International Electronic Journal of Mathematics Education
2026, 21(2), em0884
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Geometry and biocultural heritage in early childhood education

Sandra Pérez-Lisboa [1*], Cristian Mejías [1]

[1] Universidad de Playa Ancha de Ciencias de la Educación, CHILE
*Corresponding Author: sandra.perez@upla.cl

Citation: Pérez Lisboa, S. R., & Mejías, C. (2026). Geometry and biocultural heritage in early childhood education. *International Electronic Journal of Mathematics Education, 21*(2), em0884. https://doi.org/10.29333/iejme/18610

ARTICLE INFO

Received: 05 Jan 2026

Accepted: 13 Apr 2026

ABSTRACT

This study aimed to identify the biocultural geometric space in children from two rural schools in the commune of San Felipe, Chile, through a pedagogical approach based on the visualization level of Van Hiele's theory. The methodology followed a Design-Based Research framework, implementing learning experiences contextualized within the local biocultural heritage. The sample was homogeneous and consisted of 25 preschool children, with participant observation and semi-structured interviews as instruments. The analysis revealed that the children developed skills in spatial orientation and organization and were able to identify 2D and 3D shapes in their surroundings. An initial understanding of attributes such as faces, vertices, and edges was observed. The results confirm that interaction with environmental elements and teacher mediation encourage geometric thinking. It is concluded that biocultural heritage is a powerful educational resource for re-signifying the learning of geometry in early childhood education.

Keywords: Van Hiele, biocultural, heritage education, geometry

INTRODUCTION

Learning development in young children is stimulated cognitively through the connection between prior knowledge and new understandings that emerge from their learning experiences. These experiences gradually enable children to recognize notions of space and time, helping them make sense of phenomena and shapes associated with their heritage through processes of identification, communication and classification.

Vanegas (2018) argues that the discoveries and explorations children engage in support their progressive integration into the environment. Similarly, Markovits and Patkin (2021) suggest that recognizing solid shapes related to biocultural heritage enhances spatial orientation, which in turn lays the foundation for logical reasoning and geometric understanding in the early years of life.

Biocultural heritage, understood as a form of connection to ancestral territory, consists of "all those objects, places, and local expressions" (Prats, 2005, p. 23). This understanding is deeply mediated by daily interactions with the environment (Berkes & Turner, 2006). According to Pérez-Lisboa (2020), the various experiences children encounter in their daily lives allow them to distinguish, make sense of, and explain this heritage, driven by their innate curiosity and ability to ask questions.

Heritage can be defined as "the set of material and immaterial resources that provide information about the preferences, knowledge and skills of ancestral societies" (Salazar et al., 2005, p. 162). Biocultural heritage develops through the interaction between local ecological knowledge, institutions, traditions, beliefs, practices, and cultural perspectives (Berkes & Turner, 2006). In educational settings, this dimension supports the assimilation of knowledge when integrated into pedagogical activities designed within educational institutions (Bonilla, 2020).

The diversity of environments in which children live fosters the development of meaningful connections between physical space and the tools provided by geometric thinking. This connection enables them to interpret their surroundings, model them, interact with them, and navigate them with purpose (Cabanne & Ribaya, 2009). The space perceived by people never has a symmetrical character, some objects are located above, others below, some further away, others closer, some on the right, others on the left, inside a cube or outside of it (Pérez-Lisboa et al., 2024). Along these lines, Alsina et al. (1997) suggest that shapes, objects, and patterns found in children's biocultural heritage offer a concrete invitation to engage in geometric experiences from an early age.

Geometry learning in early childhood, as proposed by Van Hiele (1999), begins through playful activities such as drawing sequences, creating mosaics, or folding paper. These experiences enrich children's visual structures and consolidate knowledge of shapes and their attributes. However, this is a complex and lengthy process, as it requires active exploration and internalization of the environment (Canals, 1997), mediated by spatial organization (the relationships between objects) and spatial orientation (awareness of one's own body in space), along with the development of basic concepts (Alsina, 2006). Alsina also emphasizes that didactic proposals for introducing geometric contents should start from the immediate environment, addressing one-, two- and three-dimensional concepts (line, surface, space) through activities involving body movement, object manipulation, direct experimentation, and graphic or artistic representation (p. 153). Finally, as noted by Öcal and Halmatov (2021), geometric thinking involves the ability to define, understand and classify shapes; establish relationships among them; reason; make conjectures; visualize and represent them graphically.

Geometry, understood as the study of space, is not limited to a linear and sequential analysis of dimensions, but should be approached within meaningful and situated contexts (Giménez, 1997). This way, strategies that integrate mathematical and geometric objects into the environment promote processes of understanding, questioning and investigation (Mejías, 2019). Observing biocultural heritage not only contextualizes geometric learning but also provides an opportunity to challenge the traditional paradigm of geometry teaching at the elementary and secondary education levels. This paradigm has favored a linear progression from two-dimensional shapes to three-dimensional ones. (Ministerio de Educación, 2023).

In this context, observing heritage buildings and analyzing the rich biocultural legacy enable children in early childhood education to develop relevant geometric skills. These skills may begin with the understanding of three-dimensional geometric solids and then progress toward their representation in two-dimensional plans.

Within the Chilean curriculum framework, the teaching of geometry in early childhood education falls under the core area of Mathematical Thinking, as established in the Curricular Bases for Early Childhood Education. This area includes several learning objectives aimed at the transition levels (children aged 4 to 5), such as communicating the position of objects and people, representing objects, identifying attributes of 2D and 3D shapes, and describing the steps followed in problem solving. (Subsecretaría de Educación Parvularia, 2018, p. 99).

Although curricular frameworks include learning objectives related to geometry in early childhood, there is considerable scientific evidence indicating that both geometric thinking and spatial reasoning remain undervalued in early mathematics education (Davis & The Spatial Reasoning Study Group, 2015; Rittle-Johnson et al., 2018; Sarama & Clements, 2009). While mathematical knowledge begins to develop at an early age, the academic and cognitive skills that support it are not always properly identified and fostered. (Rittle-Johnson et al., 2018).

Several studies have shown that mathematics teaching in early childhood education often focuses primarily on basic arithmetic content and has insufficient knowledge of teaching techniques and methods (Gonulates & Gilbert, 2023). Preschool teachers have little basic knowledge about shapes and solids and also encountered difficulties naming polygons, which should be familiar and are part of children's learning objectives (Markovits & Patkin, 2021). In this context, the adult's role is essential, as they must be able to follow children's interests and/or guide their attention to spatial elements present in everyday activities and play (Davis & The Spatial Reasoning Study Group, 2015).

There is also evidence supporting children's ability to identify two-dimensional representations of three-dimensional shapes, as well as to recognize 3D shapes themselves (Öcal & Halmatov, 2021). The National Council of Teachers of Mathematics (NCTM, 2000) states that, from early childhood education through second grade, students are able to name, distinguish, build, and compare three-dimensional objects, as well as describe their properties and parts.

Building on this, Blömeke et al. (2019) emphasize the importance of ensuring that initial teacher education provides meaningful and successful experiences in teaching geometry. Similarly, Orba and Develi et al. (2015) stress the need to incorporate specific and formal training in this area so that future educators can foster effective geometric skills in their students from the earliest years.

Given the background presented, this study implements a didactic proposal aimed at teaching geometry to five-year-old children, based on the first level of Van Hiele's model (1957) – visualization – and using biocultural heritage as central axis. The intervention was carried out in two rural schools in the commune of San Felipe, a territory characterized by its natural diversity and traditions rooted in Indigenous cultures.

The general objective of the study was to identify manifestations of biocultural geometric space in five-year-old children from rural settings in the commune. To achieve this, the following specific objectives were established:

(1) To identify similarities and differences in elements of biocultural heritage, and

(2) To distinguish physical properties of these elements.

The proposal was implemented through pedagogical visits and excursions to emblematic heritage sites in San Felipe. These activities enabled learning experiences integrated with the three core areas established by the Subsecretaría de Educación Parvularia, (2018) for early childhood education: Exploration of the natural environment, understanding of the sociocultural environment, and mathematical thinking. This strategy seeks to reframe the negative preconceptions often associated with mathematics—particularly geometry—which are frequently perceived by students as abstract, monotonous, and lacking practical relevance (Rocard, 2007).

Given that children in early childhood education begin to engage with essential geometric objects, this proposal operationalizes geometric visualization through the recognition and analysis of biocultural and heritage spaces. To this end, playful learning experiences were designed, including the manipulation of three-dimensional solids, the construction of two- and three-dimensional shapes, and the resolution of contextualized problems - thus contributing to the development of geometric thinking from a situated and culturally meaningful perspective.

METHOD

Design-based research (DBR) is a methodology that enables the testing of interventions in real-world learning contexts. This approach is led by educational researchers with the aim of creating practical and sustainable interventions (Easterday et al., 2018). Interventions designed under this methodology are structured around essential components for their implementation (Howell et al., 2021).

The development of the proposal involved researchers working alongside a multidisciplinary team composed of two scholars specializing in mathematics education for early childhood and two preschool teachers, representing diverse perspectives within the field of education (Collins, 1999). The implementation of the proposal was organized according to the phases outlined by Easterday et al. (2018). The first two phases — focus and understand — were carried out beforehand, allowing for the analysis of the educational context and a clear definition of the proposal.

The phases

Define

In this stage, early childhood educators were introduced to the general foundations of Van Hiele's model through training sessions led by members of the academic, where they learned the visualization level. At this level, children have a global perception of geometric figures, paying almost exclusively attention to physical properties such as identifying sides or angles and individual perception of the figure (Van Hiele, 1957).

Conceive

The early early childhood educators planned the pedagogical experiences based on the level of visualization proposed by Van Hiele, organized into learning phases. These phases are: Information, Guided Guidance, Explicit Guidance, Free Guidance, and Integration. In each phase, the teacher's work and their proposed approach are highlighted, along with its intentions. These plans were reviewed and enriched through critical dialogue with the team of researchers and trainers.

Build

From October to December, the educators implemented designed pedagogical experiences. During this process, continuous adjustments and improvements were incorporated. The proposal was structured around the five learning phases of the Van Hiele model:

1) In the information phase, the children were introduced to the new topic, and the preschool teachers determined their prior knowledge.
2) In the guided orientation phase, the children solved activities and problems in the learning experiences to learn the basic content of the new topic.
3) In the explicitation phase, the children verbally expressed their ideas and problem-solving methods, debated, questioned, and constructed learning, incorporating new mathematical terms.
4) In the free orientation phase, the children worked on the learning experiences with activities and problem situations to deepen their knowledge of the new topic and their use of the new type of reasoning.
5) During the integration phase, the preschool teachers and the children conducted a comprehensive review of everything they had learned. This review was intended to foster metacognition.

Test

The proposal was implemented from March to October of the following year. Its implementation was monitored through systematic observations. In November, semi-structured interviews were conducted with the participating children to collect qualitative data about their experience.

Present

Finally, the results were shared by the researchers in various academic forums, contributing to reflection and improvements in the teaching of geometry in early childhood education.

The sample in this study was homogeneous, as it included all five-year-old children enrolled in the higher transition level at two rural schools in the commune of San Felipe. The sample consisted of 25 participants in total: 16 from one school and 9 from the other.

The instruments used for data collection were participant observation and semi-structured interviews. Participant observation is a technique that allows the researcher to have "direct experiences with the participants and the environment" (Hernández et al., 2014, p. 417). Based on these experiences, the researcher was able to observe how the children identified their geometric space. Semi-structured interviews, in turn, are a tool that allows for "gathering more information about the desired topics" (Hernández et al., 2014, p. 418), which enabled the researcher to determine whether the participants were able to identify shapes and point out their similarities and differences.

Ethical Considerations Prior to Data Collection

To carry out this study, the instruments were reviewed and approved by the university's scientific and ethical committee. Once approval was obtained, informed consent was sent to the parents and legal guardians of the participating children. Furthermore, prior to each observation during the implementation of the proposal, informed assent was requested from each child. This procedure was also followed before conducting the semi-structured interviews.

Context of the Study

The commune of San Felipe is located 18 km from the Andes Mountain Range and, prior to the regionalization process in Chile, was the capital of the former jurisdiction of Aconcagua. It currently has a population of approximately 86,000 inhabitants and is known for its rich biocultural heritage. Among its most notable historic landmarks are the San Francisco de Curimón Church and Convent, the San Francisco El Almendral Convent, and the Mardones Mansion, home of the San Felipe Club. In addition, it is home to the San Felipe History and Archaeology Society Museum, the Serranía El Ciprés Nature Sanctuary, and a remarkable biodiversity, including endangered species such as the culpeo fox, quiques and yacas.

Within this context, the rural schools considered in this study are public institutions under the authority of the Municipal Department of Education Administration (DAEM) of San Felipe. They are located in rural areas of the commune: one in the locality of El Asiento and the other in Algarrobal. These schools serve as meaningful educational settings for implementing contextualized pedagogical proposals that integrate the biocultural heritage of the territory into teaching and learning processes.

ANALYSIS AND DISCUSSION

Based on the observations conducted at the two participating rural schools, we proceeded with the analysis of the observation records, complemented by the responses from the semi-structured interviews carried out with five-year-old children. The analysis was structured around the general objective of the research: To identify the biocultural geometric space in children of this age group. In addition, two specific objectives were addressed:

(1) To identify the similarities and differences within the biocultural heritage and

(2) To distinguish the physical properties of that heritage.

To analyze the qualitative data, the software Atlas.ti was used, enabling the systematic organization, coding, and interpretation of both the observations and interviews. The observations were classified into two analytical codes: "Movement through space" and "2D and 3D shapes". The semi-structured interviews, in turn, were coded under the categories "Naming 3D shapes" and "Attributes", in order to examine the children's understanding and use of geometric language.

Within the code "Movement through space", it was observed that, through teacher-guided activities, children explored the concept of topological space by changing position and engaging in movements such as running, jumping and moving sideways. During these experiences, spatial concepts such as "right" and "left" were introduced, helping children become familiar with ideas of location and orientation, as well as recognize similarities and differences in their movements. The activities also incorporated spatial vocabulary related to the understanding of absolute and relative space, including terms like "inside", "outside", "up", "down", "on", "under" and "between". These actions reinforced the development of spatial thinking by encouraging children to engage with their physical environment and use their motor skills connected to spatial orientation and organization, such as walking along straight or curved lines, moving with accuracy, and recognizing spatial relationships in their immediate surroundings.

These actions demonstrate a progressive understanding of spatial boundaries, as well as the ability to recognize and classify elements based on their location, reflecting an initial grasp of fundamental spatial concepts. In addition, young children are able to perceive differences and similarities within topological space, which is essential for identifying components of the biocultural heritage.

It is important to emphasize that these activities integrate spatial learning with bodily movement and play, promoting a meaningful didactic approach. According to Alsina (2006), experiences involving spatial organization and orientation support children's assimilation of their environment during early childhood. This ability to assimilate enables children to describe direction and distance, as well as perform more complex movements such as rotating, turning, and moving through space (NCTM, 2000).

Moreover, the use of gestures and spatial vocabulary enhances the development of spatial reasoning, encouraging active participation in the construction of geometric concepts (Davis & The Spatial Reasoning Study Group, 2015).

In relation to the code "3D and 2D Shapes", it was observed that children from both educational institutions were able to describe the physical attributes of objects, such as size, shape, and thickness. Likewise, they were able to identify perceptual differences between objects, distinguishing those that were larger or smaller, as well as those that characterized thinness or width, for Öcal and Halmatov (2021), it involves classifying shapes and establishing relationships between them. These experiences encouraged children to identify and describe three-dimensional objects in an intuitive way, with emphasis on their function and physical properties, driven by curiosity and spontaneous formulation of questions (Pérez-Lisboa et al., 2024).

A meaningful practice takes place when early childhood educators organize walks around the streets near the school, allowing children to observe, identify, and describe the geometric attributes of three-dimensional shapes, such as faces, edges, vertices, and bases. During these pedagogical outings, students take photographs of the objects they identify. Later, back in the classroom, the educators present these images, encouraging the children to make connections between the objects observed in the urban environment and those presented in the classroom. This diversity of physical space and then, with the tools provided by geometric space, they can appropriately interpret it (Cabanne & Ribaya, 2009). This strategy enhances visual understanding, spatial awareness, and the ability to describe three-dimensional shapes. According to Bonilla (2020), this type of activity also fosters the construction of meaningful knowledge connected to the cultural and material heritage of the surrounding environment.

In relation to the responses given by the children during the semi-structured interviews, when asked about the names of 3D shapes, most of them replied as follows:

Girl 3 The trunks of the trees are cylinders.

Girl 21 The pine trees we saw have a pyramid on top.

Boy 10 The roof of a house had a pyramid.

Boy 22 The garbage can at school is shaped like a parallelepiped.

Boy 7 There is a car that is looks like a cube.

Girl 25 The hills are like pyramids.

Girl 17 Oranges are spheres.

The children from both schools showed the ability to recognize basic geometric attributes of a cube, identifying that this three-dimensional shape has six faces, twelve edges and eight vertices. These findings suggest an initial understanding of the concepts of face and base. However, some difficulties were observed in the use of specific geometric terms. A few children forgot the word "vertices" and replaced it with more colloquial expressions like "points". Despite these language limitations, most of them were clearly able to tell the difference between faces, edges, and vertices.

Regarding details of the observations, only one student from one of the schools was unable to recall the concept of "faces", while two students from the other school did not correctly identify "vertices" and "edges". These findings are consistent with those reported by Öcal and Halmatov (2021), who note that preschool children are generally able to recognize the geometric properties of three-dimensional shapes and distinguish between vertices, edges, and faces - though this may vary depending on their familiarity with technical vocabulary.

In the analysis of the semi-structured interviews about geometric attributes, some children made the following observations:

Boy 2 The cube has eight vertices.

Girl 8 The parallelepiped has 12 edges, some are long and others are shorter.

Boy 18 The cylinder has a circular base.

Boy 6 There are pyramids that have a square base and another base shaped like a triangle.

Girl 16 The sphere has no vertices or edges, and it's all round.

Boy 19 The sphere doesn't have different faces; it's all curved.

As the learning experiences progress, the observation records show that young children begin to differentiate geometric shapes and their properties, and to recognize their value in relation to their immediate surroundings. These learning activities support the development of a more structured understanding of geometric forms, allowing students to identify these shapes in real-world contexts. This learning process also contributes to a growing appreciation of the biocultural environment, integrating mathematical knowledge with the observation and valuing of the world around them.

The responses from the interview reveal the following:

Childern 4 and 19 The windows and doors have the shape of a parallelepiped.

Boy 20 The school tiles look like cubes.

Girl 11 There were cylinders on a fence.

In addition, the children begin to apply geometric concepts in their lives, identifying patterns and similarities between abstract shapes and objects in their everyday environment. They also describe similarities and differences between 3D shapes, as evidenced by their responses during the interview:

Girl 12 The sphere has no vertices, but the cube does, and it isn't circular.

Boy 13 The cube and the parallelepiped have squares and 12 edges.

Boy 1 The pyramid has one vertex at the top and four at the bottom; the cube has eight identical ones.

Girl 15 The cylinder doesn't have faces like the cube.

Boy 14 The pyramid has five faces, and the cube has six.

At both educational institutions, young children demonstrated the ability to identify key attributes of three-dimensional geometric shapes while modeling spheres, cubes, and parallelepipeds, as well as constructing various three-dimensional representations. Observations made during the activities revealed that, when invited by the early childhood educators to describe their creations, all participating students were able to correctly identify the three-dimensional shapes they had made and also to recognize and name the two-dimensional shapes present in their work. Only three children showed difficulty using specific geometric vocabulary, omitting terms such as "parallelepiped", "sphere" (referred to as "ball"), and "edges" (referred to as "lines"). These experiences proved to be meaningful for developing their skills in observing, classifying, and describing geometric attributes.

Additionally, the implementation of learning experiences in diverse settings – including the classroom, the schoolyard, nearby squares, and field trips beyond the school – enabled active exploration of both two- and three-dimensional geometric shapes. The presence of biocultural heritage in these spaces, expressed through both natural elements (such as plants) and human-made structures, provided a concrete foundation for establishing connections between geometric concepts and everyday objects in the children's environment. This integration fosters young children's understanding and appreciation of their natural and cultural surroundings (Alsina et al., 1997).

CONCLUSIONS

This research demonstrates that using biocultural heritage as a pedagogical resource allows for the reframing of mathematical learning - particularly geometry - from a situated and culturally relevant perspective. Through excursions and visits to emblematic heritage sites in the city, it was possible to create experiences that contributed to transforming traditional preconceptions about the subject, moving beyond reductionist views focuses solely on measurement and calculation. As Fensham and Harlen (1999) pointed out, the teaching of science, mathematics, and geography has long faced the challenge of developing pedagogical approaches that meaningfully connect with students' interests and experiences. While this situation has influenced levels of motivation toward these disciplines, there is now a growing emphasis on pedagogical approaches that promote greater contextualization, relevance, and active student participation.

In this context, the implemented pedagogical approach focused on early childhood education, a stage at which children begin to become familiar with essential geometric shapes. The planned activities took place in both natural and built environments, incorporating play, sensory experiences, and the manipulation of real objects to construct and explore two- and three-dimensional shapes. In doing so, the activities promoted spatial visualization and strengthened skills such as identifying, describing, and classifying geometric shapes, along with the progressive use of increasingly precise mathematical language.

It is important to highlight that this proposal was not limited to a Euclidean view of school geometry focused solely on measurement and distance. On the contrary, the approach was broadened to incorporate topological and projective dimensions, allowing children to develop a broader and richer understanding of space rooted in both everyday and cultural contexts. This perspective integrated various forms of geometric visualization which, by being connected to the immediate environment, supported meaningful learning (Alsina, 2006; NCTM, 2000).

Additionally, it was observed that preschool children demonstrate strong potential for developing geometric skills when engaged in learning situations rich in exploration and interaction. The transfer between experiences outside the classroom and school-based activities was facilitated by intentional teacher mediation, which effectively connected mathematical knowledge to the students' sociocultural context (Bonilla, 2020; Pérez-Lisboa et al., 2024).

From a formative perspective, the findings highlight the importance of strengthening both initial and ongoing training for early childhood educators, particularly in relation to pedagogical approaches that recognize territory and culture as foundational elements of mathematical learning. As Markovits and Patkin (2021) stated, recognizing the value of context in mathematics education is key to designing more inclusive, contextualized, and transformative proposals.

In sum, biocultural heritage not only contributes to the development of geometric thinking from an early age, but also offers an opportunity to promote a mathematics education that is more closely connected to everyday life, identity, and the cultural diversity of the territory.

Author contributions: Both authors contributed equally in all parts of the study and the writing of the article. All authors have agreed with the results and conclusions.

Acknowledgments: The authors would like to thank the preschool teachers who implemented the pedagogical proposal and all the children who participated in the study

Funding: The authors declare that this research was funded by the General Directorate of Research of the University of Playa Ancha, Chile.

Ethical statement: The authors stated that ethical approval for this study was granted by the Ethics Committee of the University of Playa Ancha (CEC-UPLA) on May 10, 2023, under approval code No. 09-2023. Written informed consents were obtained from the participants.

AI statement: The author(s) stated that they did not use artificial intelligence.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Alsina, Á. (2006). *Didáctica de la geometría para la educación infantil y primaria*. Graó.

Alsina, Á., Burgués, A., & Fortuny, J. M. (1997). *La geometría en la educación infantil*. Graó.

Berkes, F., & Turner, N. J. (2006). Knowledge, learning and the evolution of conservation practice for social–ecological system resilience. *Human Ecology, 34*(4), 479-494. https://doi.org/10.1007/s10745-006-9008-2

Blömeke, S., Busse, A., Kaiser, G., König, J., & Suhl, U. (2019). The role of pedagogical content knowledge in early mathematics education: A longitudinal study of prospective teachers. *ZDM – Mathematics Education, 51*(3), 325-336.

Bonilla, M. (2020). Educación patrimonial y desarrollo cognitivo en la infancia. *Revista Iberoamericana de Educación, 83*(1), 145-162.

Cabanne, M., & Ribaya, C. (2009). *Didáctica de la matemática: Un enfoque integral para el nivel inicial*. Paidós.

Canals, M. (1997). *La matemática en la educación infantil*. ICE-Horsori.

Collins, A. (1999). The changing infrastructure of education research. In E. Lagemann, & L. Shulman (Eds.), *Issues in education research: Problems and possibilities* (pp. 289-298). Jossey-Bass.

Davis, B., & The Spatial Reasoning Study Group. (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. Routledge. https://doi.org/10.4324/9781315762371

Easterday, M., Rees, L., & Gerber, E. (2018). Design-based research process: Problems, phases, and applications. In J. L. Plass, R. E. Mayer, & B. K. Schneider (Eds.), *Handbook of learning sciences* (2nd ed., pp. 151-169). Routledge.

Fensham, P. J., & Harlen, W. (1999). School science and public understanding of science. *International Journal of Science Education*, *21*(7), 755-763. https://doi.org/10.1080/095006999290417

Giménez, J. (1997). *Didáctica de la geometría en la educación primaria*. Síntesis.

Gonulates, F., & Gilbert, A. (2023). Pre-service early childhood teachers' mathematical pedagogical content knowledge: Emphasis on geometry. *Early Childhood Education Journal, 51*, 1245-1257.

Hernández, R., Fernández, C., & Baptista, P. (2014). *Metodología de la investigación*. McGraw-Hill Interamericana.

Howell, E., Lo, J., & Fahey, M. (2012). Design-based research in education: The challenges and promises of collaboration between researchers and practitioners. *Educational Research: Theory and Practice, 32*(1), 1-13.

Markovits, Z., & Patkin, D. (2021). Preschool in-service teachers and geometry: Attitudes, beliefs and knowledge. *International Electronic Journal of Mathematics Education, 16*(1), Article em0619. https://doi.org/10.29333/iejme/9303

Mejías, C. (2019). Contextualización del aprendizaje geométrico en la formación inicial docente. *Revista de Didáctica de la Matemática, 34*(2), 87-105.

Ministerio de Educación. (2023). *Priorización curricular: Orientaciones para los niveles de enseñanza básica y media* [Curricular prioritization: Orientations for basic and secondary education levels]. Gobierno de Chile.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. NCTM.

Öcal, T., & Halmatov, M. (2021). 3D geometric thinking skills of preschool children. *International Journal of Curriculum and Instruction, 13*(2), 1508-1526.

Orbay, K., & Develi, H. (2015).Pre-school Students' Informal Acquisitions Regarding the Concepts of Point and Straight Line. *International Electronic Journal of Elementary Education*. 7(3), 319-332.

Pérez-Lisboa, S. (2020). *Estrategias didácticas de las educadoras de párvulos del nivel transición mayor y las habilidades científicas desarrolladas por los niños y niñas en las escuelas municipales de la comuna de San Felipe* [Tesis doctoral en educación]. Universidad Metropolita de Ciencias de la Educación.

Pérez-Lisboa, S., Mejías- Zamorano, C., Repetto, A., Mattiello, G., Basulto, C., & Salgado, L. (2024). *Resignificaciones geométricas a través de observaciones patrimoniales bioculturales en el Valle de Aconcagua* [Geometric resignifications through biocultural heritage observations in the Aconcagua Valley]. Eliva Press.

Prats, L. (2005). *Concepto y gestión del patrimonio cultural* [Concept and management of cultural heritage]. Ariel.

Rittle-Johnson, B., Zippert, E., & Boice, K. (2018). The roles of patterning, spatial, and numeracy skills in early mathematics learning. *Early Childhood Research Quarterly, 46*, 166-178.

Rocard, M. (2007). *Science education now: A renewed pedagogy for the future of Europe*. European Commission.

Salazar, N., Londoño, M., Guevara, S., & Vergara, H. (2005). El patrimonio cultural desde una nueva perspectiva. *Contaduría Universidad de Antioquia*, *46,* 159-168.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge. https://doi.org/10.4324/9780203883785

Subsecretaría de Educación Parvularia. (2018). *Bases curriculares de la educación parvularia*. Ministerio de Educación, Gobierno de Chile.

Van Hiele, P. M. (1957). *El problema de la comprensión. En conexión con la comprensión de los escolares en el aprendizaje de la geometría* [The problem of comprehension: in relation to school students' understanding in learning geometry] [Tesis doctoral]. Universidad Real de Utrecht, Países Bajos. https://www.uv.es/aprengeom/archivos2/VanHiele57.pdf

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics., 5*(6), 310-316. https://doi.org/10.5951/TCM.5.6.0310

Vanegas, J. (2018). El entorno como espacio de aprendizaje en la educación infantil [The environment as a learning space in early childhood education]. *Revista Colombiana de Educación,75*, 85-104.