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Knowledge profiles on numbers for teaching by primary education pedagogy students

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ABSTRACT

The study investigates the knowledge profiles of elementary pre-service teachers (PST) concerning numbers and operations before their professional classroom practice. By validating an instrument through exploratory and confirmatory factor analyses, it identifies and categorizes the PST' performance into distinct profiles based on the mathematical knowledge for teaching model. The results reveal that PST demonstrate varying levels of content and pedagogical knowledge. Those with high pedagogical knowledge effectively anticipate student difficulties and employ suitable teaching strategies. In contrast, future teachers with intermediate knowledge can identify errors and explain basic concepts but struggle with advanced strategies. Elementary PST with low pedagogical knowledge face significant challenges in anticipating learning issues and lack effective teaching methods. The study emphasizes the necessity for personalized teacher training to ensure comprehensive and specialized mathematical knowledge that aligns with professional standards, crucial for improving educational outcomes in mathematics.

Keywords: mathematical knowledge for teaching, pedagogical content knowledge, teacher education, numbers

INTRODUCTION

For over three decades, understanding the nature of mathematics teacher knowledge has been a topic of great interest and effort among researchers in mathematics education. Various conceptualizations and models have been developed, allowing this cognitive aspect of teachers' dispositions to be studied (Amador & Weston, 2024; Ball et al., 2008; Blömeke et al., 2015; Louie & Zhan, 2022; Rowland et al., 2005; Shulman, 1986). The mathematical knowledge for teaching (MKT) model, developed by Ball et al. (2008), proposes a structure of six subdomains that expands Shulman's (1986) approach to content knowledge and pedagogical content knowledge (PCK). The MKT model has served as a framework for numerous studies focused on analyzing its nature, measuring this knowledge, and evaluating its impact on student learning. While there is no consensus on the extent and complexity of the relationship, recent studies have shown that teachers' knowledge positively predicts student achievements in various assessments (Charalambous et al., 2020; Chen et al., 2020; Jan et al., 2023; Warrah et al., 2020). In this context, research has highlighted that teachers with a high level of MKT not only possess a deep understanding of mathematical concepts but also can translate this knowledge into pedagogical practices that significantly enhance student learning (Ekmekci & Serrano, 2022; Hill et al., 2008; König et al., 2021; Mello Román & Hernández Estrada, 2019). Specialized knowledge allows teachers to design and execute more effective lessons, use advanced pedagogical strategies, and foster a classroom environment that promotes critical thinking and problem-solving among students (Charalambous, 2016; Phelps et al., 2020). It is therefore vital that pre-service teachers (PST) develop an MKT that they can efficiently utilize in instructional situations, thereby advancing the quality of teaching and educational outcomes in mathematics (Santagata & Lee, 2021).

The specific characteristics of knowledge that a graduate from a pedagogy program in Chile should achieve have been established in the standards of the teaching profession, recognizing the need for this knowledge to be specialized in the sense proposed by Ball et al. (2008) (Ministerio de Educación de Chile [Chilean Ministry of Education] [MINEDUC], 2021). Under law 20.903, initial teacher training programs have the responsibility to provide opportunities for deepening and extending the mathematical knowledge acquired during the school stage, in addition to developing types of knowledge that are exclusive to teaching and learning (Gaona et al., 2024). Consequently, they must establish instructional trajectories that consider the starting points of their students and allow them, through various milestones, to achieve MKT that meets the required standards (Godoy et al., 2021). Training institutions implement mandatory diagnostic assessments in pedagogy careers, which primarily focus on basic mathematical skills, competencies, and knowledge (Giaconi et al., 2022). Therefore, the information that training programs have about the MKT profile of PST is often scarce or non-existent (Gaona et al., 2024). This makes it difficult to make effective decisions in training future teachers, limiting and conditioning learning opportunities (Muhtarom et al., 2019; Verdugo Peñaloza et al., 2021).

Although the literature reports instruments for measuring the MKT of pedagogy students, it also highlights the need for these instruments to adapt to the curricular and cultural characteristics of each country (Hambleton & Zenisky, 2011; Saadati et al., 2024). In this regard, this article seeks to contribute to the discussion about the characteristics of future teachers' knowledge and how their response patterns are grouped in specific instructional and mathematical situations in the area of numbers and operations, using MKT as an explanatory model. To address these issues, the following research questions are formulated:

1. How are PSTs' responses clustered when responding to an instrument based on the MKT model in the area of numbers?
2. What are the profiles of PST based on the dimensions of the MKT model in the numbers area?

Answering these questions will provide information on the composition of knowledge for teaching in the area of numbers for primary PST in Chile, considering their response patterns to various items designed for the six subdomains of the MKT model and in relation to the topics of the Chilean national curriculum. Furthermore, knowing the profiles of PST according to the MKT model provides valuable information for decision-making regarding their training trajectory, ensuring that it allows them to achieve the MKT established in the standards of the teaching profession. On the other hand, the adaptation and construction of items to measure the MKT in the area of numbers is a significant contribution to the field and the Chilean context.

THEORETICAL FRAMEWORK

Mathematical Knowledge for Teaching

A significant focus in mathematics education is the knowledge that a mathematics teacher should have for their teaching practice (Ball et al., 2008; Blömeke et al., 2015; Carrillo-Yañez et al., 2018; Charalambous et al., 2020; Hill, 2010; Ribeiro et al., 2022). One of the first researchers to conceptualize teacher knowledge for teaching was Shulman (1986), who proposed the distinction between two domains: content knowledge and PCK. Shulman (1986) noted that within the content knowledge

> "it is not only necessary to understand that something is so; the teacher must also understand why it is so, on what basis its justification can be asserted, and under what circumstances our belief in its justification may weaken or even be denied" (p. 9).

Shulman (1987) described PCK as a "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8).

Ball et al. (2008) argue that the issues identified by Shulman (1986, 1987) are key for research on teaching and teacher education. In this regard, various models have been proposed that address the characterization of the knowledge a mathematics teacher needs to enhance their students' learning. For example, the Didactical-Mathematical Knowledge model proposed by Godino (2009), which interprets and characterizes teachers' knowledge from three dimensions: the mathematical dimension, the didactic dimension, and the meta-didactic-mathematical dimension. On the other hand, Carrillo-Yañez et al. (2013) propose the specialized mathematical knowledge for teaching model. This model suggests that all knowledge used by the teacher is specialized, dividing it into two major domains: the mathematical domain and the didactic content knowledge, which are further subdivided into three subdomains each. It also considers, at the center, beliefs about mathematics and its teaching and learning. Other researchers, such as Schoenfeld and Kilpatrick (2008), propose the proficiency theory model, which distinguishes seven dimensions: knowing school mathematics in depth and breadth; understanding students as thinking individuals; understanding students as learners; designing and managing learning environments; developing classroom norms and supporting classroom discourse as part of "teaching for understanding"; building relationships that support learning; and reflecting on one's own practice.

The MKT model, proposed by Ball et al. (2008), has been one of the most utilized in research on the mathematical knowledge required for effective teaching (Buschang et al., 2012; Charalambous, 2016; Delaney, 2012; Hiebert et al., 2019; Hill et al., 2008; Krauss et al., 2008). The model organizes the necessary knowledge to carry out the work of teaching mathematics. In addition to being based on Shulman (1986), this model emerges directly from the study and analysis of teacher practice, distinguishing between the different types of knowledge needed for teaching mathematics. It proposes content knowledge divided into three subdomains:

1. Common content knowledge (CCK), which refers to the mathematical knowledge and skills that other professionals also use in their professional work, i.e., CCK is not exclusive to teaching. For example, in response to the question "What is 0/7?", others who know mathematics might answer correctly; accountants need to calculate and reconcile numbers but do not need to explain why, or when multiplying by 10 people might describe it as "adding a zero" (Ball et al., 2008).
2. Specialized content knowledge (SCK), which refers to the mathematical knowledge and skills unique to teachers. Teachers need to respond to students' questions about the "why"; how to explain and justify mathematical ideas (e.g., why you invert and multiply to divide fractions).
3. Horizon knowledge, which refers to the knowledge of the trajectory of a mathematical content throughout various educational stages, as well as intra- and extra-mathematical connections (Sosa & Carrillo-Yañez, 2010).

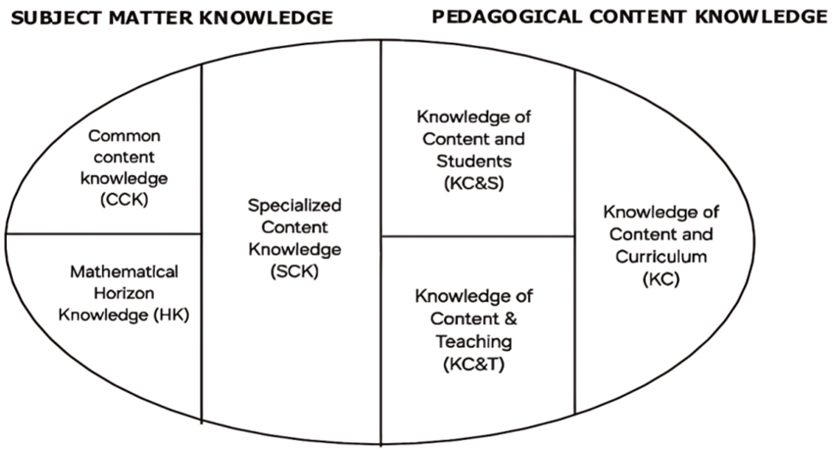

Figure 1. Domain and subdomains of the MKT model (adapted from Ball et al., 2008, p. 403)

On the other hand, PCK is also divided into three subdomains:

1. Knowledge of content and students (KC&S), related to knowledge that combines understanding of students and mathematics. Teachers must anticipate student responses and difficulties in tasks. When choosing an example, teachers must predict what students will find interesting and motivating. They must be able to interpret their students' reasoning (Ball et al., 2008).
2. Knowledge of content and teaching (KC&T), which combines knowledge about teaching and mathematics. Mathematical teaching tasks require mathematical knowledge for their design and sequencing. Teachers choose which example to start with and which examples to use to deepen their students' understanding, evaluating the advantages and disadvantages of the representations they use to teach a concept and identifying what the different methods and procedures offer for instruction. For example, knowing different viable instructional models for place value (Ball et al., 2008).
3. Knowledge of the curriculum (KC), see **Figure 1**, which refers to knowledge about the level of the courses to which the contents belong, the curricula, and curricular materials (Shulman, 1986).

The study of the mathematical knowledge necessary for both PST and in-service teachers is fundamental for the development of training programs; however, this task presents significant complexities. Various studies have implemented a variety of tools to assess this knowledge. For example, Gutiérrez et al. (2016) analyzed the mathematical knowledge of future primary school teachers in Spain, using both closed and open questionnaires from the international "teacher education and development study in mathematics" (TEDS-M). The results were classified according to the type of numerical problem, the curricular level of the mathematical content, and the cognitive domain evaluated in each question. In another context, Ball et al. (2008) from the "learning mathematics for teaching" (LMT) project at the University of Michigan developed specific items to assess the mathematical knowledge of primary school teachers. These items have been subsequently adapted and used in various educational research, such as the work of Charalambous et al. (2020), who evaluated teachers' knowledge of advanced common content (aCCK) and SCK using items from the Massachusetts test for educator licensure (MTEL) for primary and the LMT project form. Additionally, the study by Hill et al. (2005) at the University of California, Los Angeles, highlighted the importance of a deep understanding of mathematical concepts in teaching practice. This study developed a series of assessments measuring teachers' ability to connect mathematical concepts with effective pedagogical methods.

In more recent projects, work is being done on updating and expanding the tools for assessing mathematical knowledge. For example, the "mathematical knowledge for teaching measures: Refreshing the item pool" project at Harvard University seeks to better align assessment instruments with current standards and develop a more robust and flexible online survey platform (Harvard University, n. d.). Likewise, the TASK project and the CADRE initiative are innovating in the creation and improvement of items and in adapting the delivery systems of assessments, to meet the current needs of educational environments and curricular standards (Harvard University, n. d.).

In the domain of numbers and operations, Gutiérrez et al.'s (2016) study evaluated the mathematical knowledge of future primary school teachers in Spain, with a specific focus on numbers and operations. The findings revealed that, although the participants demonstrated an adequate grasp of basic content from primary and early secondary education, they struggled with more complex concepts such as ratio, proportion, percentage, and the interpretation of simple fraction subtractions in word problems. While they understood the density of rational numbers, their preparation in advanced mathematics was found to be inadequate. Similarly, in Chile, Pincheira et al. (2023) identified that 40 PST exhibited limited knowledge when designing mathematical tasks related to patterns, addressing only partial aspects of subject knowledge and pedagogical content. This finding aligns with de Almeida et al. (2021) study, which also emphasized the need to enhance the mathematical training of future educators to effectively meet curricular demands.

Further complementing these findings, Parra-Fica et al. (2020) observed that the self-study manuals created by future teachers predominantly focused on the first two years of primary education, featuring largely decontextualized activities aimed at counting up to 1,000. This trend mirrors the observations of Depaepe et al. (2015), who reported that PST' mathematical knowledge was conceptually limited, particularly in the area of fractions. These results are consistent with the international comparative study TEDS-M, which ranked Chile near the bottom in initial teacher education in mathematics (Tatto et al., 2012).

Additionally, the INICIA assessment (MINEDUC, 2014) revealed significant gaps in both disciplinary and pedagogical knowledge among graduates of primary education programs. Lo (2020) further illustrated that while future primary teachers possessed adequate general content knowledge, they demonstrated significant deficiencies in their understanding of both content and students, underscoring the need to strengthen these areas in teacher training. This issue is closely related to the observations of Zakaryan and Ribeiro (2016), who highlighted the decline of effective practices in teaching rational numbers during initial teacher training in Chile, suggesting substantial improvements are needed. Finally, Ruz et al. (2023) conducted a study on teachers' didactic-mathematical knowledge, emphasizing the importance of developing deep and specialized knowledge that enables future educators to effectively address the challenges their students face in learning mathematics.

METHODOLOGY

This research has two primary aims: to validate specific knowledge profiles essential for teaching numbers in Chilean primary education (basic education) and to explore these profiles among prospective teachers. A quantitative, descriptive, and relational methodology was chosen to achieve these aims. This approach allows for both a description of prospective teachers' current numerical competencies and an examination of the relationships between these competencies and variables related to their initial mathematical training.

Sample

The study population consists of PST from two universities in Chile, both with initial teacher training programs for Basic Education. The selected sample is composed of 116 PST, who had already completed courses on the teaching and learning of numbers and their operations, providing them with a relevant theoretical and practical base for the study. Participation was voluntary, which determined the use of a purposeful and non-probabilistic sampling for the selection of participants. In addition, the corresponding authorizations were obtained from the authorities of both training programs, ensuring the institutional support necessary for the execution of the study. Before beginning data collection, each participant signed an informed consent, which was rigorously reviewed and approved by the Ethics Committee of Universidad de Chile. This process ensures compliance with the necessary ethical standards and reinforces the methodological validity of the research.

Procedure

The assessment was conducted remotely during the first semester of 2021. A link was distributed via institutional email to each student, which directly redirected to the online questionnaire. This email included details about the purpose of the research and the importance of the study. The average duration to complete the questionnaire was 51 minutes (DS = 5.8 minutes). The responses collected at the end of the questionnaire formed the database used for subsequent analyses.

Instrument Development

For the development and construction of the assessment instruments, an exhaustive analysis of the programs of the subjects related to numbers or the teaching of numbers at the two participating institutions was started. Using the constant comparison technique, a central method in grounded theory according to Strauss and Corbing (2002), a panel of experts was formed consisting of six academic teacher trainers, each with at least five years of experience (Fram, 2013).

The Delphi methodology was employed by the panel to reach consensus on the curricular objectives addressed by the subjects (Hasson et al., 2000). This iterative process consisted of at least five cycles and continued until data saturation was achieved, culminating in the development of a specification matrix that provided the foundation for designing evaluation instruments.

The development of multiple-choice items with non-unique selection options by the panel was undertaken on the basis of the aforementioned foundation. The purpose of this development was to ensure that each item accurately reflected the curricular objectives that had been previously agreed upon. The items were then subjected to review by teacher trainers from the same institutions as the students in the sample. The purpose of this review was to ensure contextual relevance and appropriateness. The trainers evaluated the items for clarity and relevance, and their feedback informed the final adjustments, thus ensuring that the instruments met both pedagogical and contextual requirements.

In both processes and to ensure expert agreement, Aiken's (1985) coefficients were calculated, yielding an average value of 0.87. Items scoring below the threshold of 0.75 were discarded to uphold high standards of quality and accuracy in the evaluation measures, ensuring a reliable alignment with the established evaluation objectives. Following a rigorous validation process, this matrix was condensed and presented in **Table 1**.

Data Analysis

Data analyses were conducted in five stages, employing various statistical techniques:

1. First stage: Content validity was assessed through expert evaluation. This step verified the validity of the relationship between the items and the dimensions of MKT, as well as the contents established in the selected and adjusted school curriculum through the specification matrix.
2. Second stage: In order to investigate the dimensionality of the proposed measurement instrument, exploratory factor analysis (EFA) was employed. This statistical technique examines the interrelationships among observed variables to identify latent constructs (factors) that account for the observed covariation. Items exhibiting high loadings on the same factor are considered to measure a common underlying dimension.

Table 1. Condensed matrix

Level	Category and objectives	MCK			PCK		
		CCK	HK	SCK	KC&S	KC&T	KC
	Comprehension of place value						
Third	LO5. Identify and describe units, tens, and hundreds in numbers from 0 to 1,000, representing quantities according to their place value, using concrete, pictorial, and symbolic materials.	P12					
	Comprehension and representation of fractions						
Fourth	LO8. Demonstrate understanding of fractions* with denominators 100, 12, 10, 8, 6, 5, 4, 3, and 2: Explaining a fraction represents part of a whole or group and a place on the number line • Describing situations for using fractions • Showing fractions can have different representations						P5
	Unit fractions						
Sixth	LO5. Demonstrate understanding of fractions and mixed numbers • Representing these numbers on the number line	P15					
	Comparison of fractions						
Fifth	LO7. Demonstrate understanding of proper fractions • Comparing proper fractions with equal and different denominators concretely, pictorially, and symbolically				P14		
	Problem-solving						
Third	LO9. Demonstrate understanding of division within up to 10 × 10 tables • Creating and solving problems involving distribution and grouping						P33
Seventh	LO3. Solve problems involving multiplication and division of fractions and positive decimals concretely, pictorially, and symbolically (manually and/or with educational software)	P19					
	Counting						
First	LO1. Count from 0 to 100 by 1s, 2s, 5s, and 10s, forward and backward, starting from any number below 100						P24
	Reading and writing numbers/representation of numbers						
Fourth	LO1. Represent and describe numbers from 0 to 10,000 • Reading and writing them • Representing them concretely, pictorially, and symbolically						P28
	Meaning and representation of multiplication and division						
Fourth	LO5. Demonstrate understanding of multiplying three-digit numbers by one-digit numbers: • Using strategies with or without concrete materials • Using multiplication tables • Estimating products • Using the distributive property of multiplication over addition						P29
	Properties of operations: Inverse operations						
Fourth	LO6. Demonstrate understanding of division with two-digit dividends and one-digit divisors • Utilizing the relationship between division and multiplication						P32

3. Third stage: Having identified factors through EFA, confirmatory factor analysis (CFA) was employed to rigorously test the hypothesized factor structure. CFA examined the extent to which observed variables (scale items) loaded onto the predefined latent factors, thus confirming the underlying relationships and dimensionality. This involved assessing the fit between the observed covariance matrix and the covariance matrix implied by the model. A good fit provided evidence for the robustness of the identified dimensionality and strong support for the instrument's construct validity.
4. Fourth stage: Reliability analysis was conducted to assess the internal consistency of each latent variable. This analysis enabled the determination of the extent to which items within each scale measured the same underlying construct. Cronbach's alpha was used as the primary measure.
5. Fifth stage: In conclusion, latent class analysis (LCA) was utilized to identify discrete latent profiles of performance within the framework for mathematics instruction, encompassing both disciplinary areas and dimensions. This statistical technique categorizes individuals into mutually exclusive latent classes based on their observed response patterns, thereby unveiling heterogeneity in performance related to the specified dimensions and areas of focus.

Model fit for the EFA and CFA was evaluated using several indices. Absolute fit was assessed using the Chi-square statistic (χ^2) and root mean square error of approximation (RMSEA), while incremental fit was assessed using the Tucker-Lewis index (TLI) and comparative fit index (CFI). The Bayesian information criterion (BIC) was also considered. Following Hu and Bentler (1999), TLI and CFI values above .90 indicated acceptable fit, and values above .95 indicated excellent fit. RMSEA values below .05 indicated good fit, and values between .05 and .08 indicated reasonable fit (Schermelleh-Engel et al., 2003).

RESULTS

Reliability Relationship of Test Items

Initially, a reliability analysis of the test was conducted using Cronbach's alpha coefficient, which reached a value of .84 for the entire test (see **Table 2**).

Table 2. Cronbach's alpha reliability coefficients and item-test correlation

Item	Alpha	r.cor	Item	Alpha	r.cor	Item	Alpha	r.cor
P1	.84	.138	P19_D	.84	.123	P28_C	.83	.555
P2	.83	.302	P19_E	.84	.164	P28_D	.83	.343
P3	.83	.333	P20	.84	.147	P28_E	.83	.521
P4	.84	.185	P21	.84	.137	P29_A	.83	.327
P5	.83	.419	P22	.83	.315	P29_B	.83	.309
P6	.84	.057	P23_A	.84	.233	P29_C	.83	.527
P7	.83	.261	P23_B	.83	.334	P29_D	.83	.545
P8	.84	.088	P23_C	.83	.398	P29_E	.84	.080
P9	.84	-.037	P23_D	.84	.079	P30	.83	.325
P10	.84	.009	P23_E	.83	.285	P31	.83	.386
P11	.84	.162	P24_A	.83	.530	P32_A	.83	.551
P12_A	.83	.363	P24_B	.84	.208	P32_B	.84	.208
P12_B	.83	.374	P24_C	.84	-.067	P32_C	.83	.461
P12_C	.83	.312	P24_D	.83	.312	P32_D	.83	.504
P12_D	.84	.061	P24_E	.83	.369	P32_E	.83	.371
P12_E	.83	.277	P25	.84	.206	P33_A	.83	.514
P13	.83	.382	P26_A	.83	.273	P33_B	.83	.600
P14	.83	.433	P26_B	.84	.101	P33_C	.84	.020
P15	.83	.435	P26_C	.84	.209	P33_D	.84	.010
P16	.83	.310	P26_D	.83	.244	P33_E	.83	.596
P17	.84	.203	P26_E	.83	.272	P24_C	.84	-.067
P18	.83	.357	P27	.84	-.061	P24_D	.83	.312
P19_A	.83	.242	P28_A	.83	.278	P24_E	.83	.369
P19_B	.84	.187	P28_B	.83	.525	P25	.84	.206
P19_C	.84	.143	P24_C	.84	-.067	P26_A	.83	.273

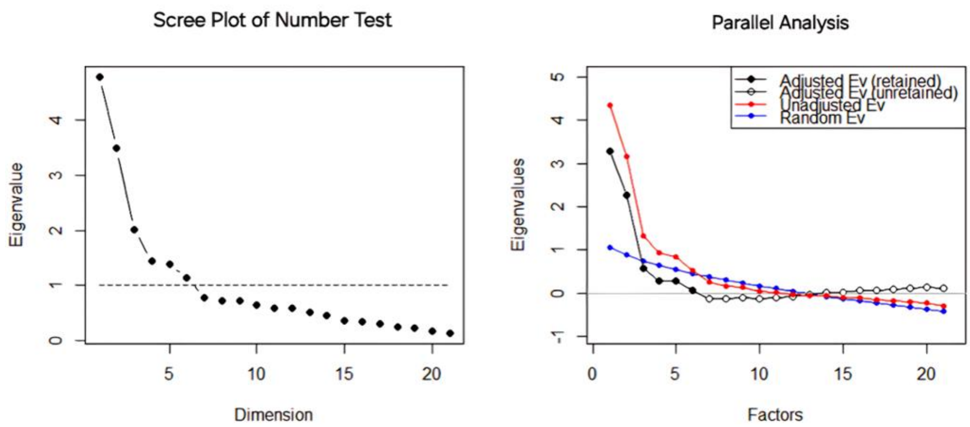

Figure 2. Scree plot and parallel analysis performed on the correlation matrix (Source: Authors' own elaboration)

This value is considered very good according to the standards established by Novick and Lewis (1967). However, specific items (P6, P8, P9, P10, P12_D, P23_D, P24_C, P26_B, P27, P29_E, P33_C, and P33_D) were identified that had a low correlation with the total test, with coefficients less than or equal to < .10. The removal of these items resulted in an increase in the Cronbach's alpha coefficient to .86, indicating an improvement in the internal consistency of the test. With the remaining items, a factor analysis will be conducted to evaluate the underlying structure of the dimensions of mathematical knowledge.

Exploratory Factor Analysis

The Kaiser-Meyer-Olkin (KMO) index of sampling adequacy and the Bartlett's test of sphericity (significant at 99%) for the sample meet the minimum necessary limits, allowing for the performance of a factor analysis (KMO = .54; Bartlett χ^2 [N = 116, gl = 1,596] = 3,194.998, $p < .001$) (Tabachnick et al., 2007). Initially, an EFA was conducted using the maximum likelihood (ML) estimation method and Varimax rotation. The number of factors to retain was determined using Cattell's scree test and parallel analysis. The scree test suggested retaining eight factors, while the parallel analysis indicated the retention of twelve factors; however, four of these factors explained low variability. Therefore, it was decided to analyze an eight-factor model. The fit indices for the sample were considered low (χ^2 [N = 116, gl = 1,168] = 3,195, $p < .25$; TLI = .673; RMSEA = .049). Subsequently, items with factor loadings less than .40 or that exhibited cross-loadings were removed, resulting in the elimination of 36 items. Afterward, both the scree test and the parallel analysis suggested retaining five and six factors, respectively, for the reduced test of 21 items (see **Figure 2**). A new calculation of estimates for five and six factors was performed using the 21-item base (see **Table 3**). It was observed that in the six-factor model, the TLI index was overestimated (1.014), and one of the factors only explained 4% of the total variance, suggesting that this model be discarded. On the other hand, the five-factor model showed acceptable levels of fit, explaining 51% of the accumulated variance.

Table 3. Summary of estimates for five- and six-factor extractions for 21 items

Factors	Proportion of variance explained by factor								
	χ^2 (df)	RMSEA	TLI	MR2	MR1	MR5	MR4	MR3	ML6
5	1,027.29 (115)	.038	.952	.15	.11	.11	.07	.07	–
6	1,027.29 (99)	.000	1.014	.15	.11	.04	.10	.06	.08

Table 4. Factor loadings of items for the five-factor model

Topic	χ^2 (df)	CFI	TLI	RMSEA	Item	Factor loading
NUM1	5.883 (5)	.997	.994	.039	P5	.363***
					P12_A	.754***
					P12_B	.912***
					P12_C	.771***
					P12_E	.901***
NUM2	1.252 (2)	1.000	1.012	.000	P14	.344***
					P33_A	.728***
					P33_B	.879***
					P33_E	.877***
NUM3	0.255 (2)	1.000	1.043	.000	P24_E	.512***
					P28_B	.732***
					P28_C	.793***
					P28_E	.699***
NUM4	11.121 (5)	.890	.780	.103	P15	.512**
					P19_A	.579**
					P19_D	.607**
					P19_E	.415**
					P29_B	.359**
NUM5	0.000 (0)	1.000	1.000	.000	P32_A	.721***
					P32_C	.563***
					P32_D	.810***

Note. *p < .05; **p < .01; & ***p < .001

Confirmatory Factor Analysis

Estimations were conducted for the retained five-factor model using the ML extraction method. The complete test showed levels of fit considered as regular (χ^2 [N = 116, df = 189] = 342.055, p < .001; CFI = .830; TLI = .811; RMSEA = .084). Consequently, the test was composed of 21 items, distributed across five latent constructs. The final factor loadings of the items are presented in **Table 4**. **Table 4** presents the results from the CFA for the five topics (NUM1 to NUM5). Fit indices for models NUM1, NUM2, NUM3, and NUM5 are excellent, with CFI and TLI values close to or equal to 1 and RMSEA indicating a good fit (low to nil values). Specifically, NUM1 shows good fit with χ^2 (5) = 5.883, CFI = .997, TLI = .994, and RMSEA = .039; items P12_B and P12_E have the highest factor loadings, representing well the understanding of the place value of natural numbers and the concept of fractions. NUM2 also displays a perfect fit with χ^2 (2) = 1.252, CFI = 1.000, TLI = 1.012, and RMSEA = .000; items P33_B and P33_E are the most representative, related to the comparison of proper fractions and solving division problems of natural numbers. NUM3 shows perfect fit indices with χ^2 (2) = 0.255, CFI = 1.000, TLI = 1.043, and RMSEA = .000, with item P28_C standing out. NUM5 presents a perfect fit with χ^2 (0) = 0, CFI = 1.000, TLI = 1.000, and RMSEA = .000; item P32_D is the most representative, related to understanding division of numbers with two-digit dividends and one-digit divisors. However, the model for topic NUM4 shows less adequate fit with χ^2 (5) = 11.121, CFI = .890, TLI = .780, and RMSEA = .103; the items in this group relate to the multiplication and division of fractions. Overall, the test shows that the topics have items with significant factor loadings (*p < .05, **p < .01, ***p < .001), although some items in NUM4 contribute less to the factor.

Internal Consistency

The complete test shows internal consistency with a Cronbach's alpha of .80, with reliability values ranging from .76 (underestimation) to .92 (overestimation). The NUM1 dimension shows the highest reliability, with a range between .68 and .86. The reliability of the NUM2 dimension varies from .59 to .81. The NUM3 dimension has a reliability that fluctuates between .58 and .79. On the other hand, the NUM4 dimension shows a reliability range from .49 to .66. Finally, the NUM5 dimension presents the lowest internal consistency, with reliability values ranging from .49 to .62.

Performance Profiles by School Disciplinary Area

From the LCA to the correct responses of the items remaining after validation, the two-class model achieves the best fits for each of the factors, and the indices and probabilities of belonging to each class are observed in **Table 5**. The probability of belonging to class 1 is 71%, and to the second class is 29%.

Class 1, comprising 71% of students, shows high probabilities (greater than .5) of correctly answering most of the items, except for P19_E and P29_B, which are related to the division of whole numbers by fractions and understanding the multiplication algorithm of three-digit numbers by one-digit. In contrast, class 2, comprising 29% of the PST, shows low probabilities of correct responses (less than .5) for all items. Specifically, in class 1, items in NUM1 and NUM2 elicited high probabilities of correct

Table 5. Performance profiles by school disciplinary area

Topic	Item	AIC	BIC	G2	χ^2 (df)	Probility class 1 (71%)	Probility class 2 (29%)	Difference class 1 & class 2
NUM1	P5	523.5	553.8	19.27	23.5 (2)	.74	.38	.36***
	P12_A					.88	.09	.79***
	P12_B					.98	.07	.91***
	P12_C					.93	.16	.77***
	P12_E					.94	.02	.92***
						82%	18%	64%
NUM2	P14	410.7	435.5	5.58	3.6 (6)	.66	.19	.47***
	P33_A					.91	.09	.82***
	P33_B					.94	0	.94***
	P33_E					.96	.04	.92***
						87%	17%	60%
NUM3	P24_E	343.8	368.6	3.74	3.2 (6)	.92	.40	.52***
	P28_B					.95	.19	.76***
	P28_C					.94	.04	.90***
	P28_E					.92	.13	.79***
						32%	68%	-36%
NUM4	P15	591.0	621.2	28.77	46.3 (20)	.95	.35	.60***
	P19_A					.78	.12	.66***
	P19_D					.62	.07	.55***
	P19_E					.36	.09	.27***
	P29_B					.24	.06	.18***
						71%	29%	42%
NUM5	P32_A	364.6	383.9	2.91	2.9 (0)	.99	.35	.64***
	P32_C					.86	.30	.56***
	P32_D					.89	.05	.84***
						87%	32%	55%

Note. *p < .05; **p < .01; & ***p < .001

Table 6. Performance profiles regarding the mathematical content knowledge dimension

MKT dimension	MKT subdimension	Items	Probility class 1 (54%)	Probility class 2 (29%)	Probility class 3 (16%)
MCK	CCK	P12_A	.86	.12	.91
		P12_B	.97	.09	1
		P12_C	.93	.14	.94
		P12_E	.93	0	1
		P15	.45	.44	1
		P19_A	.13	.33	1
		P19_D	.07	.26	.77
		P19_E	.09	.23	.35
	SCK	P14	.57	.38	.99

Note. MCK: Mathematical content knowledge; CC: Common content; & SC: Specialized content

responses (0.74-0.98 and 0.66-0.96, respectively), indicating strong comprehension of place value in the decimal number system, the ability to identify and model fractions, and proficiency in solving division problems within mathematical tasks. In NUM3, probabilities are also high in class 1 (.92 to .95), signifying adept capabilities in identifying and fostering the enumeration and representation of natural numbers. It is noted that the NUM3 factor has a lower proportion of future teachers in class 1 than in class 2, indicating a greater challenge in this topic. Despite the reduced consistency exhibited by NUM4, the items retain a moderate probability of falling within class 1 (.36 to .95). The items pertain to the comprehension of fractions and mixed numbers, the resolution of problems involving the multiplication and division of fractions and positive decimals, and the understanding of multiplying three-digit numbers by one-digit numbers. NUM5 demonstrates exceedingly high probabilities (.86 to .99), indicating a commendable comprehension of dividing numbers with two-digit dividends by a single-digit divisor.

Performance Profiles Regarding the Framework of Mathematical Knowledge for Teaching

From the LCA to the correct responses of the items that remained after validation and grouped by the dimensions of MKT, the three-class model achieves the best fits for the items of the first sub-dimension (BIC = 1,127.43; AIC = 1,047.57; G2 = 166.59; χ^2 [87, n = 114] = 560.96). The probability of belonging to class 1 is 54%, to the second is 29%, and to the third is 16%.

Based on the information from **Table 6**, the results of **Table 7** are interpreted, as follows: PST in class 1 have a solid understanding of concepts such as the place value of natural numbers and fractions (P12), understanding and representation of fractions (P15), and comparison of proper fractions (P14). However, they show difficulties in solving complex problems related to the multiplication and division of fractions (P19). Students in class 2 need significant reinforcement in all topics, as they do not show probabilities greater than .5 on any item. Class 3, although the smallest, demonstrates high performance in all topics, excelling in advanced understanding of fractions and solving complex problems. In particular, only the NUM4 (items P15 and P19) factor has a lower proportion of students in class 1 than in class 2, suggesting that multiplication and division of fractions are particularly challenging areas even for students with overall better performance.

Table 7. Performance profiles regarding the PCK dimension

MKT dimension	MKT subdimension	Items	Probility class 1 (81%)	Probility class 2 (19%)
PCK	KC&T	P33_A	.91	.14
		P33_B	.94	.05
		P33_E	.96	.09
		P24_E	.92	.53
		P29_B	.11	.14
	KC	P5	.69	.40
		P28_B	.96	.39
		P28_C	.91	.44
		P28_E	.88	.53
		P32_A	.90	.25
		P32_C	.75	.38
		P32_D	.71	.21

For the second group of items determined by the validity analysis and referring to PCK, the LCA determines that the model with two classes achieves the best fits (BIC = 1,300.06; AIC = 1,231.22; G2 = 359.10; χ^2 [91, n = 114] = 7191.72). The probability of belonging to class 1 is 81%, and to the second class is 19%.

With an 81% probability of membership, students in class 1 show high probabilities (greater than .7) of correctly answering most of the items, with probabilities ranging from .69 to .96. In the sub-dimension of KC&T, items P33_A, P33_B, and P33_E show high probabilities of correct responses (.91 to .96), indicating a strong understanding of the content and teaching of fractions and division problems. The item P24_E also has a high probability (.92), suggesting a good understanding of the representation of natural numbers. However, the item P29_B presents a low probability (.11), indicating difficulties in recognizing errors in the multiplication of a three-digit number by a one-digit number and in selecting the best strategy to guide students. In the sub-dimension of KC, items P28_B, P28_C, and P28_E show high probabilities (.88 to .96), reflecting a strong understanding in the representation and description of numbers. Items P32_A, P32_C, and P32_D also show high probabilities (.71 to .90), indicating a good understanding of division with two-digit dividends and one-digit divisors.

On the other hand, with a 19% probability of membership, students in class 2 show low probabilities of correctly answering most of the items, with probabilities ranging from .05 to .53. Items in the sub-dimension KC&T, such as P33_A, P33_B, and P33_E, show low probabilities of correct responses (.05 to .14), indicating difficulties in understanding the content and teaching of fractions and division problems. However, the item P24_E has a moderate probability (.53), suggesting a partial understanding of the representation of natural numbers. In the sub-dimension KC, items P28_B, P28_C, and P28_E show low probabilities (.39 to .53), indicating that there are considerable difficulties in establishing a connection between the curricular learning objectives (LO) and the content related to the representation of numbers in the monetary system and the lecture and representation in the value positional table of natural numbers. Items P32_A, P32_C, and P32_D also show low probabilities (.21 to .38), reflecting challenges in understanding division with two-digit dividends and one-digit divisors.

In the context of the LO declared by the MINEDUC (2021), these results indicate that PST in class 1 have a solid understanding of concepts such as the place value of natural numbers ranging from 0 to 1,000, in addition to the concept of fractions (P12), the understanding and representation of fractions (P15), and the comparison of proper fractions (P14). They also show a good understanding in solving division problems (P33) and representing natural numbers (P24). However, students struggled with item P29_B (KC&T), which required identifying a multiplication error and selecting an appropriate teaching strategy. They also had challenges in establishing a connection between the curricular LO and the content pertaining to natural number representation in item P28 within the sub-dimension KC. Conversely, students in class 2 need significant reinforcement in all these topics, as they do not exhibit probabilities greater than .5 on most items, except for item P24_E (related to content and teaching knowledge) and item P28_E (related to curricular knowledge [KC]), indicating general difficulties in understanding mathematical content and its teaching.

DISCUSSION

Our statistical analyses result in a robust assessment tool for measuring the MKT of elementary PST in the area of numbers and Operations. The test comprises 21 items grouped into five topics, presented in the section above, with significant factor loadings. The first factor (NUM1) incorporates elements of CCK and SCK. It assesses prospective teachers' ability to identify, describe and represent units, tens and hundreds (P12_A and P12_B), as well as to understand and justify concepts related to fractions, such as their representation on the number line (P12_C and P12_E). The items are linked to the fundamental understanding of the decimal system and fractions. The second factor (NUM2) encompasses items pertaining to KC&S and KC&T. The items that have been emphasized (P33_A, P33_B, and P33_E) serve to evaluate the capacity to foresee PST challenges in the comparison of fractions with distinct denominators and in the resolution of division problems, thereby ensuring the utilization of suitable didactic representations. This serves to demonstrate an integrated understanding of mathematical content and its pedagogical application. The third factor (NUM3) is predominantly associated with KC. The designated items (P28_B, P28_C, and P28_E) evaluate the prospective teachers' capacity to comprehend, articulate and depict natural numbers in various forms (concrete, pictorial and symbolic). This proficiency is imperative for the integration of mathematical content with the curricular objectives of the initial cycles of primary education. The fourth factor (NUM4) encompasses elements of SCK and KC&T. The items (P15, P19_A, P19_D, and P19_E) measure the ability to solve problems involving multiplication and division of fractions, along with the representation and justification of their results. It is vital to understand these advanced operations in order to design didactic strategies that allow students to overcome common conceptual difficulties. The fifth factor (NUM5) integrates CCK, SCK, and KC. The items (P32_A, P32_C, and P32_D) evaluate the comprehension of the association between multiplication and division, in addition to the capacity to depict and resolve problems with two-digit dividends and divisors of one. This expertise is paramount in facilitating students' development of effective computational strategies and conceptual comprehension. In view of the aforementioned considerations, this study makes a significant contribution to the field by presenting an instrument with robust internal consistency. This enables teacher educators and teacher education programs to reliably assess the MKT of teacher candidates in the domain of numbers.

The profiles obtained through statistical analyses are of significant value, as they facilitate comprehension of the MKT exhibited by pedagogy students following completion of courses on the teaching and learning of numbers and their operations. Previous studies have predominantly analyzed performance in terms of the percentage of correct and incorrect answers; however, our approach enhances this understanding by employing LCA. This methodological choice offers significant advantages, chiefly in that LCA allows us to identify distinct performance profiles by uncovering latent subgroups within the data, which would otherwise remain obscured in traditional analyses. By expanding beyond the binary framework of correct and incorrect answers, LCA provides a more nuanced perspective on student knowledge and competencies, capturing variations in their responses and understanding patterns. This methodological approach mirrors the intricacies inherent in the MKT framework, facilitating a more profound examination of the multifaceted dimensions of MKT and offering actionable insights for customized instructional strategies.

Regarding knowledge strictly related to content, the three identified classes show different facets of CCK and SCK. The first group demonstrates fluency in alternative forms of representing the organization of unit types (hundreds and tens) that make up a quantity, which is fundamental to CCK for understanding how the decimal numbering system works. Furthermore, these future teachers display adequate levels when needing to access CCK to represent the result of a fraction multiplication on a number line. Additionally, since the test also evaluates that content knowledge which is unique to teachers (SCK; Ball et al., 2008), we can see that this group could distinguish between valid and invalid explanations for comparing fractions with different denominators. However, both in CCK and SCK, it is noticeable that items addressing rational numbers have notably lower probabilities than for natural numbers. Moreover, the items significantly more challenging for students in this profile are those that present problems of division or multiplication of rational numbers for them to discern whether they are represented by a given symbolic division. This common knowledge is crucial as it will support future teaching skills like the correct selection of multiplicative problems and discrimination of correct strategies and representations in students. Thus, our work reaffirms what previous research has shown regarding the needs in mathematics teacher training (Lo, 2019; Ruz et al., 2023; Zakaryan & Ribeiro, 2016).

As shown in the results, the profile corresponding to the second-class evidence common and specialized knowledge that needs to be extensively strengthened. In addition to the weaknesses described for the first group in items involving fractions, the second group adds difficulty with the multiplication of fractions, their representation on the number line, and the comparison of improper fractions. In the context of professional teaching practice, this could translate into deficiencies in identifying correct answers for school tasks and, more importantly, in discriminating between valid strategies and justifications for operating and representing rational numbers. The findings of this study on the mathematical knowledge of future teachers in Chile are consistent with previous research in various educational contexts. For example, the study by Gutiérrez et al. (2016) revealed that future teachers in Spain have an adequate command of basic contents but show weaknesses in more complex concepts such as ratio, proportion, and fractions, both conceptually–by confusing fractions with whole numbers–and procedurally–by extending the procedures and algorithms from natural or whole numbers to fractions (Lo, 2020; Van Steenbrugge et al., 2014). Even though the items that Gutiérrez et al. (2016) analyzed are focused on mathematical knowledge with a major focus on proportional thinking, overall, their findings also unveil the deficiencies in PST' arithmetic knowledge. These similarities suggest that challenges in mathematics teacher training may be common across different geographical contexts. In particular, the consistent difficulty across the three classes in our study with word problems of division or multiplication of rational numbers aligns with Depaepe et al.'s (2015) result. They found that the most difficult MCK item in their instrument was a word problem of multiplication of fractions with only 34% of correct responses. In their study and in this present study, the sample of student teachers have already taken a course that addressed the type of knowledge and concepts measured in the instruments. Our research reinforces the call to support PST in learning how to model complex word problems that involve rational numbers.

An encouraging aspect of our results is that the third class in mathematical knowledge excels with high performance on all items. This indicates that both common and specialized knowledge are successfully engaged in responding to items addressing contents of natural and rational numbers, however, the probability of belonging to this class is less than 20%.

Secondly, the profiles of pedagogy students concerning PCK involve items addressing pedagogical decisions and instructional movements in mathematics (KC&T) and items directly related to the curricular presence of mathematical concepts (KC). The two identified classes clearly portray the strength in the subdimensions of KC&T and KC for the first class and deficiencies in the same for the second class. Students from the first group excel when discriminating between tasks that allow or do not allow evaluating the level of achievement of an LO of division in the third grade and counting in the first grade. This type of knowledge, corresponding to KC&T, is directly applicable in the daily teaching practice of designing procedures and assessment instruments. Moreover, this class shows high performance in KC, when asked about mathematical ideas that students should know and be able to use in the first elementary school cycle. Taken with discretion, it is encouraging that the second class has a probability of membership of only 19%, as these students perform poorly on all items. However, it is important to remember that these items only evaluate pedagogical knowledge of mathematics, and successful instructional actions require accessing both types of MKT. It is noteworthy that both classes, in the KC&T subdimension, show great difficulty in discriminating instructional strategies to support fourth-grade students in the conceptual understanding of multiplying a single-digit number by a three-digit number.

In the Chilean context, studies like those by Pincheira et al. (2023) and Parra-Fica et al. (2020) have reported that teachers in training show limited knowledge in specific areas such as the understanding and representation of fractions and the multiplication and division of numbers. These findings provide a valuable context for interpreting our results, underscoring the persistence of these weaknesses in teacher training in Chile. The consistency of these results with the findings of our study highlights the urgent need to implement more effective and personalized teaching strategies in pre-service teacher education programs, responding to specific diagnostic and monitoring processes.

Furthermore, the LMT project by Ball et al. (2008) highlighted the importance of measuring and developing both KC&T and KC&S. Our study confirms this need by identifying specific weaknesses in these areas. For example, difficulties in solving complex division problems and understanding fractions and their teaching reflect the need for a more robust approach in developing KC&T in teacher training in Chile. In this regard, Zakaryan and Ribeiro (2016) highlighted good practices and missed opportunities in the teaching of rational numbers, suggesting areas for improvement in teacher training. Our findings, which identify specific deficiencies in the understanding of fractions and solving complex problems, reflect this need for improvement. These results indicate that, like in other contexts, pre-service teacher education programs in Chile must focus on providing richer and more contextual learning experiences that directly address the areas of knowledge where future teachers show difficulties.

Implications

From a policy perspective, it is essential that initial teacher training programs in Chile incorporate systematic mechanisms to assess and improve the MKT of future teachers. Law 20.903 establishes the responsibility of training programs to provide opportunities for deepening and extending mathematical knowledge, which is crucial to ensure that future teachers can meet the required standards for effective mathematics teaching. The implementation of mandatory diagnostic assessments could be an important step towards identifying weaknesses in mathematical knowledge and applying appropriate interventions to strengthen areas of need.

Given the reliability and validity indices demonstrated by the instrument, it can be asserted that the instrument used in this study is suitable for assessing the MKT of future teachers. The high internal consistency indices, such as Cronbach's Alpha, and the results from the CFA and LCA, show that the selected items effectively measure the dimensions of MKT relevant for teaching. The research by Charalambous et al. (2020), which assessed the advanced common content knowledge (aCCK) and SCK using items from the MTEL and the LMT project, also supports these findings. Our study, which uses adapted and validated items to measure MKT, aligns with these methodological approaches, underscoring the importance of using robust assessment tools to identify strengths and weaknesses in the mathematical knowledge of future teachers.

Limitations

Despite the significant findings, this study has some limitations. First, the sample was limited to students from two universities in Chile, which may not fully represent the diversity of educational contexts in the country. Additionally, the assessment was conducted remotely due to COVID-19 pandemic restrictions, which could have affected student participation and the accuracy of their responses. Finally, although the instruments used were validated and showed high reliability indices, there is always the possibility of response biases from the participants.

CONCLUSION

The present study addressed the following research questions: firstly, how do student teachers' responses cluster when using an instrument based on the MKT model in the area of numeracy? Secondly, what are the profiles of student teachers according to the dimensions of the MKT model in the area of numeracy? Through LCA, clear groupings were identified in the students' responses, revealing three main profiles. The first profile comprises students who demonstrate a robust grasp of fundamental concepts, such as place value and fraction representation, though they encounter challenges with advanced operations, including multiplication and division of fractions. The second profile encompasses students with limited knowledge in the majority of the evaluated subjects, underscoring the necessity for targeted formative interventions. The third profile, which constitutes a minority, is characterized by its exceptional performance across all dimensions, reflecting a sophisticated comprehension of the subject matter and its pedagogical applications. These results not only demonstrate the distribution of competencies among students but also provide a solid foundation for designing training strategies that address their specific needs and enhance their professional development in the area of numbers.

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REFERENCES

Aiken, L. R. (1985). Three coefficients for analyzing the reliability and validity of ratings. *Educational and Psychological Measurement, 45*, 131-142. https://doi.org/10.1177/0013164485451012

Amador, J. M., & Weston, T. L. (2024). A review of analytic frameworks for noticing in mathematics and science: Comparing noticing frameworks across disciplines and over time. *International Journal of Science and Mathematics Education, 22*, 1739-1760. https://doi.org/10.1007/s10763-024-10452-8

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Blömeke, S., Gustafsson, J. E., & Shavelson, R. J. (2015). Beyond dichotomies: Competence viewed as a continuum. *Zeitschrift Fur Psychologie, 223(1)*, 3-13. https://doi.org/10.1027/2151-2604/a000194

Buschang, R. E., Chung, G. K. W. K., Delacruz, G. C., & Baker, E. L. (2012). *Validating measures of algebra teacher subject matter knowledge and pedagogical content knowledge*. National Center for Research on Evaluation, Standards, & Student Testing, UCLA. https://doi.org/10.1080/10627197.2012.697847

Carrillo-Yañez, J., Climent, N., Contreras, L. C., & Muñoz-Catalán, M. D. C. (2013). Determining specialised knowledge for mathematics teaching. In *Proceedings of the CERME* (vol. 8, pp. 2985-2994).

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-Gonzales, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teachers specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236-253. https://doi.org/10.1080/14794802.2018.1479981

Charalambous, C. Y. (2016). Investigating the knowledge needed for teaching mathematics: An exploratory validation study focusing on teaching practices. *Journal of Teacher Education, 67*(3), 220-237. https://doi.org/10.1177/0022487116634168

Charalambous, C. Y., Hill, H. C., Chin, M. J., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: Exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*(6), 579-613. https://doi.org/10.1007/s10857-019-09443-2

Chen, C., Sonnert, G., Sadler, P., & Sunbury, S. (2020). The impact of high school life science teachers' subject matter knowledge and knowledge of student misconceptions on students' learning. *CBE Life Sciences Education, 19*(1). https://doi.org/10.1187/cbe.19-08-0164

de Almeida, M. V. R., Ribeiro, M., & Fiorentini, D. (2021). Mathematical specialized knowledge of a mathematics teacher educator for teaching divisibility. *Revista de Investigación en Didáctica de la Matemática, 15*(3), 187-210. https://doi.org/10.30827/pna.v15i3.15778

Delaney, S. (2012). A validation study of the use of mathematical knowledge for teaching measures in Ireland. *ZDM Mathematics Education, 44*, 427-441. https://doi.org/10.1007/s11858-012-0382-5

Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G., Verschaffel, L., & Van Dooren, W. (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school teachers. *Teaching and Teacher Education, 47*, 82-92. https://doi.org/10.1016/j.tate.2014.12.009

Ekmekci, A., & Serrano, D. M. (2022). The impact of teacher quality on student motivation, achievement, and persistence in science and mathematics. *Education Sciences, 12*(10), Article 649. https://doi.org/10.3390/educsci12100649

Fram, S. (2013). The constant comparative analysis method outside of grounded theory. *The Qualitative Report, 18*(1), 1-25. https://doi.org/10.46743/2160-3715/2013.1569

Gaona, J., G. Miranda, D., Vergara, A., Ramírez, P., & Menares, R. (2024). Evaluation of didactic disciplinary standards of preservice mathematics teachers in Chile: Building an indebted teaching staff? *Education Policy Analysis Archives, 32*. https://doi.org/10.14507/epaa.32.7997

Giaconi, V., Gómez, G., Jiménez, D., Gareca, B., Durán del Fierro, F., & Varas, M. (2022). Evaluación diagnóstica inicial en la formación inicial docente en Chile y su relación con contextos institucionales [Initial diagnostic assessment in initial teacher training in Chile and its relationship with institutional contexts]. *Pensamiento Educativo, 59*(1), Article 00104. https://doi.org/10.7764/pel.59.1.2022.4

Godino, J. D. (2009). Categorías de análisis de los conocimientos del profesor de matemáticas [Categories of analysis of the knowledge of the mathematics teacher]. *UNIÓN-Revista Iberoamericana de Educación Matemática, 5*(20). https://union.fespm.es/index.php/UNION/article/view/1063

Godoy, T. C. G., Reyes-Santander, P., & Ayarza, R. O. (2021). Mathematical knowledge and overall practice in initial teacher education of early childhood teachers. *Revista Brasileira de Educação, 26*. https://doi.org/10.1590/S1413-24782021260061

Gutiérrez, A. G., Gómez, P., & Rico, L. (2016). Conocimiento matemático sobre números y operaciones de los estudiantes de magisterio [Mathematical knowledge of numbers and operations of teaching students]. *Educación XX1, 19*(1), 135-158. https://doi.org/10.5944/educxx1.15581

Hambleton, R. K., & Zenisky, A. L. (2011). Translating and adapting tests for cross-cultural assessments. In D. Matsumoto, & F. J. R. van de Vijver (Eds.), *Cross-cultural research methods in psychology* (pp. 46-74). Cambridge University Press. https://doi.org/10.1017/CBO9780511779381.004

Harvard University. (n. d.). *Measures of effective teaching: Refreshing the framework*. Center for Education Policy Research at Harvard University. https://cepr.harvard.edu/mkt-measures-refresh

Hasson, F., Keeney, S., & McKenna, H. (2000). Research guidelines for the Delphi survey technique. *Journal of Advanced Nursing, 32*(4), 1008-1015. https://doi.org/10.1046/j.1365-2648.2000.t01-1-01567.x

Hiebert, J., Berk, D., Miller, E., Gallivan, H., & Meikle, E. (2019). Relationships between opportunity to learn mathematics in teacher preparation and graduates' knowledge for teaching mathematics. *Journal for Research in Mathematics Education, 50*(1), 23-50. https://doi.org/10.5951/jresematheduc.50.1.0023

Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 41*(5), 513-545. https://doi.org/10.5951/jresematheduc.41.5.0513

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400. https://doi.org/10.5951/jresematheduc.39.4.0372

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal, 6*, 1-55. https://doi.org/10.1080/10705519909540118

Jan, D., Riaz, A., & Yaseen, M. (2023). The impact of pedagogical knowledge and teaching practices of teachers on students' achievement in public schools of Pakistan. *Pakistan Journal of Social Research, 5*(2), 469-478. https://doi.org/10.52567/pjsr.v5i02.1098

König, J., Blömeke, S., Jentsch, A., Schlesinger, L., née Nehls, C. F., Musekamp, F., & Kaiser, G. (2021). The links between pedagogical competence, instructional quality, and mathematics achievement in the lower secondary classroom. *Educational Studies in Mathematics, 107*, 189-212. https://doi.org/10.1007/s10649-020-10021-0

Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and con- tent knowledge: Validation of the COACTIV constructs. *ZDM Mathematics Education, 40*(5), 873-892. https://doi.org/10.1007/s11858-008-0141-9

Lo, W. Y. (2020). Examining the mathematical content knowledge of pre-service primary teachers at the highest primary school level in Hong Kong. *Teacher Development, 24*(3), 315-332. https://doi.org/10.1080/13664530.2020.1758760

Louie, N., & Zhan, W. (2022). A socio-ecological framework for research in mathematics education. *Journal for Research in Mathematics Education, 53*(5), 365-371. https://doi.org/10.5951/jresematheduc-2022-0003

Mello Román, J. D., & Hernández Estrada, A. (2019). Un estudio sobre el rendimiento académico en matemáticas [A study on academic performance in mathematics]. *Revista Electrónica de Investigación Educativa, 21*. https://doi.org/10.24320/redie.2019.21.e29.2090

MINEDUC. (2014). INICIA 2014 assessment report: Diagnosis of graduates from primary and secondary education teaching programs. *Ministerio de Educación de Chile*. https://www.mineduc.cl/wp-content/uploads/sites/19/2015/11/Presentaci%C3%B3n-Resultados-INICIA-2014.pdf

MINEDUC. (2021). Estándares orientadores para la formación inicial docente [Guiding standards for initial teacher training]. *Ministerio de Educación de Chile*. https://www.cpeip.cl/estandares-formacion-docente/

Muhtarom, M., Juniati, D., & Siswono, T. (2019). Examining prospective teachers' belief and pedagogical content knowledge towards teaching practice in mathematics class: A case study. *Journal on Mathematics Education, 10*(2), 185-202. https://doi.org/10.22342/jme.10.2.7326.185-202

Novick, M. R., & Lewis, C. (1967). Coefficient alpha and the reliability of composite measurements. *Psychometrika, 32*, 1-13. https://doi.org/10.1007/BF02289400

Parra-Fica, J., Samuel, M., & Díaz-Levicoy, D. (2020). Design of self-instruction textbooks on number and operations by preservice elementary school teachers: A preliminary Study. *Acta Scientiae, 22*(2), 2-28. https://doi.org/10.17648/acta.scientiae.5536

Phelps, G., Howell, H., & Liu, S. (2020). Exploring differences in mathematical knowledge for teaching for prospective and practicing teachers. *ZDM Mathematics Education, 52*(2), 255-268. https://doi.org/10.1007/s11858-019-01097-x

Pincheira, N., Alsina, Á., & Acosta, Y. (2023). Preservice teachers designing mathematical tasks on patterns: Context, cognitive demand, and skills. *Uniciencia, 37*(1), 24-43. https://doi.org/10.15359/ru.37-1.2

Ribeiro, M., Policastro, M., Caldatto, M., & Almeida, A. (2022). Interpretative knowledge of prospective kindergarten and primary teachers in the context of subtraction. *Acta Scientiae, 24*(3), 1-31. https://doi.org/10.17648/acta.scientiae.6767

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281. https://doi.org/10.1007/s10857-005-0853-5

Ruz, F., Ubilla, F. M., & Giaconi, V. (2023). Characterising the didactic-stochastic knowledge of future mathematics teachers: The case of Chile. *Acta Scientiae, 25*(4), 95-132. https://doi.org/10.17648/acta.scientiae.7698

Saadati, F., Larrain, M., Bastian, A., Felmer, P., & Kaiser, G. (2024). Adapting german instruments and validating their use to measure chilean mathematics teachers' professional competence. *Journal of Curriculum Studies, 56*(4), 413-431. https://doi.org/10.1080/00220272.2024.2322494

Santagata, R., & Lee, J. (2021). Mathematical knowledge for teaching and the mathematical quality of instruction: A study of novice elementary school teachers. *Journal of Mathematics Teacher Education, 24*(1), 33-60. https://doi.org/10.1007/s10857-019-09447-y

Schermelleh-Engel, K., Moosbrugger, H., & Müller, H. (2003). Evaluating the fit of structural equation models: Tests of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research Online, 8*(2), 23-74. https://doi.org/10.23668/psycharchives.12784

Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics. In S. Llinares, & O. Chapman (Eds.), *International handbook of mathematics teacher education: Volume 2* (pp. 321-354). Brill Sense. https://doi.org/10.1163/9789087905460_016

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1). https://doi.org/10.17763/haer.57.1.j463w79r56455411

Sosa, L., & Carrillo-Yañez, J. (2010). Caracterización del conocimiento matemático para la enseñanza (MKT) de matrices en bachillerato [Characterization of mathematical knowledge for teaching (MKT) of matrices in high school]. In M. M. Moreno, A. Estrada, J. Carrillo, & T. A. Sierra (Eds.), *Investigación en educación matemática XIV* (pp. 569-580). SEIEM.

Strauss, A. L. & Corbin, J. (2002). *Bases de la investigación cualitativa: Técnicas y procedimientos para desarrollar la teoría fundada* [*Bases de la investigación cualitativa: Técnicas y procedimientos para desarrollar la teoría fundada*] (1. ed.). Editorial Universidad de Antioquia.

Tabachnick, B. G. (2007). *Experimental designs using ANOVA*. Thomson/Brooks/Cole.

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., Ingvarson, L., Reckase, M., & Rowley, G. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA teacher education and development study in mathematics (TEDS-MM)*. International Association for the Evaluation of Educational Achievement.

Van Steenbrugge, H., Lesage, E., Valcke, M., & Desoete, A. (2014). Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? *Journal of Curriculum Studies, 46*(1), 138-161. https://doi.org/10.1080/00220272.2013.839003

Verdugo Peñaloza, A., Tejada Fernández, J., & Navío Gámez, A. (2021). Valoración de la formación de los estándares pedagógicos según estudiantes de pedagogía [Assessment of the formation of pedagogical standards according to pedagogy students]. *Perfiles Educativos, 43*(171), 119-137. https://doi.org/10.22201/iisue.24486167e.2021.171.59216

Warrah, S. M., Musa, A. F., & Ibrahim, A. O. (2020). Determine the relationship between teacher knowledge and students' academic performance using teacher commitment as mediator. *Quantum Journal of Social Sciences and Humanities, 1*(5), 1-12. https://doi.org/10.55197/qjssh.v1i5.25

Zakaryan, D., & Ribeiro, M. (2016). Conocimiento de la enseñanza de números racionales: Una ejemplificación de relaciones [Knowledge of teaching rational numbers: An exemplification of relationships]. *Zetetiké, 24*(3), 301-321. https://doi.org/10.20396/zet.v24i3.8648095