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Impact of problem-based learning on the interdisciplinary skills of upper secondary school mathematics students

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ABSTRACT

This study explores the impact of problem-based learning methodology on high school students. To do so, an activity based on the logistic function was designed, connecting learning with real and contextualized problems. The research involved 42 students from the scientific-technological and social sciences (SS) specializations. A mixed questionnaire was used, consisting of 26 closed Likert-type questions, which measured the impact of the problem, the methodology, its practical application, and the students' awareness, along with 3 open-ended questions that provided a more comprehensive view of the students' perceptions. Positive results were obtained in both groups, with averages ranging from 3 to 5 in all measurable items, though the effects were more moderate in the SS group. The results suggest that PBL enhances motivation and learning in mathematics and strengthens the value of this discipline, although its implementation should be adapted to the context and specific needs of the students.

Keywords: problem-based learning, teaching and learning of mathematics, interdisciplinary learning, GeoGebra, logistic function

INTRODUCTION

Mathematics plays a vital role in the development of students' cognitive skills as it helps them process information, solve problems, and make decisions efficiently (Arroyo & Pallasco, 2025). These skills encompass key aspects of mathematical education, such as memory, logical reasoning, analytical ability, and creativity (Godino et al., 2019). For this reason, it is essential to provide effective and efficient mathematics teaching and learning that promotes meaningful learning and critical thinking (Organization for Economic Co-operation and Development [OECD], 2023). In general, mathematics is often perceived by students in high school as an abstract subject with little connection to practical applications or real-life situations (Wen & Dubé, 2022). This may be because, for many years, mathematics education has been characterized by a clear focus on memorizing formulas and procedures that students barely manage to understand (Barrantes, 2003). However, this is changing significantly due to advances in educational innovations that are increasingly being implemented in classrooms.

It is crucial, therefore, to create contextualized environments for debate and reflection that help students understand the practical application of various mathematical concepts studied in the classroom (Reyes et al., 2019). In this regard, mathematics teachers must be well-prepared and seek methodological strategies that enhance the value of mathematics in society and promote integrated education that helps students think critically and meaningfully (Goos et al., 2023). A good strategy to achieve this is to use problem-based learning (PBL) as an active methodology from an interdisciplinary perspective (e.g., Domènech-Casal, 2018; Rehmat & Hartley, 2020; Santillán-Aguirre et al., 2023; Satrústegui & Mateo, 2023) because it combines a global and practical view of what scientific work entails (Muñiz-Rodríguez et al., 2016; Savin-Baden & Major, 2004). Of course, as Smith et al. (2022) point out, it allows students to understand the relationship between what they are learning, how they are learning it, and its potential applications in the real world.

Problem-Based Learning With Interdisciplinary Approach

The learning derived from this approach can be considered as "constructive, self-directed, collaborative, and contextualized" (Dolmans et al., 2005, p. 39), as it is a comprehensive approach in which, among other things, students actively build their knowledge through practice, manage their own learning by developing self-management and problem-solving skills, collaborate and interact with their peers to share ideas, and apply what they have learned in real-world situations (El Sayary et al., 2015). For

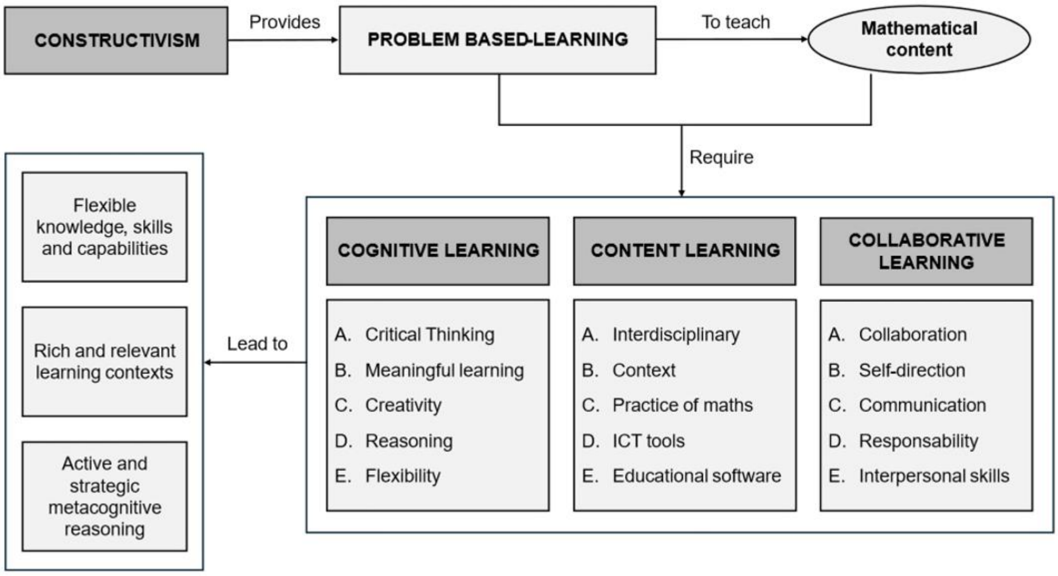

Figure 1. Conceptual framework of the PBL methodology (Source: Authors' own elaboration)

this reason, this methodology is being widely adopted in various fields and educational contexts to promote students' critical and analytical skills in authentic learning situations (Domènech-Casal, 2018; Pazos-Yerovi & Aguilar-Gordón, 2024).

In this regard, Escribano and Del Valle (2008) point out that PBL involves a series of fundamental cognitive processes, the key elements of which are:

(a) the learning context, which sets the stage in which students face the problem,

(b) the activation of prior knowledge, which allows students to connect what they already know with new learning, facilitating the understanding and application of concepts, and

(c) the acquisition of new knowledge, which occurs through the study and reflection on the presented problem, promoting a deeper and more meaningful understanding.

These components do not work in isolation; rather, they are interrelated, helping students develop a more comprehensive and critical view of the problem while also reinforcing their cognitive skills, such as problem-solving, reasoning, and decision-making (Sukayasa et al., 2025).

Furthermore, when applying methodologies with this focus, it is also important to consider the use of digital environments as part of the learning process, as they significantly enhance the teaching of mathematics (Ortiz et al., 2025). For example, searching for information online is fundamental in the process of solving activities within the PBL framework, as students will need to find relevant data and extract accurate, validated information to complete their tasks properly (Cruz et al., 2021). It is also important to be familiar with dynamic mathematics software for educational use, such as GeoGebra, which is an excellent resource that helps students better understand mathematical concepts at any educational stage (Yohannes & Chen, 2021).

Therefore, the main purpose of PBL is to support individuals in their process of developing flexible mental frameworks that adapt to the surrounding environment, strengthening their investigative and reasoning skills (Pazos-Yerovi & Aguilar-Gordón, 2024). The development of these competencies requires a series of interventions in the teaching-learning process, which will allow the achievement of the expected objectives in an active and self-regulated manner through the independent selection and use of resources. From this perspective, the role of the teacher shifts to that of a learning facilitator, guiding students not only in acquiring knowledge but also in developing relevant skills and attitudes for both their academic field and daily life (Smith et al., 2022).

In line with the research of El Sayary et al. (2015) and Smith et al. (2022), we present in **Figure 1** a conceptual framework illustrating the relationship between the PBL methodology and the STEAM approach for integrated, interdisciplinary education.

To analyze the impact of this methodology, we propose an activity for 16-17-year-old high school students with scientific-technological and socio-scientific profiles. This activity focuses on the study of logistic function, as we believe their mathematical content is crucial for studying the behavior of many social and scientific phenomena.

The Logistic Function

The logistic function was introduced by Verhulst (1838) in a series of articles published between 1838 and 1847 as part of his effort to study population growth. Verhulst (1838) proposed the differential equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right), \quad (1)$$

to model the growth of a population P at time t, where r is the growth rate and K is the carrying capacity (or saturation or limiting capacity).

The equation for the population P is solved by integration:

$$\int \frac{dP}{P\left(1-\frac{P}{K}\right)} = \int r\,dt, \tag{2}$$

whose general solution is

$$P = P(t) = \frac{K}{1+e^{-(rt+C)}}, \tag{3}$$

where C is an integration constant. The inflection point of the function occurs when $P(t) = K/2$, that is, when the exponent $rt + C$ in the general solution equals zero. This implies that

$$t = t_0 = -\frac{C}{r}. \tag{4}$$

Thus, replacing t_0 in $P(t)$, we get

$$P(t) = \frac{K}{1+e^{-(rt+C)}} = \frac{K}{1+e^{-r\left(t+\frac{C}{r}\right)}} = \frac{K}{1+e^{-r(t-t_0)}}, \tag{5}$$

for all $t \in \mathbb{R}$. However, Verhulst (1845) wanted to express $P(t)$ considering the population size at time $t = 0$. Therefore, by taking $P_0 = P(0)$, we get

$$P_0 = \frac{K}{1+e^{-C}}, \tag{6}$$

where

$$C = Ln\left(\frac{P_0}{K-P_0}\right). \tag{7}$$

Finally, the solution to the equation proposed by Verhulst (1845) is

$$P_L(t) = \frac{K}{1+e^{-\left[rt+Ln\left(\frac{P_0}{K-P_0}\right)\right]}}, \tag{8}$$

for all $t \in \mathbb{R}$. Simplifying and regrouping terms, we obtain

$$P_L(t) = \frac{KP_0}{P_0+(K-P_0)e^{-rt}}. \tag{9}$$

Verhulst (1845) referred to this function as the logistic function (or logistic curve). Verhulst's (1845) work was widely criticized by others, including himself, which led to it being largely ignored by the scientific community. However, the logistic function was reintroduced several decades later by different authors without knowledge of Verhulst's (1845) work (Lloyd, 1967) to study the growth of individual animals and plants (Robertson, 1908), microorganisms (McKendrick & Kesava, 1911), or, as Verhulst (1845) did, to study human population growth (Pearl & Reed, 1920). Eventually, Verhulst (1845) was recognized as the precursor of the logistic function (Pearl, 1922), and since then it has been commonly used as an effective model for population growth in various scientific fields (Bacaër, 2008).

Now, if we take $K = 1, r = 1$ and $t_0 = 0$ in $P(t)$, we obtain the function

$$P_S(t) = \frac{1}{1+e^{-t}}, \tag{10}$$

defined on the set of real numbers (**Figure 2**). This function is the normalized case of the logistic function, commonly known as the sigmoid function or standard logistic function (Dekain, 2018; Kyurkchiev & Markov, 2018). It is a real-valued differentiable function of a real variable that satisfies the following properties:

1. Horizontal asymptotes at $P_S = 0$, as $t \to -\infty$, and at $P_S = 1$, as $t \to \infty$.
2. A simple non-negative first derivative: $P_S'(t) = \frac{dP_S}{dt}(t) = P_S(t)[1 - P_S(t)]$.
3. The second derivative: $P_S''(t) = \frac{d^2P_S}{dt^2}(t) = P_S'(t)[1 - 2P_S(t)]$.
4. Inflection point: $\left(0, \frac{1}{2}\right)$.
5. Symmetry around the inflection point: $P_S(t) + P_S(-t) = 1$.

As shown in **Figure 2**, the initial stage of the function's growth is exponential, and as saturation begins, the growth becomes approximately linear, eventually levelling off at maturity.

The logistic function is commonly used in artificial intelligence for image classification, audio signal processing, or neural networks; in machine learning as a logistic regression model for spam detection or financial fraud; in biology, to describe the growth curve of populations of organisms, drug dose-response, or oxygen saturation in hemoglobin; in physics, to describe phenomena with saturation behavior such as material magnetization; in ecology and chemistry, to model predictions of rainfall,

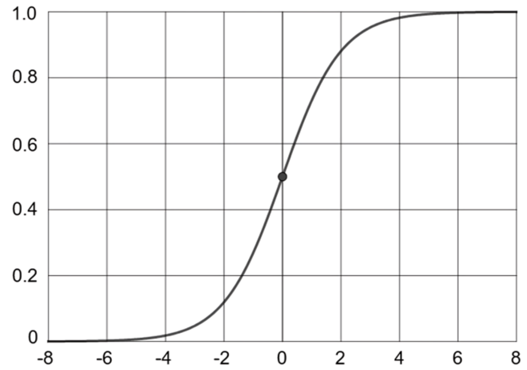

Figure 2. Sigmoid curve: $y = \frac{1}{1+e^{-x}}$ (Source: Authors' own elaboration, using GeoGebra software)

crop yields, or soil salinity; in medicine, for detecting heart diseases or modelling tumor growth; and even in economics and finance, to model situations where growth is limited by resources, such as market saturation (Rządkowski & Sobczak, 2020).

Specifically, the designed activity centers on the study of an epidemic's evolution over time. Given the COVID-19 pandemic, we believe this topic can generate interest and curiosity among students and is a way of demonstrating the social and scientific value of mathematics.

Research Questions

This study aimed at answering the following research questions:

1. What is the students' perception of the impact and practical application of the PBL methodology? Has it helped them recognize the importance of mathematics in studying other disciplines?
2. Are there significant differences between groups of students with different scientific fields of specialization?

METHODOLOGY

Approach

A descriptive mixed-methods approach was adopted in this study, integrating both quantitative and qualitative data to obtain a more comprehensive understanding of the educational impact of the proposed activity. Specifically, a questionnaire was administered consisting of a Likert scale with five response levels, aimed at collecting quantitative data, together with three open-ended questions designed to gather qualitative insights into students' perceptions and experiences. Both types of data were analyzed independently using appropriate analytical procedures and subsequently integrated at the interpretation stage.

The quantitative analysis allowed for the identification of general patterns, tendencies, and group differences, while the qualitative analysis contributed to a deeper understanding of students' perceptions, experiences, and interpretations of the activity. The integration of both data strands enabled triangulation of results and supported a more nuanced interpretation of the findings than would be possible using a single methodological approach. It is worth noting that the questionnaires were administered afterward and completed anonymously by all participants to avoid socially desirable responses.

Context and Participants

The activity was carried out in a public secondary school in Madrid (Spain) during May and June 2024, across two 50-minute sessions, right after the students had finished the topic on derivatives' applications. Specifically, the activity was conducted in two eleventh grade high school mathematics classes, from different specializations: science and technology (ST) and social sciences (SS), both taught by the same teacher. In total, 19 ST students and 23 SS students participated in the experience (total sample size N = 42).

The teacher who conducted the activity holds a degree in mathematics, a master's degree in teacher training in mathematics, and has seven years of experience teaching mathematics in public schools in Spain, from the first year of secondary education to the second year of high school. He is currently a member of a school project on STEM education, organising and carrying out projects and activities with this approach. Therefore, we consider his profile ideal for this research. Specifically, his role was to introduce and guide the activity, address questions, and interact with the class when necessary, allowing students to build their own learning.

Data Collection Instruments

The assessment instruments used were the PBL activity designed by the researchers as the methodological tool and a questionnaire administered to the students after the activity, aiming to measure the emotional and cognitive impact the activity had on them.

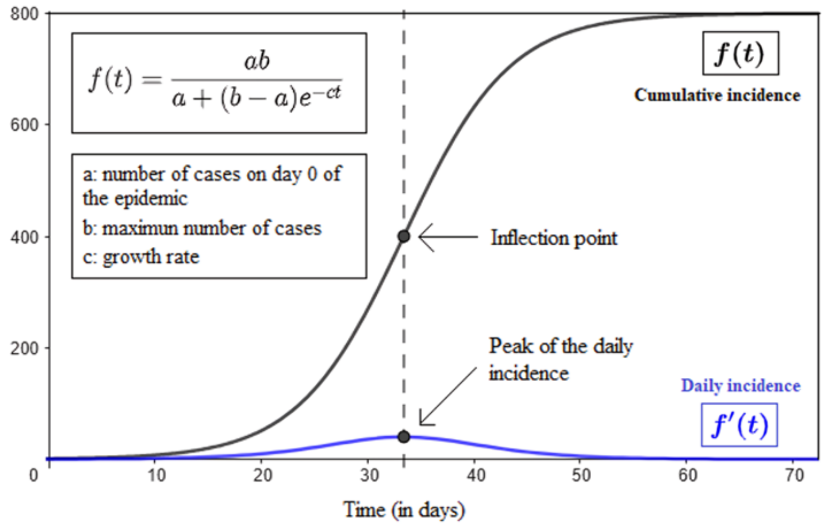

Figure 3. Example of a logistic function to study the evolution over time of an epidemic: cumulative and daily incidence for a = 1, b = 800, and c = 0.2 (Source: Authors' own elaboration, using GeoGebra software)

Activity design

The main objective of the activity was to understand how to mathematically model the evolution over time of the total number of cases generated by an epidemic within a population. To achieve this objective, it was essential to understand the mathematical function that studies this phenomenon (logistic function), the parameters that govern it, and its main properties. During the activity, students worked with concepts such as cumulative incidence, daily incidence, growth rate, equilibrium situation, peak of maximum infections, and saturation of spread, among others (**Figure 3**).

The activity was divided into three parts:

1. **Introduction:** The teacher introduces and contextualizes the activity.
2. **Formulation:** Students study the sigmoid function analytically.
3. **Investigation and conclusions:** Students search for information online and use GeoGebra to analyze a real-world epidemic case modelled using the logistic function.

The logistic function might be difficult to understand without a graphical representation. GeoGebra allows students to visualize how the function changes with different given parameters. This helps us to understand concepts such as the shape of the curve, the inflection points, and the behavior of the function as the parameters vary. This is particularly useful for enabling students to internalize and directly interact with the function, making the learning process more interactive and accessible.

Specifically, the introduction and formulation were carried out during the first session, while the investigation, conclusions, and closure of the activity took place during the second. To promote collaborative work, students worked in pairs or groups of three.

Questionnaire

The questionnaire was designed based on previous research studies on attitudes towards mathematics (Auzmendi, 1992; Martínez-Artero & Checa, 2020; Stelzer et al., 2020) and the evaluation of PBL (Nufus & Mursalin, 2020; Saá-Rojo et al., 2022), to ensure clarity, relevance, and consistency in data collection. A preliminary version of an ad hoc Likert-scale questionnaire with 26 items and five response levels was developed: 1 = totally disagree, 2 = disagree, 3 = indifferent, 4 = agree, and 5 = totally agree. In particular, for item construction, the focus was on ensuring the validity of the items related to the variable being measured, allowing for easy control of response effects. Furthermore, an ambivalent option was included among the response choices to avoid forcing respondents into a position of either satisfaction or dissatisfaction (this prevents errors in the data caused by lack of understanding).

The questionnaire was distributed to two experts in mathematics education who agreed to participate in the content validation process. This process involved two iterative rounds of review:

1. In the first round, the experts assessed the relevance, clarity, and coherence of the 26 initial items. All items were considered relevant to the objectives of the study and were therefore retained; however, several items were rewarded to improve clarity and to minimize potential response bias. In particular, items were formulated using clear and accessible language to reduce the likelihood of respondents selecting the neutral option due to misunderstanding rather than genuine ambivalence. An additional objective check was conducted to ensure that the items were discriminative, meaning that they were not answered uniformly by all participants.
2. In the second round, the final version of the questionnaire was established. The 26 items were grouped into four thematic dimensions: educational impact (activity assessment), awareness, methodological impact, and practical application (**Table 1**). This grouping was theoretically driven and based on

Table 1. Questionnaire items

Factors	Item
A. Educational impact	1. The activity is useful and beneficial to my learning.
	2. The subject matter provides me with new knowledge.
	3. The proposal is interesting and relevant.
	4. I find it a novel activity.
	5. Using GeoGebra allows me to clarify concepts that I did not understand before.
	6. The activity helps me see how different subjects are connected.
	7. I consider it an activity that enhances the value of mathematics.
	8. The activity fosters my curiosity for topics that integrate mathematics with other disciplines.
B. Awareness	9. I am interested in the use of the mathematical concepts I learn.
	10. I am aware that mathematics is necessary for the study of other disciplines.
	11. I am aware of the value of mathematics in science and society.
	12. I am aware of the influence of mathematics on research and decision-making.
	13. I am aware of how important mathematics is to my future both academically and personally.
C. Methodological impact	14. PBL makes classes more interesting, dynamic, engaging and/or motivating.
	15. I have noticed improvements in my ability to solve complex problems thanks to PBL.
	16. This approach helps me better understand theoretical concepts.
	17. PBL has increased my confidence in tackling mathematical problems with an interdisciplinary approach.
	18. PBL has enhanced my analytical and deductive skills.
	19. These types of activities enhance my ability to research.
D. Practical application	20. PBL helps develop practical skills in mathematics.
	21. PBL helps me develop digital skills and improve my computational thinking.
	22. PBL has a significant impact on my understanding of the concepts studied.
	23. PBL allows me to apply theoretical knowledge in real-life practical situations.
	24. I find the practical approach of PBL useful and satisfying to consolidate my learning.
	25. PBL allows me to learn in a more effective and meaningful way.
	26. I see the value of applying this type of methodology more often.

Table 2. Cronbach's alpha

Factor	Cronbach's alpha ST	Cronbach's alpha SS
Educational impact	0.875	0.934
Awareness	0.804	0.952
Methodological impact	0.900	0.874
Practical application	0.819	0.800
Total	0.824	0.836

(a) the constructs identified in previous research on attitudes towards mathematics and PBL (Nufus & Mursalin, 2020; Saá-Rojo et al., 2022; Stelzer et al., 2020) and

(b) the consensus reached by the two experts during the validation process regarding the conceptual coherence of the items within each dimension.

The internal consistency of the questionnaire was examined using Cronbach's alpha coefficient, calculated both for each thematic dimension and for the overall scale. No item recoding was required, as all items were designed to reflect positive attitudes. Given the different academic orientations and profiles of the ST and SS students, Cronbach's alpha was computed separately for each group in order to assess the reliability of the instrument within each context (**Table 2**). The resulting coefficients indicate high internal consistency for all dimensions and for the total scale, with alpha values ranging from 0.80 to 1.00.

Regarding construct validity, no exploratory or confirmatory factor analysis was conducted. Although such analyses are commonly used to examine the internal structure of questionnaires, the relatively small sample size (N = 42), and especially the division into two subgroups (ST: n = 19; SS: n = 23), would have limited the robustness and interpretability of factor-analytic results (Field, 2018). For this reason, and in line with methodological recommendations for small samples (de Winter et al., 2009), construct validity was addressed through theoretical grounding, expert validation, and internal consistency analysis rather than through factor analysis.

For the quantitative analysis of the results, the open-source statistical software R and Microsoft Excel were used. Specifically,

(a) descriptive analysis was conducted using frequency percentages, means, and standard deviations (SDs) to summarize and organize the questionnaire data and identify patterns and relationships, and

(b) comparative analysis of independent samples between the ST and SS groups was conducted using the non-parametric Mann-Whitney U test (two-tailed hypothesis) to determine if there were significant differences at a 5% significance level, without assuming any distribution of the data.

Open-ended questions

The open-ended questions, of a qualitative nature, were designed as an extension of the Likert scale questionnaires to gather a broader understanding of the students' opinions and perceptions on certain aspects. Depending on the nature of the question

Table 3. Results of the descriptive analysis for the factor "educational impact"

Item	ST group					SS group					
	Mean	SD	%3	%4	%5	Mean	SD	%2	%3	%4	%5
1	4.10	0.621	15.79	63.16	21.05	3.83	0.491	-	21.74	73.91	4.35
2	4.37	0.597	5.26	52.63	42.11	3.78	0.422	-	21.74	78.26	-
3	4.11	0.658	15.79	57.89	26.32	4.13	0.694	-	17.39	52.17	30.43
4	3.90	0.567	21.05	68.42	10.53	4.09	0.733	-	21.74	47.83	30.43
5	3.84	0.602	26.32	63.16	10.53	3.35	0.775	17.39	30.43	52.17	-
6	4.21	0.535	5.26	68.42	26.32	3.39	0.783	17.39	26.09	56.52	-
7	4.26	0.562	5.26	63.16	31.58	4.17	0.650	-	13.04	56.52	30.43
8	4.42	0.507	-	57.89	42.11	3.70	0.635	-	39.13	52.17	8.70
Average	**4.15**					**3.80**					

and the interest of the respondent, the answers may vary greatly in terms of length and depth, which will enrich the results. Specifically, the following questions were asked:

1. What advantages and disadvantages do you think using this type of methodology in math class has? Justify your answer.
2. What do you think of the activity? Has it contributed to your learning? (regarding originality, relevance, creativity, structure, resources, etc.).
3. What is your opinion about the role of the GeoGebra tool in this activity?

The qualitative data from the open-ended questions were analyzed using inductive thematic analysis (Braun & Clarke, 2006). The process followed a systematic four-step procedure:

1. **Familiarization with the data:** All responses were read multiple times to gain a comprehensive understanding of the content, identify patterns, and note initial impressions.
2. **Generating initial codes:** Meaningful units of text, including words, phrases, and sentences, were coded manually. Each code captured a single idea or aspect of the students' perceptions.
3. **Developing themes (or categories):** Related codes were grouped into preliminary themes that represented broader patterns in students' perceptions. Exemplary quotes were selected to illustrate each theme, ensuring transparency and traceability from raw data to interpreted results.
4. **Reviewing themes:** Themes were reviewed to ensure internal consistency and clear distinction between them. Overlapping codes were reassigned or merged, and themes that lacked sufficient data were discarded.

Additionally, coding reliability was enhanced by cross-checking the codes among two independent researchers. Discrepancies were discussed until consensus was reached. This detailed approach ensured that the qualitative analysis was systematic, transparent, and rigorous, providing rich evidence to complement the quantitative findings.

RESULTS AND DISCUSSION

Quantitative Analysis

All participants answered all items of the questionnaire. The results are presented in tables and grouped by factors and groups (the item numbers align with those in **Table 1**). For the descriptive analysis, good results are considered when the average score is between 3 and 4 on a 1-5 scale, and excellent results are those where the average exceeds 4. Scores below 3 are indicators of poor or moderate results. An SD close to 0.5 indicates that most of the responses for an item are clustered around the group's average, implying little variability in students' answers within each group (they tend to agree with each other). Furthermore, significant differences between groups are identified with a 5% confidence level if the U-value from the Mann-Whitney U test is below 140.5, or equivalently, if the p-value is less than 0.05 (or the Z-score is below -1.95847).

Regarding the "educational impact" category (**Table 3** and **Table 4**), the average score for the ST group is 4.15, and for the SS group, it is 3.80, indicating generally good results. It is worth noting that in both groups, more than half of the students agree that the activity is useful, interesting, innovative, and beneficial to their learning. Also notable is the high level of curiosity about connecting mathematics to other disciplines, especially in the ST group. This perception is not as pronounced in the SS group, where almost 40% of the students report not feeling that interest. Specifically, 43.48% of the SS students believe the activity has only somewhat helped them understand this connection. The most moderate results came from item 5 about GeoGebra, where there were greater indifferences, particularly in the SS group, where 17.39% disagreed with the statement that the tool helped clarify concepts they previously didn't understand.

This moderately positive result is consistent with previous studies, such as those by Recio et al. (2019) and Yohannes and Chen (2021), who note that the impact of these tools greatly depends on how they are integrated into the teaching process. The fact that the averages are not exceptionally high in this case suggests that while students find GeoGebra useful, the tool may not be fully used in every case, possibly due to its familiarity or the teaching approach used (Recio et al., 2019). This interpretation is further supported by the qualitative data, in which several SS students reported initial difficulties when interacting with GeoGebra, despite recognizing its potential usefulness.

Table 4. Mann-Whitney U test for the factor "educational impact"

Item	U-value	Z-score	p
1	177.5	-1.02346	0.30772
2	110.5	-2.71659	0.00652*
3	213.5	0.11372	0.91240
4	185.5	0.82129	0.41222
5	149.5	-1.73104	0.08364
6	100.5	-2.96930	0.00298*
7	204.5	-0.34115	0.72786
8	96.0	-3.08301	0.00208*

Note. *The result is significant at $p < 0.05$

Table 5. Results of the descriptive analysis for the factor "awareness"

Item	ST group					SS group					
	Mean	SD	%3	%4	%5	Mean	SD	%2	%3	%4	%5
9	4.69	0.671	10.53	10.53	78.95	3.00	0.674	21.74	56.52	21.74	-
10	4.63	0.496	-	36.84	63.16	3.87	0.757	-	34.78	43.48	21.74
11	4.74	0.452	-	26.32	73.68	3.91	0.733	-	30.43	47.83	21.74
12	4.26	0.452	-	73.68	26.32	3.83	0.650	-	30.43	56.52	13.04
13	4.79	0.631	10.53	-	89.47	3.09	0.949	30.43	39.13	21.74	8.70
Average	**4.62**					**3.54**					

Table 6. Mann-Whitney U test for the factor "awareness"

Item	U-value	Z-score	p
9	28.0	-4.80142	< 0.00001*
10	100.0	-2.98193	0.00288*
11	87.5	-3.29781	0.00096*
12	140.5	-1.95847	0.05000
13	40.0	-4.49817	< 0.00001*

Note. *The result is significant at $p < 0.05$

As seen in **Table 4**, the Mann-Whitney U test shows significant differences between the groups on items 2, 6, and 7 with a significance level of 5%. In line with the results in **Table 3**, the central theme of the activity seems to have provided more new knowledge to the ST students than to the SS students, with a clear discrepancy between the groups, particularly in the percentage measures for values 3 and 5 on the scale. This is especially noteworthy because students with a clear inclination towards experimental science or technology tend to have more mathematical abstraction and analytical thinking skills than those with a preference for SS, where abstraction tends to be more conceptual (Zúñiga-Tinizaray & Marín, 2024). Furthermore, the activity seems to have helped clarify concepts more for ST students than for SS students, with an almost one-point difference in the average for item 6. However, the variability in responses is more significant in this case, with a tendency toward value 4 on the scale. Finally, item 8 shows the greatest significance, with much better results in the ST group regarding interest in interdisciplinary activities.

Regarding the "awareness" category (**Table 5** and **Table 6**), the ST group has an average of 4.62, while the SS group has an average of 3.54, indicating very good results for ST and much more moderate results for SS. The differences between groups are more apparent here.

As shown in **Table 6**, four of the p-values from the Mann-Whitney U test are well below 5%, and one is right at the threshold of significance. First, there is greater response variability in the SS group, especially for item 13 (SD close to 1). While SS students are unsure about the importance of mathematics in their daily lives and future academic careers, 90% of ST students are strongly in agreement with this statement. Additionally, the stark contrast between the two groups regarding the interest in understanding the applications of the mathematical concepts they learn stands out. While 90.47% of ST students highly value this item, 21.74% of SS students report no interest, and 56.52% are indifferent. These differences are also reflected in the open-ended responses, where ST students more frequently referred to the relevance of mathematics for real-world and scientific contexts, whereas SS students tended to describe mathematics as more abstract or distant from their academic interests. Overall, questions 10, 11, and 12, concerning the significance of mathematics in research and other fields, are valued positively in both groups, but there is a clear discrepancy, with higher ratings in the ST group compared to the SS group.

In the "methodology" category (**Table 7** and **Table 8**), the average for the ST group is 4.01, while for the SS group, it is 3.48. There is not much variability in responses, but **Table 8** shows that there are differences between groups in four of the items assessed, though this time they are less noticeable, except for items 17 and 18, where more significant changes are observed. The results show that the PBL methodology in the SS group did not seem to provide enough confidence to approach interdisciplinary mathematical problems, with 21.74% disagreeing and 73.91% being indifferent. However, the ST group showed more promising results, with scores distributed between 3 and 4 on the scale. Moreover, both groups appear to have benefited from enhanced analytical and deductive skills, though the ST group showed more favorable results. The same is true for research skills, information-seeking abilities, and solving complex problems (item 15 and item 19), though there is consensus in both groups with average scores between 3.4 and 3.9.

Table 7. Results of the descriptive analysis for the factor "methodological impact"

Item	ST group					SS group					
	Mean	SD	%3	%4	%5	Mean	SD	%2	%3	%4	%5
14	4.37	0.496	-	63.16	36.84	3.78	0.671	-	34.78	52.17	13.04
15	3.68	0.478	31.58	68.42	-	3.39	0.499	-	60.87	39.13	-
16	4.21	0.713	15.79	47.37	36.84	3.74	0.541	-	30.43	65.22	4.35
17	3.58	0.507	42.11	57.89	-	2.83	0.491	21.74	73.91	4.35	-
18	4.26	0.653	10.53	52.63	36.84	3.48	0.511	-	52.17	47.83	-
19	3.95	0.524	15.79	73.68	10.53	3.65	0.487	-	34.78	65.22	-
Average	**4.01**					**3.48**					

Table 8. Mann-Whitney U test for the factor "awareness"

Item	U-value	Z-score	p
14	118.5	-2.51443	0.01208*
15	154.5	-1.60468	0.10960
16	138.5	-2.00901	0.04444*
17	81.5	-3.44944	0.00056*
18	89.0	-3.25991	0.00112*
19	162.0	-1.41515	0.15560

Note. *The result is significant at $p < 0.05$

Table 9. Results of the descriptive analysis for the factor "methodological impact"

Item	ST group					SS group					
	Mean	SD	%3	%4	%5	Mean	SD	%2	%3	%4	%5
20	4.37	0.496	-	63.16	36.84	3.70	0.635	8.70	13.04	78.26	-
21	4.37	0.496	-	63.16	36.84	3.83	0.984	8.70	30.43	30.43	30.43
22	4.16	0.688	15.79	52.63	31.58	4.13	0.694	-	17.39	52.17	30.43
23	4.21	0.713	15.79	47.37	36.84	3.44	0.507	-	56.52	43.48	-
24	4.16	0.688	15.79	52.63	31.58	3.52	0.511	-	47.83	52.17	-
25	4.37	0.496	-	63.16	36.84	3.91	0.668	-	26.09	56.52	17.39
26	4.68	0.478	-	31.58	68.42	4.30	0.559	-	4.35	60.87	34.78
Average	**4.33**					**3.83**					

Table 10. Mann-Whitney U test for the factor "awareness"

Item	U-value	Z-score	p
20	108.0	-2.77977	0.00544*
21	150.5	-1.70577	0.08726
22	214.0	-0.10108	0.92034
23	94.5	-3.12092	0.00180*
24	112.5	-2.66605	0.00758*
25	140.0	-1.97111	0.04884*
26	142.0	-1.92057	0.05486

Note. *The result is significant at $p < 0.05$

Finally, regarding "practical application" (**Table 9** and **Table 10**), the ST students agree that the PBL methodology helped them develop practical and digital skills, improve their computational thinking, and learn mathematics effectively and meaningfully, with an average of 4.37 points (items 20, 21, and 25). However, SS students rated these items lower, with an average below 4, and there was considerable variability in responses to item 21. Additionally, there is a similar percentage distribution in both groups regarding the high impact of understanding the mathematical concepts involved in the activity (item 22).

Clear discrepancies between groups appear in item 23 and item 24, as while 84.21% of ST students agree that PBL helped consolidate their learning by connecting theory to practice in real-world situations, around 50% of SS students were indifferent. Nevertheless, the results are generally favorable, especially for item 26, where 100% of ST students and 95.65% of SS students believe that PBL should be applied more frequently in math classes. However, while students recognize the benefits of PBL, research by Smith et al. (2022), Santillán-Aguirre et al. (2023) or Satrústegui and Mateo (2023) indicate that such positive perceptions are not always present at the outset, making it crucial to conduct more classroom activities with this approach to ensure its impact becomes significant.

After conducting the descriptive and quantitative analysis, good results were observed in both groups across all factors, with averages ranging from 3 to 5 on all items. Specifically, there is a significant concentration around a median of 4.26 for the ST group, with half of the responses falling between 4.09 and 4.38, and a median of 3.76 for the SS group, with half of the responses falling between 3.42 and 3.91 (**Figure 4**).

Note that these results tend to be more moderate in the SS group, which could be due to the technical and quantifiable nature of the problem. Social phenomena typically require a more subjective approach, with less immediate results than scientific phenomena. This pattern is consistent with previous research indicating that although active methodologies like PBL have the

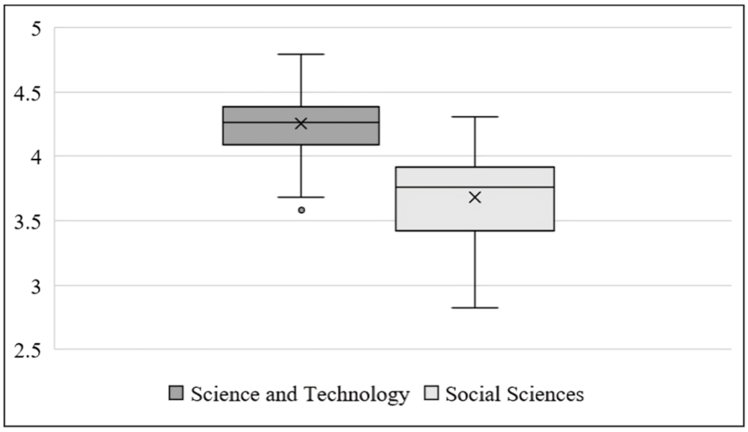

Figure 4. Quantitative analysis of the set of items in both groups (Source: Authors' own elaboration)

potential to improve student motivation and learning, the effects may not always be as pronounced as expected, depending on factors such as the class context, students' familiarity with the methodology, its implementation, and the specific design of activities (Arévalo-Duarte et al., 2024; Yew & Goh, 2016).

In general, items related to the perception of usefulness and interdisciplinary connections received moderately high ratings, indicating that students recognize the value of PBL in understanding how mathematics is applied in other areas and real life. Previous research, such as Hmelo-Silver (2004), Páez (2017), Cadena-Zambrano and Nuñez-Naranjo (2020) or Santillán-Aguirre et al. (2023) have shown that PBL can facilitate the integration of mathematics with other disciplines and promote knowledge transfer to practical contexts, fostering a more holistic understanding. According to Alreshidi and Lally (2024), students may perceive these connections more clearly as they become more familiar with the PBL, but initially, they may feel uncertain about the applicability of mathematical concepts to other areas, which could explain the moderate results, especially in the SS group.

Items related to motivation and problem-solving confidence, as well as the impact of PBL on skills development, also showed a high-moderate pattern of responses in both groups. In this case, the methodology has proven effective in enhancing students' motivation and confidence, but the degree of effectiveness may depend on factors such as activity design, teacher support, and student involvement in the process (Rahmatjati et al., 2023). Intermediate averages might reflect a positive response but with awareness that not all aspects of PBL are equally or effectively applied in all contexts.

Regarding the overall impact of the methodology, Cuong et al. (2025) have highlighted that PBL has a positive impact on student understanding, but its effects may be more pronounced with continuous implementation and well-structured curricular design. Students tend to feel more motivated when activities are engaging and clearly related to their interests and needs, which is not always achieved in every application of the methodology (Wen & Dubé, 2022). The moderate results in the SS group reflect this variability in implementation.

Qualitative Analysis

The qualitative analysis of the open-ended questions provided deeper insight into students' perceptions of the PBL activity and helped contextualize the quantitative findings. Students' responses were analyzed using inductive thematic analysis and organized into four main categories: advantages, disadvantages, perceptions of the activity, and GeoGebra Tool (**Table 11**). Within each category, patterns were identified separately for the ST and SS groups, allowing similarities and differences between both profiles to be examined in detail.

Overall, the qualitative data reveal a generally positive perception of the activity, particularly among ST students, while also highlighting specific difficulties and reservations, especially within the SS group. These findings help explain the moderate-to-high questionnaire scores and the greater variability observed in certain items.

Advantages

One of the most frequently mentioned advantages in both groups was collaborative work. Students valued the opportunity to work in pairs or small groups, as it allowed them to share ideas, discuss strategies, and support each other during the problem-solving process (Hmelo-Silver & DeSimone, 2013). This aspect was particularly emphasized by ST students, who highlighted how group discussions contributed to deeper reasoning and understanding: "Working in a group helped me understand things better because if I didn't get something, someone else explained it" (ST student). Similarly, SS students appreciated the social and cognitive support provided by group work, noting that it encouraged participation and reflection: "You have to think more because everyone says their opinion and you can compare ideas" (SS student).

Another prominent advantage, especially in the ST group, was the use of real-world problems, highlighting how PBL helps students connect theory to everyday situations, making learning more relevant (Smith et al., 2022). Many students highlighted that modelling an epidemic made mathematics feel more meaningful and relevant: "It's more interesting when maths is connected to real things, not just exercises from the book" (ST student). This approach is also reflected in group SS, which points out the practical application of the concepts learned, suggesting that PBL allows them to apply what they have studied to concrete situations: "I liked that it was about something real and not just numbers without meaning" (SS student).

Table 11. Process of coding and categorization of responses

Categories	Codes (ST group)	Codes (SS group)
1. Advantages	– Greater reasoning – Group work – Real-world problems – Exploring new ideas – Structured exercises	– Research – Group work – Practical application – They make you think more
2. Disadvantages	– Little information – Lack of teacher support	– Insecurity – Slower process – Lack of teacher support
3. Perception	– Different from usual – Conceptual clarity – Innovative – Current – Initial confusion – Unfamiliar concepts	– Difficult – Interesting – Information overload – Current – Creative – Different – Very long
4. GeoGebra	– Useful – Further exploration – Experimentation – Visual – Dynamic	– Difficult at first – Very helpful – Visual – Interactive

Additionally, ST students frequently mentioned the opportunity to explore new ideas and approaches, linking the activity to creativity and innovation in learning. Several students highlighted that the open nature of the task allowed them to go beyond routine exercises, stating, for example, “We could try different ideas and not just do the same type of exercises as always” or “It lets you think in another way, not only follow steps” (SS students). These perceptions reflect a greater openness to creative exploration within the learning process (Boyce & Agyei, 2024; Simanjuntak et al., 2021). SS students, on the other hand, emphasized that the activity made them think more, associating PBL with the development of critical thinking skills. Some students explicitly noted the cognitive demand involved, commenting that “You really have to think about what you are doing, not just apply a formula” or “It’s harder, but it makes you reason more than normal maths classes”. This perception aligns with previous research highlighting the role of PBL in fostering higher-order thinking skills (Arévalo-Duarte et al., 2024; Pazos-Yerovi & Aguilar-Gordón, 2024; Satrústegui & Mateo, 2023).

Disadvantages

Despite the positive aspects, students in both groups identified several disadvantages. The most common concern was the lack of teacher support, which some students perceived as disorienting. While ST students mentioned this occasionally, it was more prominent among SS students: “Sometimes we didn’t know if we were doing it right because the teacher didn’t explain everything step by step” (SS student). Feelings of insecurity and uncertainty were particularly evident in the SS group, where students reported difficulty managing the autonomy required by the activity: “I felt a bit lost at the beginning because we weren’t used to working like this” (SS student). These difficulties could be related to the search for information and the flexibility in solutions within PBL, which does not provide students with the security and clarity they might receive in more traditional teaching methods.

SS students also described the activity as a slower process, requiring more time and effort than traditional lessons. Some reported information overload, especially when searching for data or trying to understand new concepts: “There was too much information and it was hard to know what was important” (SS student). ST students, in contrast, pointed out insufficient information as a limitation, expressing frustration when data or guidance were not immediately available: “Sometimes there wasn’t enough information, and we had to look for everything ourselves” (ST student).

Although these difficulties were perceived negatively by some students, they are closely related to the core principles of PBL, which intentionally involve open-ended problems and limited initial information to foster research, autonomy, adaptability in responses and the development of critical, creative, and problem-solving skills (Cuong et al., 2025).

Perceptions of the activity

Regarding their overall perception of the activity, students from both groups agree that PBL is different from what they are used to, showing that this methodology represents a significant shift from their previous learning experiences. This aspect is particularly valued by students in group ST, who also mention that the activity was innovative and current, allowing them to engage with more relevant content: It’s not like normal maths classes. It feels more connected to real life (ST student). However, several ST students also acknowledged initial confusion, mainly due to unfamiliarity with the PBL methodology: “At first I didn’t really understand what we were supposed to do, but later it made more sense” (ST student). SS students, on the other hand, feels that the logistic function studied was difficult and had not been covered in class before: “The function was hard because we hadn’t seen it before, so it was confusing” (SS student). This perception added an extra level of difficulty to the activity and possibly caused frustration, especially if students do not feel prepared to face complex concepts (Tsybulsky & Oz, 2019). At the same time, many SS students recognized the activity as interesting and creative, despite its difficulty: “It was hard, but also interesting because it wasn’t the usual kind of exercise” (SS student).

Some students from SS group also mentioned that the activity felt too long or that they experienced an information overload, which may indicate that the time and volume of material provided during PBL can be overwhelming for some, especially if they are not accustomed to such an autonomous approach. This reinforces the idea that, while PBL fosters greater autonomy and a more active learning experience, it also requires good time management and adequate planning by the instructor to prevent students from feeling overloaded (Bosica et al., 2021).

GeoGebra

Finally, both groups identified GeoGebra is a visual and interactive (or dynamic) tool. This is especially useful when working with the logistic function (or any other parametric function), as students can quickly see how the function evolves as its parameters change. The interactivity of the tool allows students to manipulate these parameters directly and observe how the characteristics of the graph change. This type of visual exploration makes abstract concepts, such as the behavior of the function in different contexts, more tangible and easier to understand (Suratno & Waliyanti, 2023). The results seem to indicate that the students in the ST group felt more comfortable with the idea of exploring the tool, as they emphasized the need to experiment with adjusting the function's parameters: "When you change the parameters and see how the graph changes, it's much easier to understand" (ST student). In contrast, some students in the SS group reported more initial difficulties with the tool, which is understandable if they are not familiar with it: "At the beginning GeoGebra was hard to use because I didn't know how it worked" (SS student). Nevertheless, most of them recognized its usefulness for visualization and comprehension: "Even if it was difficult at first, it helped me understand the function better" (SS student). These perceptions align with previous research showing that digital tools like GeoGebra can enhance conceptual understanding, provided that students receive sufficient guidance and time to become familiar with the mathematical concepts involved in the problem (Ramadhani & Narphila, 2018; Recio et al., 2019; Septian et al., 2020).

In general, the qualitative findings help to explain and contextualize the quantitative results obtained in this study. The more positive perceptions expressed by ST students regarding real-world relevance, autonomy, and experimentation are consistent with their higher average scores across all questionnaire factors. Conversely, the difficulties and insecurities reported by SS students help contextualize their more moderate ratings and greater response variability. This convergence between quantitative and qualitative evidence strengthens the interpretation of the results and supports the robustness of the mixed-methods design, providing a more comprehensive understanding of the educational impact of the PBL activity.

CONCLUSIONS

In conclusion, the results of the quantitative analysis, with averages ranging from 3 to 5, indicate that students generally hold a positive view of the PBL methodology, particularly when it incorporates an interdisciplinary approach and the use of technological tools such as GeoGebra in mathematics learning. Students appear to appreciate the opportunity to connect mathematical concepts with real-world applications and other subjects, which can enhance both understanding and engagement. However, the effects still appear to be moderate, especially among students with an SS orientation, who may face greater challenges in engaging with highly analytical or quantitative content. Previous research supports the idea that these methodologies and technological tools have the potential to enhance student comprehension and motivation, but their effectiveness depends on proper and consistent implementation. This finding aligns with previous research suggesting that the effectiveness of PBL and technology-enhanced learning depends on multiple factors, including the design of the activity, the prior knowledge and motivation of students, and the extent to which the methodology is consistently and systematically implemented (Yew & Goh, 2016; Alreshidi & Lally, 2024).

In this regard, it is likely that the most significant benefits of PBL will emerge through continuous practice, the structured integration of interdisciplinary content, and sustained guidance and support from teachers (Cuong et al., 2025). The teacher's role is particularly crucial in scaffolding learning, providing timely feedback, and helping students navigate challenges while maintaining their autonomy (Bosica et al., 2021; Rahmatjati et al., 2023). Therefore, ongoing refinement of PBL strategies, careful adaptation to the needs and context of each student group, and deliberate use of digital tools such as GeoGebra are essential to maximize its impact on learning and ensure a more meaningful, engaging, and effective educational experience (Recio et al, 2019; Suratno & Waliyanti, 2023; Yohannes & Chen, 2021).

From the qualitative analysis, it is evident that while PBL offers significant advantages–including the development of reasoning, problem-solving, collaboration, and practical application skills (Hmelo-Silver, 2004; Smith et al., 2022)–it also presents notable challenges. Students often report feelings of insecurity, difficulty managing autonomy, limited teacher support, and initial struggles with unfamiliar concepts or abstract functions (Tsybulski & Oz, 2019). Despite these challenges, the overall perception of the methodology is positive; students recognize its value as a dynamic, innovative, and relevant approach to learning mathematics. The success of this methodology, however, depends on achieving an appropriate balance between fostering student independence and providing sufficient guidance to support effective problem-solving and conceptual understanding. When this balance is achieved, PBL has the potential not only to improve academic outcomes but also to enhance students' motivation, confidence, and capacity for interdisciplinary thinking, preparing them for more complex and authentic learning situations in both academic and real-world contexts.

Limitations and Future Lines of Research

It can be noted that measuring a change in students' attitudes is not sufficient with just one PBL implementation. It is advisable to carry out at least one per semester throughout the academic year. However, this was not possible due to the structure of the teaching program at the educational institution where the research was conducted. For this reason, we focused on measuring students' perceptions of the experience and evaluating the potential of the methodology in groups with different academic profiles, leaving the attitudinal and affective domain aspects for future work.

On the other hand, within each study group, there may be significant variability in students' previous experiences and skills. Individual differences, such as the level of competence in mathematics, familiarity with tools like GeoGebra, or even personal motivation toward the problem, could have influenced the students' responses, which may affect the generalization of the results within each group. Although the problem was designed in line with the specialization areas of both groups, students with scientific-technological and socio-scientific profiles may have different interests and approaches to the activity. This self-selection bias could influence how they perceive the problem, as students' expectations might not be homogeneous due to their prior interests in science or the social applications of mathematics. Moreover, while open-ended questions are useful for collecting detailed opinions, they are subject to students' interpretation and subjectivity. Furthermore, some students may not be sufficiently reflective or detailed in their responses, which could result in missing important nuances regarding their experience with the PBL methodology.

Finally, to implement activities from this perspective in the mathematics classroom, it is very important for the teacher to be well-versed in both the mathematical content and the pedagogical content involved, and, of course, to know how to relate it to the knowledge of the other disciplines. For this reason, analyzing the teacher's knowledge structure is essential for the successful implementation of the activity, as the teacher will be the one guiding and supporting students through their learning process. In this regard, as a future line of short-term research, we aim to explore how this methodology influences the professional development and pedagogical skills of teachers and how it may impact the teaching and learning process of mathematics.

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AI statement: The authors stated that no generative AI or AI-based tools were used in this study.

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