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Exploring the creative potential of mathematical tasks in teacher education

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ABSTRACT

Creativity is a cross-cutting ability that is highly valued in today's society. Therefore, it should play an important role in education in general and in mathematics in particular. This requires teachers to create appropriate learning opportunities that allow creativity to flourish in students, helping them to develop their mathematical potential. In particular, the use of rich and challenging tasks can encourage fluency, flexibility and originality as three of the essential dimensions of creative thinking. Based on these assumptions, we developed a qualitative study with 19 elementary pre-service teachers to identify the dimensions of creativity revealed by these participants when solving challenging tasks, as well as their ability to recognize these dimensions in written tasks' resolutions. Ultimately, we aimed to identify potential tasks that contribute to the development of creativity in future teachers. Data was collected mainly through written productions. Preliminary results suggest that the tasks used have creative potential, with participants demonstrating some of the dimensions of creativity. Flexibility was identified as the most challenging dimension for them to identify.

Keywords: creativity, multiple-solution tasks, problem-solving, problem posing, teacher education

INTRODUCTION

Nowadays, it is not enough to be proficient in computation, memorizing facts and procedures, or solving routine problems. While these skills are important, the abilities to understand and define problems, produce multiple solutions or paths towards a solution, seeking for the most elegant or efficient ones, justify conclusions, and communicate results are even more important. These abilities are not innate; they can be cultivated and nurtured if teachers provide students with appropriate learning opportunities to unlock their creative, innovative and critical potential.

Creativity has been gaining increasing interest in current educational research, making it crucial for teacher educators to find ways to introduce this new component into mathematics classrooms. This means that we must be concerned not only with what students do but also with what teachers do. As Shulman (1986) refers, knowing for oneself is different from knowing how to teach. Therefore, we must provide future teachers with situations and tasks that allow them to confront their mathematical knowledge with that of their students. Specifically, tasks that challenge students to find multiple solutions, primary those related to problem-solving and problem posing, are essential. These tasks the used of require essential mathematical concepts, while fostering the development of fluency, flexibility and originality as dimensions of creativity (e.g., Leikin, 2009; Leikin & Guberman, 2023; Silver, 1997; Vale & Barbosa, 2015, 2023).

Considering creativity an inherent characteristic of mathematical knowledge we assume that it can be effectively encouraged among students at various school levels (Silver, 1997). So, teaching that does not provide opportunities for students to be creative denies them the chance to develop their mathematical skills and to appreciate this subject. Creative thinking is considered one of the essential abilities that all students are expected to master during the learning process (e.g., OECD, 2021). Several organizations (OECD, 2021, 2022) report that creativity positively influences students' interest and academic achievement, leading to better results by engaging them in innovative and meaningful experiences. Additionally, creativity enhances a range of other skills, such as problem-solving skills and career success.

Although this has been a theme in mathematics education for several decades, it is not yet a reality in our classroom. This can happen for different reasons, three of which we can identify. One has to do with a traditional view of mathematics teaching and learning, which is still dominant in mathematics classes, where the focus is on memorization and practice. In this approach, the teacher explains the subject mainly through procedures, rules and algorithms and the students reproduce what they observe, doing repetitive exercises. This context is not conducive to developing students' creative thinking, which requires active learning and the use of multiple-solution tasks. On the other hand, teachers may not be familiar with the concept of creativity, despite its reference in the curriculum, and therefore may not feel comfortable addressing it in their lessons. Also, some teachers, due to personal reasons or their curriculum and/or pedagogical perspective, may prefer not to use creative approaches because they do not see them as necessary. So, it seems important to address this issue during initial and continuing teacher training, understanding future teachers' performance in tasks that contribute to the development of creativity. Furthermore, we believe that creativity implies seeing things in new ways or seeing things with new eyes to analyze old situations. It entails the ability to challenge assumptions, break boundaries, take risks, recognize patterns, imagine, reflect, make new connections and seize opportunities when dealing with a problem. These are broad qualities of creativity, but it is difficult to provide a simple and general definition of this concept. Thus, we will focus our study on creativity in relation to problem-solving tasks.

Based on these ideas we developed an ongoing study, which follows an exploratory qualitative and interpretative approach (e.g., Erickson, 1986; Miles & Huberman, 1994), with pre-service elementary teachers (PSTs) in their training course. In this course, PST had a didactical experience focused on creativity grounded on the exploration of rich and challenging tasks, which we believe contributed to the development of their creative thinking. In particular, we aim to identify the dimensions of creativity in future teachers' work when solve challenging tasks, as well as how they recognize some of those dimensions when analyzing the written productions of students on similar tasks. Additionally, it is important to identify tasks with the potential to be used in all classes to foster mathematical creativity.

THEORETICAL FRAMEWORK

The Mathematics Classroom

In a mathematics classroom setting, learning is highly dependent on the strategies used by the teachers and the tasks they offer. The tasks that teachers select for their classes are fundamental in defining and highlighting their work. The design of these tasks influences the type of activity provided, the responses elicited from students, and the level of discussion produced. This allows teachers to introduce new ideas and skills while challenging students to think differently (e.g., Stein & Smith, 1998). Meaningful discussions and reflections only arise when teachers have a deep knowledge of the subject they teach, which is essential for student learning and enables teachers to follow and support students' reasoning. The use of questioning is a powerful tool to support students as creative thinkers. This is only possible if teachers are themselves comfortable with this kind of work, have creative tasks to offer their students, and possess didactical resources and teaching strategies to explore and discuss these tasks with them. Thus, it is critical that teachers can take advantage of all the potential contained in a task. To do this, they need opportunities to explore and solve these tasks in the same way they will explore them with their own students. In this scope of task exploration, we give special attention to patterning tasks in visual contexts. Such importance comes from the fact that visualization is not merely used for illustration; it is recognized as an important component of reasoning, deeply involved with the conceptual rather than just the perceptual (e.g., Rivera, 2011; Vale & Barbosa, 2023). Often, it is easier to communicate a concept though a visual image, because it is quickly understood and retained longer than a sequence of words. The figurative features of a task can help students overcome some of the constraints in their mathematical knowledge on concepts and procedures, enabling them to successfully solve a problematic situation.

It is generally recognized that teachers' knowledge impacts the mathematics training and students' learning. This means teachers play a fundamental role in the teaching and learning process, not only based on their knowledge but also in the way he/she uses that knowledge and how it works in the classroom. Understanding teachers' knowledge continues to be an emergent issue, with ongoing debate about whether knowledge beyond content knowledge is crucial for effective mathematics teaching (Lannin et al., 2013). It is consensual that it is not only the quantity of mathematics covered in instructional programs that is important, but also the opportunities given to future teachers to understand this knowledge meaningfully. This allows them to unpack and consolidate that knowledge, enabling them to reach all students. So, our teacher education programs are designed to offer experiences that develop both knowledge of mathematics and knowledge of mathematics teaching in PSTs, as these are often considered interconnected (e.g., Lannin et al., 2013; Oliveira & Hannula, 2007; Ponte & Chapman, 2015).

There is common agreement that teachers need to possess sound content knowledge as this affects both what they teach and how they teach it (Ponte & Chapman, 2015). Ma (1999) reinforces this idea by arguing that, to be effective, teachers must understand and explain mathematical concepts, provide connections, and explain the rationale behind mathematical relationships. They need a profound understanding of fundamental mathematics, particularly in the subject they teach. Thus, PSTs need this knowledge, but it does not emerge solely through experiences they bring from k-12 schooling, as these are inadequate for teaching elementary school mathematics (e.g., Lannin et al., 2013; Ma, 1999). Instead, such knowledge must be gained through teacher education programs, professional development and classroom experiences.

Having a strong knowledge alone does not guarantee that we have an efficient teacher (Ponte & Chapman, 2015). However, this knowledge is connected to self-confidence, which impacts teachers' ability to face a class with all the associated challenges. For example, it is challenging to integrate students with varying levels of mathematical knowledge into different contexts that allow them to develop their mathematical ideas, reasoning and problem-solving strategies, as well as to enjoy solving mathematical tasks.

Teacher efficacy is connected to a teacher's level of confidence. According to Burton (2004), a teacher's confidence stems from the convergence of beliefs about oneself and one's efficacy in the mathematics classroom. Research (e.g., Graven, 2004) has shown a positive relation between a teacher's confidence and teaching effectiveness. In other words, the more I improve my mathematical knowledge, the better my teaching practice will be, leading to improved student achievement.

This confidence of the teacher contributes to expertise and appears to be connected to the dimensions of creativity, fluency and flexibility. The more fluent and more flexible a teacher is, the better he/she can foster students' creativity. According to Ponte (2007), this flexibility is very important for teachers during exploratory teaching, promoting active learning in students. In this environment teachers must diversify the tasks proposed to the students that allow them to access mathematical contents and develop mathematical processes such as experimenting, conjecturing, generalizing, communicating, and creating.

The development of creative thinking skills is influenced by several factors, namely individual internal factors and environmental factors. Teachers can act, mainly on environmental factors, as curriculum, school atmosphere, study groups, tasks, instructional strategies (Lucas & Spencer, 2017). Therefore, creativity can be developed if teachers provide the right learning opportunities, playing an important role in developing students' creative thinking skills.

Concerning the dimensions of creativity, as mathematics teacher educators, we emphasize the importance of future teachers having a solid content knowledge in this area. They should not only identify their own creative potential through creative tasks' productions but also develop skills to recognize the creative dimensions in their students' work. Teachers have significant responsibility in the way mathematics is taught and learned, making them pivotal in promoting creative thinking in the classroom (Aiken, 1973). As previously stated, individuals' beliefs about a certain idea influence their actions and behaviors, so teachers' perceptions about creativity in mathematics significantly influence students' activity and engagement in the classroom. Some curricular disciplines are often perceived as offering less opportunities to foster creativity than others, with mathematics frequently included in this group. Bolden et al. (2010) stress the importance of discussing with teachers (pre- and in-service teachers) their beliefs about creativity in mathematics, to understand how these perceptions, influence teaching strategies and classroom practices. Teachers need more than a basic understanding of creativity; they must also understand that the characteristics or attributes of creativity can depend on the subject and context they are dealing with. Therefore, it is essential that, in particular, PST training promotes reflection on this theme.

The Nature of the Tasks–Challenging and With Multiple-Solutions

As previously stated, research (e.g., Doyle, 1988; Stein & Smith, 1998) shows that tasks greatly influence the way students learn. Therefore, it is important to have good or rich mathematical tasks. According to the National Council of Teachers of Mathematics (NCTM, 2014), a task is considered good when it has key characteristics, such as: it introduces fundamental mathematical ideas effectively; poses intellectual challenges for all students; promotes creativity; enables multiple approaches or solutions; and encourages discussion. According to Stein and Smith (1998), a task is a segment of the classroom activity designed to develop specific mathematical ideas and concepts, and its nature shapes the type of learning produced. Mason and Johnston-Wilder (2006) define a mathematical task as what students are asked to do, which could involve a calculation, manipulation of symbols, various representation, or translating word problems. We consider that a task is anything used by the teacher in the teaching and learning process of mathematics to illustrate ideas and engage students in solving a situation (e.g., exercises, problems, investigations, questions, definitions, demonstrations, projects, constructions, games, reports), provoking students' learning.

Challenge is an important variable in the mathematics classroom because students can quickly loose motivation and interest in "routine" classes. Some may even struggle to learn, unless they are appropriately challenged (Holton et al., 2009; Vale & Barbosa, 2023). Mathematics is engaging, useful, and creative, but it only allows for engagement and creativity when students are attracted to and challenged by the tasks with which they are faced (e.g., Barbeau, 2009). Challenge is an idea related to creativity. Barbeau (2009) defines a challenging task as a question that is intentionally posed to attract students to engage students in seeking a solution while simultaneously enhancing their comprehension and knowledge of a given topic. The term 'challenging task' typically describes a task that is engaging and possibly pleasant, but not necessarily easy to solve or accomplish. Such tasks should actively engage learners in developing a diversity of ways of thinking and learning approaches. Challenging tasks often require students to connect mathematical concepts or procedures, considering different representations, perspectives or applications. Such challenges demand flexibility and imagination in response to the situation (Barbeau, 2009). In a mathematics classroom, challenge is important as it prevents students from becoming disengaged and bored very easily with 'routine' classroom tasks. Also, if teachers don't use challenging tasks, it is likely, as a result, to hold back the brightest students (Holton et al., 2009).

Challenge is often connected to problem-solving. In fact, for some problem-solving theorists (e.g., Polya, 1973; Silver, 1997), a problem is a mathematical task that challenges students to solve it when they lack the necessary knowledge of essential procedures or algorithmic tools and when no standard method of solution is evident. Consequently, the solver must engage in thought and analysis of the situation, possibly relating a variety of factors, and create mathematical actions to reach the answer. A problem is considered challenging when it involves significant affective demands, such as curiosity, imagination and creativity, making it interesting and enjoyable, though not necessarily easy to solve (Freiman et al., 2009). Challenge is a key feature that promotes mathematical learning, so it is necessary to develop students' mathematical potential through an appropriate level of mathematical challenge (Leikin & Guberman 2023). Tasks are considered rich or good because they allow students to learn by choosing between different mathematical areas and use various mathematical and non-mathematical skills in an integrated, creative and meaningful way (Vale & Barbosa, 2023). It is the challenge embedded in the task that drives mathematical learning (Leikin & Guberman, 2023). For Becker and Shimada (1997), the term rich task is used when we want to engage students in a complex mathematical activity through that task. These tasks are considered rich because they offer students opportunities to learn by choosing from a wide set of mathematical and non-mathematical skills, integrating them in a creative and purposeful manner. Rich learning tasks not only involve students in the underlying activity, but also encourage them to carry out these actions in a natural, balanced and purposeful way (Flewelling & Higginson, 2003). This contrasts with the more traditional tasks that over-emphasize the use of manipulation and transformation, asking students to follow prescribed steps to reach expected outcomes, and limiting the opportunities to explore alternatives and be creative. Such tasks normally tend to keep students dis-engaged and often feel like routine work.

Tasks should enable students to explore, make mistakes, reflect, and expand into new related areas, allowing them to display their abilities in different ways, such as verbally, geometrically, graphically, algebraically, numerically. Nevertheless, the teacher, as the facilitator and orchestrator of challenges in the classroom, must be mindful of specific considerations. For example, adequate challenges should be used with both mathematically proficient and less advanced students. The same task can be scaffolded and structured differently for different students, proposing challenges at different levels. The main responsibility of teachers is to create rich learning opportunities for students to demonstrate not only what they know but also what they are able to do with that knowledge. Therefore, among the different tasks used in mathematics learning, we privilege problem-solving tasks that have mathematical challenge embedded and can be approached and solved through multiple solution strategies and required mathematical insight to solve them (Leikin & Guberman, 2023; Leikin & Lev, 2007; Leikin et al., 2013; Vale & Barbosa, 2023). These tasks facilitate the development of mathematical knowledge and encourage flexibility and globally creativity in the individual's mathematical thinking (Leikin et al., 2013; Polya, 1973; Vale & Barbosa, 2013, 2015; Vale et al., 2012).

Creativity and Creative Thinking

Creativity and problem-solving

Examining research on the definition of mathematical creativity, we found that there is a lack of consensus, since there are numerous ways to express it. Many authors (e.g., Krutetskii, 1976) relate creativity with giftedness and sometimes use the terms as synonymous. But, within educational contexts, research on creativity has largely been concerned with the creative enhancement of the quality of teaching and learning for all students. The term creativity is usually used to refer to the ability of producing new ideas, approaches or actions. It begins with curiosity, engaging students in exploration and experimentation tasks where they can express their imagination and originality (e.g., Barbeau, 2009). Leikin et al. (2013) view creativity as a mental process that includes the generation of new ideas or concepts, or the result of new relations between existing ideas or concepts. In contrast, Ervynck (1991) associates mathematical creativity with advanced mathematical thinking, defining it as the ability to pose important mathematical questions and discover essential connections among them. Sriraman (2005) considers creativity as the search for all kind of problems that an individual wants to solve, which is in accordance with Beghetto's (2007) perspective, who refers that it is an ability to generate new perspectives, new and meaningful ideas, new questions, and solutions to problems. Despite variations, all definitions agree that creativity requires producing new and valuable products or ideas (e.g., Lubart & Guignard, 2004).

They also recognize that the creative process requires a combination of some cognitive skills or components, such as: fluency, flexibility and originality (e.g., Leikin & Guberman, 2023). According to other authors (e.g., Leikin, 2009; Leikin et al., 2013; OECD, 2022) creativity manifests itself in the creation (individually or in group) of a product that is new and useful. It is this feature of being new that it is at the core of originality and that somehow characterizes creativity in the most common sense. Additionally, creative thinking involves the cognitive processes needed to engage in creative work, which assists students to be imaginative, develop original and effective ideas, think outside the box and solve problems (e.g., OECD, 2021, 2022).

A simple idea of creativity is presented by Woods (1982) as "the ability to think of alternatives". And the word "alternatives" is fundamental. These alternatives could relate strategies, processes, procedures, conjectures, methods, designs, constructions, interpretations, communication, etc. In rich learning tasks, students are often given the opportunity to explore alternatives, which encourages creativity. But for this to happen we need, as Haylock (1997) says, to "overcome fixation", breaking one's established mental sets of knowledge, a crucial factor to foster creativity.

It seems consensual to consider three components/dimensions of creativity: fluency flexibility and originality (e.g., Conway 1999; Guilford, 1987; Leikin, 2009; Leikin & Guberman, 2023; Mann, 2006; Silver, 1997; Torrance, 1987). *Fluency* is the ability to generate a great number of ideas. *Flexibility* is the ability to produce different ideas about the same problem and is evident when students can shift their way of thinking among solutions. *Originality* refers to the ability to create unique, unusual or novel ideas or products. In the context of mathematics classrooms, originality can be demonstrated when students develop an effective solution to a problem that is new and different to them, even if others have already solved it, and creates a novel approach (e.g., Silver, 1997; Vale et al., 2012). In addition to these dimensions, other authors (e.g., Morais, 2011; Torrance, 1987) include *elaboration* as the ability to present an extensive amount of detail used to extend or improve an idea or solution, a skill usually used with mathematicians. These components have been used to identify/evaluate evidences/traits of creativity in students. These components of creative thinking work in harmony with each other, and rarely occur isolated in the thought processes. However, we personally highlight flexibility, since it requires fluency, may involve originality, and represents an important facet of divergent thinking.

Many authors (e.g., Haylock, 1987; Leikin, 2009; Silver, 1997; Torrance, 1987) have noted that mathematical problem-solving and problem posing are closely related to creativity. Haylock (1987) further argued that integrating problem-solving and problem posing tasks, which require students to manipulate information within given problems and solve them, leads to the development of mathematical creativity. This contributes to students' divergent thinking, considered as the ability to explore news approaches, think and discover original and different ideas, adopting different perspectives, and produce numerous ideas (Cropley, 2010; OECD, 2022). Problem posing can be a powerful strategy to enhance problem-solving skills and to have good problem solvers. We advocate for multiple-solution tasks as proposed by Leikin (2009), arguing that these are the tasks that consider different solutions to the same problem, allow the use of different representations and involve different properties of a mathematical concept (Vale et al., 2012). The most successful and proficient problem solvers are those capable of applying a diversity of approaches (Conway, 1999).

As previously mentioned, creativity is closely related to problem-solving, and the process of creating problems has been described using various terms such as inventing, creating, posing, formulating or elaborating. Silver (1997) defines problem posing as either generating (creating) a new problem or reformulating an existing one. Stoyanova (1998) views problem posing as the process through which students, grounded on their mathematical knowledge and experience, construct individual interpretations of concrete situations and formulate them as new mathematical problems. From these definitions, we can understand problem posing as the creation of new problems or the reformulation of a given problem based on knowledge, mathematical experiences, and personal interpretations of concrete situations. Stoyanova (1998) identified three categories of problem posing situations:

(a) free situations, where students pose problems without any restrictions,

(b) semi-structured situations, where students are asked to write problems similar to given problems or to elaborate problems based on specific images and diagrams, and

(c) structured problem posing situations, where students pose problems by reformulating known problems or by changing the conditions or questions of a given situation.

Math problems or tasks can be created or reformulated in various ways, according to the students' level of proficiency and teaching objectives. Brown and Walter (2005) identify two problem posing strategies: the *accepting the given* strategy and the *what-if-not* strategy. The *accepting the given* strategy starts with a static situation, such as an expression, chart, condition, picture, diagram, sentence, calculation, or set of data, upon which students generate questions to create a problem, without changing the given information. In contrast, the *what-if-not strategy* involves extending the task by modifying some of the given information. This approach helps identify the problem, what is known, what is required, and the constraints implied by the problem's answer. On the other hand, Sullivan and Liburn (2002) propose an accessible method for posing open-ended questions to a specific situation, using a three-step process:

(a) *working backwards,* which includes identifying a topic, thinking of a closed question, writing down the answer and inventing a question that includes (or addresses) the answer and

(b) *adapting a standard question*, which includes identifying a topic, thinking of a standard question and adapting it to make a good question (Vale & Barbosa, 2015).

Problem posing has often been overlooked in the mathematics classroom and, particularly, in the study of problem-solving. However, it is fundamental for learning mathematics. Problem posing promotes critical thinking and a deeper comprehension of mathematical concepts by stimulating students to think creatively, elaborating new mathematical problems or scenarios.

Evaluation of creative mathematical thinking

We now approach a crucial and complex point, which is the evaluation of creativity in mathematics. Mathematical creativity is a complex and multifaceted construct, and it has been defined, evaluated and assessed in several ways. Some authors (e.g., Mann, 2006) note that there is neither a single well-accepted definition of mathematical creativity nor a consensus on the best way to evaluate it. The most common approach is to analyze students' creativity through psychometric methods. The traditional model includes operational definitions and a corresponding scoring scheme to evaluate creativity based on the three dimensions (originality, fluency, and flexibility), as initially suggested by Torrance (1987) and by other authors (e.g., Conway, 1999; Silver, 1997; Vale et al., 2012). These dimensions are proposed to analyze both problem-solving and problem posing tasks: fluency is analyzed by the number of correct responses/solutions, obtained by the student to the same task; flexibility is analyzed by the number of different solutions that the student can produce, organized into different categories, indicating a variety of different ideas about the same situation; and originality is analyzed as the statistical infrequency of responses in relation to peer group responses. The scoring scheme suggested by some authors (e.g., Leikin & Lev, 2007) has some limitations that need to be addressed. For instance, a more precise description of indicators for originality, fluency, and flexibility are needed. Additionally, even when using a scoring scheme, it would be necessary to qualitatively analyze students' performance based on their scores.

We have used the ideas of Conway (1999) and Silver (1997) in our work with students during their initial teacher training. Our primary focus is on identifying tasks that have the potential to develop or stimulate creativity in students and then finding ways for these future teachers to effectively implement these tasks in a teaching and learning environment.

To address the problem of fluency, we ask students to solve tasks in different ways. We see the potential in encouraging students to explore and exhibit various resolutions, as it helps to counter the misconception some have that the goal is simply to achieve a correct answer, regardless of whether there is a simpler or more interesting way to approach the task. For flexibility, we constructed an inductive categorization of the proposed resolutions for each task solved, which can be highlighted by some general indicators. The psychometric analysis can help to face the dimensions of creativity, but it does not exhaust its analysis.

Grounded on the previous ideas, we need an exploratory teaching approach where the teacher acts as the orchestrator (Stein & Smith, 1998), engaging students in solving and discussing challenging tasks. The teacher must have a sound knowledge about the mathematics to teach and how to teach it, and be creative, selecting good tasks for the use in mathematics classes. This involves offering students diversified, rich and challenging tasks that allow them to access mathematical content and develop mathematical processes such as to experimenting, conjecturing, generalizing, communicating and creating. By posing questions that stimulate brainstorming, reflection and discussion, teachers provide students opportunities to share ideas and clarify meanings; build coherent arguments about how and why everything works; develop a language to express mathematical ideas; and learn to understand things from the perspective of others. So, this approach fosters an environment that promotes mathematical reasoning and problem-solving with multiple and diverse solution processes, including visual ones, thereby encouraging creativity (e.g., Leikin & Lev, 2007; Leikin et al., 2013; NCTM, 2014; Vale et al., 2018).

RESEARCH DESIGN AND METHODS

Methodological Options

To conduct this research and in accordance with the problem under study, we followed an exploratory qualitative and interpretative approach methodology (e.g., Erickson, 1986; Miles & Huberman, 1994). One of the purposes of our study is to identify tasks with the potential to help teachers develop mathematics creativity in their classrooms. In other words, we aim to find rich and challenging tasks that engage students in their resolution, enabling them to use different mathematical topics and approaches, and identify the type of thinking students use that qualify as creative. Given these assumptions, it is important to identify the components of creativity that future teachers reveal when solve challenging tasks, and to understand how they recognize some of these dimensions when they analyse written productions of the same type they have performed. In particular, in this paper, we chose to study the following questions:

1. What dimensions of creativity can be identified in future teachers' work when they solve challenging tasks?
2. How do future teachers identify themselves dimensions of creativity when analyzing productions of tasks similar to those they have solved?

The Didactical Experience

The context of this study were the classes of a didactics of mathematics subject, involving nineteen PSTs, who are future elementary school teachers (ages 6-12 years old). We developed a didactical experience aimed at discussing the role of creativity in mathematics with the students. In these classes, we adopted an active learning approach as an alternative to more practices more traditional, focusing on fundamental transversal skills such as the 4Cs (critical thinking/problem-solving, creativity, communication and collaboration). The classes included teaching modules on problem-solving and problem-posing, creativity, reasoning, communication, visualization and mathematical connections - internal and external to mathematics (e.g., within mathematics, with other disciplines, and with daily reality). This experience occurred during the module about creativity, which took two lessons of three hours each.

During the classes we proposed a set of challenging tasks that incorporate ideas from problem-solving (e.g., Polya, 1973; Silver, 1997), problem posing (Brown & Walter, 2005; Silver, 1997; Stoyanova, 1998; Sullivan & Liburn, 2002) and multiple-solutions tasks (e.g., Leikin & Guberman, 2023; Vale & Barbosa, 2023). The tasks are mainly non-routine or open-ended in nature, featuring multiple solutions, with at least one insightful solution, preferentially a visual one. The tasks' solutions depend on each student's mathematical ability and general mathematical knowledge, require good organizational skills, and allowing the solver to use multiple strategies and representations. In the problem posing category, we considered semi-structured situations where students are asked to create problems, mainly from a static situation, using the accepting the given strategy (Brown & Walter, 2005) and pose good questions (Sullivan & Liburn, 2002). The tasks used along our study were designed or chosen based on the identification of one of the dimensions of creativity and with the aim of being challenging for all students. Whenever possible, we privileged figurative contexts and patterning tasks because they can act as a facilitator context for mathematical comprehension and can be used with elementary students (e.g., Barbosa, 2013; Vale et al., 2018).

This experience assumed an exploratory dynamic in class where:

(a) problem-solving was the primary work context,

(b) students were asked to analyze and discuss the proposed tasks,

(c) communication was emphasized, particularly through the use of questioning and different representations, and

(d) a set of challenging tasks was used, which elicited multiple (re)solutions within problem-solving and posing.

In a first moment, the students were asked to solve different tasks that allowed us to identify, in addition to the creative potential of the tasks, dimensions of creativity in their written productions. In a **second moment,** the students were confronted with resolutions of the tasks similar to those they had solved, to identify those dimensions themselves. They solved the tasks and then exchanged the solutions with each other for analysis (**Figure 1**).

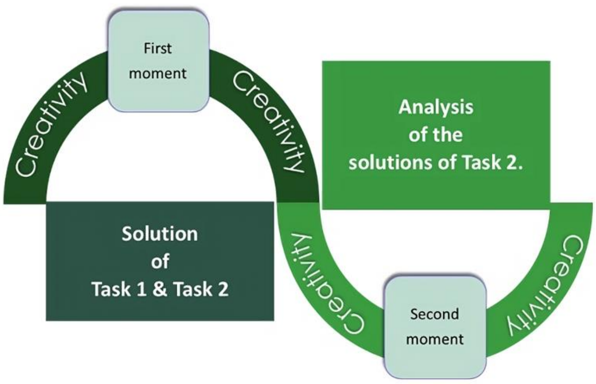


Figure 1. Main moments of the didactical experience (Source: Authors' own elaboration)

From the different tasks used with these future teachers, we selected two for this paper, which are presented in **Figure 2**.

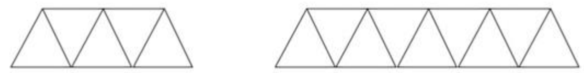
Task 1. The beams	Consider a sequence of figures made with toothpicks, from which the 3rd and 5th term are presented.  Fig. 3 Fig. 5 1. Draw the following missing figures: Fig. 1, Fig. 2, and Fig. 4, so that all of them form a growing pattern sequence. 2. Discover different ways to calculate the number of toothpicks in the nth term of the sequence.
Task 2. The wire	"We have a flexible wire 36 cm long." Formulate questions we might ask based on this situation in order to create a problem. Present as many proposals as you can.

Figure 2. Example of two tasks (Source: Authors' own elaboration)

As previously mentioned, in the first moment, the students individually solved task 1 and task 2 in as many different ways as they could. In the second moment, they analyzed in pairs some of their peers' solutions to task 2, focusing on the dimensions of creativity. **Figure 3** illustrates the instructions given to the students.

Analyze the resolutions according to the three dimensions of creativity: fluency, flexibility and originality, related to the following situation:
"We have a flexible wire 36 cm long." Formulate questions we might ask based on this situation in order to create a problem. Present as many proposals as you can.

Figure 3. Script for the analysis of the resolutions of task 2 (Source: Authors' own elaboration)

Data Collection and Data Analysis

Data was collected in a holistic, descriptive, and interpretative mode (e.g., Erickson, 1986; Miles & Huberman, 1994) during the classes with these PST. The collected data included mainly naturalistic classroom observations, recorded through free-flowing notes and written productions of the tasks.

To analyze the data, we used a qualitative and inductive approach, recurring to content analysis relying on the written productions (Miles & Huberman, 1994). We used an inductive categorization to organize the systematically and facilitate the interpretation emerged from the solutions provided by the PST. This approach was supported by all the data collected, as well as by the literature review conducted and the research questions formulated. Our analysis followed the fundamental ideas of some authors (e.g., Conway, 1999; Leikin, 2009; Silver, 1997; Vale & Barbosa, 2013, 2015; Vale et al., 2012) regarding creativity, using the same dimensions in problem posing and problem-solving: fluency, flexibility and originality. No scores were assigned to the students concerning these dimensions. Instead, we conducted an overall analysis of the presented work, considering the frequency of the most common as well as the most original responses.

RESULTS AND DISCUSSION

The 1st Moment

In the description of the first moment, we chose to present only the solution to task 1. During this moment, PSTs solved both tasks. Task 1, demanded a linear figural pattern generalization, intending that students searched for a pattern in a growing figurative sequence, identifying the visual arrangement that changes in a predictable way. They were asked to write numerical and algebraic expressions that translated the ways of seeing sequence, enabling them to generalize to distant terms. All PSTs successfully solved the first question, identifying the sequence of beams, according to the request that the sequence would have to be a growth pattern, as shown in **Figure 4**.

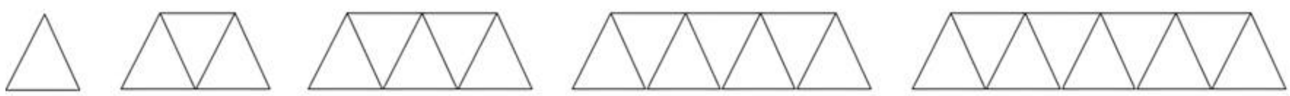


Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

Figure 4. The growing pattern sequence of beams (Source: Authors' own elaboration)

The answer to the second question, where discovering a formation law using algebraic notation and involving distant generalization becomes more complex, requires students to have some visual skills to see the configuration in different manners and relate it to prior knowledge. There are several methods of counting the toothpicks in the sequence of triangles, and each method can be translated into a numerical expression that represents the students' thinking and seeing perspective, leading them to establish a general rule. In the case of figural patterning tasks, visual perception can be an aid to reach generalization. Visual cues may be recognized in different ways. Seeing the figure as a configuration of several sub-configurations can result in a *constructive generalization*, where one sees disjunct groups of visual elements forming the initial figure; or a *deconstructive generalization*, where one identifies overlaps with elements counted more than once, requiring a subsequent subtraction (e.g., Barbosa, 2013; Rivera, 2011).

PST presented different representations, ranging from less to more formal, to solve this task. This variety of answers was encouraged by the task statement, which asked for as many proposals as possible. We will be concentrating particularly on the different manners of looking at the pattern to reach far generalization, as we believe that this is the most important feature of solving tasks of this kind, where creativity can flourish. The students arrived at a general rule through diagrams, drawings and tables, using functional reasoning that helped them achieve far generalization. This, in particular, is a linear figural pattern generalization task.

After completing the sequence in order to establish a growing pattern, all the students presented more than way to calculate the number of toothpicks for the n^{th} term of the sequence. **Figure 5** illustrates a synthesis of the most common productions used to discover the n^{th} term of the sequence arrangement, along with the corresponding expressions that translate into each way of "seeing" the pattern generalization.

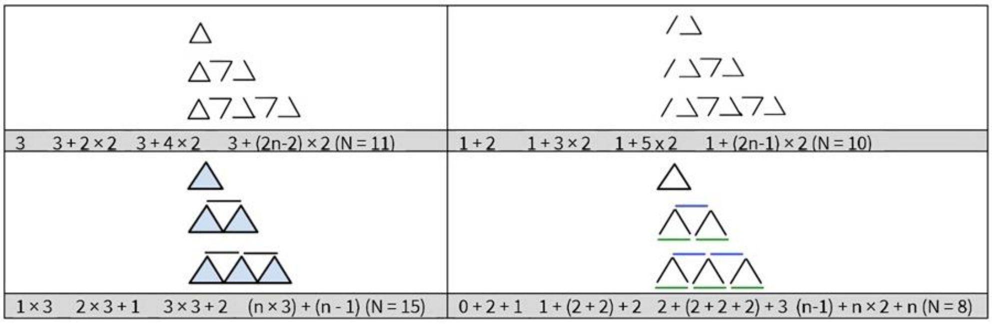


Figure 5. Synthesis of the students' most common responses (Source: Authors' own elaboration)

The examples in **Figure 6** were presented by two students who showed the same way of "seeing" the formation of the pattern, however they used different processes to generalize. In the first case, they recognized a sequence of even numbers from which 1 is subtracted. In the second case, they did not immediately identify the general expression of even numbers, so they use a similar approach and noted that in each case there is a multiple of 2.

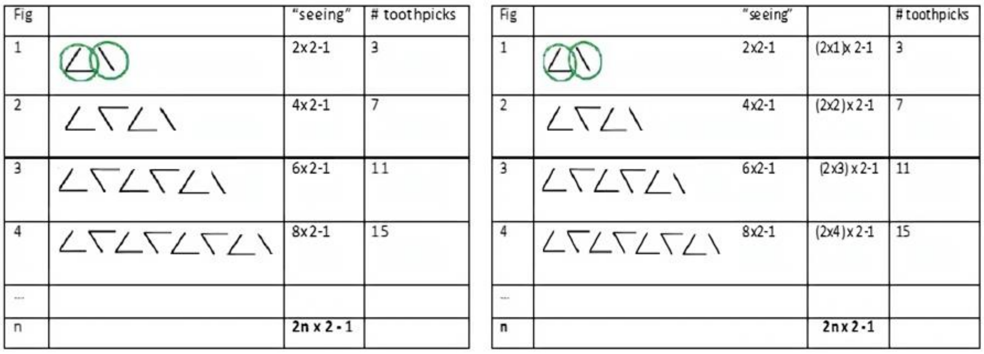


Figure 6. One way of "seeing" and two ways to identify the general expression (Source: Authors' own elaboration)

The most original responses, presented by two different students, are illustrated in **Figure 7**. The first one is considered as the most elegant solution, expressed through a simple expression. We considered the second one to be more complex because it involved deconstructive reasoning (Rivera, 2011).

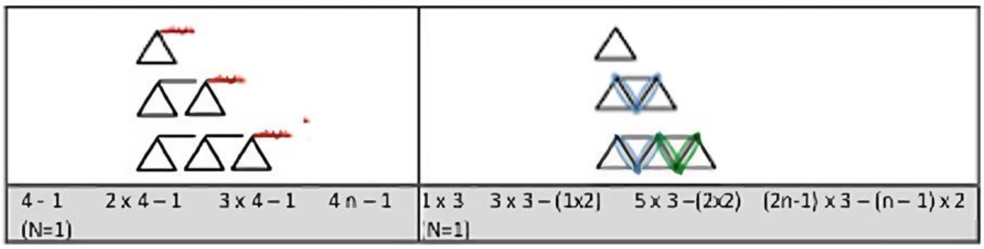


Figure 7. Summary of students' most original responses (Source: Authors' own elaboration)

Only one student used a numerical approach without any relation to the characteristics of the sequence of figures, relying solely on numerical manipulation, as seen in **Figure 8**.

# figure	1	2	3	4	...	n
# toothpicks	3 = 2x1+1	7=2x3+1	11=2x5+1	15=2x7+1	...	2x(2n-1)+1

Figure 8. The numerical approach used by a student (Source: Authors' own elaboration)

We can notice that most of the students showed fluency and flexibility. With regard to fluency, almost all the students (85%) provided more than one correct solution. Flexibility was visible in the different approaches applied to discover the nth term, according to the different categories we analyzed based on the categorization that emerged from the students' solutions to each task (**Figure 9**). However, none of the students confirmed the equivalence of the expressions obtained.

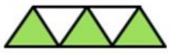
Indicators	Constructive		Deconstructive
Decomposition by sides	Fixing the 1st side, adding two sides		
	Fixing the 1st & 2nd sides, adding two sides		
	Decomposition in three parts		
Decomposition by triangles and sides	Fixing the 1st triangle, adding two sides		
	Fixing triangles, adding one side		
Decomposition by triangles			

Figure 9. Categories of students' responses (Source: Authors' own elaboration)

The 2nd Moment

In the description of the second moment, we present also the solutions to task 2 and its analysis.

The second moment was quite different from the previous one. PST were provided with the resolutions of task 2, previously solved by their peers who produced the highest number of responses. However, this task posed significant challenges for PST by two reasons, due to its nature as a problem posing task, and the fact that they mainly created one problem. To facilitate collective discussion, we ensured all participants had access to the same resolutions. Students encountered significant difficulties in analyzing the work through the dimensions of creativity. Some of the resolutions to task 2 can be found in **Figure 10**.

- How many wires of 3 cm fit in 36 cm?
- Cut the wire into five strands of different lengths. Then put them in increasing order.
- Maria has a flexible 36 cm wire to dry clothes. Knowing that each clothespin is at a distance of 6 cm from other clothespin, is it possible to place the wire?
- Sofia has a flexible 36 cm long wire. On the 1st day she used half. On the 2nd day she used a third of the wire that was left. How many meters of wire were left over?
- Find out how many rectangles of different areas can you build with a wire of 36 cm long?
- Does the wire have enough length to make a loop around a cylindrical pot with 15 cm of radius?
- Knowing that the perimeter of a rectangle is 36 cm and one side has twice the length of the other side. How long is each side?

...

Figure 10. Some resolutions of task 2 presented for analysis (Source: Authors' own elaboration)

After reading the task, students began by reading the different proposals presented, focusing mainly on determining if they made sense and had solutions. Some proposals were simple questions, while others were problems. **Figure 10** shows some examples of those responses.

They began by eliminating some proposals because they noticed that some were incorrect, such as "Joan intends to decorate two boxes with two wires of 36 cm length. One of the boxes is square and one is rectangular. What are the dimensions of the boxes?" Others were eliminated because they beyond the scope of the educational level they will be teaching, such as "What is the side of the larger hexagon you can build with a wire 36 cm long?" Next, they conducted a second reading where they attempted to create categories of analysis. This categorization was challenging and also complicated for the future teachers.

The main problem they faced was deciding on the criteria to use. Should it be based on the contents? The problem-solving strategies? The difficulties encountered? The type of problems posed? Consequently, during group discussion, they acknowledged some misunderstandings and misinterpretations of the problem statements and categorization. As a result, different students proposed different categorizations for the same resolutions, allowing them to analyze flexibility (**Figure 11**).

-Calculations -One step problems -Mathematical facts -Trial and error problems	-Perimeters -Rational numbers -Calculations -Areas -Geometric figures -Direct proportionality -Use of money	-Problems of one or more steps -Mathematical facts -Calculations	-Missing data -Calculations -Mathematical facts -One or more steps problems

Figure 11. Some categorizations of the responses to task 2 (Source: Authors' own elaboration)

Even when some categorizations included similar items, the associated problems were not the same. These difficulties were connected to flexibility, which is the most challenging dimension to identify because it involves divergent thinking and content knowledge. Concerning fluency, based on our model, all agreed that the students demonstrated fluency. However, regarding originality they did not reach a consensus because, despite the infrequency of responses, they did not even consider them "original." In conclusion, all these future teachers acknowledged that task 2 has creative potential because it stimulates many resolutions. But they also expressed that it is difficult to analyze the students' productions effectively.

CONCLUDING REMARKS

We believe that over the last decade, creativity has become a crucial skill to be fostered in the educational process, in line with the changes and needs of todays' society. (e.g., Leikin, 2009; Leikin et al., 2013; OECD, 2021, 2022). For us, creativity represents a field that we are beginning to explore in the teaching and learning process of mathematics, involving educators, teachers, future teachers and students. Many researchers have been attempting to define creativity without reaching a consensual definition. However, as Morais (2011) claims, rather than focusing on "what is creativity", it seems to be more useful to be consider "what does creativity require", because, when we talk about requirements, we necessarily talk about something operational in practice. Several authors (e.g., Leikin, 2009; Silver, 1997) suggest that problem posing is a part of problem-solving, and both are interconnected with creativity, mainly because of the challenge involved and the potential multiple processes of resolution or multiple solutions. Thus, these two abilities have to be developed in parallel, encouraging students to create their own problems (Polya, 1973; Silver, 1997) based on given situations or experiences, while also solving problems through an increasing variety of strategies. Both endeavours require divergent thinking and other components of creativity.

Preliminary results, mainly based on the productions of future teachers', suggest that PSTs reveal some dimensions of creativity, with flexibility being the most difficult dimension to identify by future teachers. This study aimed to complement some of the work we have been developing in initial teacher training, focusing on identifying tasks with potential to foster creativity based on the productions of future teachers. The tasks fall into two main categories, problem-solving and problem posing, from which task 1 and task 2 are examples. The PST's work on the proposed tasks indicates the presence of certain dimensions of creativity, suggesting that the tasks have the potential to develop these dimensions in students. Among all dimensions or components of creativity, we are especially concerned with fluency and flexibility, considering that these two dimensions are an essential component of teacher expertise. At this stage of the study, our results indicate that we mostly observed motivated students, looking for many and different resolutions. What greatly contributed to the evaluation of the three dimensions of creativity was not only the fact that the tasks had multiple resolutions but also the statement that emphasized the need to find the greatest number of answers (e.g., Vale & Barbosa, 2023; Vale et al., 2018). Tasks focused on patterning in visual contexts (e.g., task 1) were those that conducted to more resolutions, providing students with the opportunity to be more creative. This observation seems to be related to the nature of the tasks themselves, but also with previous work conducted with these students on patterns (e.g., Barbosa, 2013; Vale & Barbosa, 2013, 2015, 2023; Vale et al., 2018; Vale et al., 2012).

The analysis of the written resolutions of task 2, of which we presented a brief description, represents another aspect of our work that is still in an exploratory stage. We are studying how future teachers analyze creativity in written productions. Despite being aware of our intention to identify characteristics of the three dimensions of creativity, students showed a clear difficulty with this analysis. A possible explanation can be attributed to their lack of mathematical and didactical knowledge, and/or lack of confidence mathematics abilities. In our previous work, involving productions related to problem posing tasks, students already revealed less creativity compared to problem-solving tasks (e.g., Barbosa, 2013; Vale & Barbosa, 2013; Vale et al., 2012). Another possible difficulty may have to do with their unfamiliarity with this type of work. This is a task that is not very common for students, who are more familiar with looking at resolutions in the traditional sense, in other words, to analyze the formulation of a problem is quite different from analysing its resolution. Further reflection is necessary regarding this approach.

We all accept that the teacher plays an important role in the teaching and learning process. What teachers think, know, and do in the classroom with their students significantly impacts their teaching practice. The teaching and learning approach must provide students with opportunities to "think outside the box", but this will only be possible if teachers are convinced that creativity is "teachable" and know how to achieve that. It is essential to look for rich and challenging tasks that enhance mathematical creativity, but we also need to identify appropriate strategies to implement in the classroom so that teachers become more confident and effective in their instruction. As Ponte and Chapman (2015) suggest, we must provide teachers (and futures teachers) with opportunities to understand, appreciate, and embrace the complexity of their practice as a basis for an ongoing inquiry.

As teacher educators, we strongly believe that PSTs should become creative thinkers and recognize that they should foster the same work with their own students. Both flexibility and originality promote divergent thinking, a relevant component of mathematical thinking. Furthermore, we believe that teacher training programs should provide an understanding of the nature of mathematics and its teaching. This means that teachers must experience diverse teaching and learning experiences similar to those they are expected to use with their own students (Ponte & Chapman, 2015; Vale & Barbosa, 2015; 2023; Vale et al., 2012).

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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