

Fraction magnitude student explanations: A latent class analysis

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ABSTRACT

The study examined the types of explanations students provide for fraction magnitude problems. Student responses were coded into one of five explanation types: (a) absent, (b) faulty, (c) conceptual-partially developed, (d) algorithmic, and (e) conceptual-fully developed. When examining latent classes specific to students' explanations of their knowledge of fraction magnitude, a five-class model was the most tenable, conveying the presence of five distinct student profiles. The algorithmic class represented the largest percentage of student explanations and also revealed the strongest correlation with criterion measures. A combined algorithmic-conceptual class was not identified.

Keywords: fractions, magnitude comparison, latent class analysis, explanations, algorithmic, conceptual understanding

INTRODUCTION

Understanding fraction magnitude predicts later school achievement in mathematics (Torbeys et al., 2015). In this study, authors conduct a latent class analysis of explanation types provided by 428 fifth-grade students while responding to fraction magnitude comparison problems. Study objectives include an exploration of the possible existence of latent classes, the number and nature of these classes, and the relationship of these classes to criterion measures of student accuracy.

Numerical knowledge has a strong impact on achievement in school and later in career (National Mathematics Advisory Panel [NMAP], 2008; Schneider et al., 2018; Siegler, 2016; Siegler et al., 2012). Numerical knowledge develops along a continuum as students integrate new sets of numbers (e.g., small and then larger whole numbers, rational numbers, and integers) into their understanding of the system of real numbers (Jordan et al., 2013; Siegler, 2016). In this regard, a cross-cutting concept is magnitude (Jordan et al., 2013; Siegler, 2016; Siegler et al., 2011), which involves reasoning about the location of values on the number line (Barbieri et al., 2020). This uniting concept of numerical magnitude (whole number and fraction) has correlational, predictive, and causal relationships with both general and specific (e.g., computational) achievements in mathematics (Barbieri et al., 2020; Siegler, 2016).

As students expand their understanding of the real number system, they must consider how what they learned about one set of numbers may change with a new set of numbers (Barbieri et al., 2020; Siegler, 2016). For instance, based on their work with natural numbers, students may generalize the rule that when you multiply numbers, the product is greater than or equal to either factor. However, this rule "expires" with other sets of numbers, including rational numbers (Karp et al., 2014; Stafylidou & Vosniadou, 2004). Due to this restructuring of numerical knowledge with the introduction of rational numbers, the development of students' understanding of rational numbers is a critical aspect of overall numerical development (Siegler, 2016) as well as future success in mathematics (NMAP, 2008).

The importance of rational numbers combined with findings related to students' and adults' lack of rational number understanding (Geller et al., 2017; NMAP, 2008) highlights a "serious educational problem" (Siegler, 2016, p. 351). Studies have documented this fragile understanding of fractions in various populations, including students entering the middle grades (Resnick et al., 2016), Algebra 1 students (Hoffer et al., 2007), and students attending community college (Stigler et al., 2010). Misunderstandings about fractions are often grounded in a lack of understanding of magnitude (Jordan et al., 2013), and such misconceptions are especially prevalent among low-achieving students (Malone & Fuchs, 2017) and students with mathematics learning difficulties (Mazzocco & Devlin, 2008).

Table 1. Examples of approaches to comparing pairs of fractions (Adapted from Quebec Fuentes et al., 2025)

Approach	Example (smaller and larger)	Explanation
Same numerator	$\frac{3}{7}$ and $\frac{3}{5}$	$\frac{1}{5}$ is larger than $\frac{1}{7}$. So, $\frac{3}{5}$ are larger than $\frac{3}{7}$.
Same denominator	$\frac{3}{5}$ and $\frac{4}{5}$	For both fractions, the same size parts ($\frac{1}{5}$ s) are compared. Four of those parts ($\frac{4}{5}$) are greater than three of those parts ($\frac{3}{5}$).
Transitive	$\frac{4}{9}$ and $\frac{3}{5}$	$\frac{4}{9}$ is less than $\frac{1}{2}$ whereas $\frac{3}{5}$ is greater than $\frac{1}{2}$. Therefore, $\frac{3}{5}$ is larger than $\frac{4}{9}$.
Residual	$\frac{4}{5}$ and $\frac{6}{7}$	$\frac{4}{5}$ is $\frac{1}{5}$ less than 1, and $\frac{6}{7}$ is $\frac{1}{7}$ less than 1. Since $\frac{1}{7}$ is smaller than $\frac{1}{5}$, $\frac{6}{7}$ is larger than $\frac{4}{5}$.
Common denominator	$\frac{3}{7}$ and $\frac{3}{5}$	$\frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$ and $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$. Since 21 is greater than 15, $\frac{3}{5}$ is greater than $\frac{3}{7}$.
Cross products	$\frac{3}{7}$ and $\frac{3}{5}$	$3 \times 5 = 15$ and $7 \times 3 = 21$. Since 21 is greater than 15, $\frac{3}{5}$ is greater than $\frac{3}{7}$.

Despite the importance of understanding magnitude for numerical development, in particular, knowledge of fraction magnitude, a lack of fraction understanding, especially related to fraction magnitude, persists. Attending to students' understanding of fraction magnitude provides an opportunity to address this educational problem. The present paper, therefore, examined the types of explanations students provide for fraction magnitude problems. More specifically, students were provided with four comparisons of fraction magnitude items, prompted to identify the greater fraction, and then explain how they "figured it out." Student accuracy on these items and scores on criterion measures was analyzed along with the types of explanations provided.

Fraction Sense and Magnitude Knowledge

One recommendation in the IES practice guide: Developing effective fractions instruction for kindergarten through 8th grade (Siegler et al., 2010) is to "help students recognize that fractions are numbers and that they expand the number system beyond whole numbers" (p. 19). In this transition from whole numbers to fractions, some misconceptions stem from the application of whole number thinking to problems involving fractions (DeWolfe & Vosniadou, 2015; Mack, 1995; Siegler et al., 2011; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010). For instance, whole numbers can be represented with a single number whereas fractions are represented with two numbers and a line (Siegler et al., 2011; Stafylidou & Vosniadou, 2004). Therefore, a student misconception is viewing fractions as two whole numbers rather than numbers themselves (Cramer & Whitney, 2010; Malone & Fuchs, 2017). In order to integrate fractions and their novel characteristics into their understanding of number, therefore, students need to develop fraction sense.

Building fraction sense is multifaceted as students must develop mental images of fractions (Cramer et al., 2008; Johanning, 2011). Thus, initial work with concrete models of fractions eventually becomes internalized so that students become able to visualize fractions without physical models (Cramer & Whitney, 2010; Cramer et al., 2008). These models of fractions reinforce ideas about the referent whole (Cramer & Whitney, 2010; Cramer et al., 2008), partitioning (Johanning, 2011), unit fractions (e.g., $\frac{1}{4}$), and unit fraction iterations (e.g., $\frac{3}{4}$ is three parts of size $\frac{1}{4}$; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In addition, concrete and mental models demonstrate how the denominator of a unit fraction impacts the size of the fraction (Cramer et al., 2008, p. 491) and the relationship of a fraction to benchmarks like 0, $\frac{1}{2}$, and 1 (Fennell & Karp, 2017; Johanning, 2011).

These aspects of fraction sense support the development of students' understanding of the relative size, or magnitude, of fractions (Cramer & Henry, 2002). Magnitude knowledge is a critical component of fraction sense as it serves as a connection between whole numbers and fractions and reinforces the fact that a fraction represents a single number (Fennell & Karp, 2017; Siegler et al., 2010; Tian & Siegler, 2017). Fraction magnitude is multifaceted and includes an understanding of fraction size as determined by the interplay between the denominator and numerator (Geller et al., 2017), the relationship between the fraction and other fractions, and the size of the fraction with respect to the unit whole (Johanning, 2011). Fraction magnitude tasks include positioning fractions on a number line, using benchmarks to estimate the size of fractions, determining fraction equivalence, comparing pairs of fractions, and ordering sets of fractions (Cramer et al., 2008; Fennell & Karp, 2017; Geller et al., 2017).

The present study centered on tasks involving the comparison of pairs of fractions. Approaches to comparing fractions vary with respect to the extent to which they rely on students' fraction sense. Cramer et al. (2002) described four student-generated approaches with a conceptual orientation: same numerator, same denominator, transitive, and residual (Table 1). To implement these approaches, students use mental images and reason about unit fractions, the inverse relationship between the denominator and size of each piece, and fraction position relative to benchmarks. A more procedurally oriented approach is comparing numerators after finding a common denominator (Table 1). Although this approach is grounded in fraction equivalence and the same denominator approach (Table 1), it is often reduced to applying a rule that reinforces whole number thinking by focusing on the numerators only (Cramer & Whitney, 2010).

Another algorithmic approach to comparing a pair of fractions is the "butterfly method" for which the cross-products are determined, with the larger cross-product indicating the larger fraction (Table 1). Karp et al. (2015) argued that in the absence of conceptual understanding, this approach presents challenges for student learning. That is, it does not require students to comprehend the value of the fractions and, as a result, they may apply the strategy in inappropriate contexts (e.g., fraction multiplication).

Through the elementary grades, students' developing understanding of fractions and their magnitude supports their transition to operations with fractions (National Governors Association Center for Best Practices & Council of Chief State School

Officers, 2010; Van de Walle et al., 2019). In particular, tasks related to fraction magnitude develop prerequisite understandings for computations with fractions (Mazzocco & Devlin, 2008).

To make sense of operations with fractions, students need an understanding of magnitude, such as estimation and comparison of fractions with benchmarks, so they can evaluate the appropriateness of the results of their computations (Cramer et al., 2008; Johanning, 2011; Tian & Siegler, 2017). In contrast, if students do not understand fraction magnitude, they are not able to make meaning of computations with fractions, which then leads to an inability to judge the reasonableness of solutions (Siegler & Pyke, 2013; Stigler et al., 2010). For instance, students without magnitude knowledge may incorrectly add $\frac{2}{3} + \frac{1}{6} = \frac{3}{9}$ and not be able to determine that $\frac{3}{9}$ is not a reasonable sum since it is less than $\frac{2}{3}$, one of the addends (Cramer & Whitney, 2010). A lack of fraction sense, including an awareness of fraction magnitude, typically leads students to depend on the application of procedures rather than conceptual understanding when operating with fractions (Fennell & Karp, 2017).

Fraction Sense and Types of Knowledge

Research on the unique contribution of procedural and conceptual knowledge to students' fraction sense or fraction computation accuracy has shown that some students rely more heavily on conceptual knowledge, other students depend on procedural knowledge, while still others draw on aspects of both conceptual and procedural knowledge (Hecht & Vagi, 2012; Rittle-Johnson et al., 2001).

Procedural knowledge is algorithmic or rule-driven (Hiebert & Lefevre, 1986). "Procedural knowledge enables us to perform and use mathematical procedures, which are repeatable sequences of actions on objects, diagrams, or mathematical symbols" (Battista, 2017, p. 9). For instance, when dividing fractions, procedures such as invert and multiply may be memorized but not understood (Battista, 2017; National Research Council, 2001). The explanations associated with the application of such procedures often involve naming the procedure (e.g., keep, change, flip), reciting the steps followed (e.g., National Council of Teachers of Mathematics [NCTM], 2014), or referring to the authority of the teacher or text (Battista, 2017).

However, not everyone agrees that procedural knowledge is a rote activity, devoid of understanding. Some argue that procedural knowledge can be deep, nuanced, and flexible (Nunes et al., 2015; Star, 2005, 2007), emphasizing the distinction between the blind application of procedures and procedural flexibility (Star, 2005), or fluency (Martin, 2009) — both of which involve "conscious selection, reflection, and sequencing of steps" (Rittle-Johnson & Schneider, 2015, p. 1120). Star makes this distinction clear in his discussion of algorithms (step-by-step procedures) versus heuristics (intentional choices one makes to solve problems).

In mathematics education, a broadly accepted viewpoint is that procedural fluency develops from conceptual understanding (NCTM, 2014). Hiebert and Lefevre (1986) describe conceptual knowledge as understanding "rich in relationships" (p. 3). As discussed previously, a conceptual understanding of fractions means understanding that fractions are a subclass of numbers that all have magnitude (Geller et al., 2017; Siegler, 2016). Unless students are asked to share their reasoning about the magnitude of these rational numbers (e.g., through verbal or written explanations), the same answer may indicate varying conceptions (Ehrlén, 2009; Geller et al., 2017).

An early empirical study of the relative contribution of conceptual knowledge and procedural knowledge to fraction learning reported that conceptual knowledge development preceded, or accounted for, procedural knowledge development (Byrnes & Wasik, 1991). In order to assess the relation between procedural and conceptual types of knowledge on students' understanding of fractions, the researchers conducted two studies across students in Grades 4, 5, and 6. The 19 fourth-grade students had been introduced to the concept of fractions (but not procedures), and the 51 fifth- and sixth-grade students had received both conceptual and computational instruction on fractions. All students completed fraction problems categorized as procedural (computational skill and knowledge of the least common denominator algorithm) or conceptual (cardinality and magnitude knowledge), and researchers analyzed students' performance on one type of task as being either above or below chance with respect to how they performed on the other task type. Byrnes and Wasik (1991) concluded although conceptual knowledge revealed "developmental precedence" over procedural knowledge, it was "a necessary but not sufficient condition for the acquisition of procedures" (p. 785). Results provided early empirical evidence to support the long-standing belief in the importance of teaching conceptual understanding before introducing procedures for solving fraction problems (Cramer et al., 2008; NCTM, 2014).

In a study of fraction magnitude comparison tasks (i.e., identifying the greater of a pair of fractions), Geller et al. (2017) concluded that "conceptual explanations might be better understood as thinking strategies that get recruited in some, but not all, situations" (p. 128). They also reported that students' use of conceptual or non-conceptual explanation strategies when providing the correct answer was considerably influenced by item type (i.e., division of a positive value a by either a whole number, decimal, or variable).

More recently, researchers have reported that procedural and conceptual knowledge is bidirectional or iterative (Rittle-Johnson & Schneider, 2015; Rittle-Johnson et al., 2001), refuting earlier findings supporting the development of conceptual knowledge before procedural knowledge. In another study, Hallett et al. (2010) used cluster analysis to explore possible "learner profiles" related to students' reliance on conceptual and/or procedural knowledge when solving fraction knowledge problems coded as procedural, conceptual, or "ambiguous" (p. 398), as well as a series of "intensive quantities tasks" (e.g., probabilities or ratios) in the form of word problems (p. 397). Using residual scores to isolate one form of knowledge from another, the researchers generated five learner profiles:

- (a) Lower procedurally than predicted,
- (b) Lower conceptually than predicted,

- (c) Higher than predicted conceptual score but lower than predicted procedural score,
- (d) Higher than predicted procedural score but lower than predicted conceptual score, and
- (e) Higher than predicted conceptual score and higher than predicted procedural score.

Important to note is that students in the final cluster, who scored higher than expected across conceptual and procedural knowledge, also obtained significantly greater mean scores than any other cluster on the 40-item fraction knowledge assessment and the 30-item intensive quantities measure.

Using methodology similar to that of Hallett and colleagues (2010), Hecht and Vagi (2012) conducted a two-year longitudinal study of fourth-grade students who solved estimation, computation, and word problems all involving fractions. Using cluster analyses, these researchers found that the largest cluster of students (39% of their sample) was equally strong across procedural and conceptual knowledge with 34% favoring conceptual and 27% favoring procedural. However, when student performance was analyzed on computation problems only, the procedural knowledge cluster outperformed the other two groups of students, and this knowledge of procedures appeared to compensate for lower conceptual knowledge.

As the above studies illustrate, scholars are beginning to coalesce on the belief that conceptual and procedural knowledge, with respect to fraction understanding, is interdependent (Bailey et al., 2015; Hecht & Vagi, 2012; Rittle-Johnson et al., 2001, 2015), but exactly when and why students demonstrate a combination of conceptual and procedural knowledge with fraction tasks or a reliance on one type of knowledge over another is still unclear.

Hypotheses explaining the application of different knowledge types are diverse, and include:

- (a) General development of mathematical knowledge (Byrnes & Wasik, 1991);
- (b) Specificity and/or complexity of task demands (Geller et al., 2017);
- (c) Quality and quantity of the instruction received (Rittle-Johnson et al., 2015); or
- (d) Learning profiles of individual students (Hallett et al., 2010; Hecht & Vagi, 2012).

More than likely, the approach that students take to solving fraction problems manifests differently in different studies due to a combination of these factors. As summarized succinctly by Rittle-Johnson and Schneider (2015), “The strength of the relation varies across studies and over time, but it is clear that the two types of knowledge are often related” (p. 1124).

Varied hypotheses for explaining students’ reliance on conceptual or procedural knowledge with fraction tasks encourage the use of varied methodologies to study this phenomenon. For example, Hallett et al. (2010) created items and coded existing fraction items in an attempt to elicit different types of knowledge, whereas Hecht and Vagi (2012) used the same set of items to track student performance longitudinally, assuming that different levels of instructional exposure to fractions would prompt different approaches.

The present study considers item type and, indirectly, students’ exposure to grade-level curriculum standards to investigate the possible existence of latent classes related to students’ understanding of fraction magnitude. Items were not created or coded as conceptual or procedural a priori, but instead represented fraction magnitude problems that, in consideration of students’ grade-level instructional exposure, could prompt the use of either procedural or conceptual knowledge. The researchers asked students to solve fraction comparison problems and then explain how they determined their answers. Student explanations were subsequently coded.

Student Explanations of Fraction Magnitude Knowledge

Students’ explanations have been considered a critical aspect of the learning and teaching of mathematics (NCTM, 2000) and are reflected in two of NCTM’s process standards (Communication and Reasoning and Proof). Researchers have analyzed student explanations related to their knowledge of fraction magnitude in the areas of mathematics education (e.g., Cramer et al., 2008; Peck & Jencks, 1981), psychology (e.g., Geller et al., 2017), and special education (e.g., Crawford et al., 2019; Hunt et al., 2015), with special education researchers reporting the benefit of supported self-explanations (Fuchs et al., 2017; Schumacher et al., 2018). Explanations provided by students make explicit their reliance on a conceptually – and/or a procedurally – based understanding of fraction magnitude, thus they may be useful when designing and delivering mathematics instruction or intervention (Burns, 2011).

But not all student explanations are grounded in conceptual or procedural understanding. When students’ knowledge of fraction magnitude is underdeveloped, they often provide faulty or unfounded explanations (Crawford et al., 2019). One commonly occurring faulty explanation reflects whole-number thinking (Behr et al., 1984); that is, students think about the size of both the numerator and the denominator independently (e.g., $\frac{3}{5}$ is smaller than $\frac{4}{7}$ because 3 is smaller than 4, and 5 is smaller than 7) or the numerator or denominator in isolation (e.g., 5 is smaller than 7, so $\frac{3}{5}$ is smaller than $\frac{4}{7}$) when comparing fractions.

In their interview study of 88 Grade 6 students, Mitchell and Horne (n.d.) found that whole-number explanations were provided more frequently by students whose accuracy on fraction magnitude comparison items and equivalence items was extremely poor. Whole-number thinking is faulty with respect to fractions but not with respect to whole numbers, and in this way may be categorized as a misconception, defined as an application of a conceptual relation in an inappropriate context (Pines, 1985). Misconceptions always lead to faulty reasoning or explanations, but faulty explanations are not entirely explained by misconceptions (Pines, 1985).

Students may also provide faulty explanations even after acquiring initial fraction understanding. For instance, “gap thinking” (Pearn & Stephens, 2004, p. 434) is revealed when students compare the gaps between the numerator and denominator of two different fractions without considering the size of the relative whole. Sharing that $\frac{3}{4}$ is equivalent to $\frac{7}{8}$ because each fraction is only

Table 2. Student demographics

Characteristic	N	Valid %	M(SD)
Race			
American Indian	9	2.2	
Black/African America	22	5.3	
White	211	50.8	
Asian	15	3.6	
Native Hawaiian/P.I.	2	.5	
Hispanic	147	35.4	
Multiracial	9	2.2	
Total	415	100.0	
Gender			
Male	194	45.3	
Female	234	54.7	
Total	428	100.0	
School Setting			
Public	204	65.2	
Private	109	34.8	
Total	313	100.0	
WRAT-IV Average Percentile Rank			65.35(28.67)
WRAT-IV Average Grade Equiv.			7.7 (3.02)

Note. Valid %: percentage when missing data are excluded from the calculations

one part away from the whole is an example of gap thinking. In order for this line of thinking to be accurate, students need to clarify that $\frac{3}{4}$ is $\frac{1}{4}$ smaller than one whole and $\frac{7}{8}$ is $\frac{1}{8}$ smaller than a whole, and since $\frac{1}{8}$ is smaller than $\frac{1}{4}$, $\frac{7}{8}$ is the larger fraction. Failure to understand fractions as numbers with different magnitudes results in overgeneralization of a rule that can be used successfully only when fractions represent numbers 0 to 1 and contain identical denominators. Again, Karp et al. (2015, p. 212) described this type of overgeneralization as a rule that “expires.”

Students’ explanations, whether conceptual, procedural, faulty, or absent, provide insight into their understanding of number. The present study analyzed types of student explanations for fraction comparison problems among upper-elementary students. Explanations were coded and analyzed for the possible existence of distinct groups or latent classes of fraction magnitude knowledge and the relationship of these classes to overall accuracy scores as well as performance on criterion measures was investigated. Three research questions guided by this work:

RQ1 Do student explanations of their fraction magnitude knowledge reveal the existence of latent classes?

RQ2 What is the number and nature of these latent classes? And,

RQ3 What is the relationship of latent classes to criterion measures of student accuracy?

METHOD

Setting and Participants

Participants were drawn from two prior studies conducted by the authors. In both studies, mathematical explanations were included as an outcome variable. Study One comprised 105 elementary students attending school in Oregon or Texas, spanning four grade levels (Grade 3 [16 students], Grade 4 [13 students], Grade 5 [57 students], and Grade 6 [19 students]). Teachers reported that all students were underperforming and receiving Tier II interventions. Study Two included 418 fifth-grade students representing varied academic ability levels in public and private schools in North Texas.

The students were not randomly selected; however, the sample encompassed all fifth-grade students at the participating schools and was normative. For this study, all fifth-grade scores were extrapolated from prior studies. Data was available for 475 fifth-grade students; however, only participants with valid data for fraction items were considered. Therefore, the convenience sample consisted of 428 fifth-grade students. The sample size of 428 students was deemed sufficient for conducting the latent class analysis given the number of indicators and the expected number of latent classes, as prior simulation research suggests that samples of this size generally allow for stable estimation and reliable class detection (Nylund et al., 2007; Tein et al., 2013). The achievement levels of the students ranged from very low to very high performing according to the results from the Mathematics Computation Subtest of the *Wide Range Achievement Test-4* (WRAT-4) (Wilkinson & Robertson, 2006). More than half of the sample were female (54.7%), and approximately 50.8% identified as White. **Table 2** displays all demographic data.

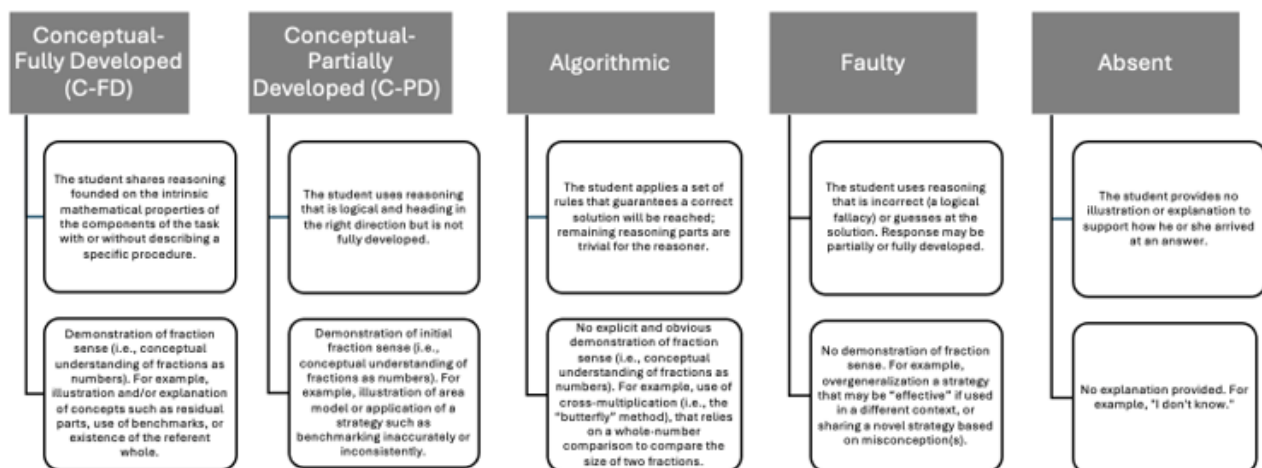
Measures

Math reasoning inventory (MRI)

The MRI uses one-on-one, in-person interviews to formatively assess mathematical reasoning (Burns, 2012). As a psychometrically validated measure of mathematical reasoning for various mathematical domains (Bernbaum-Wilmont, 2012), with the fraction items demonstrating strong internal consistency (Cronbach’s Alpha = .87), students are prompted to answer the

Table 3. MRI (Burns, 2012) fraction magnitude comparison items ranked by most to least difficult

Item	Point-Biserial (MRI)
1. Which is greater $\frac{3}{8}$ or $\frac{5}{6}$?	.44
2. Which is greater $\frac{5}{12}$ or $\frac{5}{8}$?	.46
3. Which is greater $\frac{7}{12}$ or $\frac{2}{5}$?	---
4. Carlos lives $\frac{3}{4}$ of a mile from school and Terrell lives $\frac{6}{8}$ of a mile from school. Which of these is correct? Both boys live the same distance from school. One boy lives farther from school.	.59

**Figure 1.** Types of student explanation with definitions and corresponding fraction magnitude knowledge (Adapted from Quebec Fuentes et al., 2025)

questions *and* then explain their thinking. The interviewer documents the correctness of responses and reasoning strategies used. Students are not provided with pencil nor paper during the assessment.

Questions on the MRI are reported as having point-biserial correlation coefficients in the range .38 to .61 (Bernbaum-Wilmont, 2012). This study reports on four fraction items from the MRI, with point-biserial correlation coefficients in the range .44 to .59 (Bernbaum-Wilmont, 2012). Fraction magnitude comparison items selected from the MRI are displayed in **Table 3**. The fourth item differs from the previous three in that the pair of fractions is equivalent and was presented to students in context. The item was read aloud to students and, therefore, did not require decoding skills.

Wide range achievement test-4

WRAT-4 (Wilkinson & Robertson, 2006) measures academic skills in the areas of reading, mathematics, spelling, and comprehension. The assessment is designed for individuals between 5 and 94 years of age. The math computation subtest (limited to 15 minutes without a calculator) indicates mistakes related to a lack of attention to detail (e.g., numbers out of alignment or incorrect operations) and broad skill levels ranging from basic mathematics operations to introductory algebra. The assessment has two forms (blue and green), the present study used the green form, which reportedly has adequate internal consistency with levels ranging between .92-.98.

Explanation types

Student responses were coded into one of five explanation types:

- Absent,
- Faulty,
- Conceptual-partially developed,
- Algorithmic, and
- Conceptual-fully developed.

These explanation types were developed and validated through previous studies of elementary student populations on both whole-number operation and fraction comparison problems (Crawford et al., 2019). The original framework was theoretically grounded in the extensive body of literature on conceptual knowledge and procedural knowledge, while also demarcating student explanations at a more granular level. More specifically, the researchers categorized student conceptual explanations into "fully developed" (FD) or "partially developed" (PD) in order to distinguish between thorough, and often generalizable, explanations from explanations that reflect some accurate conceptual knowledge but are not fully developed. Students' procedural explanations that relied explicitly on the use of an algorithm were coded as "algorithmic." The remaining two categories were "absent" and "faulty" (recognizing the difference between students who are not able or willing to provide any explanation of their thinking from students who provide faulty explanations that may or may not represent mathematical misconceptions). Each explanation type is defined along with an example in **Figure 1**. Responses were also scored as correct or incorrect.

Procedure

The University Institutional Review Board approved all study procedures. The researchers recruited schools through an email sent to administrators or teachers who had been identified as possible collaborators (through their involvement in previous research studies, professional development efforts, or both). The email provided an overview of the study, participant requirements, anticipated amount of student time required to participate in the study, and an initial letter to parents. Packets containing a letter outlining study details and parental consent forms were provided to schools that were interested in participating. All students who had a consenting parent gave assent prior to the start of data collection.

The first two authors conducted a two-day training for the research staff focused on assessment administration and data collection. This training included practicing the standardized WRAT-4 script to ensure adherence to the test protocol, as well as practicing administration of the MRI. Research staff then conducted practice administrations of the MRI with graduate students, following all data collection procedures exactly: Reading the prompts, recording student responses verbatim, and coding the reasoning strategy within the MRI system. To evaluate each staff member's competence, all practice sessions were audio recorded, and transcribed responses were reviewed by the first two authors to verify 100% accuracy. Refer to the upcoming Coding section for details about training with respect to coding the recorded reasoning strategies.

The MRI was administered in one-on-one sessions about 30 minutes in length. For each session, the researcher read the item to the student, the student reasoned and answered orally without the aid of any tools (e.g., paper, pencil, or calculator), and the researcher documented the student's response into the MRI portal. The researcher followed up with the question:

How did you figure that out?

The researcher then documented the explanation word for word into the MRI website.

WRAT-4 was administered in a group setting. Students were provided with the WRAT-4 green form and instructed to place it face down until given further instruction. Research staff followed the WRAT-4 protocol, by reading the directions verbatim, setting the timer for 15 minutes, and informing students when to turn over the protocol. After time had elapsed, students were instructed to turn over the protocol and place their pencils down. Research staff immediately collected all protocols.

Coding

After extensive training on the scoring of MRI items using the original MRI strategy codes, an inter-reliability analysis of approximately 20% of the items revealed 98% agreement (determined by agreements divided by the sum of agreements and disagreements) across two research teams consisting of two raters each. Research staff also engaged in additional training in the operational definitions and coding of the explanation types (see Measures; [Figure 1](#)). Following training, research staff practiced coding student data from a previous study. Once research staff reached 90% accuracy, they were able to engage in coding data for the current study. Two raters independently coded each item. Inter-rater reliability was determined for all the items with a 91% agreement. To resolve coding disagreements, the original coders first met to discuss and attempted to reconcile differences. If consensus was not reached, a third rater (lead author) -and, when needed, a fourth rater (third author) - adjudicated the response and determined the final code, thereby maintaining reliability and rigor in the coding process (Guest et al., 2012; Lombard et al., 2002).

Data Analysis

The researchers employed Mplus v7.4 (Muthén & Muthén, 1998-2016) to match latent class analysis (LCA) models to the group of four fraction comparison problems. LCA (Lazarsfeld & Henry, 1968) identifies unobserved, or latent, subgroups, or classes, of a population by portraying subgroups of participants who present common patterns pertaining to a set of indicators (in the case of the present study, fraction comparison problems). LCA is beneficial for investigating unmeasured differences (i.e., unobserved heterogeneity) in a population in the field of behavioral science and as a statistical approach for ascertaining educational profiles (Collins & Lanza, 2009; Muthén, 2004). The following includes descriptions of how the analysis addresses the assumptions of model identifiability, adequate class separation, and local independence, consistent with recommended practices for LCA (Collins & Lanza, 2009).

The LCA for the present study addresses the main question:

What is the number and nature of the latent classes?

In other words, LCA serves as a descriptive analytical approach to reveal the number of different latent classes (i.e., heterogeneity of the sample) and define profiles of problem-solving approaches, that is, the nature of the latent classes. As an exploratory analysis with only partial hypotheses about the number and nature of latent classes, the approach extracted latent classes until non-tenable solutions were indicated by model indices.

To identify and extract latent classes, the models were evaluated using three main criteria:

- (a) Whether the optimal log likelihood (LL) value was consistently reproduced,
- (b) The Bayesian Information Criterion (BIC) adjusted for sample size, and
- (c) The statistical significance of the bootstrapped likelihood ratio test (BLRT).

Additionally, researchers checked the entropy values to gauge how accurately the models' classified cases, although this metric did not influence the final decision on the number of classes.

Table 4. Percentages of explanation types used for each item

	Algorithmic	Conceptual FD	Conceptual PD	Faulty	Absent	Missing
Fractions						
Item 1	23.8%	7.7%	26.2%	19.2%	7.0%	16.1%
Item 2	21.5%	10.3%	25.5%	19.2%	9.6%	14.0%
Item 3	19.2%	2.3%	21.0%	28.3%	10.5%	18.7%
Item 4	42.5%	6.5%	15.7%	22.9%	6.5%	5.8%

Table 5. Probabilities for the five-class LCA model

	Replicated LL	BIC	$\Delta -2LL$	BLRT df	p-value	Entropy
2-Class Model	Yes	3936.67	455.03	16	<.01	.781
3-Class Model	Yes	3785.75	199.98	17	<.01	.846
4-Class Model	Yes	3685.35	149.46	17	<.01	.837
5-Class Model	Yes	3654.09	80.31	17	<.01	.831
6-Class Model	Yes	3673.67	29.48	17	.05	.848

To ensure that the global maximum of the likelihood function was identified rather than a local maximum, all LCA models were estimated using multiple sets of starting values. Specifically, each model was specified with 200 random initial starting values and 50 final-stage optimizations, resulting in 50 replications for each model to verify the stability of the log likelihood function. Parameter estimates from models that failed to replicate the maximum LL value were deemed unreliable and were not retained for interpretation.

The BIC was employed as an index of model fit for the LCA analyses. Although there are no universally accepted cutoff thresholds for interpreting BIC values, lower BIC scores indicate a better fit to the data. Accordingly, BIC values are expected to decrease as model complexity increases, such as when additional latent classes are specified. The BLRT was used to compare models with k classes to those with $k-1$ classes; a significant BLRT result suggests that the model with more classes provides a significantly better fit than the more parsimonious alternative. For example, comparing a two-class solution to a one-class solution, a significant BLRT statistic indicates that the two-class model more accurately represents the data than a single-class model, which assumes a homogenous population without distinct subgroups.

When group membership is predicted rather than directly observed, the uncertainty associated with classification can be quantified using the entropy statistic, which indicates how effectively the indicator variables distinguish among latent classes (Asparouhov & Muthén, 2018). Entropy values range from 0 to 1, with higher values signifying clearer separation between classes. Although models with higher entropy are generally preferred due to greater classification precision, entropy alone does not reflect the overall fit of the model to the data. Therefore, in the present study, entropy was not used as a criterion for determining the number of classes.

Finally, local independence is an assumption underlying the validity of LCA results. Local independence assumes that indicators are uncorrelated after accounting for the latent variables. This assumption was tested by checking the magnitude and statistical significance of residual bivariate chi-square tests using the TECH10 output command (Muthén & Muthén, 1998-2016). The final model met the assumption of local independence.

Relation of Latent Classes to External Variables

Following the identification of the optimal number of latent classes and the evaluation of entropy and local independence, the validity of the latent classes was assessed by analyzing the profiles' patterns and examining the associations between latent class membership and criterion variables (i.e., MRI and WRAT-4 scores). To estimate the relationships between latent classes and distal outcomes, the BCH method in Mplus was employed (Bakk & Vermunt, 2015; Clark & Muthén, 2009). This method tests for mean differences across latent classes while accounting for measurement errors in class assignment. Because latent classes are inherently unobserved and thus estimated with uncertainty, the BCH approach was used instead of conventional analyses that treat class membership as known (Clark & Muthén, 2009). For these analyses, the Type I error rate was set at .05.

RESULTS

Table 4 presents the percentages of each explanation type reportedly used by students when solving four fraction magnitude comparison problems. As illustrated, just under 27% of all the item responses represented algorithmic explanations, 6.7% represented conceptual, 22.4% represented faulty, 22.1% represented conceptual-PD, and 8.4% represented absent.

Table 5 summarizes the statistical indices for the LCA models. These indices include checks for consistent replication of the log likelihood value across random starting values (to verify convergence at the global maximum), the BIC, BLRT outcomes, and entropy values for solutions specifying two to six classes. The BLRT was significant at the .01 level for the models with two through five classes but non-significant for the six-class solution, indicating that the five-class model was the most tenable option with the maximum number of distinct classes. The five-class model also produced an entropy value of .83, which is relatively high (greater than .80) (Clark & Muthén, 2009), although no single standard defines an acceptable threshold for entropy.

Table 6. Probabilities for the five-class LCA model

	Class 1 (n = 122)	Class 2 (n = 48)	Class 3 (n = 80)	Class 4 (n = 63)	Class 5 (n = 115)
	Conceptual PD	Conceptual FD	Faulty	Absent	Algorithmic
Item 1					
Algorithmic	.243	.147	.042	.076	.861
Concept.-FD	.059	.853	.000	.000	.039
Concept.-PD	.616	.000	.093	.346	.080
Faulty	.061	.000	.838	.165	.020
Absent	.021	.000	.027	.413	.000
Item 2					
Algorithmic	.191	.095	.010	.073	.796
Concept.-FD	.030	.905	.000	.016	.125
Concept.-PD	.719	.000	.039	.169	.079
Faulty	.060	.000	.951	.076	.000
Absent	.000	.000	.000	.667	.000
Item 3					
Algorithmic	.075	.274	.000	.000	.943
Concept.-FD	.000	.396	.000	.000	.032
Concept.-PD	.625	.176	.000	.144	.000
Faulty	.286	.154	.969	.188	.014
Absent	.014	.000	.031	.668	.012
Item 4					
Algorithmic	.454	.226	.186	.201	.935
Concept.-FD	.012	.583	.019	.000	.026
Concept.-PD	.223	.133	.203	.226	.027
Faulty	.303	.058	.551	.217	.000
Absent	.008	.000	.041	.356	.012

Note. Probabilities over .20 are bold to help quickly identify the most common types of explanation used for each item

Table 7. MRI fraction comparison scores by latent class and pairwise comparison with effect sizes

	Class 1	Class 2	Class 3	Class 4	Class 5
	Conceptual PD	Conceptual FD	Faulty	Absent	Algorithmic
Mean (S.E.)	3.14 (0.09)	3.47 (0.13)	1.81 (0.11)	2.18 (0.14)	3.80 (0.08)
Class 1	–	$\chi^2 = 4.47^*$ $w = .10^a$	$\chi^2 = 87.18^{***}$ $w = .45$	$\chi^2 = 31.11^{***}$ $w = .27$	$\chi^2 = 28.93^{***}$ $w = .26$
Class 2	–	–	$\chi^2 = 95.47^{***}$ $w = .47$	$\chi^2 = 44.61^{***}$ $w = .32$	$\chi^2 = 4.30^*$ $w = .10$
Class 3	–	–	–	$\chi^2 = 4.18^*$ $w = .10$	$\chi^2 = 222.64^{***}$ $w = .72$
Class 4	–	–	–	–	$\chi^2 = 97.60^{***}$ $w = .48$

Note. Each item was scored as correct or incorrect, for a possible total of 4.

*p < .05

***p < .001

^a Cohen's w values indicate effect size for each pairwise comparison: small = 0.10, medium = 0.30, large = 0.50 (Cohen, 1988).

The statistical results of the LCA supported the identification of five distinct profiles, which were then qualitatively analyzed to explore patterns in mathematical explanations across items. **Table 6** lists the probabilities with which an individual would provide a particular type of explanation for each of the four fraction items conditional on their latent class. For example, individuals classified in Latent Class 1 were estimated to have a .616 probability of giving a conceptual-PD explanation for item 1 and a .243 probability of giving an algorithmic explanation. Based on the response patterns shown in **Table 6**, the latent classes were labeled as conceptual-PD, conceptual-FD, faulty, absent, and algorithmic. The conceptual-PD class was the largest, comprising approximately 29% of the sample (n = 122), followed by the algorithmic class at about 27% (n = 115). The faulty class accounted for 18.7% (n = 80), the absent class for 14.7% (n = 63), and the conceptual-FD class for 11.2% (n = 48).

Following the estimation of the five latent classes, their relationships with the total scores on the four MRI fraction comparison items and the WRAT-4 mathematics computation subtest were assessed. **Table 7** lists the mean score (and standard error) for the MRI fraction comparison problems for each of the five latent classes as well as each pairwise comparison between the latent classes. Individuals in the algorithmic latent class obtained the highest mean score on the MRI items (M = 3.80), followed by those in the conceptual-FD class (M = 3.47), conceptual-PD class (M = 3.14), absent class (M = 2.18), and faulty class (M = 1.81). Inferential comparisons indicated that all latent classes differed significantly from one another at the .05 significance level. Specifically, students in the algorithmic class scored significantly higher than those in all other classes; the conceptual-FD latent class scored higher than the conceptual-PD, absent and faulty latent classes; and so forth. Effect sizes for the pairwise chi-square tests, calculated as Cohen's w (Cohen, 1988), ranged from 0.10 to 0.72, indicating small to large magnitudes of association among the latent classes.

Table 8 lists the mean WRAT-4 score (and standard error) for each of the five latent classes as well as each pairwise comparison between the latent classes. The mean WRAT-4 scores showed that students in the algorithmic class performed the highest

Table 8. WRAT scores by latent class and pairwise comparison with effect sizes

	Class 1	Class 2	Class 3	Class 4	Class 5
	Conceptual PD	Conceptual FD	Faulty	Absent	Algorithmic
Mean (S.E.)	109.24 (1.65)	78.11 (10.95)	96.73 (1.74)	97.55 (2.08)	130.75 (5.16)
Class 1	–	$\chi^2 = 8.15^{**}$ $w = .14^a$	$\chi^2 = 28.68^{***}$ $w = .26$	$\chi^2 = 20.52^{***}$ $w = .22$	$\chi^2 = 13.28^{***}$ $w = .18$
Class 2	–	–	$\chi^2 = 2.82$ $w = .08$	$\chi^2 = 3.05$ $w = .08$	$\chi^2 = 20.29^{***}$ $w = .22$
Class 3	–	–	–	$\chi^2 = 0.09$ $w = .01$	$\chi^2 = 38.47^{***}$ $w = .30$
Class 4	–	–	–	–	$\chi^2 = 34.97^{***}$ $w = .29$

* $p < .05$; *** $p < .001$; ^a Cohen's w values indicate effect size for each pairwise comparison: small = 0.10, medium = 0.30, large = 0.50 (Cohen, 1988).

($M = 130.75$), with lower average scores observed for the conceptual-partially developed class ($M = 109.24$), the absent class ($M = 97.55$), the faulty class ($M = 96.73$), and the conceptual-fully developed class ($M = 78.11$). Inferential analyses indicated that the algorithmic class scored significantly higher than all other classes, while the conceptual-PD class scored significantly higher than the conceptual-FD, faulty, and absent classes. Finally, the faulty and absent latent classes did not differ significantly from one another or from the conceptual-FD latent class. Effect sizes for the pairwise chi-square tests, calculated as Cohen's w (Cohen, 1988), ranged from 0.01 to 0.30, indicating small to medium magnitudes of association among the latent classes.

DISCUSSION

The present study built upon existing research by exploring students' developing understandings of fraction magnitude through an analysis of the explanations they provide for fraction comparison problems; specifically, examining the relationship of their explanations with the accuracy of their responses and analyzing scores on criterion measures. The reported findings provide a deeper understanding of students' fraction magnitude knowledge.

When examining latent classes specific to students' explanations of their knowledge of fraction magnitude, a five-class model was the most tenable, conveying the presence of five distinct student profiles. Current theory and previous research findings suggest the possible existence of a combined latent class of procedural (algorithmic) and conceptual knowledge (Bailey et al., 2015; Hallet et al., 2010; Hecht & Vagi, 2012; Rittle-Johnson et al., 2015); yet findings of the present study did not uncover a latent class of this nature. Instead, algorithmic, conceptual-PD, and conceptual-FD emerged as distinct classes as did absent and faulty. These statistical results of the LCA were supported by the finding of significant differences across all five classes on total correct scores obtained on the MRI fraction comparison problems (with the algorithmic class scoring the highest). The significant difference between the algorithmic class and all other classes on the WRAT-4 criterion measure adds additional support, as does the significant difference between the conceptual-PD class and the remaining three classes (including conceptual-FD).

Unexpectedly, no significant difference was found between WRAT-4 scores and conceptual-FD, faulty, and absent latent classes. This latter finding is perplexing and does not lend support to a five-class model. One explanation may be that fully developed conceptual understanding is not necessary in order to complete the type of procedural items on the WRAT-4 computation subtest. In fact, Hecht and Vagi (2012) found that as students moved from fourth grade to fifth grade, a greater percentage of students were in the procedural cluster. The nature of instruction could also impact such results; students who are taught to follow procedures may not be successful in the long run but may outperform students taught conceptually on similar tasks in the short term (Liljedahl, 2020). Another explanation could be that the timed nature of the WRAT-4 computation subtest (15 minutes) did not support a fully developed conceptual approach. Further, students with conceptual understanding may not have developed procedural fluency as measured by the WRAT-4 (Rittle-Johnson & Schnieder, 2015).

Explanations coded as algorithmic accounted for the largest percentage of explanation types (27%), a finding that differs from prior studies reporting that a combined conceptual and procedural cluster was the most prevalent type followed by conceptual, which was larger (Hecht & Vagi, 2012). The reliance on algorithmic explanations may be due, in part, to several reasons that have been acknowledged in prior research, such as varied instructional approaches and/or exposure (Burns et al., 2010; Hecht & Vagi, 2012; Rittle-Johnson et al., 2015), a stall in the learning progression that occurs with developing fraction magnitude understanding (Cramer et al., 2008; Johanning, 2011; NCTM, 2014), or the complexity (or, in the case of the problems solved in this study, lack of complexity) of problem type (Byrnes & Wasik, 1991; Geller et al., 2017).

Another reason for the high occurrence of algorithmic explanations may stem from the fact that when using the MRI protocol, students were prompted to explain "how" they arrived at a particular answer, and this response was coded. Student responses may be qualitatively different when their answers are followed by a "why" inquiry. For example, after providing an incorrect answer, students might respond differently when asked "Why do you think that $\frac{3}{5}$ is greater than $\frac{4}{7}$?" as opposed to "How did you figure that out?"

In the present study, asking students "How?" appeared to generate responses that were more algorithmic than conceptual in nature. Initial analyses of student explanations in response to "Why?" questions in a study of students' written explanations to the same fraction problems (Crawford et al., 2023) are revealing qualitatively different responses than those captured in the current study. Others agree that "Why?" questions may be more effective for generating richer student explanations (Best, 2013).

The explanation type that was least likely to be used was conceptual-FD (6.7%) – another finding that does not align with the findings of Hecht and Vagi (2012). However, combining explanation types coded as reflecting fully developed conceptual understanding with those coded as partially-developed conceptual understanding resulted in approximately the same percentage as responses coded as algorithmic (29 vs. 27%). Arguably, although conceptual understanding of number magnitude is critical for an understanding of more advanced mathematics (Barbieri et al., 2020; Jordan et al., 2013; Siegler, 2016), it may not always be required for fraction comparisons. The verbal explanation requirement in this study was intended to capture students' magnitude knowledge; however, perhaps the nature of the fairly simple comparison problems prompted students to rely on a memorized algorithm (LCD or cross multiplication) that requires knowledge of multiplication facts mastered by most fifth-grade students. When studying student performance on fraction magnitude comparison tasks (i.e., identifying the greater fraction in a pair), Geller et al. (2017) came to a similar conclusion, reporting that conceptual explanations might need not be used in every case and that that students' procedural or conceptual explanations were largely influenced by complexity of item type.

Limitations

Several limitations should be acknowledged in the present study. First, participants were drawn from two states using a convenience sample. Although the students attended public and private schools in both states and were heterogeneous with respect to race and ethnicity, lack of random sampling across a larger population limits generalizability. Therefore, the findings may best generalize to similar school contexts with comparable student demographics.

Second, participating schools varied in terms of their adopted and/or enacted curriculum, and the instruction received by students was not taken under consideration. The lack of investigation into these and other potential moderators on students' explanations of their understanding of fraction magnitude limits interpretation of the results. As one teacher noted during the study, "It would be interesting to give the teachers the same assessment and compare their explanation types with those of their students," highlighting the possible influence of teachers' own knowledge and instructional practices on student reasoning. Future studies should account for instructional context when investigating how students explain fraction magnitude to better understand the conditions under which these profiles emerge and how generalizable they are across different classrooms and curricula.

Third, the use of only four items in determining the existence of latent classes may have failed to capture the full range of fraction magnitude knowledge. Further, the four items were limited to comparing two fractions, and did not include other established methods for measuring student understanding of the magnitude of fractions (e.g., sequencing three or more fractions from smallest to largest) that may have prompted different types of explanations.

CONCLUSION

The results presented here offer implications for teachers as well as for the research community. A primary implication for teachers is the potential presence of latent classes to be considered when planning and providing instruction to individual students. These findings echo that of Burns (2011), who reported positive effects for mathematics interventions designed intentionally with use of data from students' baseline skill levels. Specifically, Burns found that providing an individually aligned mathematics intervention targeting each student's weak area (conceptual or procedural) was more beneficial than the type of intervention (conceptual or procedural) used. For example, a student who demonstrated low conceptual knowledge at baseline benefited more from a procedural intervention whereas a student who demonstrated poor procedural knowledge at baseline benefited more from a conceptual intervention. A necessary first step toward designing such intervention involves collecting the data to inform instruction. Prompting students to explain their answers to fraction comparison problems is one way for teachers to collect this information. As previously mentioned, without prompting students to explain their reasoning about rational number magnitude, teachers cannot determine if identical answers reflect different levels of understanding (Ehrlén, 2009; Geller et al., 2017).

A second implication, worth considering by teachers, may be those algorithmic explanations, the explanation coded the most in the present study and the explanation type most closely correlated with external measures, provide the foundation for later mathematical reasoning, and that "success in mathematics needs to be grounded in well-learned algorithms as well as understanding of the concepts ... The challenge, as always, is balance. 'Mindless algorithms' are powerful tools that allow us to operate a higher level" (Ross, 1998, p. 253). And the challenge "of a mathematics teacher is to help students develop the flexibility to move back and forth between the abstract and the mechanical" (Ross, 1998, p. 253).

Implications for researchers include that the results of the LCA performed in this study categorized algorithmic and conceptual explanations into separate classes supporting the existence of different learner profiles as related to the use of procedural and conceptual approaches to understand fraction magnitude. In this way, findings support previous work on the profiles of learners (Hallett et al., 2010; Hecht & Vagi, 2012). However, our findings did not support a "procedural plus conceptual" learner profile, which, as discussed previously, has been theoretically and empirically reported in the past. Implications of these findings for researchers include the need for consistent use of operational definitions across studies, systematic use of similar items to study fraction magnitude knowledge, and the importance of a deeper understanding of the interplay between development, curriculum, and instruction.

In conclusion, an LCA conducted on student explanations of their fraction magnitude knowledge revealed the existence of five latent classes. The algorithmic class contained the largest percentage of student explanations and also revealed the strongest correlation with criterion measures. A combined algorithmic-conceptual class was not identified.

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