more adjustable to a student's individual comprehension speed. Lectures in asynchronous and face-to-face classes could not, meaning that students needed to immediately comprehend the instructor's speech on the first attempt. We calculated the instructor pace of speech for four face-to-face or synchronous class sessions and found that pace of speech in these lectures varied from 2.41 to 3.67 words per second, or between 145 and 220 words per minute. Mr. Bergman, an instructor of a large face-to-face pre-calculus class had the fastest speech at 220 words per minute, nearly 65 words per minute faster than the 154 words per minute of typical English-speaking broadcast journalists (Leetaru, 2019).

That some instructors speak quickly was unsurprising, but in searching each transcript for continuous instructor speech we discovered that there was also variability with the length of stretches of continuous speech for each instructor. For example, Ms. Ives's longest recorded stretch of continuous speech was 28 seconds spanning 69 words over three clauses, whereas Mr. Bergman's longest stretch was 41 seconds spanning 154 words and 10 clauses. These long stretches of continuous speech add another dimension beyond rate of speech to the listening demands required in university mathematics classrooms.

Consensus on the effects of rate of speech on comprehensibility for second language learners has not yet been reached (Chang, 2018), but Conrad (1989) found that while native English speakers could recall nearly 100% of speech at 216 words per minute–similar to the pace of the fastest observed instructor--advanced-level non-native speakers only had 64% comprehension at that same speed. This suggests the possibility that the international students in Mr. Bergman's lecture, even with advanced level English proficiency, may have not comprehended over a third of the spoken content he delivered due to speed alone.

Student Oral Language Production

Oral language production by students in both face-to-face and synchronous online classes was exclusively voluntarily and typically required students to self-select and ask or answer a question in a whole-class setting. Ms. Campbell invited students to engage in small group work with their peers for one class activity, but students could still opt to work independently, meaning that it was quite possible for a student to remain silent for every observed class. Outside of Ms. Campbell's class, student speech occurred only in instructor-student interactions and typically took the form of asking and answering questions. Students in synchronous sessions occasionally typed questions into the online chat, but student-initiated questions in face-to-face sessions often involved students raising their hands and waiting until acknowledged by the instructor.

Required Speech

Student oral participation, though individually voluntary, was collectively required during the lesson. In other words, participation in a student-instructor exchange by at least one member of the class was expected. This participation was most often observed through one-word responses to teacher-initiated question, seen in the exchange below:

> Ms. Campbell: Zero. Now, at this stage, do I just add them all up and that's the degree of my polynomial?
>
> Student A: No.
>
> Ms. Campbell: What would be?
>
> Student A: Highest.
>
> Ms. Campbell: I would take the highest, which would be what?
>
> Student A: 11.
>
> Ms. Campbell: 11. So, the degree of this polynomial is going to be eleven, because what we do is we find the degree of each term, and we take the highest one. So to find the degree of a term, you need to add all the exponents on variables, and the degree of a polynomial you just take the highest one (March 2).

Teacher-initiated exchanges such as these were more common in classes with low enrollment but were also present in large classes with more than 100 students:

> Ms. Dai: Which property could be used to solve *log 3* to the *x* over *log 3* to the fourth?
>
> Student A: Six.
>
> Ms. Dai: There are two logarithms, so it's different from the example problem. Number nine is better (February 26).

Questioning sequences such as these in which the instructor asked a series of questions to the class often seemed to serve the purpose of moving the pedagogy forward and gauging student comprehension by eliciting steps in mathematical processes, to be immediately called out by a self-selecting student. The questions posed in Ms. Campbell's first and second turns (i.e., do I just add them all up; what would be?) and Ms. Dai's first turn (i.e., which property could be used?) focused on an aspect of the procedure, either for determining the degree of a polynomial or for calculating a logarithm. Ms. Campbell then asked for the correct answer to the problem (i.e., which would be what?), repeated the student's correct answer, then summarized the process in full. Even though Ms. Dai's student called out the incorrect answer, she provided the correct answer before continuing to solve the problem.

Similar questioning sequences, wherein the instructor would pose questions concerning the next step in a problem-solving sequence or the correct answer to a problem to be called out by willing students were seen in every face-to-face and synchronous class session observed. However, not all questions posed by the instructor expected an oral student response, as these questions also appeared in asynchronous classes without a student audience, for example, as Dr. Getz asked, "so what did I do?" (April 3) before re-explaining a process recorded in an instructor video and Mr. Ramsey asked, "how are we going to find that angle?" (April 13) to preface his problem-solving process. Similarly, some instructors, especially in large classes, used questions rhetorically, often providing their own immediate answers to move the explanation along, seen in the following two excerpts from Mr. Reed and Mr. Bergman, respectively.

Excerpt 1

> How fast are you moving at each point? You have to look at the slopes, look at implicit differentiation. We'll start with an example with a circle (Mr. Reed, May 26).

Excerpt 2

> When is cosine positive? Quads 1 and 4. So you find the one in quad 1. The other one is a full circle minus that angle, putting it in quad 4. So that's how this works for these two (Mr. Bergman, class, March 11).

This latter example from Mr. Bergman stands in contrast to a similar question later in the class, which expected an oral answer from students:

> Mr. Bergman: Where is sine root three over two?
>
> [0.3 second pause]
>
> [Several Ss call out pi over three]
>
> Mr. Bergman: Pi over three. Very good.

Taken together, these excerpts from different modalities and class sizes show the complexity surrounding the expectations of producing oral language in classes, where oral language is not individually required. These teacher-initiated questioning sequences, either answered entirely by the instructor or involving the students, seem to drive much of the instructor's process-oriented language, indicating that students need to recognize questions not only as potential expectations or opportunities to give oral responses, but also as important indicators of steps in a problem-solving sequence.

Explanations of Logic

Beyond describing and eliciting steps of mathematical processes, much instructor speech focused on explaining mathematical logic, both in online and face-to-face settings. These instances focused on reasoning and in many ways resembled the instructor thinking aloud about the thought processes and reasoning behind the procedural steps, seen in the example from Mr. Ramsey below:

> So, framing in this sense immediately gives us some key facts, which is that these angles alpha and beta here add up to 90 degrees. That's because we know the entire angle measure for a right triangle–for any triangle for that matter is 180 degrees, and since this angle is already 90 what's left is 90 degrees between both alpha and beta, and from this if we subtract alpha, we immediately see that beta is equal to 90 degrees minus alpha and likewise alpha is equal to 90 degrees minus beta (class, April 13).

Explanations of logic such as this were consistently used across all class modalities and in both large and small classes as all teachers seemed to, as Ms. Ives put it, "try to explain the why behind [a problem]" (Interview, Feb 28). The presence of this language in instruction was consistent with extant descriptions of mathematical language (Schleppegrell, 2007), but although the instructors regularly engaged in this practice, no recorded instance of students being asked to explain their own mathematical reasoning in similar detail was present in the data.

Vocabulary & Terminology

All the instructors fluently and consistently used mathematical terminology during both face-to-face and online classes. Only Ms. Campbell was observed explaining terminology during the observed classes, defining polynomial through its etymology of many (poly) and terms (nomial) before further explaining "and a term is something that has a coefficient connected to a variable, and we have a polynomial when our exponents on our variables are positive integers" (class, March 2). Although the new vocabulary term polynomial was defined, the other mathematical terms used to define it (i.e., coefficient, variable, exponents, and integers) were not, indicating the assumption that these other terms were already familiar to students. International students who have not yet taken mathematics classes in English may understand the presented concept but may be unfamiliar with the terms perceived to be previously known; however, comprehending these terms may be mediated by the instructors' frequent use of the board or screen.

Since classes largely involve instructors writing notation as they describe the problem-solving process of a problem on the board or screen, there are frequent opportunities to determine meaning from context. For example, many instructors circled, pointed, and underlined as they simultaneously wrote and spoke, thereby connecting terminology with its corresponding mathematical representation and providing opportunities for students to connect previously known concepts with new terms.

When asked about potentially challenging language for international students in classes, none of the instructors mentioned technical vocabulary. Instead, they referenced words such as pier, wattage, or bearing that often appeared for contextualization in word problems on homework, quizzes, and tests. These words, according to the instructors, could be challenging because of cultural reasons, as neither domestic nor international students, for example, may have nautical-related background knowledge. To address these concerns, Mr. Ramsey commented that when designing assessments, instructors would “try very hard to stick to stuff that’s as universal as we can make it. Everybody knows about grocery stores and parking lots, at least as much as possible” (interview, April 13).

Although instructors identified and addressed potentially confusing cultural language in written assessments, this same attentiveness to language was not seen in lesson delivery. Instructional language both in synchronous and asynchronous contexts contained cultural references (e.g., sprinklers and Ferris wheels), colloquial verbs (e.g., plug in, deal with, and fiddle around with), and idioms (e.g., this is cake and out of the gate) in addition to mathematical terminology. For example, as Ms. Ives narrated her problem-solving steps, she explained “we’re gonna put [this number] up in the numerator, so this just flips” (class, February 28), combining the term numerator with the common verbs put and flip in describing her process instead of using more mathematical-oriented terminology such as multiply the reciprocal.

Instances such as these suggest the importance of comprehending technical terminology as well as how common language is used in mathematical ways. Much of this language involved phrasal verbs, or two-word verbs, (e.g., plug in, figure out, flip up, add on, fiddle around with), which vary widely semantically based on the preposition (e.g., flip up, flip off). Furthermore, Mr. Elliot’s use of the word figure in “figure it out” (class, February 28) had a decidedly different semantic meaning than “the figure of a triangle” used in Mr. Ramsey’s class. In addition to put and flip, common verbs such as take, give, move, run, have, go, and get were regularly used, indicating the importance of understanding both mathematical terms (usually nouns) as well as more common language around those terms (usually verbs) in comprehending the mathematical processes the instructor explained.

Deixis

Fortunately, because mathematics instruction relies heavily on diagramming and visual representation, much potentially problematic language likely could be understood from context as the instructors wrote and drew as they explained their process. In these explanations, deictic language was particularly common as instructors connected a procedural action to a visual aid, such as when Ms. Ives explained exponent distribution: “So this is just taking it down, making this positive, and then now we can distribute that power” (class, February 28). In this example, Ms. Ives used this, it, this, then, and that to refer to the process by which a number appeared in the numerator, an exponent, an integer, the order of the process in question, and a different exponent.

Deictic language in instruction is not unusual or problematic itself, but it places unique language demands on students because of the linguistic and semiotic complexity of the context. Deictic referencing “may be a tool or a hindrance for learning given the potential for vague reference that it creates” (Hansen-Thomas & Langman, 2017 p. 123), and linguistically diverse students may face additional challenges as they seek to correctly interpret, where “here” is on a graph, for instance. Furthermore, students wishing to take notes in class must therefore listen for deictic language, watch the instructor’s movements, locate the deictic reference, and read the notation on the board and presentation slide while simultaneously writing any desired notes. Given the potential ambiguity of deictic language combined with the pace of instructor speech, possible student misinterpretation of procedural steps or logical connections seems quite plausible.

Summary

We found mathematical instructional language at the tertiary level largely to be consistent with the language Schleppegrell (2007) described in K-12 contexts, including the use of vocabulary, explanations of logic, and the multiple semiotic systems students must navigate. The data also showed multiple and competing language demands centered primarily around listening. All classes, whether large or small, face-to-face or online were primarily lecture-based, wherein instructors presented information and explained mathematical processes while completing a problem on the board or screen. Instructor speech varied in both speed as well as length of continuous speech between pauses, placing listening demands of both speed and quantity on students.

Speaking by individual students was not required in any of the observed face-to-face or synchronous classes, but oral participation by at least a few students in response to teacher-initiated questions was expected in each class. These teacher-initiated question sequences, in addition to student-initiated questions, were the primary opportunities for verbal participation in face-to-face and synchronous classes.

Mathematical terms were often used by instructors and largely assumed to be previously known given the infrequency of definitions provided. These mathematical terms were used in combination with colloquial speech, idiomatic language, and deixis as instructors explained and illustrated mathematical processes.

CONCLUSIONS

For those working with international or other CLD students preparing for or in academic programs, findings from this study suggest a very heavy and complex demand for myriad listening skills that need to be accounted for in EAP courses. Students attending face-to-face lectures, for example, must learn to listen to long stretches of sometimes fast-paced speech while simultaneously taking effective notes. Speed of speech is one of the most commonly-reported obstacles in second language comprehension (Gilakjani & Sabouri, 2016) and combining that demand with reading the semiotically mixed notation on the board and with writing that notation or any other notes can be quite taxing; however, these are skills that can be taught and simulated in EAP courses. For instance, the same technology used to slow recorded online video lectures can also be used to speed them up and prepare students for faster instructor speech they might encounter. Note-taking skills can be incorporated into instruction, and as hybrid and online courses become more available, tools such as closed captioning can also be used to assist listening comprehension.

To further aid student listening comprehension, we propose the creation of a glossary or inventory of commonly used mathematical terms assumed to be previously known from earlier courses such as integer, parallel, or cosine. This glossary could especially aid students who have not yet taken mathematics courses in English and, though mathematically familiar with integers, parallel lines, or cosines, may be unfamiliar with the corresponding mathematical terminology used by instructors. Such a glossary or inventory may allow international students to gain exposure to frequently used terms needed to engage in listening and conceptual understanding more fully. EAP and mathematics faculty, together with international students, might collaborate on appropriate inventories for each course. These inventories could be used to inform both EAP courses and the support resources available to international students.

There is no obvious need for instruction of oral participation in mathematics contexts given that none was required; however, we argue that raising CLD student awareness of students calling out short answers in response to teacher-initiated questions as a commonly used feature of university mathematics instruction be included as part of EAP curriculum. Although instruction of how to participate in these complex, nuanced, and optional exchanges is not necessary, EAP programs could prepare students to ask questions. Every instructor was willing to answer student questions in oral or written (i.e., email and chat) form, and giving CLD students encouragement and confidence to ask questions could be highly beneficial to them, especially for those students from cultures, where asking questions is not a readily-accepted aspect of classroom interaction.

EAP faculty and others involved in supporting international students academically also need to be aware of the breadth of language used for “academic” or “mathematical” purposes and the frequent use of language that could be considered colloquial or informal during instruction but that has specific meanings within mathematics discourse. EAP faculty need to model academic language as well as draw student attention to the alternate forms of academic expression used in academic instruction, such as the use of phrasal verbs within mathematics. Given the findings of this study and other studies related to the language of mathematics, there is also considerable justification for providing courses for international students that focus solely on helping students become familiar with the type of language and language demands that they will encounter in their undergraduate mathematics courses.

We did not find compelling oral language demands in our data, which appears inconsistent with standards and initiatives within United States K-12 education that emphasize the acquisition of reasoning, explaining, and other language-oriented skills. As several instructors noted, systematic factors such as material coverage, class size, and common, computer-graded assessments constrained pedagogy to lecture-style classes focused on listening, which therefore influenced the language demands placed on students. Language demands may indeed shift in more advanced courses, when class sizes are often smaller and taught by permanent faculty who can implement different pedagogical strategies and design original assessments and could involve participation and engagement in the broader mathematics practices and discourses Moschkovich (2015) described.

Therefore, we argue that a focus on mathematical oral language skills has an important place in EAP courses and other support resources. This oral development could involve collaborations with campus resources such as Math Learning Centers to provide opportunities to engage CLD students in a range of participation structures to produce the different types of mathematical discourse advocated for in K-12 education (Moschovich, 2013, 2015; Schleppegrell, 2007) and potentially needed in their future STEM study.

Beyond our analysis of language demands, an understanding of student perspectives of the language they encounter in mathematics classrooms is also needed. The second phase of this study examining student perspectives of mathematical language demands will complement our findings here and further guide the direction for student support in universities. Further research could explore oral language demands in advanced mathematics courses as well as language demands in other undergraduate general education courses across the curriculum.

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REFERENCES

Chang, A. C.-S. (2018). Speech rate in second language listening. In J. Liontas, & M. DelliCarpini (Eds.), *TESOL encyclopedia of English language teaching*. John Wiley and Sons. https://doi.org/10.1002/9781118784235.eelt0576

Clegg, J. (2007). Assessing the language demands of lessons taught in a second language. In F. Lorenzo, S. Casal, V. Alba-Quiñones, & P. Moore (Eds.), *Models and practice in CLIL* (pp. 113-128). Spanish Association of Applied Linguistics.

Conrad, L. (1989). The effects of time-compressed speech on native and EFL listening comprehension. *Studies in Second Language Acquisition, 11*(1), 1-16. https://doi.org/10.1017/s0272263100007804

Durand-Guerrier, V., Kazima, M., Libbrecht, P., Njomang Ngansop, J., Salekhova, L., Tuktamyshov, N., & Winsløw, C. (2015). Challenges and Opportunities for Second Language Learners in Undergraduate Mathematics. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero, & M Villavicencio Ubillús (Eds.), *Mathematics education and language diversity: The 21st ICMI study*. Springer, Cham. https://doi.org/10.1007/978-3-319-14511-2_5

Gilakjani, A. P., & Sabouri, N. B. (2016). Learners' listening comprehension difficulties in English language learning: A literature review. *English Language Teaching, 9*(6), 123-133. https://doi.org/10.5539/elt.v9n6p123

Halliday, M. A. K. (1978). *Language as social semiotic*. Edward Arnold.

Hansen-Thomas, H., & Langman, J. (2017). Deictics and the construction of mathematics and science knowledge in the secondary school classroom. *Classroom Discourse, 8*(2), 122-138. https://doi.org/10.1080/19463014.2017.1328698

Hora, M. T., Oleson, A., & Ferrare, J. J. (2013). Teaching dimensions observation protocol (TDOP) user's manual. *Wisconsin Center for Education Research, University of Wisconsin–Madison*. http://tdop.wceruw.org/Document/TDOP-Users-Guide.pdf

Hughes, E. M., & Powell, S. R. (2020) Mathematics is more than numbers: Forward to the special issue. *Reading & Writing Quarterly, 36*(2), 81-83. https://doi.org/10.1080/10573569.2020.1733236

International Institute of Education. (2020). Open doors fast facts 2020: International students in the United States. *Opendoors*. https://opendoorsdata.org/fast_facts/fast-facts-2020/

Israel, E., & Batalova, J. (2021). International students in the United States. *Migration Policy Institute*. https://www.migrationpolicy.org/article/international-students-united-states-2020#enrollment_numbers_trends

Kersaint, G., Thompson, D. R., & Petkova, M. (2009). *Teaching mathematics to English language learners*. Routledge. https://doi.org/10.4324/9780203894521

Leetaru, K. (2019). How fast do people speak on television news? *Forbes*. https://www.forbes.com/sites/kalevleetaru/2019/08/07/how-fast-do-people-speak-on-television-news/

Lucas, T., Villegas, A. M., & Freedson-Gonzalez, M. (2008). Linguistically responsive teacher education: Preparing classroom teachers to teach English language learners. *Journal of Teacher Education, 59*(4), 361-373. https://doi.org/10.1177/0022487108322110

Monks, J., & Schmidt, R. M. (2011). The impact of class size on outcomes in higher education. *The B. E. Journal of Economic Analysis & Policy, 11*(1), 62. https://doi.org/10.2202/1935-1682.2803

Moschkovich, J. N. (2007). Examining mathematical discourse practices. *For the Learning of Mathematics, 27*(1), 24-30. https://www.jstor.org/stable/40248556

Moschkovich, J. N. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1), 45-57. https://doi.org/10.21423/jume-v6i1a204

Moschkovich, J. N. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior, 40*, 43-62. https://doi.org/10.1016/j.jmathb.2015.01.005

NCES. (2020a). English language learners in public schools. *National Center for Education Statistics*. https://nces.ed.gov/programs/coe/indicator/cgf

NCES. (2020b). Table 29.10. High school graduates by sex and control of school. Digest of education statistics. *National Center for Education Statistics*. https://nces.ed.gov/programs/digest/d20/tables/dt20_219.10.asp

Saldaña, J. (2016). *The coding manual for qualitative researchers*. SAGE.

Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective*. Lawrence Erlbaum. https://doi.org/10.4324/9781410610317

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139-159. https://doi.org/10.1080/10573560601158461

Smith, M. K., Jones, F. H., Gilbert, S. L., & Wieman, C. E. (2013). The classroom observation protocol for undergraduate STEM (COPUS): A new instrument to characterize university STEM classroom practices. *CBE–Life Sciences Education, 12*(4), 618-627. https://doi.org/10.1187/cbe.13-08-0154

Uccelli, P., Galloway, E. P., Barr, C. D., Meneses, A., & Dobbs, C. L. (2015) Beyond vocabulary: Exploring cross-disciplinary academic-language proficiency and its association with reading comprehension. *Reading Research Quarterly, 50*(3), 337-356. https://doi.org/10.1002/rrq.104