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Knowledge for teaching early algebra in primary education: Integral analysis from the MKT

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ABSTRACT

The aim of this study is to conduct a full analysis of all domains of knowledge. involved in the teaching of algebra, from the perspective of the Mathematical Knowledge for Teaching model. To this end, a descriptive mixed-methods methodological approach was adopted, which involved the administration of the previously validated MKT-Early Algebra Questionnaire (6–12 y.o.) to a sample of 76 pre-service primary school teachers. This instrument consists of six open-ended items presenting different contextualised situations to identify the different subdomains of knowledge for teaching algebra. The main results show: a) In general, low knowledge for teaching early algebra; b) More specifically, Common Content Knowledge exhibits the highest scores, and Knowledge at the Mathematical Horizon the lowest. These overall findings are a starting point for the design of initial training programmes to help improve the knowledge gaps for teaching algebra in primary education.

Keywords: knowledge for teaching mathematics, MKT model, early algebra, pre-service teachers, primary education

INTRODUCTION

One of the main aims of early algebra is to promote the development of algebraic thinking and algebraic practices from an early age. In this sense, the challenge of teaching algebra in primary school arises from the urgent need to foster learning experiences that prepare students for the more formal study of algebra in higher grades (Cai et al., 2011). This contributes to providing coherence to the curriculum and facilitating the gradual development of algebraic thinking, allowing students to develop a deep, integrated, and meaningful understanding of mathematics (Blanton et al., 2015).

Algebraic thinking is conceived as a way of thinking, that allow progress to be made towards the central aspect that early algebra promotes, the generalization (Blanton & Kaput, 2011; Kaput, 2008) and is not restricted to working with algebraic symbolism, that is, it does not focus on the use of letters and their manipulation (Radford, 2011). From a broad perspective, guaranteeing the development of algebraic thinking in primary education considers attending to different approaches to algebraic activity (Blanton et al., 2015; Kaput, 2008):

a) Generalized arithmetic: The study of structures and the analysis of numerical relationships that result from arithmetic;
b) Equivalence, expressions, equations and inequalities: This idea includes a relational understanding of the equals sign;
c) Attending to the development of functional thinking: The study of functions, joint relationships and change;
d) Variables: Refers to symbolic notation as a linguistic tool for representing mathematical ideas succinctly.

Curriculum guidelines in various countries (e.g., Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020; Ministry of Education Singapore, 2012; National Council of Teachers of Mathematics [NCTM], 2000) have explicitly established the progressive knowledge of algebra in primary education curricula (Pincheira & Alsina, 2021a). The curricula of countries such as Australia and Singapore, for example, emphasize from the earliest educational levels the development of algebraic thinking through recognising, continuing, and creating repeating patterns with numbers, symbols, shapes, and objects, as well as identifying the repeating unit. In addition, they promote finding unknown values in numerical equations involving addition and subtraction by using the properties of numbers and operations. By contrast, the algebra standard of the United States curriculum is organized around four fundamental strands during primary education: Understanding patterns, relations, and functions; representing and analysing mathematical situations and structures using algebraic symbols; using mathematical models to represent and understand quantitative relationships; and analysing change in a variety of contexts. However, the successful implementation of curricular proposals that promote the study of algebra from an early age depends on teachers' ability to provide students with meaningful opportunities to develop age-appropriate algebraic thinking through their everyday learning experiences (Stephens et al., 2017).

According to Santagata and Lee (2021), the effects on student learning and the quality of instruction are closely related to the mathematical knowledge of the teachers. Therefore, promoting the development of algebraic thinking in students directly challenges the teaching staff, since they require broad knowledge of the subject; that is, a solid understanding of various elements and concepts associated with early algebra, as well as pedagogical knowledge that lets them present effective learning experiences to achieve this kind of thinking.

Updating both the mathematical and pedagogical knowledge associated with teaching algebra poses a challenge during the initial training of primary school teachers, who need instruction that reflects the transformation demanded by early algebra (Kim & Kim, 2022). However, on the one hand, few studies have explored teachers' knowledge for teaching early algebra (Hohensee, 2017); and, on the other hand, most studies in this research agenda have analysed partial aspects of this knowledge, focusing on some subdomain of mathematical or pedagogical knowledge (Pincheira & Alsina, 2021b). Therefore, studies are needed that integrally and exhaustively evaluate all domains of knowledge in order to teach this content standard in primary education.

Malara and Navarra (2009) assert that teachers play a fundamental role in the early teaching of algebra, since they are responsible for making decisions about its instruction. Assuming this perspective, the following research question has been posed:

RQ1 What knowledge to teach early algebra do pre-service school teachers mobilize?

To answer this question, the aim of this study is to conduct an integral analysis of all domains of knowledge for teaching early algebra of pre-service primary education teachers, considering the perspective of Ball et al. (2008) and the Mathematical Knowledge for Teaching (MKT) model.

THEORETICAL FRAMEWORK

Mathematical Knowledge for Teaching (MKT) model

The MKT model emerges from the notions proposed by Shulman (1986, 1987) in relation to knowledge of the subject and pedagogical knowledge. Ball et al. (2008), in an effort to deliver analysis tools specific to the teaching of mathematics, define mathematical knowledge for teaching as "the mathematical knowledge that teachers use in classrooms to produce instruction and student growth" (Hill et al., 2008, p. 374).

MKT considers two major domains of mathematical knowledge for teaching: Subject Matter Knowledge and Pedagogical Content Knowledge (**Figure 1**).

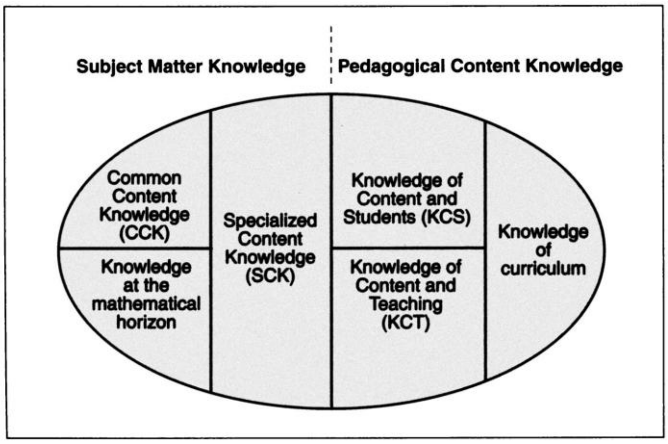


Figure 1. MKT model (Hill et al., 2008, p. 377)

Subject Matter Knowledge includes three subdomains:

1) *Common Content Knowledge* (CCK), which refers to "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p. 399), meaning it corresponds to the knowledge that can be achieved over the course of one's education, and that is possessed by anyone when doing a mathematical task;
2) *Specialized Content Knowledge* (SCK), which refers to "mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p.400), knowledge that is specific to the teacher and that is used to perform teaching tasks alluding to: "The mathematical knowledge that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill et al., 2008, p. 377-378); and
3) *Knowledge at the Mathematical Horizon* (KMH), which is described as "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p.403), which allows establishing the way in which mathematical content relates to other content in the curriculum throughout the various educational stages, and offers a vision to understand the connections between the various notions of mathematics with other sciences.

The Pedagogical Content Knowledge also consists of three subdomains:

1) *Knowledge of Content and Students* (KCS), which is defined as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content.”(Hill et al., 2008, p. 375), is the knowledge that teachers have about the knowledge of students, allowing them to predict situations and anticipate the concerns, attitudes or difficulties of the students;
2) *Knowledge of Content and Teaching* (KCT), defined as that which “combines knowing about teaching and knowing about mathematics” (Ball et al., 2008, p. 401), which integrates specific mathematical knowledge, and pedagogical and didactic aspects of the teaching processes involved in student learning; and, finally,
3) *Knowledge of Curriculum* (KCC), is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level (Ball et al., 2008), is related to the methods and approaches corresponding to the programs designed for each educational level in the area of mathematics and the materials available in relation to them.

According to Blömeke et al. (2015), the teachers’ MKT could predict perceptual interpretation as well as the ability to make decisions and teach mathematics. The description given by the subdomains of the MKT model lets us categorize the knowledge that a teacher must exhibit over the course of their practice to teach mathematics, from different teaching approaches, whether in a class-wide discussion, a written task or a questionnaire (Ball et al., 2008).

Knowledge of Early Algebra of Primary School Teachers

In recent years, some studies have shown that the knowledge of primary school teachers is not sufficient to ensure the effective teaching of early algebra.

As noted in the introduction, several preliminary studies have partially analysed the knowledge to teach algebra, focusing on some subdomain of the MKT model (e.g., Narváez et al., 2024; Pincheira & Alsina, 2021b). McAuliffe and Lubben (2013), for example, analyse SCK through the performance of a pre-service primary school teacher when designing and teaching an early algebra lesson on patterns. These authors note limitations involving the SCK, given the teacher’s difficulties guiding the students through a numerical pattern while simultaneously focusing on the function. Likewise, Wilkie (2014) analyses the mathematical knowledge of 105 in-service primary school teachers for teaching functional thinking, focusing on the subdomains SCK, KCS, KCT, and KCC. The results reveal adequate content knowledge for pattern generalization tasks; however, the pedagogical knowledge of teachers is limited, since they fail to provide adequate learning experiences involving functions, relationships, and variables. Oliveira et al. (2021) analyse the SCK involving functional thinking of 164 teachers at the start of their training program. The results reveal a lack of successful strategies to generalize functional relationships, and problems understanding and connecting the different representations of functions (Pinto & Cañadas, 2021).

Trivilin and Ribeiro (2015) analyse the CCK, SCK, and KCC stated and exhibited by ten in-service teachers on the different meanings of the equal sign. Certain limitations are noted in recognizing the notions of operation and equivalence. Difficulties are also evident in determining the implications of teaching the different meanings of the equals sign in the curriculum. More recently, Barboza et al. (2020, 2021) probe the mathematical knowledge of six in-service primary school teachers in relation to the meanings of the equal sign, delving into aspects of the SCK, KMH, KCS, and KCT. These studies report remarkable advances in the development of the mathematical knowledge of teachers, since they begin to consider relational thinking from the resignification of the equal sign, by transiting between the operational and equivalence meanings, and subsequently to the relational meaning.

Ferreira et al. (2017) identify the mathematical knowledge, more specifically the CCK, SCK, and KCS, of 14 in-service teachers when discussing tasks with algebraic potential. The results show little familiarity with core questions of algebraic thought related to the generalization of arithmetic, such as number relationships, the properties of operations and the meanings of the equal sign. However, deficiencies are noted in identifying errors and recognizing the nature of a mathematical error.

Bernardo et al. (2017) administer a questionnaire to determine the SCK that 60 pre-service teachers have to interpret student output in the context of an algebraic task. The results show the difficulties in assigning the semantic meaning involved in the students’ solution to an equitable distribution task. Likewise, Zapatera and Vega (2018) analyse the mathematical knowledge of 40 pre-service teachers in the context of pattern generalization, from CCK and SCK, obtaining as a result a low level of knowledge, since they exhibit difficulties identifying the mathematical elements used by students, and in abstracting observed regularities to interpret the characteristics of understanding generalization.

Based on research in this field, significant gaps have been identified in the knowledge of both pre-service and in-service teachers when it comes to teaching and facilitating algebraic tasks. While previous studies on teachers’ Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008) have focused on specific aspects of certain subdomains and particular elements of early algebraic activity, the main contribution of this study is, first, to address all the knowledge subdomains outlined in the MKT framework and, second, to explore the various types of knowledge related to early algebra that play a role in algebraic activity in primary education.

METHOD

In accordance with the study objective, a mixed-methods approach of a descriptive nature has been adopted, since it considers the systematic integration of qualitative and quantitative variables to interpret a phenomenon (Creswell, 2014); in this case, an analysis of the mathematical knowledge of pre-service primary education teachers for teaching early algebra.

Participants and Context

The study participants were 76 students majoring in primary education in a Spanish public university who were selected through a non-probabilistic sampling of an accidental or causal nature (Fernández et al., 2014), since the selection criterion was determined by the ability to access this group.

Of the participants, 77.6% were women and 22.4% were men, and their ages ranged between 20 and 28. In relation to the participants' previous training, 75 (98.7%) graduated from high school, 6 (7.9%) completed vocational training and 5 (6.6%) completed both.

The research was conducted in 2022 during the third academic year of the five-year degree as part of the "Mathematics II" course. In this course, pre-service teachers analyse the teaching contents and strategies of the thematic blocks on measurement, space and form, relationships and change.

It should be noted that in the previous year, the participants had taken the "Mathematics I" course, where they received instruction on teaching numbers and calculation, statistics and probability.

Design and Procedure

The data were obtained after administering the questionnaire called MKT-early algebra (6-12), described in Pincheira and Alsina (2024), which was subjected to a content validation process based on the judgement of 12 experts with extensive experience in the field of mathematics education. The criteria for selecting the experts were:

a) Knowledge about the MKT model,

b) Knowledge of the study of early algebra, and

c) Expertise in designing instruments to assess the knowledge of pre-service and in-service teachers.

An analysis of the reliability of the internal consistency of the instrument yielded a Cronbach's alpha of 0.73, adequate for education research (Cohen et al., 2011).

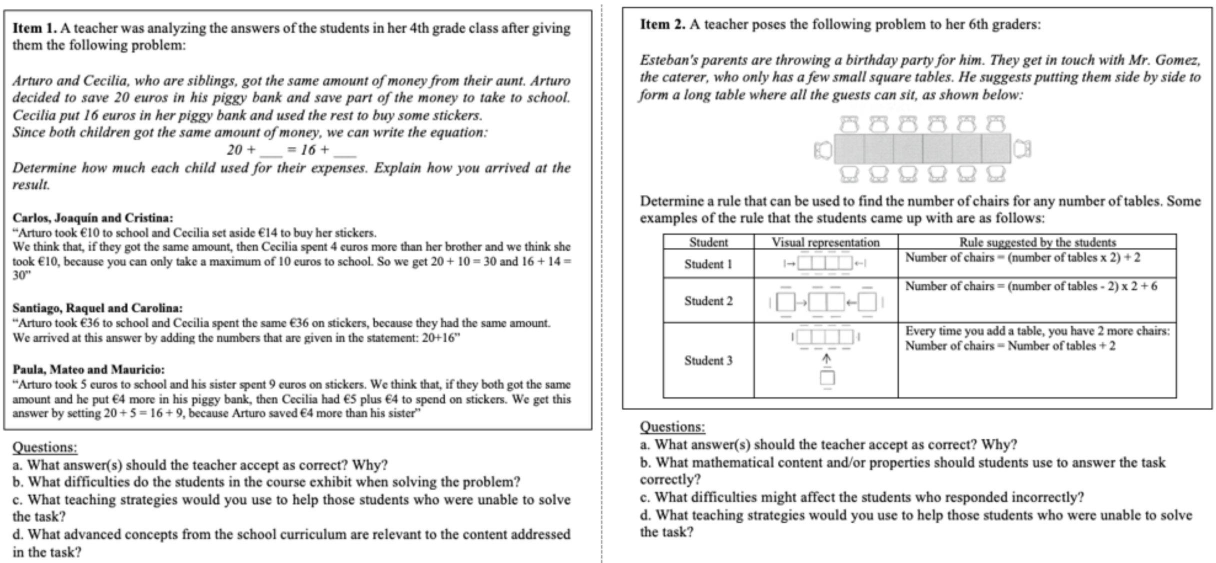
Given the limited research available on the preparation of primary school teachers for the teaching of early algebra, there is a clear need for the development of specific assessment instruments capable of rigorously characterising the mathematical knowledge that such teachers possess in relation to this content domain. In this context, the questionnaire arises from a need identified in the existing literature and is conceived as a relevant and applicable instrument for assessing mathematical knowledge of early algebra within initial primary teacher education. Its design is firmly grounded in this identified research gap, thereby reinforcing both its content validity and its significance for educational research.

The instrument consists of six open-answer items that consider various teaching situations that respond to the different approaches to algebraic activity for primary education (Blanton et al., 2015; Kaput, 2008), as shown in **Table 1**.

Table 1. Categories of knowledge mobilized by the items in the MKT-Early Algebra Questionnaire (6-12)

Algebraic activity for primary education	Questionnaire items					
	1	2	3	4	5	6
Generalized arithmetic					x	
Equivalence, expressions, equations and inequalities	x		x			x
Functional thinking		x				
Use of variables				x		

The items in the questionnaire were taken and adapted from previous research that delves into core aspects of knowledge for teaching early algebra in primary education (Barboza et al., 2020; Bernardo et al., 2017; Demonty et al., 2018; Tanisli & Kose, 2013; Ferreira et al., 2017). **Figure 2, 3** and **4** shows the questionnaire items.


Item 1. A teacher was analyzing the answers of the students in her 4th grade class after giving them the following problem:

Arturo and Cecilia, who are siblings, got the same amount of money from their aunt. Arturo decided to save 20 euros in his piggy bank and save part of the money to take to school. Cecilia put 16 euros in her piggy bank and used the rest to buy some stickers.
Since both children got the same amount of money, we can write the equation:

20 + ___ = 16 + ___

Determine how much each child used for their expenses. Explain how you arrived at the result.

Carlos, Joaquín and Cristina:
"Arturo took €10 to school and Cecilia set aside €14 to buy her stickers.
We think that, if they got the same amount, then Cecilia spent 4 euros more than her brother and we think she took €10, because you can only take a maximum of 10 euros to school. So we get 20 + 10 = 30 and 16 + 14 = 30"

Santiago, Raquel and Carolina:
"Arturo took €36 to school and Cecilia spent the same €36 on stickers, because they had the same amount. We arrived at this answer by adding the numbers that are given in the statement: 20+16"

Paula, Mateo and Mauricio:
"Arturo took 5 euros to school and his sister spent 9 euros on stickers. We think that, if they both got the same amount and he put €4 more in his piggy bank, then Cecilia had €5 plus €4 to spend on stickers. We get this answer by setting 20 + 5 = 16 + 9, because Arturo saved €4 more than his sister"

Questions:
a. What answer(s) should the teacher accept as correct? Why?
b. What difficulties do the students in the course exhibit when solving the problem?
c. What teaching strategies would you use to help those students who were unable to solve the task?
d. What advanced concepts from the school curriculum are relevant to the content addressed in the task?

Item 2. A teacher poses the following problem to her 6th graders:

Esteban's parents are throwing a birthday party for him. They get in touch with Mr. Gomez, the caterer, who only has a few small square tables. He suggests putting them side by side to form a long table where all the guests can sit, as shown below:

Determine a rule that can be used to find the number of chairs for any number of tables. Some examples of the rule that the students came up with are as follows:

Student	Visual representation	Rule suggested by the students
Student 1		Number of chairs = (number of tables x 2) + 2
Student 2		Number of chairs = (number of tables - 2) x 2 + 6
Student 3		Every time you add a table, you have 2 more chairs: Number of chairs = Number of tables + 2

Questions:
a. What answer(s) should the teacher accept as correct? Why?
b. What mathematical content and/or properties should students use to answer the task correctly?
c. What difficulties might affect the students who responded incorrectly?
d. What teaching strategies would you use to help those students who were unable to solve the task?

Figure 2. Items 1 and 2 of the questionnaire (Pincheira & Alsina, 2024)

Item 3. A teacher explains the following situation to his 5th graders:

Carlitos is a child who likes sweets. He has a box with 28 candies inside. Every day, he eats twice as many sweets as the day before. In three days, Carlitos has eaten all the sweets.

He then asks his students: How many candies did Carlitos eat each day?
Two students describe how they solved the problem.

Teresa:
"The first day Carlitos eats some of the candies, and we don't know how many... [Teresa draws a square], the second day he eats twice as many as the first, so two servings [draws two squares] ... the third day twice as many as the second, so four servings [draws four squares]. Now the twenty-eight candies are divided among the seven servings I identified, and I know the value of each serving..."

Lucas:
"I took the candies he had in the box and divided them by seven. The result is 4, which is how many candies he eats every day. The first day, then, he eats 4, the second day he eats 8, and the third day he eats 16."

Questions:
a. Solve the problem presented by the teacher. Explain your answer.
b. Explain whether or not you consider the pupils' work product to be mathematically correct. Justify the appropriateness or insufficiency of the mathematical rationale shown by the students.
c. Considering the primary school curriculum, what might be the goal of the task proposed to the students?

Item 4. Over the course of a class, the following situation is discussed:

Pedro is 4 cm taller than Clara. If Clara is "n" cm tall, how tall is Pedro?

Below is the discussion among three students:
Luis: Pedro is 4n tall
Pilar: No. Pedro is 104 cm tall.
Maria: I think Peter's height is x+4

Questions:
a. Determine Pedro's height. Explain your answer.
b. Describe the potential difficulties that led the students to answer incorrectly.
c. What teaching strategies would you use to help those students who were unable to solve the problem correctly?
d. Judging by the primary education school curriculum, what grade do you think this problem is appropriate for? Explain your answer.

Figure 3. Items 3 and 4 of the questionnaire (Pincheira & Alsina, 2024)

Item 5. A teacher asks his students to complete the following table, giving them the following instructions:
"Mark the following numerical expressions as true or false. Explain your answer"

	T	F	Explanation
24 + 37 = 37 + 24			
46 + 27 – 27 = 27			
◊ x 1 = ◊			
□ + 0 = □			

Some of the students' answers were as follows:

	T	F	Explanation
24 + 37 = 37 + 24	x		Because it's the same result, only the order changed

	T	F	Explanation
46 + 27 – 27 = 27		x	Because 46+27= makes 79 and 79 minus 27 makes 46

	T	F	Explanation
24 + 37 = 37 + 24		x	Calculations are not facts, the result never involves multiplication. So, it's wrong!

	T	F	Explanation
46 + 27 – 27 = 27		x	It is incorrect because 27-27 gives 0, so there's 46 left over

	T	F	Explanation
◊ x 1 = ◊	x		Any number x 1 is equal to the number

	T	F	Explanation
□ + 0 = □	x		The square is zero, so the result is the square.

Questions:
a. What mathematical content and/or properties should students use to answer the task correctly?
b. Describe the potential difficulties that led the students to answer incorrectly.
c. What teaching strategies would you use as a teacher to guide those students who answered the task incorrectly?
d. Judging by the Primary Education school curriculum, what grade do you think this problem is appropriate for? Explain your answer.

Item 6. A teacher writes on the board 3+2+2=5+2=7 and asks his 3rd graders to analyze whether the numerical expression is right or wrong.

Two students note the following:
"Carla explained that the expression is wrong, since not all the numbers were added together, so the final result would give 21, reading 3+2+2+5+2+7=21.
Rodrigo said the expression was right and seven was the answer".

Questions:
a. What mathematical content and/or properties should students use to answer the task correctly?
b. Describe the potential difficulties that led the students to answer incorrectly.
c. What teaching strategies would you use to help the student realize and correct her mistake? Explain your answer.

Figure 4. Items 5 and 6 of the questionnaire (Pincheira & Alsina, 2024)

These items let us investigate the mathematical knowledge involved in teaching early algebra in primary education, giving rise to a total of twenty-two questions that are based on the MKT model (Ball et al., 2008), allowing us to research the domains and subdomains that comprise it (**Table 2**).

Table 2. Domains and subdomains of the MKT model evaluated by the items that comprise the questionnaire

Questionnaire items		Content knowledge			Pedagogical knowledge of the content		
		CCK	SCK	KMH	KCS	KCT	KCC
1.	a. What answer(s) should the teacher accept as correct? Why?	x					
	b. What difficulties do the students in the course exhibit when solving the problem?				x		
	c. What teaching strategies would you use to help those students who were unable to solve the task?					x	
	d. What advanced concepts from the school curriculum are relevant to the content addressed in the task?			x			
2.	a. What answer(s) should the teacher accept as correct? Why?	x					
	b. What mathematical content and/or properties should students use to answer the task correctly?		x				
	c. What difficulties might affect the students who responded incorrectly?				x		
	d. What teaching strategies would you use to help those students who were unable to solve the task?					x	

Table 2 (Continued). Domains and subdomains of the MKT model evaluated by the items that comprise the questionnaire

Questionnaire items		Content knowledge			Pedagogical knowledge of the content		
		CCK	SCK	KMH	KCS	KCT	KCC
3.	a. Solve the problem presented by the teacher. Explain your answer.	x					
	b. Explain whether or not you consider the pupils' work product to be mathematically correct. Justify the appropriateness or insufficiency of the mathematical rationale shown by the students.		x				
	c. Considering the primary school curriculum, what might be the goal of the task proposed to the students?						x
4.	a. Determine Pedro's height. Explain your answer.	x					
	b. Describe the potential difficulties that led the students to answer incorrectly.				x		
	c. What teaching strategies would you use to help those students who were unable to solve the problem correctly?					x	
	d. Judging by the primary education school curriculum, what grade do you think this problem is appropriate for? Explain your answer.						x
5.	a. What mathematical content and/or properties should students use to answer the task correctly?		x				
	b. Describe the potential difficulties that led the students to answer incorrectly.				x		
	c. What teaching strategies would you use as a teacher to guide those students who answered the task incorrectly?					x	
	d. Judging by the primary education school curriculum, what grade do you think this problem is appropriate for? Explain your answer.						x
6.	a. What mathematical content and/or properties should students use to answer the task correctly?		x				
	b. Describe the potential difficulties that led the students to answer incorrectly.				x		
	c. What teaching strategies would you use to help the student realize and correct her mistake? Explain your answer.					x	

The questionnaire was administered during a 90-minute session of the participants' training process. The application relied on the permission and collaboration of the professor in charge of the "Mathematics II" course. To answer the questionnaire, the pre-service primary education teachers voluntarily agreed after signing an informed consent that guaranteed the confidentiality of the answers given and ensured that their answers would not affect their grade in the course.

Analysis of the Data

Out study focuses on analysing the answers of the pre-service primary school teachers on the MKT-Early Algebra Questionnaire (6-12). The analysis of the data considered quantitative and qualitative aspects. From a quantitative perspective, the variable "degree of correctness of the answers" was utilized by assigning a score of 2 if the answer is correct, 1 if it is partially correct and 0 if the answer is incorrect. These scores were assigned based on a protocol that was submitted to the judgment of experts in didactics of mathematics and early algebra, which agreed on criteria based on the relevance of the answers.

For the inferential analysis, the Wilcoxon non-parametric test was used to determine whether there is a significant difference between the knowledge subdomains. A 95% confidence level was adopted, with p-values explicitly reported to support statistical decision-making. Prior to the analysis, scores were normalized on a 0-100 scale, which facilitated comparison across dimensions and groups.

This approach enabled analytical inferences to be drawn within the sample, identifying statistically significant differences between groups and knowledge subdomains, beyond a simple description of frequencies or measures of central tendency. In addition, the analysis was conducted both at a global level and across specific subdomains, moving beyond a purely descriptive approach and allowing for a more detailed interpretation of the observed differences.

From a qualitative perspective, the answers were categorized using the content analysis technique (Krippendorff, 2013). As a result, the maximum score possible on the questionnaire was 44 points, and the minimum was 0. To ensure the reliability of the coding process, the answers to the questionnaire were successively reviewed in a cyclical and deductive manner (Bisquerra, 2009). Then, a triangulation was carried out based on continuous reviews of the responses, ending with a discussion by the authors of the disagreements involving the coding process until a consensus was reached.

RESULTS

In accordance with the objectives of the study, first, the data obtained from a general analysis of the total questionnaire scores are presented. The purpose of this general analysis is to provide an overall view of the participants' performance, while the analysis of domains and subdomains allows for a more detailed identification of the specific mathematical knowledge required for teaching. The following presents the results in relation to the domains and subdomains of Mathematical Knowledge for Teaching.

Total Questionnaire Score

To analyse the total scores on the questionnaire, the answers of the 76 pre-service primary education teachers were categorized by the degree of correctness. The scores ranged from 9 to 35 points, with no maximum scores, meaning no pre-service teacher answered all the questions on the questionnaire correctly.

The average score was 24 points, slightly above the theoretical average of 22 points, yielding a success rate of 54.5%, with a standard deviation of 5.6 points. **Figure 5** shows that the median score is the same as the average score (24 points), meaning the data distribution is symmetrical. The total scores received by the pre-service primary school teachers on the questionnaire were between 20 and 28 points and the whiskers are slightly different, with a longer lower whisker than upper whisker. The ends of the distribution are thus slightly different, meaning there are low scores that deviate from the bulk of the data. Finally, it can be seen that there are no outliers; that is, there are no extreme scores.

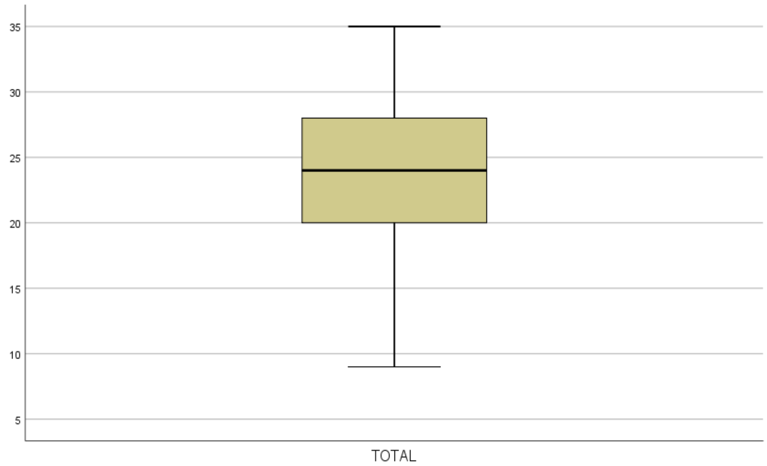


Figure 5. Distribution of total scores and median score on the questionnaire (Source: Authors' own elaboration)

Comparison of the Domains and Subdomain of Mathematical Knowledge for Teaching Early Algebra

To establish a comparison between the subdomains of mathematical knowledge to teach early algebra, the total scores for the questionnaire were recoded based on the type of knowledge as per a normalized scale of 0 to 100, since the number of items differs for each subdomain.

Figure 6 shows the scores of the pre-service primary school teachers in the different subdomains of knowledge. In general, we see that the subdomain of CCK knowledge received a higher score compared to the other subdomains of knowledge, with over 50% of the participants exceeding a normalized score of 87.5%. The subdomain of knowledge at the mathematical horizon received the lowest score on the questionnaire.

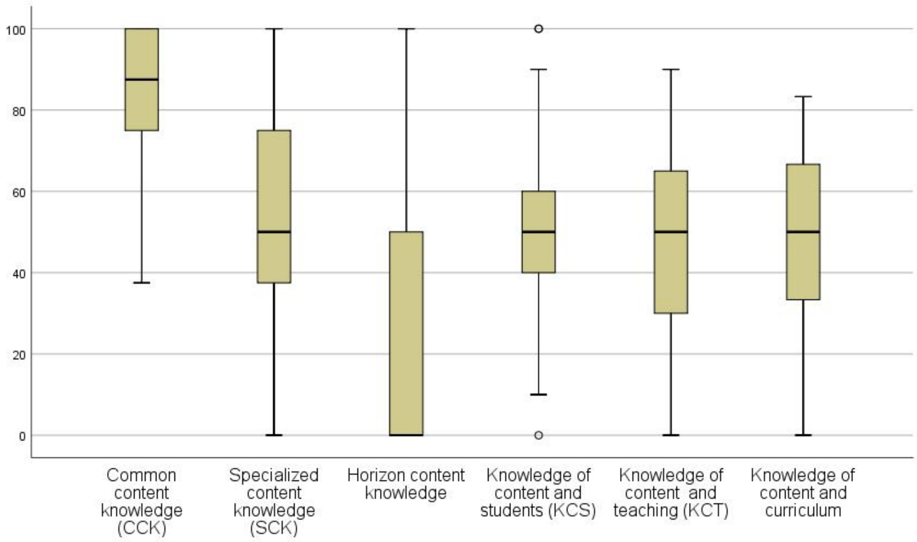


Figure 6. Distribution of normalized scores by subdomains of mathematical knowledge (Source: Authors' own elaboration)

If consider the domain of subject knowledge more specifically, we notice that the subdomain of common knowledge achieves a maximum normalized score of 100%, while the high normalized scores exhibited by the knowledge at the mathematical horizon and specialized knowledge are further removed from the bulk of the data. In the case of the latter, the median is closer to the first quartile, and therefore the normalized scores are concentrated in the lower part of the box, between 37.5% and 51.2%

Elsewhere, regarding the pedagogical knowledge of the content, the normalized scores of the subdomain of the knowledge of content and students show a symmetric distribution of the normalized scores. In addition, there are two outliers, at the upper and lower ends of the box, with minimum normalized scores that reach the minimum and maximum score, respectively.

Finally, for the subdomains of knowledge of content, teaching and curriculum, the maximum normalized scores achieved by the pre-service teachers do not exceed 90% and 83.3%, respectively. However, knowledge of content and teaching exhibits normalized scores that are slightly clustered at the top of the box, while the normalized scores for knowledge of the curriculum are concentrated in the center of the box, at around 50%.

As seen in **Figure 5**, there are differences between the scores obtained for the different subdomains of mathematical knowledge. To delve into these differences and determine if they are statistically significant, pairs of subdomains were compared by applying a non-parametric sign statistic for paired samples with a 95% confidence level, as shown in **Table 3**.

Table 3. Wilcoxon Signed-Rank test of the scores obtained in relation to the different subdomains of the MKT model

	Common content knowledge	Specialized content knowledge	Mathematical horizon knowledge	Knowledge of content and students	Knowledge of content and teaching
Specialized content knowledge	0.000	-	-	-	-
Knowledge at the mathematical horizon	0.000	0.000	-	-	-
Knowledge of content and students	0.000	0.720	0.000	-	-
Knowledge of content and teaching	0.000	0.160	0.000	0.45	-
Knowledge of the curriculum	0.000	0.812	0.000	0.724	1

The results of the Wilcoxon rank tests applied to the different subdomains of knowledge are mostly < 0.05. Therefore, the different subdomains of knowledge that were compared to one another obtain significant differences in their scores. However, the comparison between specialized content knowledge and the subdomains of knowledge of content and students, knowledge of content and teaching, and knowledge of the curriculum, show similar values. The same thing is reflected between the knowledge of content and students and the subdomains of knowledge of content and teaching, and knowledge of the curriculum, as well as knowledge of the curriculum and knowledge of content and teaching.

Subject Matter Knowledge

Common content knowledge

To evaluate the CCK, the focus is on the answers given by pre-service teachers gave when solving algebraic tasks that require understanding different types of arithmetic relationships, figurative patterns, change and the use of symbols and variables to represent mathematical situations. To analyse this knowledge, four situations were presented with potential answers from primary education students and questions of the type:

a) What answers should the teacher accept as correct? Why? and,

b) Solve the problem posed by the teacher. Explain your answer.

These questions are posed in items 1a, 2a, 3a and 4a. **Figure 7** shows the degree of correctness of the answers provided for these items.

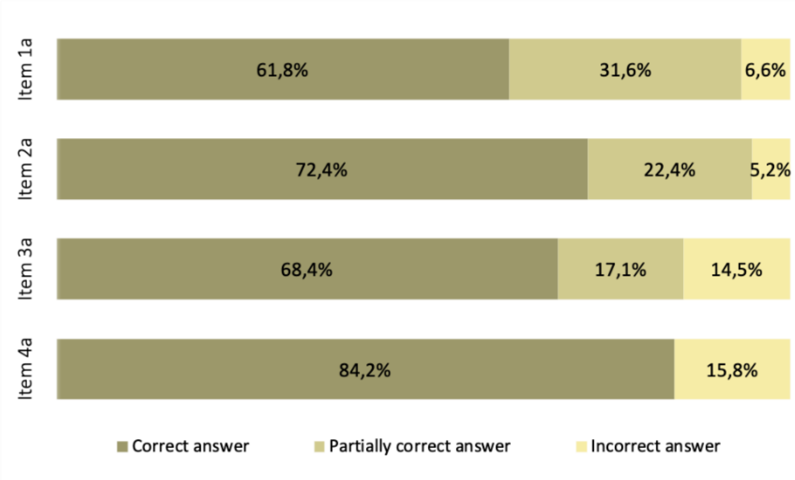


Figure 7. Distribution of answers for the CCK by degree of correctness (Source: Authors' own elaboration)

The percentage of correct answers in the CCK block exceeds 61.8%. An analysis of the answers to item 1a shows that over 50% of the teachers understood the meaning of the equal sign as an operator and expression of equivalence, when they recognized the answers by Carlos, Joaquín and Cristina, and Paula, Mateo and Mauricio as correct. The incorrect answers (31.6%) were attributed to those teachers who did not provide adequate justification as to why they accepted these answers as correct.

Regarding item 2a, 72.4% of the pre-service teachers identified as correct the answers of students 1 and 2, since they were able to determine a general rule to represent the problem based on the dependent variable "*number of chairs*" and the independent variable "*number of tables*"; while 22.4% did not justify their response or gave an inadequate justification.

In item 3a, more than 60% of the pre-service teachers solved the problem correctly, stating that on the first, second and third day, the results were 4, 8 and 16 candies, respectively. An analysis of the arguments reveals the different strategies used by the pre-service teachers to solve the situation correctly, most notably trial and error, the use of a first-degree equation and the representation of the situation in a graphic form, such as boxes, that adhere to a certain structure to distribute the candies. The pre-service teachers who gave a partial answer determined the number of candies, but did not provide any justification for solving the problem, or their argument was inadequate.

In item 4a, 84.2% of the pre-service teachers correctly identified Pedro's height as "*n* +4", showing that "*n*" represents Clara's height. In contrast, 15.8% of the participants failed to identify the indicated height.

Specialized content knowledge

The set of items focused on evaluating the SCK consider how the pre-service teachers reflect on the contents and properties that must be mobilized to solve an algebraic task, as well as the analysis and justification of mathematical situations.

To analyse this knowledge, the questionnaire proposes four items (2b, 3b, 5d and 6a) that delve into questions of the type: What content and/or mathematical properties should students use to correctly answer the task? Explain if you think the output of the students is mathematically correct or not. Justify the appropriateness or insufficiency of the mathematical rationale shown by the students. **Figure 8** shows the degree of correctness of the answers provided for these items.

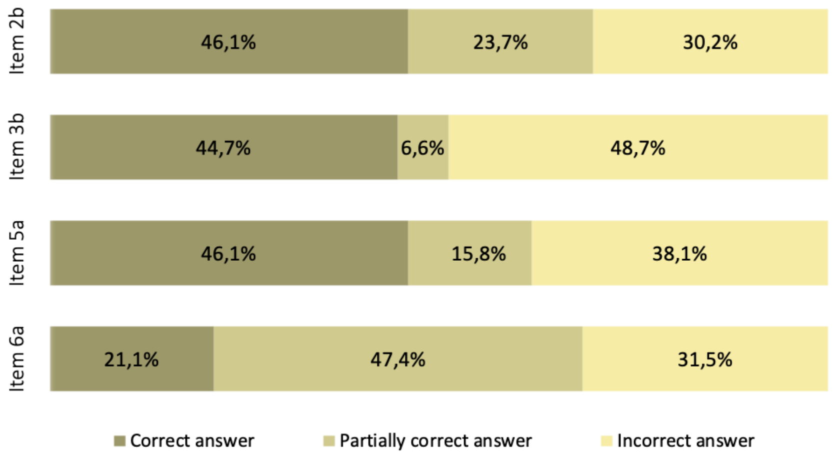


Figure 8. Distribution of answers for the SCK by degree of correctness (Source: Authors' own elaboration)

The percentage of right answers linked to SCK was below 46.1%. Note that items 2b and 5a involving the mathematical contents or properties associated with generalizing relationships through algebraic notation and understanding different types of relationships between numbers, and the construction of generalizations about their properties, respectively, exhibit a higher percentage of correct answers. They are followed by item 3b, which involves assessing the mathematical rationale expressed in a mathematical situation. Meanwhile, item 6a, related to the mathematical content on the meanings of the equal sign, exhibits greater limitations since the percentage of right answers was less than 21.1%.

An analysis of the arguments presented in item 2b shows that the majority of teachers recognized that the content involved in the teaching situation is related to discovering the rule of succession or expressing the generalization of succession. However, a high percentage of the pre-service teachers (30.2%) failed to identify the mathematical content associated with the teaching situation, and 23.7% very generically identified expressions with letters and mathematical sequences as content.

In item 3b, the arguments presented show that the majority of pre-service teachers (48.7%) failed to determine the student outputs that were mathematically correct. Only 44.7% considered Teresa's output as mathematically correct, stating that she distributed the candies according to the stipulated conditions and with help from a pictorial representation, while Lucas's output considers an intuitive resolution, without justifying the answer. 6.6% of the pre-service teachers explained which outputs they considered mathematically correct; however, they did not justify the adequacy or insufficiency of the mathematical rationale shown.

In item 5a, 46.1% of the pre-service teachers pointed out that the mathematical contents are related to the properties of the operations; more specifically, they mention the commutative property, associative property, the identity element of multiplication and the identity element of addition. 15.8% were only able to identify one of these properties, while 38.1% failed to identify the mathematical content or properties present in the task.

Finally, an analysis of the arguments in item 6a shows that a low percentage of the pre-service teachers (21.1%) identified the equality relationship, equivalence relationship and associative property contents. The majority of the pre-service teachers (47.4%) identified only one content, while a high percentage (31.5%) failed to identify the contents or properties present in the task.

Knowledge at the mathematical horizon

Item 1d enabled us to evaluate the KMH. To analyse this knowledge, the focus is on the links that teachers establish between the mathematical contents involved in the teaching situation and others proposed in the extension of the curriculum, through the question: What advanced concepts from the school curriculum are relevant to the content addressed in the task?

The teaching situation requires exploring the different meanings of the equal sign, such as its operational meaning, equivalence meaning, and relational meaning. Therefore, the participants were expected to relate the content presented in the task with the notion of first-degree equations and functions.

The results show that item 1d was one of the most difficult on the questionnaire, since only 9 pre-service teachers (11.8%) answered it correctly; 56 of them (73.7%) answered incorrectly, offering nonsensical arguments; and only 11(14.5%) gave a partially correct answer, stating that the content presented in the task is related to working with equations and problem-solving.

Pedagogical Content Knowledge

Knowledge of content and students

To assess the KCS, the pre-service teachers were asked to anticipate students' thinking in relation to errors or difficulties they might have in dealing with a given algebraic task. The questions asked that allow us to analyse this knowledge are: What difficulties do the students in the course exhibit when solving the problem? What difficulties could the students who answered it incorrectly be facing? Describe the possible difficulties that led the students to answer incorrectly. These questions are raised in items 1b, 2c, 4b and 5b of the questionnaire. **Figure 9** shows the degree of correctness of the answers provided for these items.

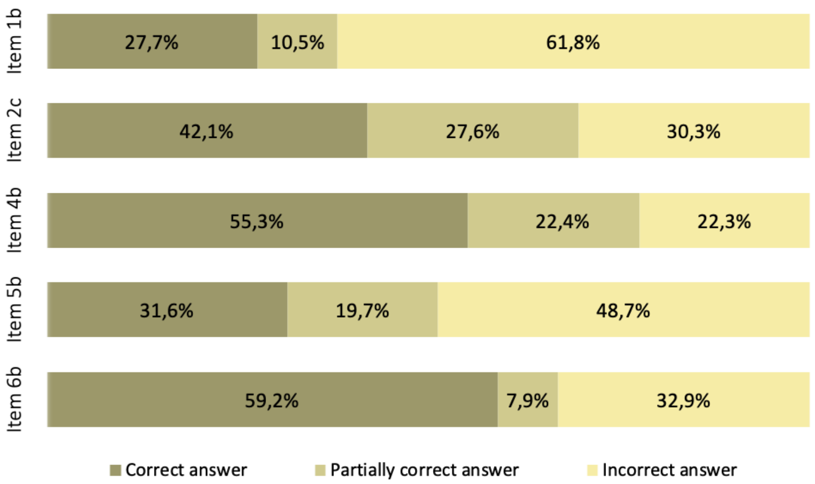


Figure 9. Distribution of answers for the KCS by degree of correctness (Source: Authors' own elaboration)

The results show a low mastery of knowledge of content and students, since the percentage of right answers was below 55.3% for all the items. More specifically, item 1b addresses the possible difficulties that fourth-graders experience when solving a problem involving the meanings of the equal sign. This item shows a high percentage of incorrect answers (61.8%), most of which mention that the difficulties presented by the students are not having enough data to solve the problem, the difficulty represented by the problem statement, and understanding the statement. Only 27.7% of the pre-service teachers responded correctly, stating that Santiago, Raquel and Carolina's answer shows a lack of understanding the equal sign as equivalence, making it difficult for the students to realize that the problem has several possible solutions. The remaining 10.5% mentioned in a very general way that the possible difficulties have to do with the understanding of the equal sign.

The arguments presented in item 2c show that 42.1% of the pre-service teachers identified that the difficulty faced by the student who answered incorrectly has to do with the representation of generalization through an algebraic expression. The arguments presented in the partially correct answers (27.6%) mention that the difficulties are centred around finding an algebraic expression or understanding the series. A high percentage of the pre-service teachers (30.3%) failed to identify the difficulties.

In item 4b, 55.3% of the pre-service teachers stated that the difficulties exhibited by the students were related to the use and interpretation of variables to represent an unknown amount, such as Pedro's height; more specifically, in the case of Luis, he interpreted the expression "4 cm higher than Clara" as "4 times higher than Clara", Pilar assumed a height that was not given, and María used another variable to represent Pedro's height. 22.4% of the participants mentioned in passing that the difficulties of the students have to do with creating an algebraic expression based on a problem, while 22.3% failed to identify the difficulties the students experienced solving the task.

The arguments presented in item 5b show that 48.7% of the participants failed to describe the possible difficulties that led students to answer incorrectly, while only 31.6% of the pre-service teachers identified as a problem the lack of knowledge of equivalence expressions and the properties of operations, and, finally, item 6b had a high percentage of right answers (59.2%) that show that the possible difficulties that led students to answer incorrectly are related to the interpretation of the equal sign; while 7.9% of the pre-service teachers only mentioned the difficulty of the addition operation, and 32.9% failed to identify any difficulties.

Knowledge of content and teaching

To respond to the set of items focused on evaluating the KCT the pre-service teachers were tested on the use of different methods and procedures that are involved in the teaching practice. This knowledge is reflected in five items on the questionnaire (1c, 2d, 4c, 5c and 6c), which consider questions such as: What teaching strategies would you use to help the students who were unable to solve the task? or, What teaching strategies would you use as a teacher to guide those students who answered the task incorrectly?

The degree of correctness of the answers given for these items is shown in **Figure 10**.

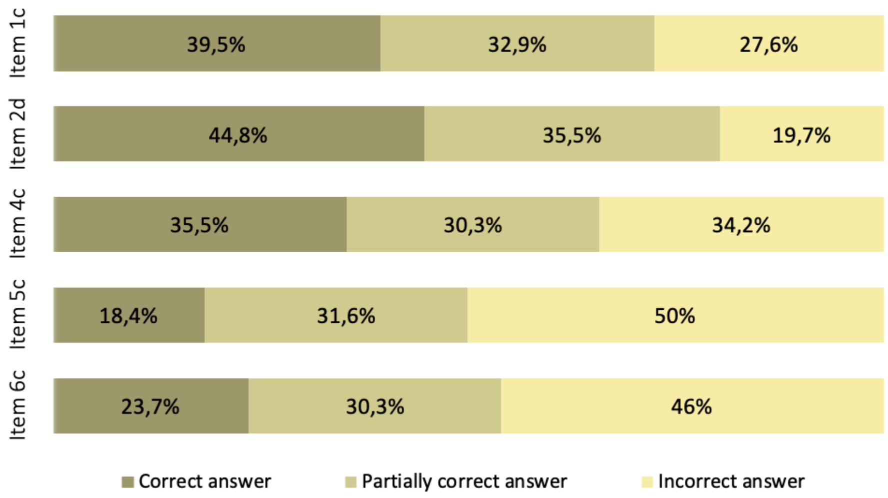


Figure 10. Distribution of answers for the KCT by degree of correctness (Source: Authors' own elaboration)

In general, the results show that the percentage of correct answers is low, since they are below 44.8% in all the items proposed. There is a high percentage of partially correct answers, where the pre-service teachers mention teaching strategies and resources that are not entirely conclusive, such as the use of manipulatives, since they do not explain how they would implement them in the classroom and why they are appropriate.

For the arguments given in the answers to item 1c, which involves understanding the meaning of the equal sign as an operator and as an expression of equivalence, 39.5% of the pre-service teachers were able to describe possible teaching strategies relevant to the task, including the use of manipulatives, such as coins, to analyse different cases where equality applies. A high percentage (32.9%) mentioned strategies on a very general level, but they were inconclusive; more specifically, they mentioned working with manipulatives, using a diagram to represent the problem presented in the task and analysing the concept of equivalence in depth, without specifying the elements and concepts proposed. The remaining 27.6% suggested meaningless strategies for dealing with the task.

Regarding item 2d, an analysis of the correct answers revealed that 44.8% of the participants proposed adequate teaching strategies to help the students who were unable to solve the problem, which involves determining a general rule from a given sequence. The main idea proposed was the use of a table of values to describe the relationship between the "number of chairs and "number of tables" variables, representing the situation live in the classroom with chairs and tables, making predictions and establishing conjectures, and checking random values for different cases once the general rule is established. As with the previous item, a considerable percentage of responses (35.5%) referred to strategies that are not entirely conclusive, mentioning, for example, the use of manipulatives to understand the sequence without detailing what materials to use. The remaining 19.7% did not mention any adequate teaching strategies.

Moving on, an analysis of the arguments presented in item 4c revealed that 35.4% of the pre-service teachers proposed appropriate teaching strategies to help those students who did not understand the notion of variable to represent an unknown constant. They included observing the difference between the heights of the students and proposing conjectures, as well as asking guided questions that prompt reflection, such as, what does it mean for Pedro to be 4 cm taller than Clara? How can we represent this idea? Does "4cm taller" mean the same as "4*n*"? What does the letter "*n*" represent in the expression provided? By contrast, 34.2% of the pre-service teachers suggested inadequate strategies to help those students who were unable to solve the task, while 30.4% offered inconclusive strategies, such as giving other examples, helping them visually by using diagrams and manipulatives, without specifying the types of examples or diagrams, or what the most appropriate manipulative would be for this situation.

For item 5c, the task that deals with the development of generalized arithmetic, 50% of the participants did not provide adequate strategies to guide those students who responded incorrectly, being the item that posed the greatest difficulty in relation to knowledge of content and teaching. Only 18.4% of the pre-service teachers gave adequate strategies, including analysing the content of the properties of the operations using manipulatives, such as scales, for example. The partially correct answers (31.6%) gave inconclusive strategies, such as providing simpler examples, analysing numerical sentences with the whole class, and the use of manipulatives was again proposed, without specifying what kind.

Finally, as in item 5c, item 6c, which focused on proposing strategies to shift from the operational meaning of the equal sign to its equivalence meaning, resulted in a high percentage of incorrect answers (46%) and a low percentage of correct answers (23.7%). The latter are directly linked to the use of manipulatives, such as Cuisenaire strips and Multilink cubes, to explain number decompositions and analyse the associative property. The remaining 30.3% of the arguments were partially correct, since they proposed generic strategies such as practicing numerical equality through meaningful activities.

Knowledge of curriculum

The set of items focused on evaluating the KCC delves into central aspects of the primary school curriculum that pertain to a certain teaching situation. To analyse this knowledge, questions were posed of the type: Judging by the current primary school education curriculum, what grade do you think this problem is appropriate for? Considering the primary school curriculum, what might be the goal of the task proposed to the students? These questions are presented in items 3c, 4d and 5d. To answer the question, the teachers were expected to bring to bear their knowledge of the contents proposed in the curriculum and its intended purpose. The degree of correctness of the answers given for these items is shown in **Figure 11**.

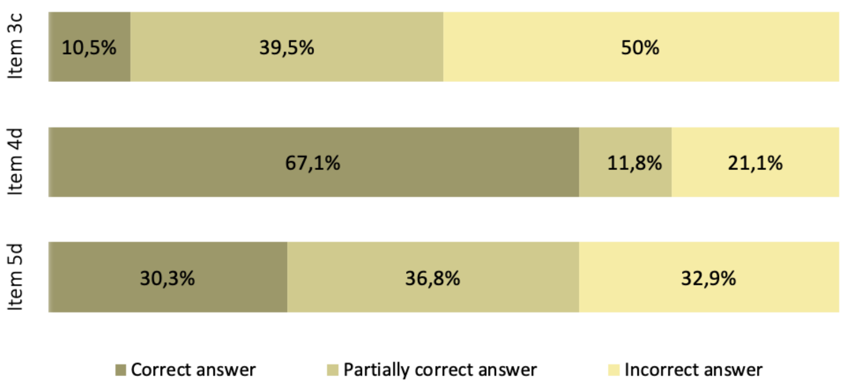


Figure 11. Distribution of answers for KCC by degree of correctness (Source: Authors' own elaboration)

The results show certain limitations in the knowledge of the curriculum, with the pre-service teachers exhibiting difficulties in item 3c to interpret the intentionality of the teaching situation. Only 10.5% said that the purpose of the task has to do with establishing relationships of equality by determining unknown variables; 39.5% stated very generically that the purpose of the task is problem solving; and 50% failed to establish the objective of the teaching situation.

Regarding item 4d, a high percentage of the pre-service teachers (67.1%) stated that the teaching situation is relevant for 5th and 6th grades, adequately justifying the relevance of the variables; 11.8% gave a partially correct answer, since they adequately identified the school level, but did not justify their response; and 21.1% failed to identify the school level to which the teaching situation is relevant.

Finally, the answers given for item 5d show that 30.3% of the pre-service teachers considered the problem appropriate for 3rd and 4th grades, justifying the adequate introduction of the properties of the operations at this level, while 36.8% identified the level, but did not justify their response. Likewise, a high percentage of the pre-service teachers (32.9%) failed to identify the appropriate school level at which this teaching situation may be presented.

DISCUSSION

Considering that the majority of preliminary studies have analysed partial aspects of teachers' knowledge to teach algebra (e.g., Narváez et al., 2024; Pincheira & Alsina, 2021b), in this study an integral analysis of all domains of knowledge of 76 pre-service primary education teachers to teach early algebra has been carried out, from the perspective of the MKT model (Ball et al., 2008). The MKT-Early Algebra questionnaire (6-12) was administered, designed and previously validated (Pincheira & Alsina, 2024), and reviewing the answers of the pre-service teachers to the different questions that comprise it.

The study delved into the domains and subdomains of the MKT model, revealing that the mathematical knowledge of the pre-service primary school teachers to teach algebra is insufficient, given the average percentage of correct answers (42.8%).

In relation to the domain of Subject Matter Knowledge, the interpretation of the results shows that the subdomain of CCK is better situated in comparison to the other subdomains of knowledge. For example, the pre-service teachers identified the general rule to establish a relationship between variables, established relationships with numerical expressions, solved problems involving the meanings of the equal sign, and so on. This agrees with other studies (e.g., Barboza et al., 2020, 2021; Ferreira et al., 2017; Trivilin & Ribeiro, 2015; Zapatera et al., 2018) where teachers successfully answered tasks involving the different meanings of the equal sign, pattern generalization and the use of variables.

The SCK reveals limitations, since the pre-service primary school teachers did not correctly identify mathematical contents and properties in tasks involving relationships between numbers and the construction of generalizations about their properties, the use of variables and relationships between covariant quantities. Along these lines, other studies identify similar results regarding specialized knowledge, where the selection of examples presented by the teachers to describe relationships that lead to generalization are not always successful (McAuliffe & Lubben, 2013), difficulties assigning the semantic meaning of division required in the solution of an algebraic task that involves an equitable distribution (Bernardo et al., 2017), the operational, equivalence and relational meaning of the equal sign in a mathematical expression (Barboza et al., 2020; 2021; Ferreira et al., 2017; Trivilin & Ribeiro, 2015), and the identification of recursive and explicit generalization strategies in the development of functional thinking (Wilkie, 2014, 2016).

As for the KMH, it exhibits major limitations since the pre-service teachers did not establish connections between the contents of a task based on the generalization of arithmetic that explores the different meanings of the equal sign - operational, relational and as an expression of equivalence - with other contents in the school curriculum. The study by Barboza et al. (2021) analyses this same area through the relationships established by teachers in tasks that involve relational thinking, finding little connection with the work required in subsequent years.

Limitations are also evident in the mastery of Pedagogical Content Knowledge. In the case of KCS, the findings indicate that the teachers had problems anticipating student errors in arithmetic generalization tasks, finding similarities with the studies of Barboza et al. (2020) and Demonty et al. (2018) and other studies that delve into possible student errors related to functional relationships (Wilkie, 2014, 2016). Likewise, in the KCT, we found that the strategies proposed by the educators to teach an algebraic task were not entirely conclusive, since they exhibited difficulties determining implications for learning number relationships and the properties of operations involving the generalization of arithmetic, as well as generalization tasks that employ covariable relationships focused on the development of functional thinking. This last point shows similarities with the results of Wilkie's (2014) study with in-service primary education teachers, which reveals the difficulties of helping students generalize the relationship between the input and output numbers of a function machine. Regarding the KCC, we observed that the pre-service primary education teachers were sometimes limited in their ability to interpret the intentionality of a teaching situation and link it with the guidelines proposed in the curriculum, as in the studies by Trivilin and Ribeiro (2015) and Wilkie (2014).

It is worth noting that this study is grounded in the MKT-early algebra (6–12) questionnaire, which was designed to systematically examine all the subdomains of the MKT model within the context of early algebra teaching. In this sense, the instrument allows for the analysis of the different types of mathematical and pedagogical knowledge involved in early algebraic activity in primary education, offering a detailed account of the knowledge required for teaching early algebra. This approach represents the main contribution of the study, as it provides a broader and more integrative perspective than previous research, which has primarily focused on specific subdomains or partial aspects of teachers' knowledge.

CONCLUSION

According to Strand and Mills (2014), teachers who introduce early algebra teaching, such as primary school teachers, are responsible for facilitating their students' ability to build their algebraic understanding. From this perspective, on the one hand, the results of the research favour a global reflection on the set of mathematical and pedagogical knowledge that pre-service primary school teachers must develop to teach early algebra; and, on the other hand, it encourages a discussion of what characteristics teacher training should consider to promote the mathematical knowledge that is demanded to teach early algebra in primary education.

In this study, it is recognized that the teaching of early algebra in primary education represents a restructuring of teaching practice (Hohensee, 2017), which thus implies changes in teacher training. Given this, it is necessary to offer training programs to primary education teachers that focus on the following areas of action: Subjects designed to analyse algebra in depth as a mathematical object, as well as its didactics. To achieve this, it is considered necessary to incorporate Blanton's (2008) guidelines on how to structure the instruction of early algebra in the primary education classroom through four elements: Represent (teachers must provide multiple ways for students to systematically represent algebraic situations); ask (pose questions that encourage students to think algebraically); listen (this involves catering to and developing student thinking); and generalize (that is, help students to develop and justify their own generalizations). Moreover, professional training tasks are needed based on teaching practice in relation to the different algebraic contents that are required to be taught in primary education, considering the general guidelines on the training of primary education teachers (Lui & Bonner, 2016), and those specific to the teaching of this content block (Alsina, 2019). These professional tasks should consider the design and implementation of early algebraic tasks focused on the ability to make and express generalizations (Kaput, 2008), as well as on presenting algebraic ideas, determining an example to express a specific algebraic approach, evaluating explanations of an algebraic nature, and asking productive mathematical questions that promote the development of algebraic thinking.

In conclusion, the analysis of our results has made it possible to identify the mathematical knowledge that must be urgently addressed in the initial training of teachers. In this regard, one limitation of the study was that evaluation of partial and initial aspects of the different subdomains of the MKT model, focusing only on the essential aspects that characterize each subdomain for the different items that comprise the questionnaire. Moreover, the study did not include follow-up interviews, which would have allowed for a deeper exploration of the participants' mathematical and pedagogical knowledge and provided a more precise validation of their responses.

In the future, therefore, we will endeavour to design further inquiries to analyse the actions that pre-service teachers employ in the primary education classroom to teach early algebra through their pedagogical practices, and to craft interviews that allow us to determine their mathematical and pedagogical knowledge for teaching algebra to young schoolchildren.

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Ethical statement: The authors stated that this work is developed within the framework of a doctoral thesis, the activities in the study were based on standard educational practices. No sensitive personal information was collected and the study involved no risk to participants. Participants were informed that the data would be used solely for academic purposes and handled securely. Informed consent was obtained from all subjects involved in the study.

AI statement: The authors stated that no generative artificial intelligence or AI-based tools were used for the writing, editing, or data analysis of this article.

Declaration of interest: The authors declare that they have no conflicts of interest to declare that they are relevant to the content of this article.

Data sharing statement: The authors declare that the participants provided their informed consent for the publication of the data.

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