

Improving students' mathematical thinking in verbally solving stochastic problems: A non-calculative approach

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ABSTRACT

Students often struggle with stochastic reasoning because instruction is dominated by calculational procedures that emphasize formula memorization rather than conceptual understanding and interpretation of uncertainty. This hinders the development of mathematical thinking, as students cannot provide solutions to questions without comprehension of reasoning procedures or probabilistic meaning. The study concentrated on deciding whether a non-calculational verbal method of solving problems in stochastics enhances students' mathematical thinking. A mixed-methods quasi-experimental design was utilized with 63 undergraduate students allocated to control and experimental groups, applying pre- and post-test measures, classroom observation, assessment rubrics, think-aloud protocols, and questionnaires. Findings indicate that the experimental group performed significantly better than the control group on post-test scores. The results also showed a large effect size (Cohen's $d = 1.86$) with $t(61) = 7.38$, $p < .001$, suggesting a significant improvement in mathematical thinking. The findings indicate that integrating non-calculational verbal reasoning activities can significantly improve students' cognitive understanding of concepts.

Keywords: non-calculational reasoning, stochastics education, mathematical thinking, probability instruction, oral problem-solving approach

INTRODUCTION

Stochastics, comprising statistics and probability, has emerged as a core element of contemporary mathematics education, especially at the post-secondary level where learners are required to interact with progressively advanced data-based problems (Dani & Al Quraan, 2023). In an uncertainty context, risk analysis, and proliferation of data, the proficiency in data analysis, interpretation, and drawing conclusions has become important rather than optional in the fields of medicine, economics, social sciences, and engineering (Sahai & Lovric, 2011; Xu et al., 2024). Therefore, statistical proficiency has evolved as an essential skill for both academic achievement and professional engagement. It also improves students' digital skills, ability to interpret results, critical thinking, and decision-making under uncertainty (Garcia & Pantaleon, 2025). Despite this increasing importance, international validation programs such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) continually demonstrate that students have difficulty with statistical thinking and probabilistic reasoning and usually perform below expectations (Arum et al., 2018; Puspitasari et al., 2019). Although learners are taught topics in stochastic process, their ability to engage actively with probability and statistics remains limited. This inconsistency raises a significant concern about the efficiency of contemporary teaching methods in promoting critical mathematical thinking in stochastics.

A major contributor to this problem is rooted in the dominant calculational model that describes much of the stochastics learning process. Teaching approaches usually focus on formula cognitive retention, structured problem-solving procedures, and mathematical manipulation, with success assessed mainly by learners' ability to calculate numerical answers correctly (Duarte & Robert Nogueira, 2019; Pjanić et al., 2025). Although these techniques can improve performance optimization, they usually do not promote conceptual knowledge or thinking flexibility in reasoning, which are important for interpreting probabilistic contexts (Ningrum et al., 2022; Tan et al., 2023). Mrope and Deogratias (2024) discovered that learners who display expertise in applying formulas may still show weakness in conceptual understanding of probability. This implies that practical knowledge does not guarantee meaningful understanding. Also, the present curricula fail to adequately address verbal reasoning and mental estimation techniques, which are essential for supporting the development of intuitive and critical mathematical thinking,

(Braithwaite & Sprague, 2021; Schreiber & Ashkenazi, 2024). Consequently, learners are usually incapable of formulating reasoning, justifying conclusions, or participating in instinctive problem-solving without depending on written computation.

In addressing these constraints, numerous instructional approaches were proposed to improve mathematical thinking in statistics education. Simulation-based learning, for instance, allows students to explore probabilistic phenomena through repeated trials and visualisation, thereby strengthening conceptual understanding (Koparan, 2019; Pfannkuch et al., 2018). Also, context-based activities and investigative learning environments have been established to promote involvement by contextualizing problems within real-life scenarios, (Hüfner et al., 2025; Melesse et al., 2025). Although these methods demonstrate substantial improvement, they do not directly target the advancement of non-numerical and mental methods of solving problems in stochastics. Specifically, the area of how students can reason about uncertainty without depending on computations is under-researched. This highlights a significant gap in existing studies: the continuous divide between learners' computational performance and their original mathematical thinking in stochastics remains insufficiently investigated. This study fills the gap by offering a non-calculational approach to stochastics problem solving. In this context, a non-calculational framework engages learners in stochastic problem solving through estimation, reasoning, logical computations, and verbal justification, without resorting to calculations or computational methods. Such an approach emphasises the articulation of thought processes, the evaluation of likelihoods based on reasoning, and the construction of arguments grounded in conceptual understanding (Ricco, 2025; Zhao & Li, 2026). The theoretical framework for this method is anchored in the view that mathematical thinking is closely connected to communication, reasoning, and reflective thinking (Hattori et al., 2025; National Centre for Excellence in the Teaching of Mathematics [NCETM], 2025; Singh et al., 2024).

By removing the need for computation, learners are compelled to engage completely with basic stochastic concepts, engage in stochastic feelings, and monitor their own cognitive procedures. This procedure is significantly different from the conventional mathematical problem-solving technique, as it focuses more on stochastic contexts interpretation skills and conceptual understanding development instead of cognitive computations.

In this regard, this study intends to examine the efficiency of a non-calculational, verbal method of solving problems in improving students' mathematical thinking in stochastics. To be specific, this study addresses the following research questions:

- RQ1** Does the application of non-calculational verbal activities in stochastics teaching significantly enhance students' mathematical thinking compared to standard teaching practices?
- RQ2** Which dimensions of mathematical thinking, such as concept-based understanding, reasoning, and problem posing are mostly affected by these methods?
- RQ3** How do students regard the non-calculational methods concerning valuable learning experience and thinking challenge?

Answering these questions will promote the theoretical and practical understanding of stochastic processes. This study is therefore rooted in the Constructivist Learning Theory, which highlights students' active development of understanding through higher-order thinking and active participation (Piaget, 1952; Vygotsky, 1978).

Theoretically, it broadens recent perspectives on mathematical thinking by highlighting non-calculational reasoning as an evaluative learning procedure. Practically, it offers an alternative instructional framework that can assist in bridging the gap between operational proficiency and real comprehension in stochastics.

METHODOLOGY

Research Design

The study employed a quasi-experimental mixed-methods design involving quantitative and qualitative approaches to assess the effectiveness of a non-calculational technique for solving stochastic problems. More precisely, the study featured two intact groups: an experimental group exposed to a non-calculational instructional method and a control group taught using traditional calculation-based methods. The quasi-experimental design was deemed appropriate because it enables comparison between groups in a natural classroom environment, where random assignment of participants is usually unfeasible, while still allowing the researcher to determine causality with reasonable confidence (Creswell & Creswell, 2018; Tharenou et al., 2007). The mixed-methods design was utilized by integrating pre- and post-test scores of quantitative data with qualitative data (student perceptions, verbal protocol analysis and classroom observations) to provide a deeper insight into how the intervention affects mathematical thinking. This method is extremely important in pedagogical research, as it documents workflow and measurable learning results (Alcaraz & Ley, 2023; Birbili, 2022). The intervention was conducted during the 2025/2026 academic session, second semester. This duration was deemed adequate to enable the students to have contact with the instructional methods and to experience observable changes in mathematical thinking. The two groups were taught within the same institutional environment and covered a similar syllabus, guaranteeing consistency while isolating the teaching methods as the main variable of interest.

Participants

The study's participants comprised 63 second-year undergraduate students who registered for the course (STA 211: Probability I) during the 2025/2026 academic session. Participants were allocated to either an experimental group ($n = 32$) or a control group ($n = 31$). The age of the participants ranged from 16 to 22 years, representing a typical undergraduate age-group, while the distribution of the participants' gender consisted of forty-three (43) males and twenty (20) females.

Table 1. Conceptual framework for stochastic reasoning calculational and non-calculational forms)

Concept	Calculational Form	Non-Calculational Form
Combinatorics	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$	Likelihood of event A occurring based on possible arrangements
Conditional Probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$	Probability that an event occurs, given that another event is already known to have taken place.
Random Events	$P(A) = \frac{\text{Frequency of Favourable outcomes}}{\text{Total frequency of outcomes}}$	Ratio of favorable outcomes to the total frequency of outcomes
Distribution	Probability density/mass functions	Shape and spread of outcomes

Participants were initially selected through a purposive sampling method from the specified course as they had requisite knowledge of basic mathematics and were actively involved in probability learning. Subsequently, learners were allocated to either the control or the experimental group to minimize errors in selections and promote internal validity (Lim & In, 2019). Participants were required to be enrolled in STA 211, attend classes regularly, and agree to participate in all study activities. Learners were removed from the study if they were significantly absent during the intervention period or had prior knowledge of advanced probability or stochastic process courses, ensuring consistent exposure to the intervention. The study strictly complied with ethical standards throughout the study duration. Formal ethics committee approval was not required by the authors' institution for this type of educational research. Participation in the study was voluntary, and consent was obtained from the participants before data were collected. Participants were assured of confidentiality, anonymity, and their right to withdraw from the study at any stage and for any reason without penalty or loss of benefits. In accordance with ethical procedures for educational research, the collected data were used solely for academic purposes.

The Non-Calculational Intervention (Experimental Group)

The new methodological approach of this study is the integration of a non-calculational intervention in solving probabilistic problems. It was suggested to include learners in solving stochastic problems through estimating, logical thinking and oral explanation instead of the conventional computational approach. The intervention concentrated on four fundamental aspects of stochastic processes, namely probability theory, permutation and combination, probability distributions and random events. Activities were carefully formulated to highlight understanding of concepts and intuitive thinking instead of algorithmic procedures.

Table 1 provides the conceptual framework directing the intervention, which distinguishes between calculational and verbal representations of key stochastics concepts. The activities employed in the intervention were deliberately non-calculational in nature. It was mandatory for participants to compare probabilities, estimate probabilities and detect errors such as the gambler's fallacy, and apply visualization methods such as sample space or tree diagrams for illustrations. These activities motivated the participants to explain the rationale behind the application of non-calculational methods instead of depending on the calculation formula. The participants were taught mainly through verbal methods such as verbalized problem-solving activities, focus group discussion and whole-class discussions. Teachers utilized the Socratic method of questioning to encourage analytical thinking, question assumptions, and assist students to develop conceptual clarity. Students were encouraged to explain their reasoning, evaluate peers' reasoning, and justify their decisions based on logical arguments. The researchers performed the intervention over twelve (12) weeks of the semester, holding two 90-minute sessions per week. This ensured continuous engagement and provided enough time for reasoning skills development. Teachers played the role of a facilitator in the experimental group rather than a solution provider. The discussion structure was influenced by the instructor instead of describing protocols. Incisive questions were asked and participants offered answers in a classroom that fosters inquiry and critical thinking. This change in instructing function is essential in improving learners' engagement and independence in mathematical thinking.

Control Group Instruction

The control group was taught using conventional teaching methods frequently utilized in higher institution stochastics process courses. This method highlights algorithmic computation, formula-driven problem-solving technique, and manipulation of mathematical expressions. The instructor used a lecture-based teaching strategy and displayed solved examples before asking learners to engage in similar examples on their own. The control group was taught using conventional teaching methods, textbooks and course-related materials. To guarantee precision, emphasis was placed on using formulas to solve problems concerning conditional probability, permutations and combinations, random events, and probability distributions.

Although concept-based analysis and interpretations were also supplied, they were mostly subordinate to data-processing techniques. To guarantee objectivity and similarity, the control and experimental groups received uniform teaching time and were taught the same course curriculum, including probability theory, permutation and combination, probability distributions and random events. The main distinction between the groups was the teaching method.

Instruments and Measures

The study employed a mixed-methods approach, incorporating qualitative and quantitative data to assess learners' mathematical thinking and educational experience. The study also adopted the Mathematical Thinking Assessment (MTA) as the primary research instrument for the pretest and posttest quantitative assessment. It was also designed to assess learners' understanding of fundamental stochastics concepts using a non-calculational reasoning approach. The MTA evaluates four major

Table 2. Sample stochastic tasks and scoring rubric

Problem	Description	Code 3 (75% and above)	Code 2 (between 50%–74%)	Code 1 (between 25%–49%)	Code 0 (Below 25%)
Problem 1	Combinatorics-based scenario	Present a clear and logically consistent verbal reasoning about possible arrangements or selections, showing a solid understanding of counting principles without relying on formulas	Display a partial understanding of arrangement logic or counting, while the explanation may be incomplete or unclear.	Displays incorrect reasoning about permutation and combinations	No attempt
Problem 2	Conditional probability	Explain the relationship between events A and B, concentrating on A given B, through a clear, logical, and intuitive argument that does not depend on standard calculations	Display a partial knowledge of conditional probability, but the explanation may not be clear.	Uses non-calculational substitution without conceptual explanation or displays misconception about conditional relationships	No attempt
Problem 3	Random events	Provide an efficient association between randomness and uncertainty, emphasizing the stochastic nature of events, their independent and unpredictable features.	Demonstrates a partial understanding of random events, but the reasoning is unclear or incomplete.	Provide misconceptions about randomness like no errors or depends on wrong computation	No attempt
Problem 4	Distribution	Provide an accurate description of the spread, behavior, or pattern of outcomes using verbal reasoning like likelihood, shape or trends without depending on calculations.	Displays partial knowledge of distribution concepts.	Provide incorrect reasoning about distributions, or depend mainly on computational descriptions.	No response

dimensions of mathematical thinking in stochastics: conceptual understanding, probabilistic reasoning, generalization, and metacognitive monitoring. The assessment structure is presented in [Table 2](#).

The research instrument comprises four open-ended activities connected with distributions, conditional probability, combinatorics, and random events. Responses were evaluated based on a four-level rubric (Code 0–3). Each learner's response or answer was scored according to four ordered levels of performance, with numerical scores ranging from 0 to 100 for conceptual understanding, probabilistic reasoning, generalization, and metacognitive monitoring respectively. The codes represent increasing levels and score over 100% for mathematical understanding and reasoning. Code 3 represents very high scores (75% and above), Code 2 represents high scores (50%–74%), Code 1 represents low scores (25%–49%), and Code 0 represents scores below 25%. Originating author of MTA was Mason et al. (2010). The framework specified specializing, generalizing, conjecturing and justifying. This framework was applied in a study conducted in Malaysia to pre-service teachers by Zeynivandnezhad et al. (2013). The study examined 96 middle-school learners and categorized their mathematical thinking using the Mason et al. five components. Recently, Aprinastuti and Amelia (2025) conducted a study involving thirty-one (31) pre-service primary-school teachers. The study administered mathematical-thinking research instrument following instruction and assessed mathematical thinking through specializing, generalizing, conjecturing, justifying and convincing. Another recent study by Çakıroğlu and Muştuoğlu (2025) that examined mathematical thinking through coding used specializing, generalizing, conjecturing, justifying and convincing as proposed by Mason et al. (2010). In the study, the five components were utilized as the analytical framework for examining students' mathematical-thinking processes. A study on mathematical thinking assessment developed for secondary school mathematical thinking test carried out by Sakaew et al. (2023) reported content-validity IOC values of 0.60–1.00, difficulty indices of 0.40–0.76, discrimination indices of 0.46–0.78, reliability of 0.97, and inter-rater agreement of 0.75–0.88.

This rubric aligns each problem directly with a stochastic concept, ensuring conceptual, non-calculational assessment. A higher score suggests greater proficiency and strong reasoning skills, while a low score is evidence of poor engagement, reliance on systematic thinking, and misunderstanding. The instrument was validated for content and construct validity by experts in measurement and evaluation and mathematics education. The overall inter-rater reliability for the rubric score was computed as 0.87, which indicates a satisfactory reliability value (Hair et al., 2019).

Qualitative data utilized in the study included Think-Aloud Protocols (TAPs), semi-structured interviews and instructional observations. The observation method concentrated on discussion trends, learners' participation and categories of reasoning strategies adopted. TAP was used to record the learners' thinking process and problem-solving experience. The responses generated were coded, transcribed and analysed.

Data Analysis

The study employed descriptive statistics to summarize participants' demographic information and to provide answers to the research questions using weighted mean score and standard deviation. The inferential statistical method was used to test the impact of the intervention applied to the quantitative data. The Shapiro-Wilk test was used to determine the normality and suitability of the data for *t*-test parametric analyses. The difference in post-test mean scores between control and experimental groups was determined with the independent-samples *t*-tests, while paired-samples *t*-tests were adopted to investigate within-groups difference between pre-test and post-test scores. For both tests, effect sizes were determined using Cohen's *d* values to measure the size of the observed differences. The result also provides a deeper insight into the results beyond statistical significance. The 0.05 level of significance was employed for all the statistical tests. Thematic analysis, which included coding, classifying, and recognizing learners' response pattern in reasoning processes was employed to analyse the qualitative data. The method enables improved understanding regarding how non-calculational techniques influence students' mathematical

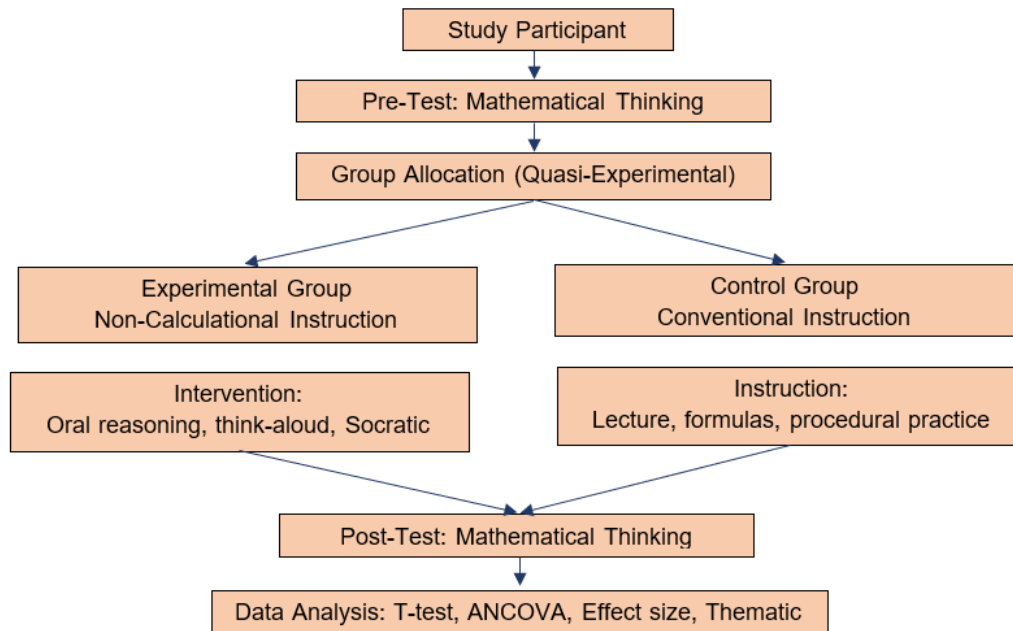


Figure 1. Research design flowchart (Source: Authors' own elaboration)

Table 3. Characteristics of participants by group (N = 63)

Variable	Experimental Group (n = 32)	Control Group (n = 31)	Total (N = 63)
Age (years)	16–22 years	16–22 years	16–22 years
Gender (Male)	Not reported separately	Not reported separately	43
Gender (Female)	Not reported separately	Not reported separately	20
Academic Year	2nd Year (STA 211)	2nd Year (STA 211)	2nd Year
Course	Probability I	Probability I	Probability I

Table 4. Descriptive statistics for pre-test and post-test scores

Groups	Test	N	Mean	SD	Min.	Max.
Experimental	Pre-test	32	42.31	8.45	25	58
Control	Pre-test	31	41.87	7.92	24	57
Experimental	Post-test	32	68.75	7.63	52	82
Control	Post-test	31	54.19	8.11	38	70

thinking. The International Business Machine (IBM) Statistical Package for the Social Sciences (SPSS) version 27 was used to analyze the quantitative data. A careful assessment is provided for the study's intervention through the combination of quantitative and qualitative data analysis, providing both statistical evidence and contextual perceptions of the students. A general diagram of the study design is shown in **Figure 1**.

RESULTS

The study's results are reported based on the research questions. The report combines findings from quantitative and qualitative analysis to offer a thorough assessment of the non-calculational method to stochastics process. **Table 3** provides the characteristics of the participants by their group. The result indicates that the study comprised 63 participants, with 32 (50.8%) assigned to the experimental group and 31 (49.2%) to the control group. Participants were aged 16–22 years and were all second-year students enrolled in STA 211 (Probability I). The overall sample consisted of 43 males (68.3%) and 20 females (31.7%). Thus, the two groups were comparable in terms of academic level, course, and age range, although males constituted a larger proportion of the total sample.

To determine foundational consistency, the control and experimental group pre-test scores were subjected to the descriptive and inferential statistical analysis. **Table 4** showed that the experimental group (Mean = 42.31, SD = 8.45) scored slightly higher than the control group (Mean = 41.87, SD = 7.92). The control group recorded scores between 24 and 57, while the experimental group generated scores between 25 and 58.

The study then employed an independent-samples t-test (**Table 5**) to assess whether the variation observed between the groups was statistically significant. The results showed no statistically significant difference between the experimental and control groups at pre-test, $t(61) = 0.21, p = .834$. Since the p-value was higher than the level of significance ($\alpha = .05$), the null hypothesis of no significant difference was not rejected. This result confirms that the experimental and control groups were statistically equal

Table 5. Independent samples t-test for group comparisons

Test	Comparison	Mean	S.D.	t	df	p	Cohen's d
Pre-Test	Experimental vs. Control	42.31 vs. 41.87	8.45 vs. 7.92	0.21	61	.834	0.05
Post-Test	Experimental vs. Control	68.75 vs. 54.19	7.63 vs. 8.11	7.38	61	< .001	1.86

Note: Mean = mean score (experimental vs. control); S.D. = standard deviation (experimental vs. control); t = calculated t-value; df = degrees of freedom; p = probability value; Cohen's d = Standardized measure of effect size. A p-value less than 0.05 indicates a statistically significant difference between the experimental and control groups. Cohen's d values of approximately 0.20, 0.50, and 0.80 represent small, medium, and large effects, respectively. The pre-test Cohen's d (0.05) indicates a negligible effect, whereas the post-test Cohen's d (1.86) indicates a very large effect.

Table 6. Paired-samples t-test of experimental group pre-test and post-test scores

Paired Measures	Mean	SD	Mean Difference	SD of Difference	SE Difference	t	df	p
Pre-Test	42.31	8.45	26.44	9.38	1.66	15.92	31	< .001
Post-Test	68.75	7.63						

Note. N = 32. Mean Difference = Post-test – Pre-test. $p < .001$.

Table 7. Sub-scale comparison (Post-test scores)

Dimension	Experimental Mean (SD)	Control Mean (SD)
Conceptual Understanding	16.85 (2.34)	13.12 (2.56)
Probabilistic Reasoning	17.42 (2.11)	12.95 (2.78)
Generalisation	17.10 (2.26)	13.45 (2.31)
Metacognitive Monitoring	17.38 (2.05)	14.67 (2.42)

at baseline. Therefore, any differences observed at the post-test stage can be justifiably attributed to the intervention, namely the non-calculational method of solving stochastic problems.

After the intervention, the experimental group achieved significantly higher post-test scores compared to the control group. The experimental group achieved a mean score of 68.75 and standard deviation of 7.63, while the control group achieved a mean score of 54.19 and standard deviation of 8.11. The results of the independent-samples t-test indicated a statistically significant difference between the two groups, $t(61) = 7.38$, $p < .001$. Cohen's d provided an effect size of 1.86, which is considered very large based on established benchmark. This finding indicates that students exposed to the non-calculational instructional method significantly outperformed those who received standard instruction. The observed effect size suggests that the intervention was both statistically significant and of practical importance in improving students' mathematical thinking in stochastics.

A paired-samples t-test (**Table 6**) was performed to investigate the pre-test to post-test improvement within the experimental group. The results showed a statistically significant increase in scores, from a pre-test mean of 42.31 (SD = 8.45) to a post-test mean of 68.75 (SD = 7.63) $t(31) = 15.92$, $p < .001$. These findings provide strong evidence that the non-calculational intervention substantially enhanced students' mathematical thinking abilities within a relatively short period. Nevertheless, these quantitative results should be interpreted with caution, in light of the limitations presented in the *Limitations* section below.

The influence of the intervention was further investigated by separating post-test scores within the four mathematical thinking dimensions, namely probabilistic reasoning, conceptual understanding, generalisation, and metacognitive monitoring (see **Table 7**). Results from the experimental group indicated that probabilistic reasoning had the highest mean score (Mean = 17.42, SD = 2.11), followed by metacognitive monitoring (Mean = 17.38, SD = 2.05), generalisation (Mean = 17.10, SD = 2.26), and conceptual understanding (Mean = 16.85, SD = 2.34). However, the control group demonstrated lower mean scores across all four dimensions, particularly in probabilistic reasoning, where the largest difference between the groups was observed. The findings further suggest that non-calculational teaching methods significantly influence all dimensions of mathematical thinking, with the highest experimental-group mean scores found in probabilistic reasoning and metacognitive monitoring. Hence, the findings indicate that the student-centered method had a positive impact on students' probabilistic reasoning.

Analysis of Qualitative Data

Classroom observations, think-aloud sessions, and student interviews offered qualitative data that provided a deeper understanding into the intervention's impact on learning. The students often reported that the non-calculational activities promoted deeper thinking and assisted them to understand concepts naturally. One student opined that, "Initially, I depend totally on formulas, but now I understand the problem's meaning." Another student was of the view that, "Describing my answer out loud assisted me identified where I was wrong and correct myself." Observational data showed that experimental group students were more actively involved in debates, challenged reasoning more often, and continued longer in problem-solving. Analysis from TAP found that continuous usage of logical reasoning and self-monitoring strategies promotes strategies and performance.

DISCUSSIONS

The findings of this study align strongly with existing research emphasising the importance of conceptual understanding and reasoning in statistics education. The significant improvement observed in the experimental group corroborates earlier work suggesting that traditional calculation-driven instruction is insufficient for developing genuine statistical thinking and significant

difference was found between the pre- and post-test mean scores of the learners in both the control and experimental groups (Camañan, 2019; Iyamuremye & Burns, 2025; Muogbo et al., 2025). Similarly, the studies by Al-Mutawah et al. (2019), Dilekli and Akkaya (2025), and Lantajo and Tipolo (2019) were in accordance with our study that concept-based understanding and probability or stochastic reasoning are very important. Also, the large effect size in the present study extends the studies by Abrenica (2025) and Dobgenski et al. (2025), who discovered that the experimental group demonstrated a statistically significant achievement in critical and mathematical thinking, with very high effect sizes. The strong gains in probabilistic reasoning are consistent with the findings of Hokor (2020) and Hokor and Sedofia (2021), who investigated probabilistic reasoning in simulation-based and inquiry-driven learning environments. Their studies compared teacher-centered and student-centered approaches to teaching probability aimed at addressing common probabilistic misconceptions.

Application of probabilistic reasoning in instructors will assist in improving their problem-solving ability in probability. This finding is supported by Wong et al. (2025), whose findings suggested that mathematics domain discourse can lead to successful outcomes and solutions to the problems. Sfard (2013) and Anderson-Pence (2017) also emphasise that studying mathematics is not only about implementation of formulas; it basically requires active involvement in discourse practices, where students generate insights through dialogue, communication, and shared reasoning. Results from the qualitative analysis found that non-calculational activities improves deeper thinking and assist students to understand concepts naturally. Furthermore, the findings on continuous utilization of logical reasoning and self-monitoring strategies promotes strategies and performance aligns with Guo (2022), which revealed that self-monitoring intervention had significant effects on techniques utilized when it was integrated into a complex intervention. Also, higher influence on academic performance was related to environmental support from numerous sources. From a cognitive perspective, non-calculational methods improve learning outcomes by reducing extraneous cognitive load and optimizing working memory capacity. Also, by removing complex calculations, learners can concentrate on understanding fundamental principles; this is corroborated by the study from Ncube and Luneta (2025).

Many interconnected mechanisms explain why the non-calculational approach is so effective. First, the need for verbal explanation compels students to express their reasoning, thereby revealing indirect assumptions. By identifying inconsistencies or gaps in understanding, this process enables self-correction and improves understanding. Expressing reasons verbally assists students to engage in metacognition, resulting in improved engagement and learning outcomes. Second, eliminating the dependence on formulas removes alternative pathways for students, preventing the superficial, memory-based procedural solving that is typical of traditional education. Conversely, fostering an improved understanding of stochastics requires moving beyond calculation to compel students to engage directly with the basic structure of the problems. Third, the success of the intervention depends mainly on collaboration. Small-group discussions provide instructional support, where learners enhance their understanding by explaining, questioning, and critiquing each other's ideas. Vygotsky's (1978) theory suggests that social interaction promotes higher-order thinking, as collaborative dialogue assist learners to progress through their zones of proximal development.

Finally, non-calculational tasks promote knowledge transfer: by concentrating on reasoning rather than specific processes, students develop flexible thinking skills that can be used in various contexts. This contrasts with process-based learning, which is usually context-based and less flexible (Jonsson et al., 2022).

The most significant gain in probabilistic reasoning indicates that the non-calculational method is exceptionally beneficial in correcting misunderstanding and improving intuitive comprehension of uncertainty. This is in line with previous research by Duke (2024) whose finding indicated that providing the numerical range in the text of the probability lexicon, improved interpretations of probability phrases. The high achievement in metacognitive monitoring underscores the emphasis on reflection and verbal explanation incorporated in the intervention. By frequently assessing their reasoning, learners developed better awareness of their thinking processes. Achievements in conceptual understanding and generalisation, though slightly reduced, were still significant, indicating that the method supports a wide range of cognitive skills. These findings indicate that non-calculational activities are very important for promoting higher-order thinking, even if they do not directly target calculation accuracy.

The findings of this study have significant implications for the teaching of probability in higher education institutions. First, non-calculational methods can be effectively incorporated into existing probability or stochastics courses without requiring substantial structural modifications. Instructors should integrate short verbal reasoning activities, group discussions, and think-aloud tasks into contemporary instructional approaches to promote conceptual understanding. Second, effective non-calculational activities should emphasize comparison, estimation, and structured peer discussion rather than purely computational methods. This suggests that activities aimed at revealing common misconceptions, such as the gambler's fallacy, are particularly important for facilitating conceptual change. Third, instructors should shift from merely transmitting knowledge to facilitating reasoning through prompting and Socratic questioning. Such approaches are essential for guiding students' mathematical thinking without directly providing solutions. Finally, assessment practices should support learning by evaluating the reasoning underlying an answer as carefully as the answer itself.

This may require the development of new rubrics and assessment methods that capture the depth and richness of students' thinking.

CONCLUSION

This study demonstrates that a verbal, non-calculational method of teaching stochastics can significantly improve learners' mathematical thinking. Learners exposed to Socratic questioning, estimation, think-aloud activities, conceptual interpretation, verbal reasoning and peer discussion exhibited substantially better post-test performance when compared with conventional,

calculation-driven instructional technique. The large effect size further validates this observed improvement to be statistically and educationally significant and meaningful. Significant performance was observed across probabilistic reasoning, conceptual understanding, metacognitive monitoring, and generalisation, with substantial improvements in reflective thinking and probabilistic reasoning.

The results from the qualitative analysis also confirmed the outcomes from the quantitative analysis. The findings indicate clearer articulation of reasoning, increased student engagement, improved self-correction, and deeper conceptual understanding among students in the experimental group.

The evidence from this study indicates that effective stochastics instruction should extend beyond application of formula and numerical calculation to deliberately enhance learners' ability to describe, substantiate, understand and assess stochastic ideas. Incorporating non-calculational reasoning strategies into existing stochastics curricula provides an effective and practical approach to enhancing mathematical thinking. Nevertheless, the small sample, single-institution setting, and short intervention period limit generalisation. Future longitudinal and multi-institutional studies are therefore needed to establish the sustainability and broader applicability of these findings.

LIMITATIONS

Although this study yields significant findings, some limitations warrant consideration.

First, the small sample size ($N = 63$), single-institutional environment and short period of the intervention (12 weeks) may impact the long-term validity and broader generalisability of the findings.

Second, the potential effect of the instructor cannot be entirely excluded especially if distinctive differences in teaching methods occurred.

Third, selection-maturation interaction (internal validity): Even if both groups appear similar on the pretest, one group may be maturing, growing, or learning at a faster rate than the other due to uncontrolled prior variables.

Fourth, pretest reactivity effect (external validity): Administering the pretest can sensitize subjects to the intervention, causing the experimental group's results to differ from what would occur if the general population received the stimulus without being previously evaluated.

Finally, although the Mathematical Thinking Assessment (MTA) demonstrated adequate reliability, further assessment is required to confirm its suitability across different contexts and populations. Addressing these limitations in future research would strengthen the research outcome for non-calculational approaches in stochastics instruction.

FUTURE RESEARCH

Future research should consider utilizing a longitudinal study to investigate whether performance in mathematical thinking is maintained over time. Multidisciplinary research could examine whether non-calculational instructional methods can be effectively transferred to other areas of mathematics such as calculus, geometry and algebra. Further studies that compare individual think-aloud methods with small-group discussion would explain the most effective verbal form for learning. Finally, studies into capacity development of teachers are very important to determine how they can effectively implement and sustain non-calculational stochastics teaching method in different classrooms.

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