INDIVIDUAL CURRICULA: TEACHERS' BELIEFS CONCERNING STOCHASTICS INSTRUCTION

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ABSTRACT. This report focuses on in-service teachers' planning of stochastic education. The theoretical and methodological settings of the research will be outlined in-depth. The methodological settings will be illustrated by research results concerning one teacher. A further main focus is to present some results concerning the planning of stochastic education conducted by 13 teachers.

KEYWORDS. Teachers' Individual Stochastics Curricula, Teachers' Beliefs, Secondary High School.

INTRODUCTION

"How teachers make sense of their professional world, the knowledge and beliefs they bring with them to the task, and how teachers' understanding of teaching, learning, children, and the subject matter informs their everyday practice are important questions that need an investigation of the cognitive and affective aspects of teachers' professional lives" (Calderhead, 1996, p. 709).

One important aspect of the research on teachers' beliefs is the conclusion that they have a high impact on students' beliefs (Chapman, 2001). This conviction is also shared by stochastic educators (e.g. Hill, Rowen & Ball, 2005). Another basic assumption in the research on teachers' beliefs is that one must accept the central role of teachers in changing or reforming mathematics education (Wilson & Cooney, 2002). Quite clearly, there is a fundamental need to understand everything that underlies the way in which mathematics teachers approach their subject before suggestions and recommendations concerning good classroom practice can be made. In order to do this, we must first investigate how teachers plan their teaching. In other words, we must reconstruct their individual stochastic curricula (in this article the term *stochastics* will be used to refer to both statistics and probability).

This research is part of a wider qualitative research project that focuses on teachers' planning of stochastic instruction (*teachers' individual curricula*), teachers' classroom practice (*teachers' factual curricula*), and the knowledge and beliefs students achieve after stochastic courses (*students' implemented curricula*). This report focuses only on the first step of the research project, namely to understand the teachers' individual stochastic curricula. We firstly intend to reconstruct the teachers' individual curricula as an argumentative system of

instructional goals and describe five clusters of instructional goals. The second objective is to construct types of teachers' individual curricula that facilitate a comparison between reform ideas concerning the stochastic instruction provided by university educators and the planning of "conventional" stochastic teachers. With regard to this objective, we describe four types of teachers' individual curricula. We also provide some evidence that there is a considerable gap between the discussion at universities and school practice.

One of the "priority questions" posed by Batanero, Garfield, Ottaviani and Truran (2000) is related to describing the "features of good practice study in statistical education". For this reason, in the report, we will discuss in depth the theoretical framework and the five-step-methodology of this research on individual curricula.

RECENT RESEARCH ON STOCHASTICS TEACHERS' BELIEFS

In 2000, Batanero et al. (2000, p. 5) posed another "priority question" citing Shaughnessy (1992): "What are teachers' conceptions of probability and statistics?" They suggested that this question "has not been sufficiently researched" (ibid.). Despite the great deal of research concerning students' reasoning about stochastics in this decade, there are still few results concerning teachers' reasoning about stochastics or teachers' beliefs about teaching stochastics. For example, Batanero et al. (2007, p. 4) mentioned that "there has not been a sustained effort in exploring, explaining and improving teachers' statistical conceptions, attitudes and beliefs."

Although research results concerning statistics education from a teachers' perspective are scarce, it is possible to find some. We can classify the existing research by introducing four dimensions, i.e. (1) the teachers' professional status, (2) the type of teachers' knowledge investigated, (3) the purpose of the analysis of empirical data, and (4) the research method. Below we summarise these dimensions, and include a research example for each dimension. Other research examples that match the specific features concerning the four dimensions will be listed additionally.

The teachers' professional status

A first classification is whether the research is concentrated on pre-service teachers or on in-service teachers. For example, Monteiro and Ainley (2006) focused on the interpretation of media graphs, by analysing the responses of 218 primary school student teachers in Brazil and England to questionnaires. Other related research has been published by Burgess (2002) and Paparistodemou, Potari and Pitta (2006).

In the same way, Pfannkuch (2006) focused on one in-service teacher and her reasoning about teaching the comparison of two distributions. Other research concerning in-service teachers has been published by Cai and Gorowara (2002), Sanchez (2002), Eichler (2004), Makar and Confrey (2004), Mickelson and Heaton (2004), Eichler (2006), Lopes (2006), Serrado and Azcarate (2006), and Watson (2006).

The teachers' knowledge

Using the classification of Shulman (1986), it is possible to distinguish research whose main goal is the teachers' content matter knowledge on the one hand, and the teachers' curricular knowledge and the teachers' pedagogical content matter knowledge on the other.

For example, Makar and Confrey (2004) looked at the content matter knowledge. They analysed the reasoning of four in-service secondary teachers concerning the comparison of two distributions. Other research concerning the teachers' content knowledge has been published by Burgess (2002), or Mickelson and Heaton (2004).

Paparistodemou, Potari and Pitta (2006) were interested in how 23 prospective teachers planned, taught and reflected upon statistics activities that would potentially develop the stochastic ideas of young children. Other research that provided results concerning the teachers' curricular knowledge or the teachers' pedagogical content matter knowledge have been published by Cai and Gorowara (2002), Sanchez (2002), Eichler (2006), Pfannkuch (2006), and Serrado and Azcarate (2006).

The purpose of the analysis of empirical data

There is a distinction between research that provides an evaluation of the teachers' beliefs based on theoretical considerations concerning statistics education, and approaches that describe the teachers' beliefs by adopting a neutral position.

For example, Mickelson and Heaton (2004) firstly described the teaching practice of one in-service third-grade teacher (Donna) concerning data and distribution, and afterwards they evaluated Donna's teaching practice basing on their normative conceptions of statistics education. Other research that evaluates teaching practices based on normative conceptions of statistics education has been published by Burgess (2002), Sanchez (2002), Makar and Confrey (2004), Monteiro and Ainley (2006), and Paparistodemou, Potari and Pitta (2006).

Serrado and Azcarate (2006) analysed the arguments of two in-service teachers to justify their reluctance to introduce probability in compulsory secondary education by adopting a neutral position. Other research with this same approach has been published by Cai and Gorowara (2002), Sanchez (2002), Pfannkuch (2006), and Eichler (2006).

The research method

Some of the existing research on teachers' beliefs concerning stochastic education uses quantitative methods. For example Monteiro and Ainley (2006), analysed the questionnaires of 218 primary school student teachers in Brazil and England.

However, most research that focuses on teachers' beliefs concerning stochastic education is qualitative, i.e. they use case studies or interpretative designs (e.g., Makar & Confrey, 2004; Mickelson & Heaton, 2004; Eichler, 2006; Lopes, 2006; Paparistodemou et al., 2006; Pfannkuch, 2006; Serrado & Azcarate, 2006).

The research project that will be discussed in this report involves a qualitative design. It focuses on in-service teachers, primarily concerning their curricular knowledge and secondarily concerning their pedagogical content matter knowledge. Finally, the aim of this research project is to describe, or rather to understand, conventional school practice.

THEORETICAL FRAMEWORK

Fundamental terms and assumptions

The research is shaped by the central terms of "curriculum" and "individual curriculum". By *curriculum* we mean teachers' conscious choices concerning mathematical contents, including the reasons for their choices.

The term teachers' *individual curriculum* is taken from a model developed by Vollstädt et al. (1999). In this model, five levels of the curriculum are distinguished. The first two levels include the *objective* curriculum, which evolves in public discourse and which is laid down in the official curriculum. The further three levels include the *individual* forming of the curriculum, i.e. the individual teacher's instructional planning (individual curriculum), the individual teacher's classroom practice (factual curriculum), and the knowledge and beliefs the students attain after school courses (implemented curriculum).

We focus on the third level of this larger model, the teachers' planning of mathematics instruction, i.e. the various individual curricula. We understand the teachers' instructional planning, in a psychological sense, as intentions of action, which cannot be observed. Action itself is understood as "the physical behaviour plus the meaning interpretations held by the actor" (Erickson, 1986, p. 126).

Both intention and action in their entirety are not observable, but are dependent on situations, as well as on individuals' interpretation of a situation. Based on this central assumption, the approach of this research is qualitative and interpretative. Moreover, the research focuses on *understanding* action as a non-observable internal process, as opposed to *explaining* an observable behaviour in a mechanistic way (this duality of *Verstehen und Erklären* – i.e. understanding and explaining is, for instance, described by Schwandt (2000) and Gadamer (1986)).

Two more general assumptions govern the research into individual curricula. These assumptions, mentioned by Groeben, Scheele, Schlee and Wahl (1988), concern the research on an individual's action. The first of them is that which Groeben et al. (1988) named "epistemologisches Menschenbild" (epistemological human idea), and involves understanding teachers as reflexive subjects who act autonomously and rationally. Furthermore, the epistemological human idea involves the assumption that the teachers construct their individual theories of mathematics, and of the teaching and learning of mathematics, in much the same way as researchers construct their theories. The second assumption is that teachers can provide insights into their own intentions of action (Pajares, 1992), while researchers have only an approximate understanding of these intentions.

Theoretical constructs

In the planning of classroom practice, individual curricula are understood as teachers' belief systems, within which "knowledge and beliefs are inextricably intertwined" (Pajares 1992, p. 325). Furthermore, individual curricula are understood to have been the outcome of a teacher's process of socialisation, including the teacher's schooling and professional experience. As the construct "individual curricula" is linked to the term "belief systems", it is anchored in two theoretical constructs, i.e. *subjective theories* and *goal-method-argumentation*.

Firstly, the construct *subjective theories* is derived from psychological research (Groeben et al., 1988). Subjective theories are defined as complex systems of cognitions (complex beliefs systems), which contain a rationale that is at least implicit. Hence, single cognitions are connected in an argumentative manner. This definition is based on the epistemological human idea (see above). Subjective theories contain:

• Subjective concepts, e.g., concerning the teachers' instructional planning, the teachers' subjective goals of instruction,

• Subjective definitions of these concepts or goals and, finally,

• The relations between the subjective concepts or goals that constitute the argumentative character of the cognitions system.

Secondly, the construct *goal-method-argumentations,* which Groeben et al. (1988) have adapted as a technique to describe subjective theories, derives from pedagogical research (König, 1975), and makes explicit the relationship of individual curricula and subjective theories. König argues that 'objective curricula' are constituted in a system of *normative* and *descriptive* sentences, where goals are connected by if-then-sentences. Just as the normative sentences represent curricular goals, the descriptive sentences represent methods for attaining a higher goal, or motivations for setting a lower goal (see figure 1).
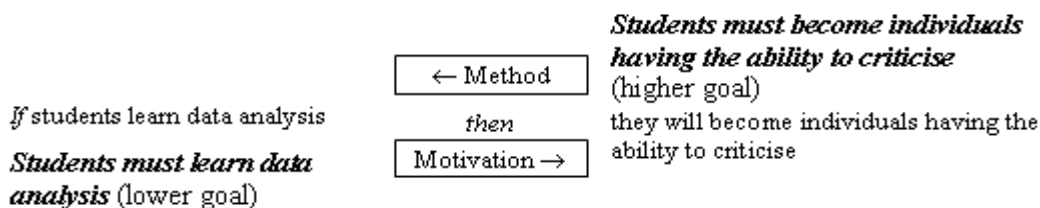
*Students must become individuals
having the ability to criticise*
(higher goal)

← Method

If students learn data analysis

then

they will become individuals having the
ability to criticise

**Students must learn data
analysis** (lower goal)

Motivation →

Figure 1. Goal-method-argumentation (example of the NCTM-standards)

With regard to the definition of the epistemological human idea (see above), the assumption is that individual curricula are constructed in much the same way as 'objective' curricula are, and for this reason include a system of normative and descriptive sentences.

METHODOLOGY

To illustrate the five-step-methodology (see figure 2) used in this research, the discussion which follows will link theoretical reflections to brief examples of empirical results.
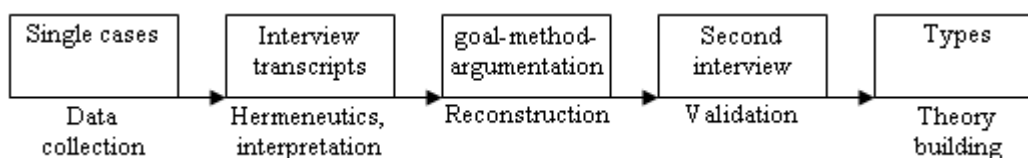
Single cases	Interview transcripts	goal-method-argumentation	Second interview	Types
Data collection	Hermeneutics, interpretation	Reconstruction	Validation	Theory building

Figure2. The five-step-methodology

Data collection

The basic methodological choice was to use case studies (Stake, 2000), where a single case is a teacher's individual curriculum. Case selection was based on *theoretical sampling* (Charmaz, 2000). We restricted the possible cases to those teachers of secondary schools (grades 7 to 13) who have experience in stochastic education. For our purpose, we decided to include the cases of 13 teachers. Data were collected by half-structured interviews that include several clusters of questions. These clusters involve subjective theories concerning:

- The content of stochastics instruction,

- The goals linked with these contents,

- The goals of mathematics instruction,

- Reflections on the nature of mathematics and of school mathematics,

- The students' views on stochastics,

- Institutional boundaries, and,

- Textbook(s) used by the teachers.

The interviews were determined by the teachers within these obligatory clusters. These clusters are the result of theoretical reflections pertaining to possible impacts on individual curricula, i.e. the analysis of the methodological issues of stochastic education. These impacts are structured according to three factors of influence on teachers' individual curricula. Firstly, there is the teachers' examination of methodological approaches, or rather the contents and goals of stochastic instruction. Secondly, there are the teachers' experiences of the results that their stochastic instruction have upon their students. Finally, there are institutional boundaries for teachers such as administrative curricula, or guidelines for stochastic curricula established by particular schools which include these schools' choices of textbooks to some degree.

The intention of these theoretical considerations is twofold: to establish a basis for interview questions and to show the *prejudice* ('Vorurteil', according to Gadamer, 1986) and *theoretical sensibility* (Charmaz, 2000) which are the prerequisites for analysing the impact of these three factors on teachers' individual curricula.

Interpretation

The interviews were taped and transcribed verbatim. Each transcript has a length of 30 to 40 pages. The first step of the analysis was to split the transcripts into episodes and label them in terms of the question clusters outlined above. A crucial step of case analysis is the sequential interpretation of episodes. While Gadamer (1986) and Schwandt (2000), in their philosophical hermeneutics, discuss understanding as a human condition, they do not propose techniques for approaching understanding.

Here, classical hermeneutics in Schleiermacher's tradition (Gadamer, 1986) proposes an approach in the form of a hermeneutic spiral (see figure 3), whose principles (Danner, 1998) were adapted for the interpretation of transcripts. These principles and the hermeneutic spiral will be described and illustrated by a brief example of a transcript.
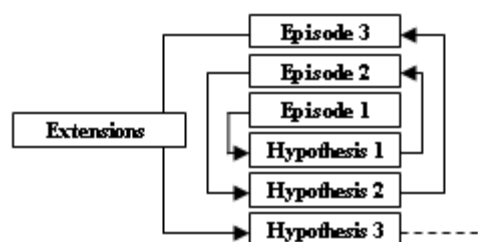

Figure 3. The hermeneutic spiral

The following episodes involve Alan, a 55-year-old secondary school teacher. Alan discusses the introduction of the term probability in grade 13, at a time when students mostly explore stochastics for the first time.

Episode 1: "In advanced courses of mathematics, you can't avoid examining frequencies. You try to make clear to students that probability is not defined. It does not yet exist but it is derived from frequencies. In my opinion it is difficult to get this point across, because when using the Laplacean experiment, it is explicit, this is where you get the rectangular distribution. However, in grade 11 to 13 I will do this the other way around. Students will have to hit on what probability is. Soon afterwards, you start with frequencies, and next it is necessary to try a lot of random experiments."

A brief interpretation of this episode is as follows: Alan describes two approaches to the term probability; statistical probability, which results as an estimate from a long series of experiments; and classical probability (Laplacean probability), which follows from reflections on the symmetry of random events. Alan's requirement for Grade 13 seems to prefer the statistical approach. This condensed interpretation yields a first hypothesis:

- **Hypothesis 1:** The statistical approach to probability is central in Alan's individual curriculum.

The interpretation of episode 1 is a prerequisite for the interpretation of further episodes. In one further episode, Alan reveals something more about the introduction of the term probability:

Episode 2: "Large numbers of random experiments will, for example, serve as simulation. However we had to restrict ourselves to, well, 100 attempts here. Someone drew a diagram, where stabilisation had not yet become visible, and when I asked for what happens later, the students showed me that they understood."

The interpretation yields: Alan's central approach to the term of probability is the classical one (he uses the dice-tossing experiment). He interrupts the evaluation of this experiment before the students are able to recognise the phenomenon of the stabilisation of frequencies. The statistical approach to probability seems to be an empirical rationale for introducing the central approach, the Laplacean probability. Hence, the second hypothesis below results from interpreting episode 2:

- **Hypothesis 2:** The classical approach to probability is central in Alan's individual curriculum. The statistical approach to probability has the function of empirically motivating the classical approach.

The interpretation of further episodes may lead to a deepening, to an extension, or to a modification of the interpretation of episode 1, or more generally, of the interpretations attained so far. In this example, the interpretation of episode 2 leads to a modification of episode 1. In general, the single interpretations are part of a global interpretation, the global interpretation affecting the parts, i.e. the single interpretations. In this way, a hermeneutic spiral evolves. Certainly, two episodes, and their interpretation will not be sufficient to develop a final hypothesis. More evidence must be gathered from other episodes, which may include concrete classroom tasks. Other crucial evidence can be gleaned, for example, by analysing the teachers' textbooks, in addition to the interpretation of the interviews. In addition to 'normal' episodes, textbook analysis may lead to deepening, extending, or modifying the interpretation attained thus far.

All interpretations yield five aspects of an individual curriculum: the content, or rather basic content-oriented goals (aspect 1), the goals of stochastic instruction (aspect 2), the goals of mathematical instruction (aspect 3), the goals concerning teachers' beliefs about how students understand the usefulness of mathematics (aspect 4) and, finally, the goals concerning teachers' beliefs about the efficiency of their own classroom practice (aspect 5).

Reconstruction of goal-method-argumentations

The next step in the methodology is to generate the structure of the reconstructed goals, that is, the goal-method-argumentations (see for example Alan's goal-method-argumentation concerning the goals of stochastical instruction in figure 4). In this scheme, the goals are arranged according to their grade of generality. Goals directly linked to specialised instructional contents stand among goals linked to clusters of instructional goals, stochastics, or mathematics in general.
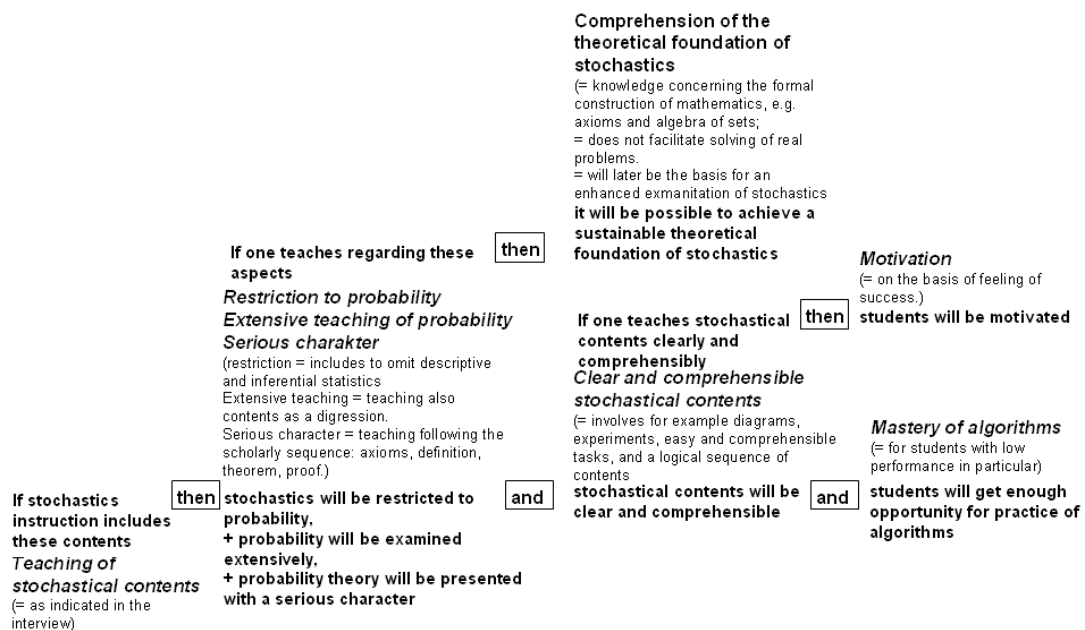

Figure 4. Goal-method-argumentation concerning the goals of the stochastic curriculum

The goal-method-argumentation includes the three aspects of a subjective theory: The teacher's individual goals (italics), the subjective definitions of these goals (smaller font description in brackets) and the if-then-sentences, to which the goals are linked. The goal-method-argumentation starts form the left bottom of the figure (see figure 1).

Validation

These formal goal-method-argumentations were developed for the five aspects of an individual curriculum, as outlined above. In general, teachers are not completely and precisely

aware of these formal structures of goals and of the relationships between them. For this reason, in particular, in the fourth methodological step, communicative validation is mandatory. The goal-method-argumentations were sent to the eight teachers along with the construction rules. After one week, these goal-method-argumentations were used as the basis for a second interview. This objective of this new interview was to reach consensus on the appropriateness of the reconstructed and formalised individual curricula.

Theory building in terms of types of teachers' individual curricula

The final methodological step provides a continuous process of abstraction and aims to identify patterns of structures or goals (Kelle & Kluge, 1999).

RESULTS

This process yields four types of individual curricula for the participants teachers, that will be termed the *traditionalists*, the *application-preparers*, the *everyday-life-preparers* and the *structuralists*. These models of individual curricula will be outlined along with quotations from the teachers in the following paragraphs.

The traditionalists

The central goal of the traditionalists concerning the stochastic curriculum is to establish a theoretical basis for stochastics (or rather probability theory). This involves algorithmic skills and insights into the abstract structure of stochastics, but it does not involve stochastic applications (aspect 2: goals of the stochastic instruction). One example is given below:

> **Alan**: "It must become clear to them [the students] first that what you can do in school is completely inadequate for [...] let's say technical problems, for problems from the field of statistics."

The goals of the stochastic curriculum are consistently embedded into the goals of the mathematical curriculum. Establishing a theoretical base for mathematics is the central goal, which does not include applications. The ultimate goal, which is understood to be the rationale of the central goal, is to have the students achieve insights into the nature of mathematics, a goal described by Alan as follows (aspect 3: goals of mathematics instruction):

> **Alan**: "Well, what is it after all, the essence of this subject. [...] One is frightened by it, one is scared to get involved in it, probably because of the complexity and because of this purely formal thing, this is something you can't grasp, that has nothing to do with reality [...] I once read a definition which sums the thing up in a nutshell, one of Hilbert's, the science of the formal systems."

To attain their central goal for the stochastic curriculum, traditionalists restrict their instructional content to probability theory. Traditionalists see descriptive statistics as a superfluous topic in stochastics. Furthermore, traditionalists restrict probability theory to sections common in many German textbooks, which will be labelled for what follows as

"classical block of probability theory". This includes the concepts of random experiment, probability, combinatorics, and the binomial distribution. Additionally, they will treat axioms and conditional probabilities as a digression from the main subject matter. This means that traditionalists spend a lot of time dealing with these concepts. However, these contents have no function concerning the other contents in the curriculum. Finally, traditionalists predominantly examine the Laplacean probability (aspect 1: content of stochastic instruction).

One of the central goals is to have students acquire algorithmic skills. For some students, in particular those with a low performance, the traditionalists declare these skills to be the only way to achieve success in mathematics.

> **Alan**: "And then you will see in such subjects that mathematics is mainly about applying algorithms, at least in the way we do it. The students, however, do need to be directed to some extent, let's say into the principal considerations about mathematics. This will not be successful with all the students, as you know from experience. Some, however, must be given the opportunity to attain, in a written examination paper, at least five points [out of fifteen] with a certain amount of effort."

Another central goal of a pragmatic nature is to enable students to graduate at the end of secondary high school (their Abitur), which is required for admission to university (aspect 4: usefulness of mathematics for the students). As for all four types of teachers' individual curricula, the traditionalists' central goal with regard to the ideals of teaching divides into two notions of efficient teaching and meaningful instruction.

> **Alan** (concerning the order of the subject matters): "Well, I'd like to say, this is how it works, if you do it. If one gives me a model which I do not know yet, I would love to let myself be convinced. But, as has been said, what is there actually works, and whether it makes sense, that is another question. Well, if it works it does not need to make sense."

Efficient teaching merely means that students work and learn mathematical contents. Whether the contents, their instructional arrangement, or the concrete problems are useful from a more advanced or educational point of view does not matter. By contrast, if the contents permit students to obtain insights into the nature of mathematics, the traditionalists will label these contents meaningful. The dualism of efficient and meaningful teaching stands for a conflict between the central goal of mathematics on the one hand, and the situation of school mathematics on the other, and this has to adapt to students' capabilities (aspect 5: efficiency of a teacher's classroom practice).

The application-preparers

The central goal of the application-preparers concerning both the stochastics and mathematics curriculum is to have students grasp the interplay between theory and applications.

> **Frank**: "Well, we have here the [mathematical] principles, now we can enlarge on that. And we can apply this and that, and that's what I mean, just as well the interplay between theory and application."

With regard to this interplay, Frank argues that mathematical theory is a prerequisite for coping with mathematical applications. For the application-preparers the concept of interplay is

central to their beliefs concerning the nature of mathematics as an abstract system on the one hand, and a technical language on the other. School mathematics should emphasise applications, so that students can make a connection to real life (aspect 2 and 3, see above).

To achieve their central goal for the stochastic curriculum, application-preparers also include the classical block of probability. They examine the classical and statistical approaches of probability on an equal footing, and as a digression the axiomatic approach to probability, but they also incorporate statistics. Descriptive statistics is an elementary preparation for probability theory. Furthermore these teachers' curriculum also contains inferential statistics, represented by a classical block (of inferential statistics). This classical block encompasses confidence intervals and hypothesis testing for the parameter of the binomial distribution. Beyond this, Frank gives a brief introduction to Bayesian statistics (aspect 1, see above).

For the application-preparers, a central goal for the students is to attain algorithmic skills. In contrast to the traditionalists, however, this means using algorithms for application, and evaluating the possibilities and limitations of algorithms in describing the world. Besides the pragmatic goal of having students succeed in graduation, the application-preparers emphasise the challenge of establishing a basis both for coping with pure mathematics and for coping with real-life mathematical problems after school (aspect 4).

> **Earvin**: "The field of practical applications [...] And I should like to think that is also the field which should, for all intents and purposes, be in the foreground in school, of course with certain safeguards [...] All the students who would like to continue this theoretically, and so on, they can do this wonderfully afterwards, outside school. School should only lay the foundations in this field so as to ensure that everybody can use it."

For the application-preparer, the two concepts of efficient and meaningful instruction are intertwined. They argue that both efficient teaching and meaningful teaching are based on teaching mathematical applications (aspect 5, see above).

The everyday-life-preparers

The goals of the stochastic curriculum of the everyday-life-preparers are fundamentally different from the two types above. The everyday-life-preparers develop stochastic methods while examining applications. The central goal of everyday-life preparers is to develop these methods in a process, the result of which will be both the ability to cope with real stochastic problems and the ability to criticise. Dave describes the process as follows:

> **Dave**: "And that's what I am trying to illustrate here as well, that you get models of approach this way, but of course become better afterwards. That you will of course somehow get quite far with relative frequency, but that if you have such problems afterwards, you will [...] advance towards reality in confidence intervals. This means showing them, as well, that mathematics, if it takes place in the applications [...] that there are quite often problems which you can solve with maths. [...] That students are enabled to better categorize mathematical models which determine our economic condition [...], and equally the uncertainties which are connected with them."

Also, the central goal of the mathematics curriculum is to prepare students to cope with real-life stochastic problems. The everyday-life preparers argue that this goal is particularly attainable for stochastics, since for many mathematical subjects it is more difficult to provide the process outlined above. The everyday-life-preparers distinguish between mathematics and school mathematics. Since an abstract structure dominates mathematics, application should dominate school mathematics, whose ultimate goal is that students are enabled to cope with real stochastic problems and that students achieve critical faculty concerning the application of stochastics in the society (aspect 2 and 3, see above).

The curriculum contents used by everyday-life-preparers to attain their central goal of the stochastic curriculum are similar to those of application-preparers. One distinction exists concerning the approach to probability, where everyday-life-preparers focus on the statistical statistics. A further distinction exists concerning the function of descriptive statistics, which is integrated into probability theory and into inferential statistics, and which has no separate role (aspect 1, see above).

For everyday-life-preparers the pragmatic goal of success in graduation exams is less significant. Students benefit from school mathematics by becoming able to cope with real problems and achieve a critical faculty (aspect 4, see above). For everyday-life-preparers the two concepts of efficient and meaningful instruction are intertwined. They argue that only meaningful instruction, that is the development of mathematical understanding through a process of exploring real problems, is efficient (aspect 5, see above).

The structuralists

The structuralists examine applications. However, they neither want to promote an interplay between theory and application (as the application-preparers do), nor do they wish to prepare students for dealing with real mathematical problems (as the everyday-life-preparers do). Structuralists understand applications as the starting point for exemplifying mathematical concepts. The central goal of their stochastic and mathematical curriculum is to encourage understanding of the abstract system of mathematics in a process of abstraction which begins with mathematical applications (aspect 2 and 3, see above).

> **Gill**: "To promote, to train the ability for abstraction. Secondly, the ability to think logically, that is, to make logical connections, and thirdly also to have [...] a little bit of fun with mathematics. [...] Well, an ability to generalize from examples. Also, to recognize structures, i.e. general structures which underlie the contents."

The curriculum contents of the structuralists are the same as those of the application-preparers (aspect 1, see above). In contrast to the other types of teachers, the central goal of structuralists is to exemplify mathematical concepts, which will facilitate the students' acquisition of a specific language, and the students' ability to think appropriately within an abstract system of assumptions and conjectures (aspect 4).

For structuralists, meaningful teaching is represented by the goals outlined for aspect 4, see above). However, similar to the considerations of the traditionalists, structuralists believe that many students are not able to manage the process successfully. For this reason, for structuralists efficient teaching is often restricted to have students acquire algorithmic skills.

DISCUSSION

As outlined above, defining types of individual curricula facilitates a systematic insight into school practice concerning the teachers' planning of stochastic teaching. On the one hand it is possible to identify the four types with some of the global beliefs studied by several researchers (e.g. Grigutsch, Raatz & Törner, 1998: instrumentalism, formalism, application and process). However, as stated by Cooney, Shealy and Arvold (1998), superficial similarities of global beliefs do not automatically indicate real similarity behind these global beliefs. With regard to the research results described in this paper, the following can be stated: contents, parts of goals and goal-method-argumentations may seem similar but there are crucial differences between the types presented. The individual beliefs behind global beliefs, i.e. the subjective theories or rather the goal-method-argumentations concerning the outlined five aspects of teachers' individual curricula (see above), do, however, provide a holistic insight into school practice concerning the teaching of stochastics.

A further aim of defining the four types of teachers' individual curricula is to facilitate a systematic comparison between school practice and conceptions of teaching stochastics as proposed by university educators. Two results of this comparison concerning the 13 teachers are as follows:

- The content (aspect 1) of the teachers' individual curricula mainly consists of the classical blocks of probability and inferential statistics. Both descriptive statistics in terms of exploratory data analysis, and Bayesian statistics, are missing in the teachers' individual curricula, although these two subject areas are incorporated in the central proposals of German university educators (e.g. Biehler, 1997; Wickmann, 2001).

- Borovcnik (1996) distinguishes six phases in mathematics education: (1) New maths in the sixties, (2) application in the seventies, (3) phenomenology after 1975, (4) open mathematics in the eighties, (5) intuition-theory from 1985, and (6) constructivism from the nineties on. In the individual curricula of the 13 teachers there exists a surprising similarity between the first three phases and types (traditionalists – new maths; application preparers – application; everyday-life-preparers – phenomenology). In contrast, the structuralists represent a mixture of these first three phases. The similarity can be explained by the fact that the graduation of six of the teachers coincided with the phases just described.

These two results give the impression that there is a considerable gap between reform ideas concerning stochastic education, proposed by university educators, and school practice – at least for the 13 teachers included in this report.

CONCLUSION

The four types described here are based upon a qualitative research approach and for this reason we cannot draw conclusions about German stochastic teachers in general. Neither it is possible to state that these four types are the sole types of existing individual curricula, nor is it possible to make any estimates about the quantity of the several types. We can, however, hypothesise that these four types are the main types to be found. Evidence for this hypothesis will be forthcoming in a research project that is currently being conducted in approximately ten percent of the German secondary high schools to investigate teachers' individual curricula using a questionnaire.

Another crucial question is how relevant teachers' individual curricula, or types of teachers' individual curricula, are for school practice. Investigating the school practice of 'conventional' teachers, and exploring the outcome of conventional school practice concerning the knowledge students attain based on conventional school practice are crucial steps in the research project concerning teachers' individual curricula, factual curricula, and students' implemented curricula. The first step of this project is outlined here; further results concerning the next steps can be found in (Eichler, 2007). In brief, these results suggest:

- That the teachers' individual curricula explain their factual curricula. Predominantly, there are no, or at most, small breaks between teachers' individual curricula and teachers' factual curricula,

- that a teacher's individual curriculum (or rather a teacher's factual curriculum) has an high impact on his students' implemented curricula. However, as a further result, we can document a phenomenon that Helmke (2007) describes as follows: a teacher's well prepared classroom practice need not necessarily yield the impact the teacher intends. Finally, the students' implemented curricula concerning different teachers are different.

Reforming stochastic education based on theoretical considerations or empirical studies with students is one important challenge of educational research. However, it is not the only challenge. To know what teachers want to teach, to know what teachers are able to teach, and finally to know what knowledge 'conventional' stochastic teaching produces in the students is another crucial but widely unrealised challenge of educational research. It is, however, in accordance with a hermeneutic truism, the first step towards reforming the teaching of stochastics: "It should be self-evident that one must first have understood what one intends to change" (Danner, 1998, p.114; translated).

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