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High school student levels of arithmetic knowledge when solving additive word problems: A case study

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ABSTRACT

The level of arithmetic knowledge of a high school student was explored when solving additive word problems considering the semantic structure and syntactic component. The methodology was qualitative and developed in four stages: the first is the selection of the participant, the second is the design of a questionnaire with twenty additive problems, the third is the application of the semi-structured interview and questionnaire for the student to solve, and the fourth is the analysis of the data to identify the arithmetic knowledge. The results show that the student is at level 4 (directional relations level) solving comparison and equalization problems. However, he had difficulty in one of the comparison problems (incorrect application of the operation), but successfully solved the change, combination and equalization problems. It is concluded that the student can reinforce the comparison structure to have arithmetic knowledge of level 4 in its entirety.

Keywords: additive word problems, arithmetic knowledge, high school student

INTRODUCTION

Problem-solving has been considered a fundamental pillar for the teaching and learning processes of mathematics; in fact, it is a necessary means to develop mathematical knowledge (Baiduri et al., 2020; Bednorz & Kleine, 2023; Cai & Rott, 2024; Krawitz et al., 2022; Ministerio de Educación Nacional [MEN], 2006; Mukuka & Alex, 2024; National Council of Teachers of Mathematics [NCTM], 2000; Prediger et al., 2025; Rodríguez-Nieto et al., 2023; Rodríguez-Nieto et al., 2024; Ufer et al., 2024; Verschaffel et al., 2020). In particular, the Ministerio de Educación Nacional [MEN] (2016) reveal that additive problem solving is proposed in various curricular materials where some problems based on additive structures are presented with their respective resolution processes as evidenced in the Basic Learning Rights (BLR).

It is worth mentioning that problem-solving opens doors to new opportunities and challenges. It not only enhances mathematical understanding but also fosters critical thinking and adaptability, skills vital beyond academic settings. By engaging with additive structures, students build a solid foundation for more complex concepts, promoting a deeper understanding of how mathematical principles are interconnected and applied in real-world contexts (Siregar et al., 2020).

Additive Arithmetic Word Problems (additive AWP) are a type of mathematical problem that seeks to strengthen the ability to solve arithmetic problems of everyday life, these additive word problems are emphasized in situations that the environment presents to us that respond to basic operations (Livaque, 2017). Authors such as Livaque (2017), Nesher (1999), Puente (1993) and Socas et al. (1997) among others, pointed out that the structure of additive AWP has a diversified classification of these problems: change, combination, comparison and matching. Next, Nesher (1999), apart from specifying the classification of additive AWP, presents some arithmetic recognition levels that allow us to identify the management of the structures, of which there are 4 levels: level 1 (Counting), level 2 (change), level 3 (part-part-whole) and level 4 (directional relationships). When facing the problem, it is necessary to study the procedure, the strategies and techniques that allowed the resolution of this in an adequate way. Pólya and Zugazagoitia (1965) suggest that to solve a mathematical problem, the Pólya method should be used, which consists of 4 steps: understanding the problem, conceiving a plan, executing the plan and retrospective vision.

In literature, various investigations have been recognized focused on the resolution of additive problems of verbal statement. Poggioli (2007) and Rodríguez-Nieto et al. (2019) reveal the widespread difficulties with the structures of the source's comparison and equalization, often due to supervision in several phases of resolution. Thus, as keywords, the order, phrases, sentences, representations and place of the unknown. Giving way to the internal taxonomy within each type of structure and question, there is the syntactic component. Livaque (2017), Pacheco (2019) and Rodríguez-Nieto et al. (2019) point out that implementing the different additive AWP in the classroom contributes positively to the teaching-learning process. It is worth noting that Pacheco (2019) in his research emphasizes that in strengthening the AWP it is necessary to make use of didactic strategies within it the technological tools, these awaken interest and motivation for the subject.

According to Jiménez (2022), in his research he points out that implementing the FEMAT program, which consists of approaches and execution of playful activities, allows improving the understanding and resolution of additive AWP. On the other hand, authors such as Pérez (2021) and Pérez (2022) postulate in their research how necessary the context, purpose and reality are when it comes to understanding and solving additive AWP, in other words, it would be to establish connections between everyday life and arithmetic. Ayala-Altamirano et al. (2022) presented that verbal arithmetic problems allow the unlimited to become a matter of thought for students. However, verbal additive problems not only give way to solving arithmetic problems, but in tasks that require the translation of natural language into a symbolic one, in this case they managed to have an algebraic character which allowed them to perform successfully.

Rojas and Sotelo (2022) in their research point out the errors that students make in activities that involve additive word problems. These difficulties include: the correct interpretation of the statement of a problem and the lack of practice in translating the verbal problem into arithmetic language. In addition, González-Caribello et al. (2022) identified difficulties in solving additive problems by second-grade high school students (**Table 1**).

Table 1. Difficulties encountered by students

Category	Subcategory	Difficulty	What does it refer to?
Processes linked to the student	Understanding the problem statement	Lack of understanding of the statement	Decodes without interpreting the overall meaning of the statement.
		Literal translation of the statement	Translate the statement literally, following the order in which the phrases contained in it are expressed and focus only on key words and values to apply a procedure without fully analyzing the information.
	Heuristics	Heuristics based on the demands of the problem	Students do not apply heuristics according to what the problem demands (mental representation of the problem and procedures).
		Algorithmic procedures	Incorrect application of an algorithm
	Metacognitive process	Reasoning and argumentation	It is the lack of analysis, justification and argumentation that students carry out when solving the mathematical problems that are proposed to them.
	Solution to the problem	Response consistent with the statement	It occurs when students, apart from obtaining the result of a procedure, do not answer the question posed in the proposed problem.
	Basic knowledge	Reading numbers	It occurs when students do not read the numbers correctly, which can lead in some cases to not understanding the statement of the problem.
Processes linked to the mathematical problem	Type of problem	Problem structure	Difficulty faced by students when the unknown is presented in different parts of the problem. That is, in some of the events or in the whole problem.
	Numeric range	Numeric range	Does not master the numerical range up to 999.
	Contents/context	No associated difficulty was found	

Rum and Juandi (2022) argue that students faced problems when calculating the arithmetic operation and interpreting the problem statement, which led them to solve it without adequately understanding the question. For example, at level 4, they have difficulties both in interpreting the problem and in performing the arithmetic calculations, creating mathematical models, and communicating their explanations and arguments. This is because they tend to forget prior knowledge and are not clear on how to apply the formula.

According to research on additive problems, it is evident that students have difficulties in solving these problems because they use operations incorrectly and do not understand the statements due to the semantic and syntactic variables they involve (Achim, 2024; Capone et al., 2021; Gabler & Ufer, 2024; Kullberg et al., 2024; Polotskaia & Savard, 2018; Riley & Greeno, 1988; Rodríguez-Nieto, 2018; Rodríguez-Nieto et al., 2023; Wee & Yeo, 2024; Wolters, 1983; Xu et al., 2024). In addition, it is important to know what level students reach and what mathematical processes they activate to achieve it, including cases of success and errors (Doz et al., 2023; Rojas & Sotelo, 2022; Roos & Kempen, 2024; Ufer et al., 2024; Verschaffel et al., 2020; Wienecke et al., 2023). Therefore, the purpose of this research is *to explore the level of arithmetic knowledge of high school students when solving additive word problems.*

THEORETICAL FOUNDATION

In the mathematical contents of primary school level, it is very familiar to hear the word problem, "a mathematical problem is defined as the statement that describes an unknown situation and of interest to the solver that contains quantitative relationships, which arises from the need to verbally express problem situations due to the impossibility of solving them without language" (Ariza et al., 2016, p. 33). The structures (additive or multiplicative) of arithmetic problems contain numerical information and written text, that is, they have verbal and numerical content in a narrative (Sabagh Sabbagh, 2008). The resolution of these must be raised in different contexts, this allows the subject to expand their thinking, capacities and mathematical skills (Livaque, 2017). They are classified as first, second- or third-degree arithmetic problems according to the number of operations necessary for their resolution, as well as the nature of the data that appear in them (Echenique, 2006).

Additive Arithmetic Word Problems (Additive AWP)

Arithmetic problems show us the different situations in our environment in which phenomena that respond to the additive field (addition and subtraction) or the multiplicative field are observed (Livaque, 2017; Riley & Greeno, 1988). Additive word problems (AWP) are solved with an addition or a subtraction and are classified based on their semantic structure and syntactic component (Rodríguez-Nieto et al., 2019; Vergnaud, 1991).

Classification of Additive AWP and Syntactic Component

The *semantic structure* refers to the meanings of the different ways of presenting a problem statement, relating its concepts and sequences specific to arithmetic (Orrantia et al., 2005; Van Dijk & Kintsch, 1983). These occur in stages, which are: change (6 problems), combination (2 problems), comparison (6 problems) and equalization (6 problems) (Pacheco, 2019; Puente, 1993). The syntactic component highlights the order, representations, characteristic expressions and length of the statement, location of the unknown, presentation of information, context of the situation, whether it is a fictitious situation or not, among others (Dröse et al., 2021; Pacheco, 2019; Rodríguez-Nieto et al., 2019), as presented in **Table 2**, **Table 3**, **Table 4** and **Table 5**.

Semantic structure of change

In the change structure, three different elements are distinguished: an initial quantity subjected to a transformation (change) that modifies it to reach a final quantity. The effect of the change can be an increase or a decrease (Rodríguez-Nieto et al., 2019) (see **Table 2**).

Table 2. Types of problems according to the semantic structure of change (Pacheco, 2019)

Subtypes	Known data	Unknown quantity	Action
Change 1	Initial and change sets	Final set	Increase
Change 2	Initial and change sets	Final set	Decrease
Change 3	Initial and final set	Change set	Increase
Change 4	Initial and final set	Change set	Decrease
Change 5	Change and final sets	Initial set	Increase
Change 6	Change and final sets	Initial set	Decrease

Semantic structure of combination

The combination structure is the meeting (or combination) problems that describe a relationship between two sets that respond to the part-part-whole scheme. The unknown of the problem may refer to the part or the whole, there are two subtypes of this category, and it is important to consider the nature of the quantities or sets (Pacheco, 2019; Rodríguez-Nieto et al., 2019) (see **Table 3**).

Table 3. Description of the combination structure (Pacheco, 2019)

Subtypes	Known data	Part (unknown quantity)	Operation
Combination 1	Set of the two "parts"	Set of the "whole"	Addition
Combination 2	Set of a "part" and set of the "whole"	Set of a "part"	Subtraction

Semantic structure of comparison

In the comparison structure are the problems that present a static relationship between two quantities, called reference quantities, comparison quantities and differences. The comparison relationship is given by words that are present in the problem statement, such as, for example, more than and less than (Cañadas & Castro, 2011; Orrantia et al., 2005) and six subtypes of this structure are established (Pacheco, 2019) (see **Table 4**).

Table 4. Description of the comparison structure (Pacheco, 2019)

Subtypes	Known data	Unknown quantity	Aumento/ disminución
Comparison 1	Referent and compared sets	Difference set	Increase
Comparison 2	Referent and compared sets	Difference set	Decrease
Comparison 3	Sets of referent and difference	Compared set	Increase
Comparison 4	Sets of referent and difference	Compared set	Decrease
Comparison 5	Compared and difference sets	Reference set	Increase
Comparison 6	Compared and difference sets	Reference set	Decrease

Equalization semantic structure

This structure is characterized by limiting the unknown to the difference between a given quantity and a desired quantity. This structure consists of three components: the equalization quantity, the compared, and the referent. Problems that present this structure usually require a physical action for one quantity to be equal to another (Cañadas & Castro, 2011). According to other studies carried out by Echenique (2006) and Sánchez and Vicente (2015), equalization problems arose from an integration of change and comparison problems, where an action is performed when comparing two quantities and then one of them is adjusted by an increase or decrease, and six subtypes of matching problems are established (Pacheco, 2019) (see **Table 5**). It is important to note that shifting and equalization problems are considered dynamic, while combination and comparison problems are static (Rodríguez-Nieto et al., 2019).

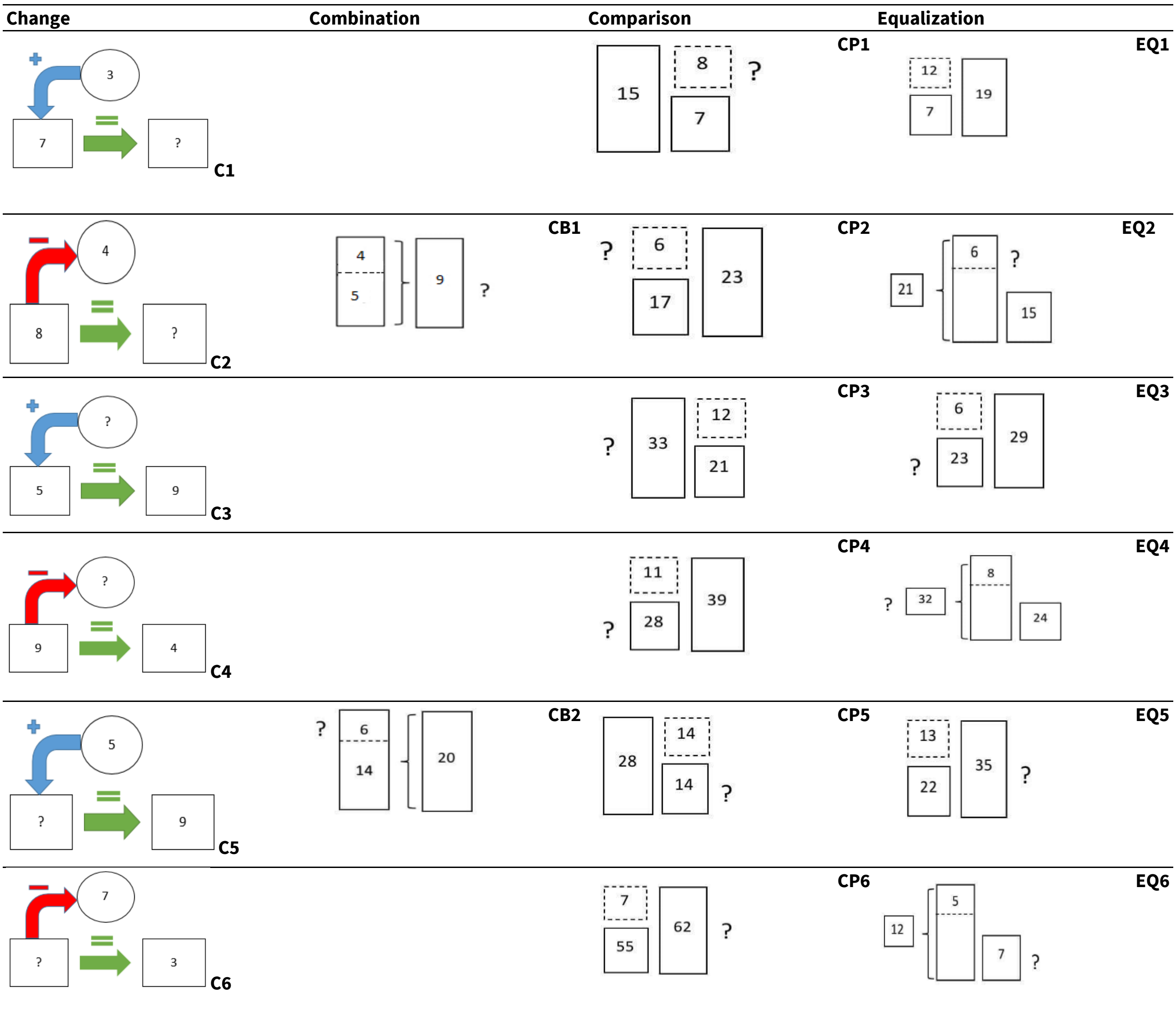
Table 5. Description of the equalization structure (Pacheco, 2019)

Subtypes	Known data	Unknown (equalization)
Equalization 1	Quantity to be equalized and referent	Quantity to be added
Equalization 2	Quantity to be equalized and referent	Quantity to be decreased
Equalization 3	Quantities of referent and equalization (adding)	Quantity to equalize
Equalization 4	Quantities of referent and equalization (decreased)	Quantity to equalize
Equalization 5	Quantity to be equalized and equalization (adding)	Referent
Equalization 6	Quantity to be equalized and equalization (decreased)	Referent

Representation of the Schemas for the Resolution of the AWP Additives

Rodríguez-Nieto et al. (2019) proposed diagrams that allow the representation and resolution of additive word problems. To do this, they use squares, rectangles, lines, arrows, circles and colors (red (subtraction), blue (addition), green (produces or total)) that indicate where the operation and relationship of the quantities are in the problem presented (see **Table 6**).

Table 6. Recommended structures for the resolution of AWP additives



Levels of Arithmetic Knowledge

Nesher (1999) establishes 4 levels of arithmetic knowledge in solving additive AWP, these are:

1) level 1 (counting),
2) level 2 (change),
3) level 3 (part-part-whole) and
4) level 4 (directional relationships), see **Table 7**.

Table 7. Levels of arithmetic knowledge in solving additive AWPs

Level	Name	Empirical knowledge	Structure activated
1	Counting	Refers to sets of adding and removing. Understanding of put, give and take which denote change in location or possession.	C1, C2, CB1
2	Change	Ability to link events by cause and effect. It refers to the amount of change.	C3, C4
3	Part-Part-Whole	A reversible scheme is available and can be used to find the unknown part in a sequence of events.	CB2, C5, C6, CP1, CP2, CP3, CP4
4	Directional relationships	Reversibility of non-symmetrical relationships. Ability to handle directional descriptions (more/less) and quantify a relationship (relative comparison).	CP5, CP6, EQ1, EQ2, EQ3, EQ4, EQ5, EQ6

Pólya's Method

In the development of mathematical skills, strategies are needed that allow the resolution of what it covers; the Pólya method is a main strategy for solving additive AWPs. Boscán and Klever (2012) point out that the application of the Pólya method (Pólya, & Zugazagoitia, 1965) to solve a problem considers four phases:

1) Understanding the problem: What is the unknown? What are the data and the conditions?
2) Conceiving a plan: Do you know a problem related to this one? Do you know any theorem that could be useful? Could you state the problem in another way? Have you used all the data?
3) Execution of the plan: Check each of the steps, can you see that the step is correct?
4) Retrospective vision: Verify the result.

The above questions allow you to identify whether the problem is being solved appropriately and if a difficulty arises, it gives you the possibility to go back and find the phase where you obtained an error that prevents you from getting on the right path (**Table 8**).

Table 8. Levels of arithmetic knowledge in solving additive AWPs

Phase	What should I do?	Questions that may arise during this phase
1) Understanding the problem	Read the problem and try to understand it.	What is the unknown? What are they asking me? What are the data? What are the conditions or restrictions that a problem has? Sufficient condition to determine the unknown? Semantics and syntax.
2) Conceiving a plan	The student or teacher uses their knowledge, imagination and creativity to develop a strategy that will allow them to find the operations necessary to solve the problem appropriately.	Have you encountered a similar problem? Have you seen the same problem posed? Do you know any math problems related to this one? Can you state the problem in another way? Can you express it in your own words?
3) Execution of the plan	The subject must implement the strategies he chose to solve the problem; this must be decided. If he does not achieve success, he must set it aside and form another one and then take it up again.	At this stage, questions or doubts do not usually arise while the process is being carried out.
4) Looking back or Retrospective analysis	It allows the students to review their work and make sure they have not made any mistakes.	Is the answer correct? Does the answer satisfy the question in the problem? Can you see how to extend your solution to a general case?

METHODOLOGY

The methodology of this research was qualitative descriptive case study (Creswell, 2014), carried out in four stages: in the first, the participants were selected, in the second, a questionnaire on additive AWP was designed, in the third stage, the questionnaire and a semi-structured interview were applied to the participant and, finally, in the fourth stage, the data were analyzed. Creswell (2014) mentions that a qualitative approach is based on the behavior of the subjects who are part of the problem under study, which allows them to interpret and characterize the reality that expresses the problem or phenomenon to be treated.

Participant and Context

One eighth-grade high school student, male and fourteen years old, from a public institution in the municipality of Repelón, Atlántico, Colombia, participated in this research. He is in the process of adapting to in-person classes because for the last two years he has been working virtually due to the pandemic generated by Covid-19, a disease caused by the SARS-CoV-2 virus. This was one of the main motivations for selecting this student because when asking him some questions about this type of additive problems (and their structures), he said that they could be substantially useful for the topics of equations and they encourage early algebra, finding the unknown quantity, etc.

Data Collection

The techniques and instruments that were considered for this research were a semi-structured interview together with a questionnaire composed of twenty additive problems guided by the theoretical foundations referring to the semantic structures and syntactic components. These qualitative data collection techniques are very specific actions, especially in the collection of information or evidence, however, these are the essential instruments in this research. Below is the questionnaire with the additive problems of verbal statements that, from the literature, are shown to be the most difficult to solve, but that contribute to the development of the mathematical problem-solving competence (Rodríguez-Nieto et al., 2019) (see **Table 9**).

Table 9. Questionnaire with additive AWPs

Change problems	Combination problems	Comparison problems	Equalization problems
(C1). Paula has 7 mangoes that she found on the tree in her house, and they gave her 3. How many mangoes does Paula have now?	(CB1). Liliana has 5 green and 4 red tokens, how many tokens does she have in total?	(CP1). Danna has 11 candies and Alfonso has 5. How many more candies does Danna have than Alfonso?	(EQ1). Carlos earns 9 dollars he will have as many as Daniela. How many dollars does Carlos have?
(C2). Paula has 8 fairy tale books and lost 4, how many books did Paula have left?	(CB2). Liliana has 20 tokens, some are blue, and others are red, if 6 are blue, how many are red?	(CP2). Carlos has 4 lollipops and Maria has 9. How many fewer lollipops does Carlos have than Maria?	(EQ2). Carlos has 10 grapes, if Ana eats 6 grapes she will have as many as Pedro, how many grapes does Ana have?
(C3). Pedro has 5 ping-pong balls; his cousin gives him some balls. Now he has 9 ping-pong balls, how many balls did his cousin give him?		(CP3). Carlos has 9 cookies; Maria has 6 more than Carlos. How many cookies does Maria have?	(EQ3). Pepe has 10 dollars and needs to earn 6 more to have as many as Daniela. How many dollars does Daniela have?
(C4). Pedro has 9 ping-pong balls, after losing some of his balls he had 5 left, how many balls did Pedro lose?		(CP4). Tatiana has 6 hens on her farm and Antonio has 4 fewer hens than Tatiana. How many hens does Antonio have in total?	(EQ4). Francisca has 15 liters of milk, and if she uses 8 liters she will have as many as Ana. How many liters of milk does Ana have?
(C5). Juliana had some stickers to fill her princess booklet, and she was given 4 stickers. If she now has 9, how many stickers does Juliana have at the beginning?		(CP5). Sheila has 7 red apples, 2 more apples than Jose. How many apples does Jose have?	(EQ5). Daniela has 20 dollars and if she spends 9 dollars she will have as many as her cousin Carlos. How many dollars does Carlos have?
(C6). Maria has some pencils and loses 7, if she now has 3 pencils, how many pencils does she have at the beginning?		(CP6). Jose has 9 apples, 3 apples less than Sheila. How many apples does Sheila have?	(EQ6). A hand fan costs $15, if it were priced at $119 more, it would cost the same as an electric fan. What is the price of the electric fan?

On the other hand, it is worth mentioning that during the videotaped interview the student spoke little in relation to the resolution of the additive problems, but afterwards he was asked some questions, and he answered as presented in the following excerpt from the transcript. Researcher-interviewer (I) and the student (S).

I : How did you feel during the activity?

S : Well, I felt good doing the activity.

I : How did you solve the exercises?

S : I solved them considering the information that was provided to me.

I : Were some exercises difficult for you?

S : Yes, because in some of them you had to find an unknown number.

I : What were your expectations about the exercises?

S : Well, I thought they were a little more difficult and with more difficulties.

I : How did you feel while doing these exercises?

S : Well, I felt good remembering how to add and subtract.

I : How do you expect the activity to be, correct or incorrect?

S : Well, I am sure that I can do it correctly because I solved them considering my experience and I also corrected them.

After explaining sections Participant and context and Data collection, it is important to mention that in qualitative research, choosing the participants first and then designing the instruments could generate biases and little novelty in the entire methodological process and the results. However, in this research, the participant was selected first because there is a current problem where it is evident that primary and secondary school students and some teachers have difficulties in solving additive word problems, and particularly the student-participant had some difficulties in solving these problems.

Later, delving deeper into the literature, it was recognized that additive problems are difficult due to their semantic and syntactic complexity and students do not pay attention to this, but rather have a mechanized and direct procedure with formulas (Rodríguez-Nieto et al., 2023). In addition, teachers do not emphasize these variables in the classroom, which does not help to mitigate the difficulties. Based on the above, the questionnaire in **Table 9** was designed, built and validated with contextualized problems based on these requirements from the literature and the selected student.

Furthermore, the questionnaire is not biased in terms of the number of problems or preferences of the situational context but is an instrument in accordance with the validated and current literature that has all the problems according to the semantic structures and syntactic component (Achim, 2024; Rojas & Sotelo, 2022). In fact, with this type of proposed problems, the student can find significant guarantees to solve problems on equations, identify the unknown place, enhance the operations of addition and subtraction in actions of increase and decrease.

Data Analysis

To analyse the data, Pólya's method was used to describe the problem-solving phases by the student. In addition, units and categories of analysis were established in accordance with the theoretical foundation referring to semantic structures and syntactic components (see **Figure 1**).

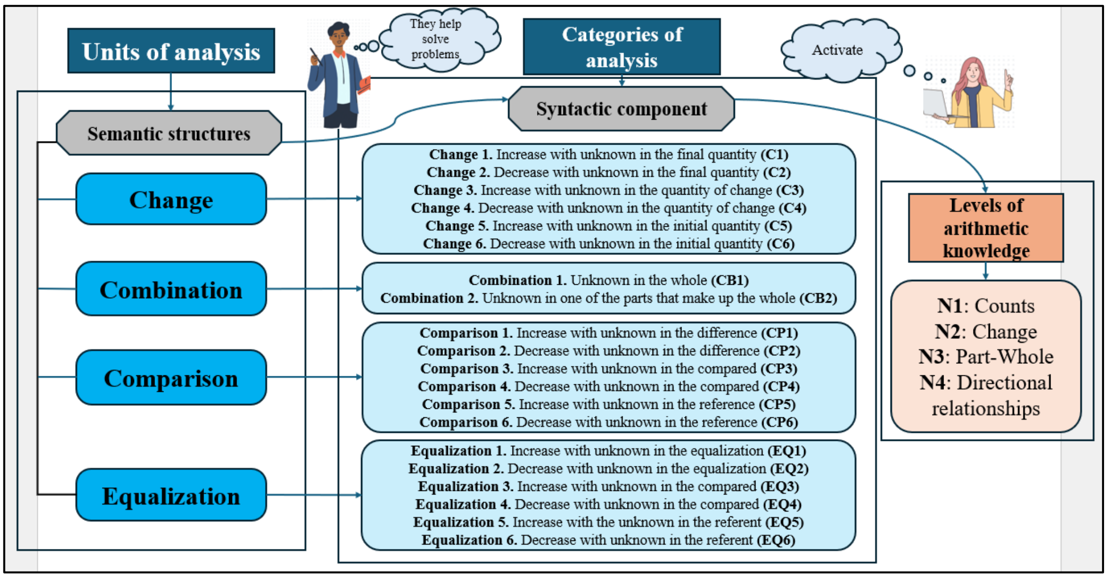


Figure 1. Units and categories of analysis (Rodríguez-Nieto et al., 2019)

The results are then presented according to the operability of the theoretical foundation and the functionality of the units and category of analysis to activate the levels of arithmetic knowledge.

FINDINGS

The results of this research show the levels of arithmetic knowledge achieved by an eighth-grade high school student when solving additive word problems. It is worth noting that the student went through the problem-solving phases proposed by Pólya and Zugazagoitia (1965) (**Figure 2**): in the first phase, the student read and understood each of the problems and extracted the data. In the second phase, he created a plan by choosing an operation that showed the relationship between the data. In the third phase, he executed the plan, performing the operation and interpreting the result. Finally, he reviewed the problems to see if he had done it well or badly (retrospective look).

Figure 2. Student solving the problems (Source: Authors' own elaboration)

Arithmetic Knowledge Levels: Level 1 (Counting)

This section shows the problems of change 1 and 2 and combination 1 solved by the student, which has allowed him to be placed in a level 1 of counting. Likewise, a scheme is proposed that dynamizes the mathematical operation performed and, in turn, is an optimal representation that allows the organization of the data underlying the problem (**Table 10**).

Table 10. Activation of level 1 of arithmetic knowledge (Counting)

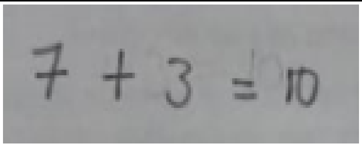
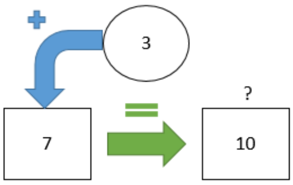
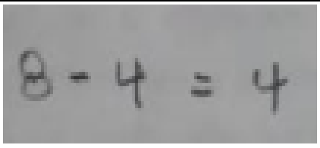
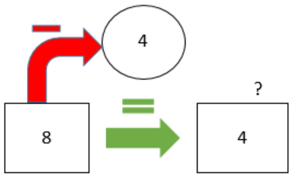
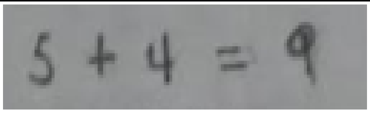
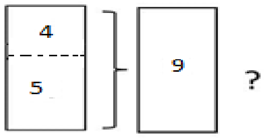
Problems/structure	Operation	Schema
C1		
C2		
CB1		

Figure 3 shows how the student solves change problems through addition and numerical representations.

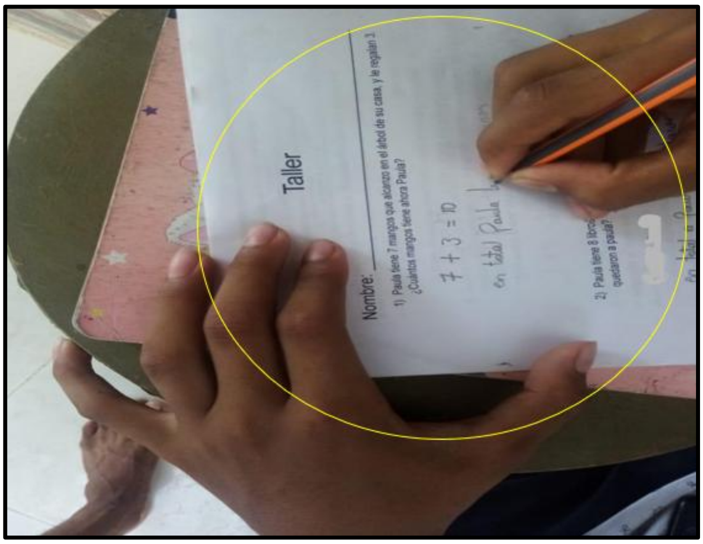


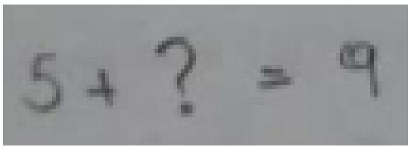
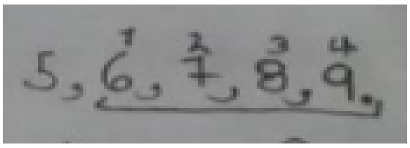
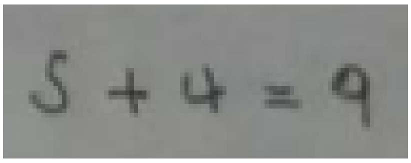
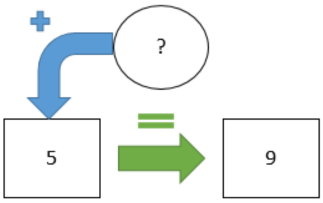
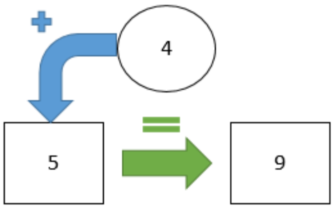
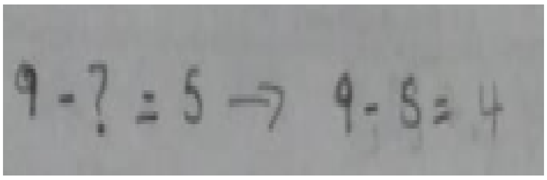
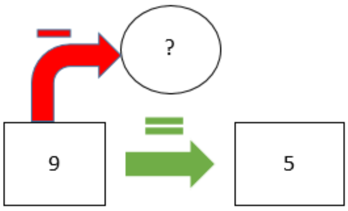
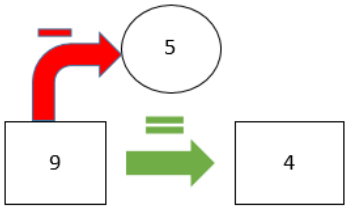
Figure 3. Evidence of problem solving for change (Source: Authors' own elaboration)

It is worth noting that the student had no problems understanding the problems with the change structure, therefore, the participant solved and passed the problems that were part of this level 1.

Level 2 of Arithmetic Knowledge (Change)

The problems belonging to level 2 of arithmetic knowledge are observed. This level has the problem situations of change 3 and 4 solved by the student, which allows him to place himself and stay at this level referring to the identification of modifications, changes as a dynamic vision of the operation and the quantities. In this same way, a scheme of the operation's functioning is proposed and, therefore, an optimal representation that allows the organization of the underlying data of the problem (see **Table 11**).

Table 11. Activation of level 2 of arithmetic knowledge (Change)

Problema/structure	Resolution	Schema
C3	The participant solved this exercise in several steps  Step 1  Step 2  Step 3	 Schema of step 1  Schema of step 2  Schema of step 3
C4	 According to the analysis, the participant applied the property of additive inverses showing it as follows: 9-? =5 Applying the property 9-? +? -5=? +5-5 Thus obtaining: 9-5=4	 

Continuing with the resolution of the problems, the student used a horizontal representation of the operations, since it seems that while he was reading the statements he was immediately extracting the data in an efficient and fast manner, as well as the identification of the corresponding operation in such a way that he was representing what he was doing (**Figure 4**).

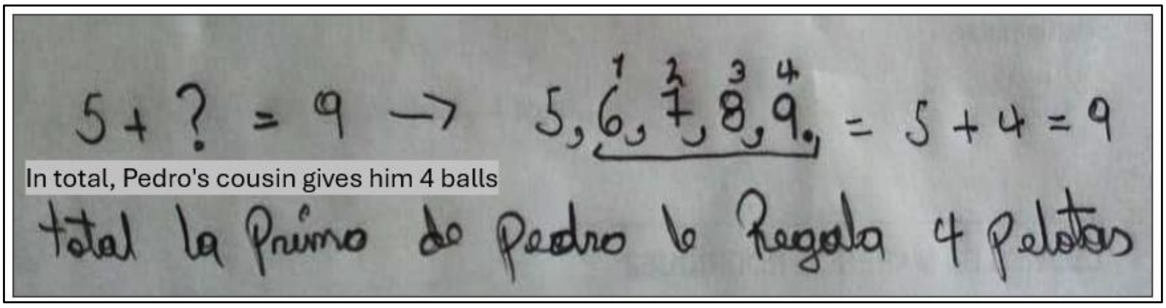


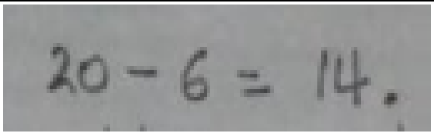
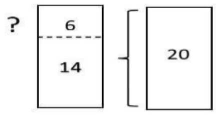
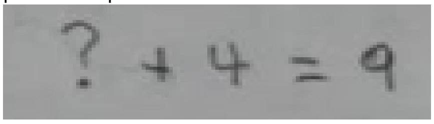
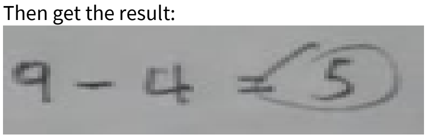
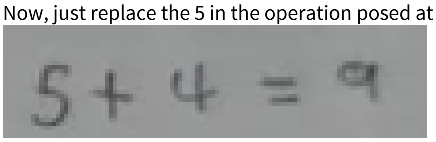
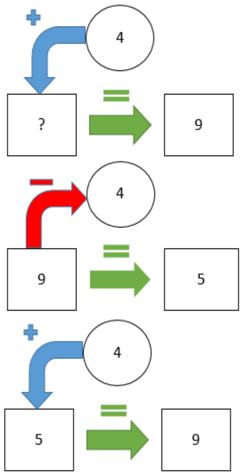
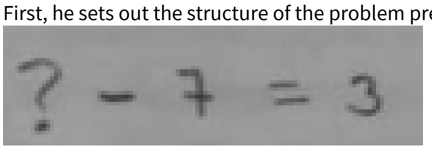
Figure 4. Written evidence of problem with C3 structure (Source: Authors' own elaboration)

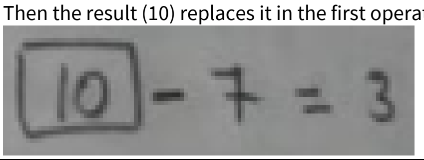
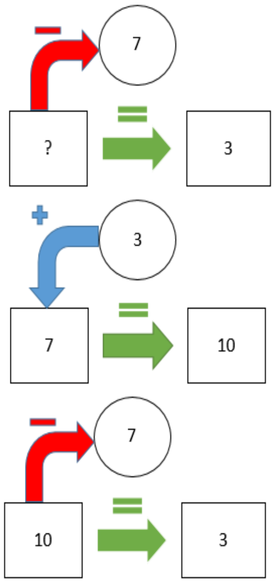
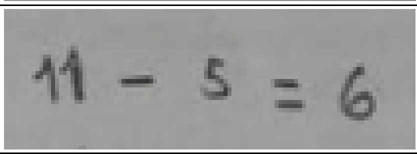
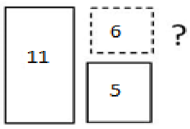
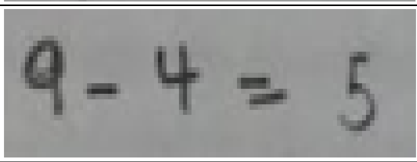
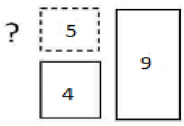
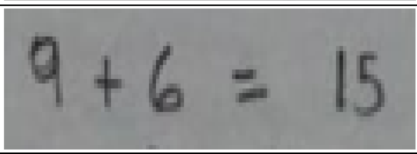
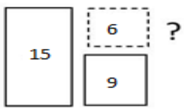
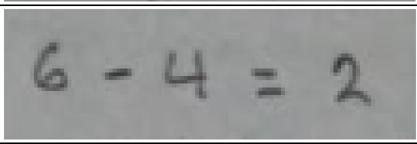
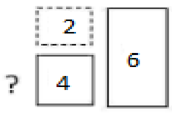
In summary, it can be confirmed that the student achieves or controls level 2 of arithmetic knowledge, without having problems with the exercises included in it.

Level 3 of Arithmetic Knowledge (Part-Whole)

For the activation of level 3 of arithmetic knowledge, the student solved problems of type CB2, C5, C6, CP1, CP2, CP3 and CP4 in a consistent manner, making modifications and comparisons that allowed him to operate with addition and subtraction. In this context, diagrams for the representation of the data of the problems as well as the difference are also shown in **Table 12**.

Table 12. Activation of level 3 of arithmetic knowledge (Part-Whole)

<table>
<tr><th>Problem/structure</th><th>Resolution</th><th>Schema</th></tr>
<tr><td>CB2</td><td></td><td></td></tr>
<tr><td>C5</td><td>In this problem the participant used the property of additive inverses and posed the operation as follows:

He is a student who used the property and obtained the following: +4-4=9-4
Then get the result:

Now, just replace the 5 in the operation posed at the beginning.
</td><td></td></tr>
<tr><td>C6</td><td>In this problem, the participant used the property of additive inverses again.
First, he sets out the structure of the problem presented.

Applying the property of additive inverses, the following is stated:
-7+7=3+7
S: “Here I used the inverses, because that property helps me find the results in a practical way”
So, it ends up like this:

Then the result (10) replaces it in the first operation posed.
</td><td></td></tr>
<tr><td>CP1</td><td></td><td></td></tr>
<tr><td>CP2</td><td></td><td></td></tr>
<tr><td>CP3</td><td></td><td></td></tr>
<tr><td>CP4</td><td></td><td></td></tr>
</table>

Continuing with the completion of the questionnaire, it was observed here that the students had difficulties in understanding the structures underlying the additive problems, however, the participant gradually adapted to these, being able to solve all the problems (see **Figure 5**).

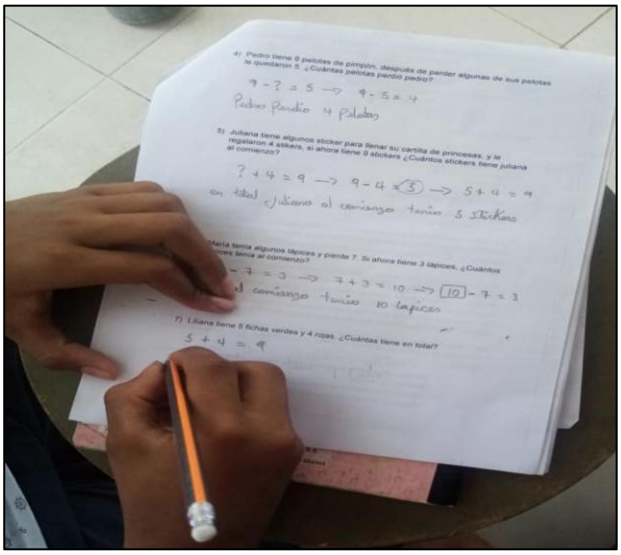


Figure 5. Evidence of problem solving that triggers level 3 (Source: Authors' own elaboration)

The student, although it took a little longer than expected, managed to correctly complete the problems that cover this level of arithmetic knowledge.

Level 4 Arithmetic Knowledge (Directional Relationships)

For the activation of this level, the student solved the problems with semantic structures and syntactic components: CP5, CP6, EQ1, EQ22, EQ3, EQ4, EQ5, EQ6 consistently and with the help of inverse relations, which allows him to operate with addition and subtraction as inverse operations and bidirectional relations (**Table 13**).

Table 13. Level 4 arithmetic knowledge (Directional Relationships)

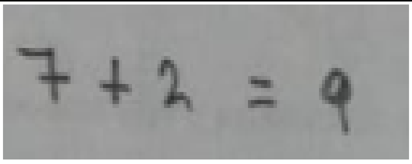
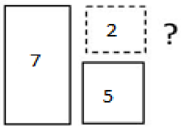
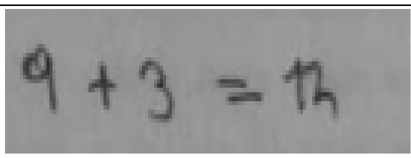
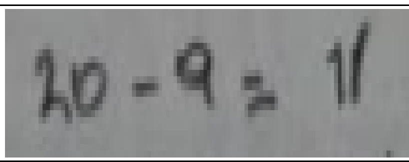
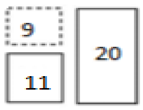
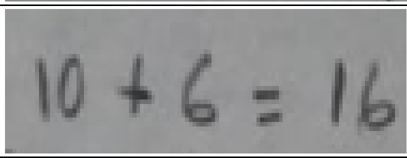
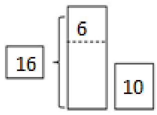
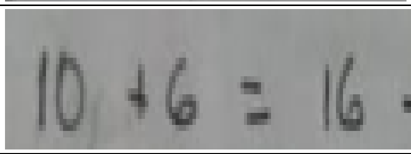
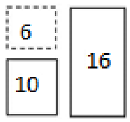
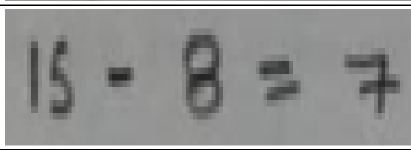
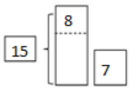
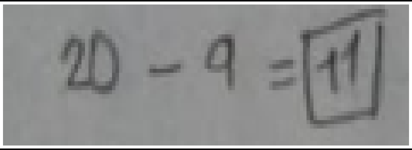
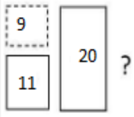
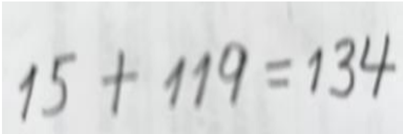
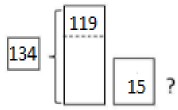
Problem/structure	Resolution	Schema
CP5		
CP6		
EQ1		
EQ2		
EQ3		
EQ4		

Table 13 (Continued). Level 4 arithmetic knowledge (Directional Relationships)

Problem/structure	Resolution	Schema
EQ5		
EQ6		

Finally, having finished with the problems posed, the student took a little more time to solve the problems corresponding to this level. However, he was unable to correctly answer problem 13, corresponding to structure CP5 (**Table 13**), referring to: *Sheila has 7 apples, 2 more apples than Jose. How many apples does José have?* where the error consisted in the representation of the operation, the participant first formulated the operation 7+2=9, but in the answer, he wrote "*José has a total of 5 apples*". The final answer is correct, but it does not match the operation performed, although the correct operation would be 7 – 2 = 5 or 5 + 2 = 7, giving the answer corresponding to the problem (**Figure 6**).

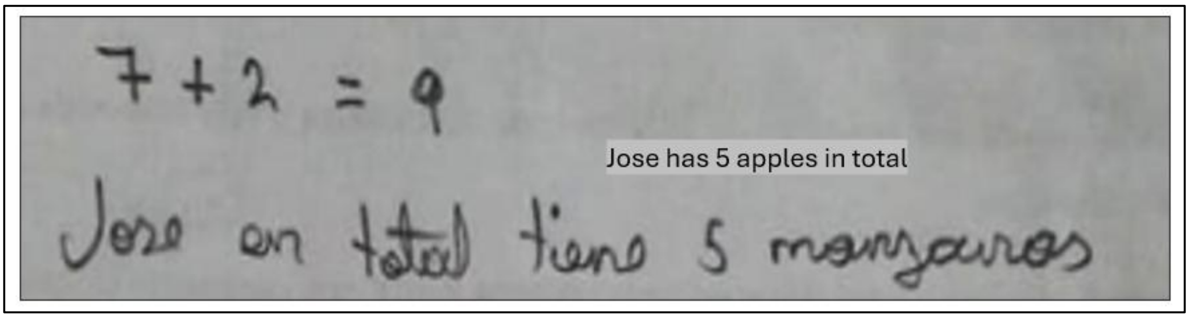


Figure 6. Evidence of the problem being solved inconsistently (Source: Authors' own elaboration)

After finishing solving the problems, the data obtained was analyzed, distinguishing each of the problems in **Table 14** by their respective level of arithmetic knowledge.

Table 14. Analysis of the proposed additive AWPs

Problems/structure	Correct (✓) /Incorrect (X)	Level of arithmetic knowledge
P (C1)	✓	Fulfills level 1 conditions
P (C2)	✓	Fulfills level 1 conditions
P (C3)	✓	Fulfills level 2 conditions
P (C4)	✓	Fulfills level 2 conditions
P (C5)	✓	Fulfills level 3 conditions
P (C6)	✓	Fulfills level 3 conditions
P (CB1)	✓	Fulfills level 1 conditions
P (CB2)	✓	Fulfills level 3 conditions
P (CP1)	✓	Fulfills level 3 conditions
P (CP2)	✓	Fulfills level 3 conditions
P (CP3)	✓	Fulfills level 3 conditions
P (CP4)	✓	Fulfills level 3 conditions
P (CP5)	X	Does not fulfill the level 3 conditions because it did not adequately resolve the problem.
P (CP6)	✓	Fulfills level 4 conditions
P (EQ1)	✓	Fulfills level 4 conditions
P (EQ2)	✓	Fulfills level 4 conditions
P (EQ3)	✓	Fulfills level 4 conditions
P (EQ4)	✓	Fulfills level 4 conditions
P (EQ5)	✓	Fulfills level 4 conditions
P (EQ6)	✓	Fulfills level 4 conditions

Reviewing the problems, as shown in **Table 14**, an inconsistency was found in the resolution of problem 13 (CP5) that had the statement (*Sheila has 7 apples, 2 more apples than José. How many apples does José have?*) which the failure consisted of the representation of the operation, the participant first formulated the operation 7 + 2 = 9, but in the answer, he placed "*José has a total of 5 apples*". The final answer is correct, but it does not agree with the operation performed, although the correct operation would be 7-2 = 5 giving the corresponding answer to the problem.

Unlike previous research, in this study the student only had difficulties solving a problem due to an experienced inconsistency. For example, this inconsistency is like the translation type errors reported by Rojas and Sotelo (2022) because he made an incorrect translation of data into an arithmetic language which caused him to solve the problem incorrectly. Furthermore, this inconsistency is associated with the causes of the students' difficulties as reported by González-Caribello et al. (2022) emphasizing the lack of understanding of the statement.

In summary, it could be said that the participants in the activity managed to correctly solve 19 of 20 problems posed correctly; according to the results obtained, the student is at level 4 (directional relations). The strategies used by the participant were modelling and numerical sequence. The modelling strategy consists of using objects (chips, sticks, marbles, etc.) or fingers to model the action, that is, to represent the elements of the sets and to carry out the actions described in the problem with them.

Impact of This Research on Other Academic Environments

Good morning, I am Author X and I am part of the arithmetic didactics course. This project is the result of it, which is titled solving additive problems by an eighth-grade student. Our purpose is to carry out a task of 20 additive AWPs, in which the question is asked of a student, who correctly solves 19 of 20 problems using concepts and strategies that allowed him to solve these exercises. It was evident that additive AWPs provide teaching-learning from another perspective and in a didactic way (**Figure 7**).

Figure 7. Evidence of the participation of researchers in an educational fair (Source: Authors' own elaboration)

DISCUSSION AND CONCLUSION

This article explored the levels of arithmetic knowledge achieved by a high school student when solving additive word problems, which is essential for their training in problem solving using basic operations and contributes to algebraic thinking in the topics of linear equations. These results also demonstrate that the student can solve comparison and matching problems, which is difficult as reported by some researchers (González-Caribello et al., 2022; Orrantia et al., 2005; Rodríguez-Nieto et al., 2019; Rojas & Sotelo, 2022).

It is important to highlight that the student in this research had difficulties solving a problem with a semantic structure of comparison 5 since the chosen operation does not match the appropriate operation and does not relate the data consistently. This type of error from the perspective of Rojas and Sotelo (2022) is an incorrect translation of data into an arithmetic language and an incorrect application of addition that coincides with the typology of errors suggested by González-Caribello et al. (2022). Given this situation, we propose that we continue to promote this type of mathematical problem with students of different school levels, encourage their use with in-service teachers, and use the diagrams to represent and organize the problem data.

Furthermore, these errors are not only made by the student who participated in this research, but in Achim (2024) it was identified that children with language development disorder (LD) have difficulties solving verbal and word addition problems because they focus only on the use of algorithms and manipulative material, and classifies it as a task that depends on language and requires children to understand the text and identify the semantic relationships between the quantities of the problem to solve them successfully. Therefore, this type of research that addresses mathematics at early ages can still be extended, which undoubtedly projects towards other school stages.

It should be noted that, unlike the results of this research, there are other works such as that of Roth et al. (2025) who state that, in the framework of solving additive word problems, it is still not clear whether number processing occurs after text processing or whether both occur simultaneously, which marks a new way of addressing these problems because it is always common to read and understand and then use the numbers and symbols.

One of the limitations of this research is the sample selected, but other works can be carried out with complete courses with more students and even videotaping the teacher in the teaching and learning process of additive problems. Another limitation is the lack of generalization of the results obtained because it was only done with a single student, so it would be interesting to consider a larger population that presents the problems in problem solving and apply statistical methods for the analysis of performance, connections, argumentation, critical reading, errors, among others.

Finally, for future research we recommend posing single-stage and multi-stage problems where several semantic structures are considered and connected in problem solving. In addition, it would be interesting for students to solve challenging problems that involve different types of additive problems (verbal, numerical, graphical, pictorial) and that are related to each other and that favor mathematical understanding.

Author contributions: CAR-N: conceptualization, data curation, writing – original draft; **JDC-J & ALS-R:** writing – review & editing, formal analysis; **BMC-R & JDB-V:** literature review, resources; **SS & ACI:** methodology, investigation, supervision, writing – review & editing. All authors have agreed with the results and conclusions.

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Ethical statement: The authors stated that both the participants and the educational institution were informed that this article has educational objectives that are not rooted in economic or political processes, and they voluntarily agreed to participate. The authors further stated that they are privileged to have the institution's willingness to conduct research projects and professional internships for pre-service mathematics teachers, where the fundamental role of research, data collection, and analysis is emphasized.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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