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Exploring professional learning opportunities of teachers who teach mathematics during post-lesson discussions in lesson study

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ARTICLE INFO	ABSTRACT
	This study analyses the interactions in post-lesson discussions in lesson studies of two groups of teachers who teach mathematics in elementary schools, as well as the factors that influence their development, aiming to understand how these interactions foster opportunities for professional learning. The qualitative and interpretative investigation was based on content analysis of transcripts from two post-lesson discussions. The results show a predominance of cumulative talk and occasional occurrences of exploratory talk, in which opportunities for learning were observed related to knowledge of teaching practice and knowledge of students and their learning processes. These opportunities are produced under specific discursive and interactional conditions, strongly influenced by the facilitator's actions and contextual factors such as the participants' professional culture. The findings indicate that professional learning opportunities require intentional facilitation, characterized by a guiding role that involves the deliberate use of specific and probing questions, references to lesson planning and classroom evidence, and other timely interventions that support exploratory interactions among teachers. **Keywords:** lesson study, teachers who teach mathematics, post-lesson discussion, professional learning, interthinking

INTRODUCTION

Lesson study is a professional development process that originated in Japan, where it has been practiced for more than a century and is deeply embedded in the professional culture of teachers, particularly those who teach mathematics (Stigler & Hiebert, 1999). It offers meaningful opportunities such as making sense of educational ideas within the context of practice, shifting perspectives on teaching and learning, adopting the students' point of view, and benefiting from collaborative support among colleagues (Takahashi et al., 2006). In this professional development process, teachers engage in guided discussions about teaching, moving through the stages of study, planning, enacting (and observing) a lesson, and post-lesson discussion, always focusing on students' learning. Furthermore, teachers themselves lead professional and curricular development work that enables systematic dissemination and improvement of teaching practices (Takahashi et al., 2006).

The post-lesson discussion stage plays a significant role in creating professional learning opportunities within the lesson study process, as it is during this phase that teachers collaboratively reflect on students' learning in the lesson that has been taught and observed. In a post-lesson discussion, teachers draw on data collected during the lesson and work collectively to understand what occurred, identify factors that influenced students' learning, and propose improvements for future teaching practices.

Although this phase of reflection is one of the core foundations of lesson study in promoting professional learning, recent research (e.g., Bae et al., 2016; Brosnan, 2014; Kager et al., 2022; Karlsen & Helgevold, 2019; Lee & Tan, 2020; Myers, 2012) has shown that, in practice, productive post-lesson discussions are not always achieved. Several factors may contribute to this, such as issues with data collection during the lesson or participant interactions that do not foster an in-depth analysis of students' learning.

Interactions, in fact, play a crucial role in learning within a discussion group. According to Littleton and Mercer (2013), when participants engage in dialogue around a shared goal, they can co-construct knowledge through interthinking, knowledge that could hardly be achieved individually. The authors point out that different types of interactions can either promote or constrain interthinking. Thus, studying interactions becomes a relevant approach to investigating how discussions among teachers unfold in the context of lesson study (Clivaz et al., 2023a, 2023b), as it may yield insights that help identify both the potential and the

limitations of professional learning opportunities in this process. Moreover, the literature has emphasized that the quality of these interactions depends not only on teachers' participation but also on the facilitator's role, whose mediation can either foster or restrict the collective construction of meaning (Kager et al., 2022).

In this context, we analyse the interactions produced during post-lesson discussions involving two groups of teachers who teach mathematics from two schools in southern Brazil, as well as the factors that influence their development. The aim is to understand how these interactions foster professional learning opportunities, thus contributing to the reflection on how to strengthen teacher professional development through lesson study.

Teachers' Professional Knowledge and Professional Learning Opportunities

Research on teachers' professional knowledge expanded significantly following the work of Shulman (1986, 1987). This author initially identified three categories of knowledge: Content knowledge, curricular knowledge, and what he termed pedagogical content knowledge. Later, he added other categories related to general pedagogy, learners, educational contexts, and also to the aims, purposes, and values of education.

Grounded in this work, in the case of teachers who teach mathematics, Ponte (2012) proposed a model of didactical knowledge, which is strongly oriented towards professional practice. This model comprises four dimensions:

1) Knowledge of mathematics for teaching,

2) Knowledge of students and their learning processes,

3) Knowledge of the curriculum and its management, and finally,

4) Knowledge of teaching practice.

The latter is considered as the core of teachers' professional knowledge, encompassing lesson planning, the design and adaptation of mathematical tasks, and the creation of a learning environment that fosters mathematical communication. The dimension of teaching practice brings together the other dimensions, indicating that they act jointly within the context of professional practice.

Teachers' didactical knowledge is rooted in professional practice. According to Ponte (2012), although it benefits from contributions arising from teacher education processes, this knowledge is essentially constructed through experience and validated by its effectiveness in solving concrete teaching problems. Thus, teachers' collaborative and reflective work around practical situations is essential for building new knowledge and deepening what they already know.

The concept of professional learning opportunity presented by Ribeiro and Ponte (2019) aligns with the nature of this model. These authors define such opportunities as "collective moments in which practicing teachers work and discuss mathematical and didactical situations in order to amplify their professional knowledge for teaching" (Ribeiro & Ponte, 2019, p. 50). More specifically, Darling-Hammond and McLaughlin (2011) indicate that these opportunities engage teachers in concrete tasks of teaching, assessment, observation, and reflection on practice. In their view, a professional learning opportunity is grounded "in the inquiry, reflection, and experimentation conducted by the participants themselves" (p. 82), within a collaborative process that is centred on and derived from teachers' work with their students, supported by modelling, guidance, and collective problem solving in practice.

Therefore, a professional learning opportunity is an intentional situation in which teachers, working collaboratively, may investigate, reflect upon, test, and refine their pedagogical practice. Although such opportunities can be promoted, they do not in themselves guarantee that a group of teachers will develop or deepen their didactical knowledge. This process depends largely on the quality of the interactions established within the group, which may be fostered by collective moments of discussion, as considered through the concept of interthinking.

Interthinking

Interthinking is a conceptual framework that considers verbal interactions as opportunities for collective learning (Wegerif & Mercer, 1997). From this perspective, dialogue is understood as a form of thinking together: "Mainly by using spoken language, people are able to think creatively and productively together. We call this process 'interthinking' to emphasize that people do not use talk only to interact, they interthink" (Littleton & Mercer, 2013, p. 1). By combining individual understandings to solve collective problems, members of a group have the opportunity to overcome challenges, share knowledge, and reason together (Littleton & Mercer, 2013).

According to Littleton and Mercer (2013), when a person talks about an experience shared with others in a discussion, this can stimulate the thinking of the other participants. They, in turn, draw on relevant knowledge to make sense of what they have heard and contribute new ideas, in a continuous and collective process of meaning construction. This dynamic constitutes a way of creating knowledge. Language is used to build shared meaning from experience, allowing two or more people to develop an understanding that none of them could reach alone. Thus, individual and collective thinking complement one another, as two or more people provide a broader source of experiences, knowledge, and skills, thereby contributing to group learning.

To understand how these interactions manifest in practice, they can be classified into three types of discourse: disputational, cumulative, and exploratory (Fisher, 1993; Littleton & Mercer, 2013; Mercer, 1995; Wegerif & Mercer, 1997):

• Disputational talk is characterized by disagreement among participants and individual decision-making, without significant use of constructive arguments. The environment is competitive rather than cooperative (Littleton & Mercer, 2013).

- Cumulative talk occurs when participants seek to agree on the purpose of the discussion but without adopting a critical stance toward the subject under consideration. Ideas are accumulated through repetition and confirmation (Mercer, 1995; Wegerif & Mercer, 1997) or through small, superficial increments (Fisher, 1993). Although information is shared, this type of discourse is limited because it does not lead to critically grounded knowledge (Wegerif & Mercer, 1997).

- Exploratory talk involves the joint construction of ideas and critical discussion, with reasoned challenges or agreements. Group members offer justifications or alternatives (Mercer, 1995; Wegerif & Mercer, 1997), provide relevant information, ask questions, and give reasons for their statements (Littleton & Mercer, 2013). Knowledge and reasoning are made visible to all participants, making this the most effective type of interaction for group learning (Littleton & Mercer, 2013; Wegerif & Mercer, 1997).

Among these three types of interaction, exploratory talk stands out. Such interactions are a central tool for identifying and expanding teachers' professional learning opportunities, since interthinking is not limited to a simple exchange of ideas but depends on the creation of conditions that foster exploratory dialogue. This requires ensuring equitable participation, open sharing of information, and reasoned justification of opinions. By adopting a critical and questioning stance toward students' learning, participants in the discussion can not only better understand students' learning processes and needs but also collaboratively construct more grounded and effective pedagogical alternatives.

Lesson Study and Post-Lesson Discussions

Lesson study typically begins with the formulation of an aim for students' learning which leads to a study phase, during which the group identifies students' difficulties, analyses curricular documents, solves tasks, anticipates obstacles, and defines objectives. In the planning phase, the lesson task for the research lesson is developed, anticipating students' reasoning, strategies, representations, and difficulties and observation is prepared. The research lesson is taught by one of the teachers and observed by the others, who collect data on interactions, difficulties, problem-solving strategies, reasoning, and mathematical communication. Finally, during the post-lesson discussion, the group analyses the collected data, evaluates the positive and negative aspects of the lesson, reflects on students' learning, and proposes improvements for future lessons (Lewis, 2016).

According to Ponte et al. (2014), lesson study provides teachers with opportunities to deepen their knowledge of mathematical content, curriculum, types of tasks and their impact on students' learning, lesson organization models, and communication strategies, particularly encouraging an exploratory approach. Although the process may seem straightforward, it creates conditions for teachers to engage in inquiry and the development of their own practice, reflecting on students' learning and on pedagogical practices specific to the content (Ni Shuilleabhain & Clivaz, 2017). Murata (2011) highlights that this process places the teacher at the centre of professional activity, fostering understanding of students' learning based on classroom experiences and dialogue with colleagues.

The post-lesson discussion is the moment of synthesis and critical analysis where theory and practice are more deeply articulated. In this stage, what was anticipated during the study and planning phases is compared with what actually occurred in the research lesson. Typically, teachers seek to understand students' learning and how their teaching influenced it, based on observations and collected data. This phase constitutes a collective opportunity for reflection on practice, analysing results related to lesson design, materials used, and implementation strategies (Gutierez, 2015; Ono et al., 2013). It allows the lesson study process to be revisited, taking into account aspects related to teaching, content, and students' ways of thinking, thereby promoting teacher professional development (Richit et al., 2021). The post-lesson discussion is a fundamental moment in the lesson study process, as it integrates collected evidence, critical analysis of practices, and the identification of paths for improving teaching and students' learning in mathematics.

However, carrying out productive post-lesson discussions in order to promote teacher learning is far from straightforward. Several studies highlight challenges in developing post-lesson discussions (Bae et al., 2016; Brosnan, 2014; Kager et al., 2022; Karlsen & Helgevold, 2019; Lee & Tan, 2020; Myers, 2012). Brosnan (2014) and Karlsen and Helgevold (2019) note that the quality of discussions is often compromised by the lack of qualified data on students' learning. Moreover, interactions can limit learning opportunities when the group normalizes inadequate practices, celebrates success without understanding the reasons behind students' learning, or gets sidetracked into parallel discussions, as well as when there is a lack of commitment to group objectives and the facilitator's actions do not promote deeper exploration of the topics under discussion (Karlsen & Helgevold, 2019). Similarly, Wood (2017) warns that teachers may simply share and reconfirm practices considered positive, without effectively understanding how students' learning occurs. To overcome these challenges, Kager et al. (2022) emphasize the importance of a clear discussion structure, the facilitator's active work to maintain focus and prevent premature conclusions, and participants' ability to reflect critically.

The potential of the post-lesson discussion to promote professional learning for teachers depends on multiple factors. Thus, although it is part of a broader process, the post-lesson discussion represents a point of great potential for teacher professional development. When well conducted, it goes beyond mere information sharing and becomes a space for collaborative inquiry, in which teachers deepen their understanding of the effects of their pedagogical choices and collectively construct alternatives to improve students' learning.

The Facilitator's Role in Collaborative Learning in Lesson Study

As highlighted by Ponte et al. (2025), the facilitator's role is fundamental in creating collaborative learning environments within lesson study. These authors indicate that facilitators should adopt an active stance in developing clear and stimulating communication among participants, adapting experiences to teachers' needs and promoting open discussions about challenges

that emerge from the classroom. In this sense, facilitators are responsible for enabling the sharing of perceptions, providing formative feedback, and encouraging shared responsibilities and collective decision-making.

Clivaz et al. (2023a) also consider the facilitator's role as central to developing interactions that promote professional learning within a group of teachers. Beyond encouraging the sharing of experiences, these authors highlight the importance of questioning, challenging, and confronting ideas from different participants, to create opportunities for deepening meaning and developing shared knowledge (Clivaz et al., 2023b). In this way, they emphasize the intellectual role of the facilitator in stimulating critical thinking.

Kager et al. (2022) add an organizational dimension, related to how the discussion is conducted. Specifically analysing post-lesson discussions, these authors argue that facilitators should continuously support the group in establishing interaction norms, managing available time, maintaining focus and scope of the discussion, and preventing premature conclusions that do not take into account available data and knowledge. This operational dimension complements the critical dimension, ensuring that interactions occur under conditions that foster meaningful learning.

Similarly, Littleton and Mercer (2013) observe that exploratory talk tends to occur more frequently when participants explicitly recognize a set of "rules" that favour the discussion of ideas, namely:

1) Free expression of ideas and opinions aimed at supporting the group's discussion purpose;

2) Recognition of the importance of all participants, regardless of relative status;

3) Sharing all potentially useful information without assuming that others already possess it;

4) Providing justifications whenever possible, creating a basis for group analysis; and

5) Questioning other members to share or add ideas and information, requesting further clarification when necessary.

In this context, the facilitator's role, beyond promoting an open space for identifying problems and critical argumentation, is also to make these necessary conditions explicit and to manage them for the proper functioning of the discussion.

Research Questions

In this article, the interactions produced during post-lesson discussion sessions with two groups of teachers who teach mathematics are analysed, as well as the factors that influence their development. The aim is to understand how these interactions foster professional learning opportunities. Based on the theoretical and empirical framework, three research questions are defined:

RQ1 How are the interactions in post-lesson discussions characterized in relation to types of talk?

RQ2 How do the facilitator and other contextual factors influence the development of interactions?

RQ3 What professional learning opportunities emerge from the interactions in post-lesson discussions, and how are they produced?

Research Methodology

This is a qualitative study with an interpretative, exploratory, and analytical approach, which allows researchers to position themselves and engage with their impacts and transformative potential (Creswell, 2014), in this case regarding classroom practices and teacher and student learning. Considering the study's objective, this methodology is appropriate, as it values the process over the product, which, in this context, relates to the collective construction of knowledge.

Two post-lesson discussions from two lesson studies conducted in southern Brazil, in two elementary schools, one private and one public, between June and August 2024, were analysed. The lesson studies were initiated as part of a professional development course for teachers who teach mathematics, offered by a public university in the region. Two groups were organized: One for early grades (1st to 4th grade) and another for upper grades (5th to 9th grade).

Both post-lesson discussions were facilitated by a university teacher (the second author of this article), who also acted as the facilitator during the previous phases of the lesson study. The facilitator holds a degree in Mathematics Teaching, a PhD in Education with a focus on teacher education, and has completed postdoctoral research in Didactics of Mathematics. Although the facilitator had limited experience in leading lesson studies, she had extensive experience in initial teacher education at the university level and has facilitated several other in-service teacher professional development activities. The post-lesson discussions were supported by a facilitation guide based on the usual lesson study structure. The teacher who taught the lesson first shared her perceptions, followed by observers' comments on students' learning. The group then discussed specific aspects such as students' difficulties, solving strategies, and mathematical communication. Throughout the process, the facilitator adopted an orienting and questioning role, using the facilitation guide flexibly.

Lesson Study 1 (LS 1) involved a group of five teachers identified as Sara (1st grade), Eliane (1st grade), Luana (2nd grade), Carla (1st grade teacher assistant), and Marisa (1st–4th grade teacher assistant), in addition to Researcher B (A master's student in mathematics education, whose research focused on lesson study as part of her master's dissertation). Carla and Marisa did not participate in the post-lesson discussion session. All teachers worked in the same private school and had never participated in lesson studies. The professional development course comprised eight sessions of approximately two hours each. The research lesson was conducted in the 2nd grade (7-year-old students) by Luana, on the topic "natural numbers," aligned with the Brazilian National Common Core Curriculum. The lesson aim was "to compare and order natural numbers up to the hundreds, recognizing place value."

Lesson Study 2 (LS 2), conducted in a public school, involved three teachers from different schools: Elisa (5th grade), Laura (6th–9th grade, who missed the post-lesson discussion session), and Pedro (6th–9th grade). Researcher B, other master's student in

mathematics education (MEM), and an undergraduate research assistant (URA) also participated. This professional development course comprised eight sessions of approximately three hours each. The research lesson topic was defined based on students' difficulties related to the place value of the decimal number system. Teachers selected a 5th-grade class for the lesson, with the aim "to identify the place value of digits by solving addition and multiplication operations."

In both lesson studies, teachers collaborated during the study and planning phases. Due to scheduling conflicts, they could not observe the research lessons in person. To minimize this limitation, the lessons were video-recorded, and the facilitator selected clips for collaborative analysis during post-lesson discussions. In LS 1, the post-lesson discussion occurred two days after the research lesson, while in LS 2, it took place the following day. Both discussions were audio-recorded and transcribed for analysis.

Content analysis (Bardin, 1997) was conducted based on the two post-lesson discussion transcripts. Transcripts were segmented into episodes through careful reading, with each change in the topic of the discussion marking the start of a new episode. Different units of analysis were used according to each research question. Initially, data were examined with respect to the first research question through deductive analysis, guided by the three types of talk proposed by Littleton and Mercer (2013). For this research question, the unit of analysis was the episode. Episodes that did not address teaching or students' learning were classified as "other" and excluded from subsequent stages of analysis.

Episodes were categorized as cumulative, exploratory, or disputational talk based on explicit criteria derived from the definitions presented in the theoretical framework (Littleton & Mercer, 2013). Cumulative talk was identified when participants shared observations or evaluations without questioning, justification, or critical engagement. Exploratory talk was identified when participants jointly examined ideas through questioning, justification, consideration of alternative interpretations, and reference to classroom evidence. Disputational talk was identified when interactions were characterized by disagreement among participants and individual decision-making without constructive argumentation.

Two researchers (the first and second author of this article) independently analysed and coded each episode according to the types of talk. Categorization results were then compared, and inter-rater reliability was calculated using Cohen's kappa, which indicated substantial agreement between the two coders (κ = 0.79). Disagreements were subsequently discussed by the two coders and the third author, until full consensus was reached on all episodes.

The analysis for the second research question, concerning the role of the facilitator, was conducted using both deductive and inductive approaches. In this case, the unit of analysis was the facilitator's turns of talk within each episode. The episodes were analysed collaboratively by the authors, aiming to identify the actions that emerged in each facilitator intervention. Data were organized by type of talk, and the facilitator's actions within each type of interaction were examined for patterns. This analysis considered actions identified in the theoretical framework, including both intellectual and organizational dimensions, while remaining attentive to other potential facilitator actions. An overall inductive analysis was also conducted, taking a broader view of the formative process, participants, and contextual conditions to identify additional factors influencing the development of interactions.

For the third research question, concerning professional learning opportunities, the unit of analysis was again the episode. Both deductive and inductive approaches were applied, drawing respectively on the didactical knowledge model (Ponte, 2012) and the concept of professional learning opportunities (Darling-Hammond & McLaughlin, 2011; Ribeiro & Ponte, 2019).

Different strategies were adopted to enhance the trustworthiness of the findings. Independent coding by two researchers, followed by collaborative discussion and consensus-building, contributed to analytical rigor. In addition, two of the three authors involved in the analysis had no involvement in the lesson studies and did not know the participating teachers, which helped reduce potential bias related to familiarity with the context or participants.

The study adhered to ethical research standards, ensuring participants' rights (Brasil, 2016) and preserving anonymity. The project was approved by the university research ethics committee responsible for the professional development course, and all participating teachers signed an informed consent form.

RESULTS

The results are presented below, following the three research questions.

Types of Talk in Post-Lesson Discussions

The first research question concerns the characterization of the post-lesson discussions in terms of the types of discourse. **Table 1** presents the results of the categorization of the two analysed sessions, including cumulative and exploratory discourse, as well as the category "other." There were no disputational episodes.

As shown in **Table 1**, the results are very similar across the two discussions, in which cumulative talk predominates. In both sessions, there were also more episodes in which other aspects were discussed, compared to episodes characterized by exploratory talk about teaching and student learning, which together account for only 11 episodes across both sessions (approximately 17% of the total). Episodes classified as "Other" were not considered in the analysis of the results, as they primarily concerned organizational or logistical aspects and, therefore, did not contribute to address the research questions. Examples of episodes featuring cumulative and exploratory talk are presented below.

Table 1. Types of talk in post-lesson discussions

	Post-lesson discussion 1	Post-lesson discussion 2	Total
Cumulative talk	20	17	37
Exploratory talk	5	6	11
Other	9	7	16
Total	34	30	64

Source: Prepared by the authors (2025)

Episode 1 (LS 1)

In this episode, the facilitator revisits the lesson aim and asks whether it was achieved based on the task and the way the lesson was conducted. Luana, the teacher who taught the lesson, responds, along with other teachers in the group:

Luana   I believe so.

Sara   And when I asked, they recognized it.

Eliane   It was well explored.

Initially, Luana affirms that the objective was met but does not provide any evidence to support her conclusion. Sara and Eliane agree but also do not mobilize relevant knowledge or evidence. The dialogue concludes between Luana and the facilitator:

Luana   There were 19 numbers. Several numbers.

Facilitator Yes. No, it was only this excerpt that we focused on [referring to the video clip]. Each one had…

Luana   Each group presented and changed positions three times. Each wrote.

Thus, although all agree that the lesson aim was achieved, the only evidence cited refers to the number of digits correctly identified by students. Other aspects, such as students' reasoning or mathematical communication, are not discussed, which could help assess whether all, or at least most, students understood the topic. Nor are there analysis of how the task and the way the lesson was conducted contributed to this result. Therefore, this episode is characterized as cumulative talk.

Episode 2 (LS 1)

The episode begins with the facilitator asking about the communication that took place among students during the lesson, both between them and with the teacher. The teachers respond, referring to evidence of students who were able to clearly express their reasoning (Eliane and Sara), as well as aspects related to the students' age and language used during the lesson (Sara):

Eliane   A lot, right? Look at that student working with decimals.

Luana   I think so.

Sara   And even when they were listening to their peers. Given their age, their way of speaking, when they were exchanging ideas, others were also taking part. There was a real exchange among them.

Eliane   The way they were composing and sharing ideas, their communication was really good too.

Then, there is a slight shift in focus, but the dialogue is resumed by Eliane and concluded by the facilitator:

Eliane   But the communication among them, I thought it was really good.

Facilitator Yes, they weren't arguing.

Eliane   I was going to say that too, it's quite a respectful class.

Facilitator They weren't trying to impose themselves. Some were a bit slower, and others patiently explained, very bright students.

Thus, although there were no disagreements in this dialogue and the group strongly agreed that mathematical communication had been successful during the lesson, the teachers supported their conclusion with both specific and general evidence about students' work, constructing a critical discussion grounded in relevant information. This process characterizes the episode as exploratory talk. However, the exploration could have gone further toward a deeper understanding of the aspects that supported students' communication, such as the nature of the task and how the lesson was conducted.

Episode 3 (LS 2)

This episode, which occurred during the second post-lesson discussion, arose from the identification of students' difficulty in working collaboratively while solving the task. The facilitator suggested that organizing the students' independent work in pairs

could be more productive than working in larger groups. The teachers then identified the students the facilitator was referring to, and she continued:

Facilitator   They seemed to start helping each other a bit, because at the beginning, I think they were kind of like… each one was solving it on their own.

Next, the episode concludes with an intervention by Elisa (the teacher who taught the lesson), in which she commented that her classes have been characterized by individualism, followed by a brief exchange with Researcher B:

Elisa   They have this thing, which I hadn't seen so strongly in other classes, that they're already competitive at this age. But they have this thing, like… "I have to be the first to finish, I have to be the best, I can't make mistakes." They really have that. I don't know if it's an age-related thing, because normally all classes have it, but in this group, it's much stronger […].

Researcher B   Like individualism?

Elisa   Yes, they're very much like that. […] I notice there's a lack of camaraderie in this class. […]. It's a behavioural issue, you know? They're very resistant to what we say to them […].

Thus, there is clear agreement between the facilitator and the teacher who taught the lesson regarding the students' difficulty in working collaboratively. However, the suggestion offered by the facilitator is ultimately set aside. The teacher attributes this difficulty mainly to the students' personal characteristics, and the group does not return to the proposed alternative nor discuss other possibilities that might lead to a didactic solution to the identified problem. Therefore, this episode is characterized as cumulative talk.

Episode 4 (LS 2)

The facilitator begins the episode by asking the group about the time allocated to students during independent work, suggesting that they took longer than originally planned. Elisa then mentions that the class had a short snack break, although they were quick to return. After a brief exchange on that, the group begins to discuss the students' independent work more directly. Elisa suggests that the students' difficulty in working collaboratively may have caused the delay in completing the task, and there seems to be general agreement on this:

Elisa   I think the time we gave them to complete the task wasn't really the problem. The issue was that it took them a while to realize they could have helped each other more.

Facilitator   Yes, and there was a moment when Elisa said, "Okay, let's move on, now you need to write down your answers, that's the next part of the lesson. Let's make those records and then we'll go to the discussions."

URA   I think it took a while to get going, right? After that, it flowed much better.

Elisa, however, considers that the amount of time given to students was appropriate, since in every group there were students who managed to complete the entire task, and the facilitator also agrees with her:

Elisa   Yes, because in every group there was someone who finished all the questions. Just as there were others who didn't finish everything. One didn't do any at all, didn't solve any.

Facilitator   The one from [Student J], right?

Elisa   Yes, and there were those who did all the questions.

Facilitator   Several managed to do so.

Next, Pedro briefly shifts the focus of the discussion, but following another intervention by the facilitator, the group returns to the original topic, emphasizing that the beginning of the task required a lot of time. The discussion progresses toward a conclusion, with contributions from both the facilitator and Elisa, noting that the work developed more smoothly once the groups began to interact better, also encouraged by the teacher's guidance.

Facilitator   I think it was like URA said, it took them a while to realize that… Well, they didn't rush exactly, but at first, they started off on their own.

Elisa   On their own.

Pedro   Just with her guidance.

Facilitator   Then, when they noticed what their peers were doing…

Elisa	That's when I started insisting with them: "You can share. One did it this way, talk to the others. Help each other."

At this point, the undergraduate research assistant (URA) intervenes, pointing out that this is precisely the idea behind the teaching approach used in the lesson, the exploratory teaching approach, which Elisa corroborates:

URA	I think that's part of it. It was good that you let them think on their own at first before stepping in. That's the goal, before you go in saying, "You have to do this", or start giving directions, the idea is to let them explore for a bit, and then, if it doesn't work out, you intervene.
Elisa	Yes, exactly. You have to see what they can do on their own. You can't just give them everything ready…

Thus, in this episode, the group identifies a problem related to the amount of time students spent on independent work, which was longer than initially planned. The group agrees that the beginning of the task took more time due to the students' difficulty in interacting, which could have led them to conclude that this form of organization did not favour learning. However, the group recognizes and argues that once the students managed to work more collaboratively, encouraged by the teacher, the completion of the task was successful. Furthermore, the group's perspective on the identified problem shifts when Elisa argues that the time was, in fact, appropriate, as it allowed a significant number of students to complete the task.

The episode concludes with a significant intervention by the undergraduate research assistant, who revisits the teaching approach used to emphasize that the way the independent work was organized and conducted proved to be appropriate. Thus, this episode is marked by well-founded argumentation among the group members, who drew on both evidence from the lesson and knowledge of the exploratory teaching approach to reach a shared understanding, characterizing it as exploratory talk. However, the group could have further deepened the discussion by examining, for instance, how the nature of the task might have influenced the identified issue.

Facilitator Actions Influencing the Development of Interactions

In exploratory episodes, the facilitator's actions were primarily to guide the discussion and question the teachers about a specific topic, in addition to revisiting aspects of lesson planning or lesson evidence and suggesting an idea or providing arguments on the topic (**Table 2**).

Table 2. Facilitator actions in exploratory episodes

Action	Number of episodes (LS 1)	Number of episodes (LS 2)
Guiding toward a specific topic	5	6
Asking a specific question	5	4
Revisiting lesson planning or classroom evidence	5	4
Offering a suggestion or argument	2	2
Asking a general or yes/no question	0	1
Making general comments or statements	0	1
Other interventions	0	3

Source: Prepared by the authors (2025)

All exploratory episodes began with an intervention from the facilitator, who, in almost all cases, sought to guide the discussion toward a specific topic or issue, usually through specific questions. This type of questioning also occurred during the development of the episodes. The category "other interventions" includes less frequent actions, such as identifying a difficulty, attempting to maintain the focus of the discussion or prevent the group from reaching a premature conclusion, and establishing interaction norms.

The episodes presented in the previous subsection illustrate some of these actions. In Episode 2, for instance, the discussion was initially guided toward the students' mathematical communication through a specific question, followed by the facilitator's argument on this aspect in her final intervention. In Episode 4, several actions were evident from the beginning, including guiding and questioning about the time allocated for students' independent work, referring to the lesson plan and classroom evidence (for example, by describing the teacher's actions), as well as arguing and helping to maintain the focus of the discussion.

Regarding the cumulative episodes, the results are presented in **Table 3**.

Table 3. Facilitator actions in cumulative episodes

Action	Number of episodes (LS 1)	Number of episodes (LS 2)
Guiding toward a specific topic	10	11
Asking a specific question	3	6
Revisiting lesson planning or classroom evidence	10	11
Offering a suggestion or argument	3	4
Asking a general or yes/no question	7	2
Making general comments or statements	5	1
Other interventions	1	4

Source: Prepared by the authors (2025)

It can be observed in **Table 3** that, of the 37 cumulative episodes, only 21 were initiated through the facilitator's guidance. Similarly, the facilitator referred to the lesson plan or classroom evidence in 21 of the 37 episodes. The questions asked were evenly distributed between those focused on specific aspects and those of a more general nature. There were 12 cumulative

episodes (six in each session) that began with an intervention from another participant, and in six of these, the facilitator made no relevant intervention.

Episode 3, presented in the previous subsection, illustrates that in a cumulative interaction the facilitator guided the group to discuss students' collaborative work, offering a suggestion, while the classroom teacher normalized the identified issue by attributing it to students' characteristics. The facilitator did not take any action that could have promoted a deeper discussion of the problem or of Elisa's intervention, and the episode ended at that point.

In another cumulative episode, presented below (Episode 5), the facilitator guided the discussion toward the teaching resource used in the lesson, through a question:

Facilitator	One thing that's interesting about the children, and I was thinking about this, is that there weren't many teaching resources, but what a difference it made to hand them the numbers, right?
Luana	Yes, they liked it.
Facilitator	It made quite a difference.
Luana	Yes.
	[…]

It can be seen that the teacher simply agrees with the idea, stating that the students liked the material. The dialogue continues with interventions from other participants, who also express agreement. It is worth to note that the facilitator's action consisted of a question that could be answered with "yes" or "no," not promoting a discussion on how the teaching material helped students learn mathematics.

Clear differences were observed in the facilitator's actions across exploratory and cumulative episodes. In exploratory episodes, the facilitator more frequently posed specific questions and revisited lesson planning intentions or classroom evidence to challenge participants' interpretations and prompt further elaboration. These actions often invited participants to explain, justify, or reconsider their ideas. In contrast, cumulative episodes were more often prompted by general questions, including yes/no questions, as well as comments or general statements aimed at confirming or acknowledging participants' contributions, allowing ideas to be accumulated without deeper examination. A further relevant difference is that in several cumulative episodes the facilitator did not intervene, whereas in all exploratory episodes the facilitator played an active role.

Contextual Factors that Influenced the Development of Interactions

Some contextual factors appear to have contributed to the interactions among participants. In Group 2, certain factors seem to have had a positive influence. One of them was that Elisa consistently demonstrated openness to critique, inviting other participants to identify problems and offer suggestions. Another positive factor was the participation of the undergraduate research assistant and two master's students, who contributed questions, observations, and arguments that were often relevant to the group's discussion.

There were also conditions that may have limited the quality of the interactions in the discussions analysed. First, both groups were participating in a lesson study for the first time, which may have meant that, initially, they lacked a clear understanding of their active role in this type of professional learning process. Second, due to scheduling and logistical constraints, only the facilitator and the university participants were able to observe the lessons. Although excerpts from the video recordings of the lessons were reviewed during the post-lesson discussions, opportunities for direct observation and data collection were limited.

Finally, a factor that seems particularly significant relates to the participants' professional culture. In this regard, the episodes analysed show that teachers rarely disagreed with each other's ideas, tending to maintain a climate of harmony, even in exploratory episodes. Moreover, Elisa was naturally more active in the Lesson Study 2 discussion, yet she often sought to normalize the problems by attributing them to individual or collective student difficulties, as observed in Episode 3. This tendency limited the group's ability to develop a deeper understanding of the situation.

Opportunities for Professional Learning in Post-lesson Discussions

From the inductive analysis of the transcripts of the two post-lesson sessions, we identified several situations with potential to promote professional learning among the participants. We found opportunities related to teachers' knowledge of students and their learning processes but mainly associated with knowledge of classroom practice.

Episode 2 (LS 1), discussed earlier, begins with the facilitator's intervention, in which she questions the group about the mathematical communication that occurred during the lesson:

Facilitator	[…] How was the communication among the students? And between the teacher and the students? Was there any negotiation of meanings? Do you think they negotiated? Were they able to construct some mathematical meanings?

As previously analysed, this intervention generated an exploratory episode in which the teachers discussed and argued, drawing on evidence, about the communication that took place. This situation represents an opportunity to deepen knowledge of teaching practice, as it enabled participants to analyse concrete teaching and learning situations and to reflect collaboratively

on what occurred during the lesson. Although the discussion did not advance to a deeper level, the group had the opportunity to explore how the task and the instructional strategy promoted mathematical communication and to consider implications for future teaching practice.

In another episode (LS 2), which occurred at the beginning of the session, while participants were still sharing their observations of the lesson, Elisa raised a discussion about a challenge she often faces in promoting collaborative work among students:

> Elisa　　　Another thing I found quite difficult, and this is already a characteristic of the class, is that they don't know how to work in groups. I've tried several times to have them work together, and yesterday, because there were other people in the classroom, they were quiet. Otherwise, for them, working in groups is synonymous with chaos.

This episode provided a meaningful opportunity for the group to discuss alternatives to the identified problem and potentially deepen their pedagogical content knowledge, either regarding students and their learning processes or aspects of teaching practice knowledge. However, the episode did not progress beyond the sharing of observations or ideas, thus constituting a cumulative discourse with limited impact on teachers' professional learning.

In the same session (LS 2), later on, the facilitator created a professional learning opportunity through questions about the students and their work:

> Facilitator　　　So, do you think they had difficulties or found it easy? What could we have done differently in the planning? Could it be something in the task, or in how the lesson was conducted […]?

In this situation, the teachers could have discussed aspects related to knowledge of students and their learning processes, by identifying and exploring students' difficulties and considering how these could be addressed through the teacher's actions. Moreover, the situation offered potential to deepen knowledge of teaching practice, by discussing the design and adaptation of tasks, elements of the instructional strategy used, and aspects of lesson planning, comparing what was planned with what actually occurred in the classroom. Following this intervention, the group revisited and discussed the difficulty previously identified by Elisa regarding students' group work, reflecting on a possible adaptation of the task to promote more collaborative work among students.

Next, another situation that represents a professional learning opportunity is presented. In Episode 1 (LS 1), the facilitator directed the discussion to the final phase of the lesson, when the teacher sought to systematize the mathematical concepts studied, which initiated a dialogue between her and Luana (the teacher who taught the lesson):

> Facilitator　　　Regarding the mathematical knowledge, I think Luana managed to bring it up and revisit it. She was able to systematize it, and during the discussion, you also called on them [to participate] …
>
> Luana　　　Yes, and the whole time I tried to make them communicate, right? "What number is this? What is the value of the number?"
>
> Facilitator　　　That also makes a huge difference, being prepared to make those interventions.

In this episode, the group had the opportunity to discuss, based on classroom observation, aspects related to teaching strategies and the teacher's actions to foster a communicative environment with the students. Thus, this represents a moment with potential to deepen knowledge of teaching practice. However, the group did not take advantage of this opportunity, as the subsequent interventions consisted merely of sharing observations, without advancing toward a more in-depth reflection on the topic.

DISCUSSION

The results of this study show that interactions during the post-lesson discussions were characterized mainly by cumulative discourse and, to a lesser extent, by exploratory discourse, with no instances of disputational discourse. The predominance of cumulative discourse shows that, although participants openly shared ideas and observations, they tended to limit themselves to confirming one another's perceptions, without often advancing toward a collective process of knowledge construction. When participants engage in exploratory discourse, they are more likely to learn collaboratively through the grounding and justification of ideas (Littleton & Mercer, 2013). In this study, the exploratory moments focused on issues relevant to the teaching and learning of mathematics, but they occurred only sporadically. Our analysis, based on the concept of interthinking within a lesson study context, yielded results consistent with other research that has highlighted difficulties in developing post-lesson discussions that are productive for teacher learning (Bae et al., 2016; Karlsen & Helgevold, 2019; Wood, 2017).

Regarding the facilitator's role, the findings indicate that her actions were decisive for the quality of the interactions. In all exploratory episodes, the facilitator played an active role, steering the discussion, posing specific questions, and revisiting evidence or aspects of the lesson planning. However, the facilitator's actions did not always lead to exploratory discourse. In cumulative episodes, her interventions were largely general in nature, and in some cases, she made no significant contribution. It appears, therefore, that the facilitator's use of specific, focused questioning is the most promising approach. These results reaffirm

the importance of the facilitator's role in lesson study (Kager et al., 2022; Ponte et al., 2025), particularly regarding its intellectual dimension (Clivaz et al., 2023a, 2023b), in supporting the group to build shared knowledge and critically reflect on teaching practices.

Beyond the facilitator's actions, other contextual factors also proved significant for the quality of interactions, including the participants' professional culture. There was a noticeable lack of critical stance (Kager et al., 2022), an essential element for promoting productive interactions. The absence of disagreement in exploratory episodes suggests a concern with maintaining a harmonious atmosphere which, while positive for collaboration, may significantly limit the need for justification and reasoning, thereby reducing opportunities for constructing new knowledge.

Despite these challenges, post-lesson discussions emerged as important moments for promoting professional learning opportunities, as discussed by Ribeiro and Ponte (2019) and Darling-Hammond and McLaughlin (2011). These opportunities were mainly related to the dimensions of knowledge of teaching practice and knowledge of students and their learning processes. The analysis showed that teachers were placed in situations that encouraged reflection on the teaching and learning of mathematics through the analysis of classroom evidence, as well as discussions of possible improvements for future lessons. These moments represented professional learning opportunities; however, many of them were not fully capitalized upon, remaining at the level of cumulative discourse. Even when discussions became exploratory, they could have been developed more deeply.

The results indicate that professional learning opportunities arise from certain discursive and interactional conditions created during the discussions, particularly when the facilitator poses specific questions, refers to classroom evidence and guides the group to reflect on teaching and learning. Such actions create moments conducive to collaborative reflection, which can be taken up in different ways. When participants justify their ideas, question assumptions, and connect arguments to evidence, these opportunities are more effectively realized, giving rise to exploratory interactions (Littleton & Mercer, 2013). Thus, both the facilitator's role and the nature of the interactions are key to transforming moments of sharing into situations of collaborative construction of professional knowledge (Clivaz et al., 2023b; Ribeiro & Ponte, 2019).

CONCLUSION

The findings of this study show that the post-lesson discussion is a rich moment in terms of professional learning opportunities for teachers who teach mathematics, with the quality of interactions being a central element for their effective use. These opportunities arise from the facilitator's actions and the interactional conditions established within the group, particularly when specific questioning, reference to classroom evidence, and reflection on teaching and student learning are promoted.

However, the predominance of cumulative discourse (Mercer, 1995) indicates that, although a collaborative environment was created and teachers had the opportunity to reflect through experimentation and the analysis of their own teaching evidence, the groups found it difficult to move beyond the simple sharing of ideas and observations. As a result, exploratory interactions, capable of promoting the construction of new knowledge through argumentation, justification, and the joint building of meaning, occurred only sporadically.

Thus, the deepening of teachers' professional knowledge in post-lesson discussions depends not only on the creation of learning opportunities but also on how these opportunities are taken up through high-quality interactions within the group (Clivaz et al., 2023b). Consequently, it is necessary to create conditions that foster engaged discussion about teaching and student learning, recognizing the facilitator's essential role in this process. The facilitator should invest in intentional mediation that promotes the grounding of ideas, the justification of opinions, and the constructive confrontation of perspectives, elements that, as Kager et al. (2022) highlight, are indispensable for developing post-lesson discussions that effectively foster meaningful professional learning. Therefore, strengthening professional learning within lesson study requires not only recognizing the potential of post-lesson discussions but also creating conditions that stimulate the emergence of exploratory interactions, an essential condition for the collective development of teachers' professional knowledge.

This study contributes to research on lesson study by addressing challenges commonly identified in post-lesson discussions (e.g., Kager et al., 2022; Karlsen & Helgevold, 2019). By providing an empirically grounded analysis of post-lesson discussions, the study shows how different types of talk are associated with specific facilitation actions, offering insight into how exploratory interactions can be fostered. In doing so, it advances understanding of teacher learning by illustrating how professional learning opportunities emerge when discussions are oriented toward students' strategies and instructional decisions. Moreover, the study contributes to the literature on facilitation by characterizing intentional facilitation as a leading and questioning role that involves specific questioning, the use of lesson planning and classroom evidence, and other timely interventions to sustain exploratory talk.

A limitation of this study is that it analysed only two post-lesson discussions that took place in similar contexts. Examining a larger number of sessions and diverse settings could yield results with a greater variety of exploratory interactions, leading to a deeper understanding of how to enhance the effectiveness of professional learning opportunities within lesson study.

Ethical statement: The authors stated that the study was approved by the Ethics Committee Plataforma Brasil on September 23, 2023, under approval code (73316423.3.0000.5317) and approval reference searchable by the study title "Estudo de Aula como processo formativo de professores que ensinam Matemática na Educação Básica". Written informed consents were obtained from the participants. Ethical approval for the project is available at: https://plataformabrasil.saude.gov.br/login.jsf.

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REFERENCES

Bae, C. L., Hayes, K. N., Seitz, J., O'Connor, D., & DiStefano, R. (2016). A coding tool for examining the substance of teacher professional learning and change with example cases from middle school science lesson study. *Teaching and Teacher Education, 60*, 164-178. https://doi.org/10.1016/j.tate.2016.08.016

Bardin, L. (1997). *Análise de conteúdo* [Content analysis]. Edições 70.

Brasil. (2016). *Conselho Nacional de Saúde. Resolução 510, de 7 de abril de 2016* [Resolution No. 510, dated April 7, 2016]. Diário Oficial da União, Poder Executivo, Seção 1, (98), 44-46.

Brosnan, A. (2014). Introducing lesson study in promoting a new mathematics curriculum in Irish post-primary schools. *International Journal for Lesson and Learning Studies, 3*(3), 236-251. https://doi.org/10.1108/IJLLS-09-2013-0050

Clivaz, S., Batteau, V., Pellet, J.-P., Bünzli, L.-O., Daina, A., & Presutti, S. (2023a). Teachers' mathematical problem-solving knowledge: In what way is it constructed during teachers' collaborative work? *The Journal of Mathematical Behavior, 69*, Article 101051. https://doi.org/10.1016/j.jmathb.2023.101051

Clivaz, S., Daina, A., Batteau, V., Presutti, S., & Bünzli, L.-O. (2023b). How do dialogic interactions contribute to the construction of teachers' mathematical problem-solving knowledge? Construction of a conceptual framework. *International Journal for Lesson & Learning Studies, 12*(1), 21-37. https://doi.org/10.1108/IJLLS-03-2022-0031

Creswell, J. W. (2014). *Investigação qualitativa e projeto de pesquisa: Escolhendo entre cinco abordagens* [Qualitative inquiry and research design: Choosing among five approaches] (3thª ed.). Penso.

Darling-Hammond, L., & McLaughlin, M. W. (2011). Policies that support professional development in an era of reform. *Phi Delta Kappan, 92*(6), 81-92. https://doi.org/10.1177/003172171109200622

Fisher, E. (1993). Distinctive features of pupil‐pupil classroom talk and their relationship to learning: How discursive exploration might be encouraged. *Language and Education, 7*(4), 239-257. https://doi.org/10.1080/09500789309541363

Gutierez, S. B. (2015). Teachers' reflective practice in lesson study: A tool for improving instructional practice. *Alberta Journal of Educational Research, 61*(3), 314-328. https://doi.org/10.11575/ajer.v61i3.56087

Kager, K., Jurczok, A., Bolli, S., & Vock, M. (2022). "We were thinking too much like adults": Examining the development of teachers' critical and collaborative reflection in lesson study discussions. *Teaching and Teacher Education, 113*, Article 103683. https://doi.org/10.1016/j.tate.2022.103683

Karlsen, A. M., & Helgevold, N. (2019). Lesson study: Analytical stance and depth of noticing in post-lesson discussions. *International Journal of Lesson and Learning Studies, 8*(4), 290-304. https://doi.org/10.1108/IJLLS-04-2019-0034

Lee, L. H. J., & Tan, S. C. (2020). Teacher learning in lesson study: Affordances, disturbances, contradictions, and implications. *Teaching and Teacher Education, 89*, Article 102986. https://doi.org/10.1016/j.tate.2019.102986

Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM Mathematics Education, 48*, 571-580. https://doi.org/10.1007/s11858-016-0792-x

Littleton, K., & Mercer, N. (2013). *Interthinking: Putting talk to work*. Routledge. https://doi.org/10.4324/9780203809433

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners*. Multilingual Matters. https://doi.org/10.21832/9781800418288

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L.C. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics* (pp. 13-24). Springer. https://doi.org/10.1007/978-90-481-9941-9_1

Myers, J. (2012). Lesson study as a means for facilitating preservice teacher reflectivity. *International Journal for the Scholarship of Teaching & Learning, 6*(1), Article 15. https://doi.org/10.20429/ijsotl.2012.060115

Ni Shuilleabhain, A., & Clivaz, S. (2017). Analyzing teacher learning in lesson study: Mathematical knowledge for teaching and levels of teacher activity. *Quadrante, 26*(2), 99-125. https://doi.org/10.48489/quadrante.22948

Ono, Y., Chikamori, K., & Rogan, J. M. (2013). How reflective are lesson study discussion sessions? Developing an instrument to analyze collective reflection. *International Journal of Education, 5*(3), 52-67. https://doi.org/10.5296/ije.v5i3.3847

Ponte, J. P. (2012). Estudiando el conocimiento y el desarrollo profesional del profesorado de matemáticas [Studying the knowledge and professional development of mathematics teachers]. In N. Planas (Ed.), *Teoría, crítica y práctica de la educación matemática* (pp. 83-98). Graó.

Ponte, J. P., Martins, M., Quaresma, M., Gomes, P., & Rodrigues, B. (2025). Needed skills and knowledge and challenges of the mathematics lesson study facilitator: A scoping systematic literature review. *International Journal for Lesson & Learning Studies*, *14*(4), 415-430. https://doi.org/10.1108/IJLLS-01-2025-0021

Ponte, J. P., Quaresma, M., Baptista, M., & Mata-Pereira, J. (2014). Os estudos de aula como processo colaborativo e reflexivo de desenvolvimento profissional [Lesson study as a collaborative and reflective process of professional development]. In J. Souza, & I. Cevallos (Eds.), *A formação, os saberes e os desafios do professor que ensina matemática* (pp.61-82). CRV.

Ribeiro, A. J., & Ponte, J. P. (2019). Professional learning opportunities in a practice-based teacher education programme about the concept of function. *Acta Scientiae, 21*(2), 49-74. https://doi.org/10.17648/acta.scientiae.v21iss2id5002

Richit, A., Ponte, J. P., & Quaresma, M. (2021). Teachers' professional learning revealed in lesson study research. *Bolema: Boletim de Educação Matemática, 35*, 1107-1137. https://doi.org/10.1590/1980-4415v35n70a26

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching foundations of the new reform. *Harvard Educational Review, 57*(1), 1-23. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Stigler, J., & Hiebert, J. (1999). *The teaching gap*. Free Press.

Takahashi, A., Watanabe, T., & Yoshida, M. (2006). Developing good mathematics teaching practice through lesson study: A US perspective. *Tsukuba Journal of Educational Study in Mathematics, 25*(1), 197-204.

Wegerif, R., & Mercer, N. (1997). A dialogical framework for investigating talk. *Computers and Talk in the Primary Classroom*, 49-65.

Wood, K. (2017). Is there really any difference between lesson and learning study? Both focus on neriage. *International Journal for Lesson and Learning Studies, 6*(2), 118-123. https://doi.org/10.1108/IJLLS-02-2017-0008