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MODESTUM

Does a stance on nos/otrx persist? Investigating beginning secondary mathematics teachers who have participated in an after-school mathematics club

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ABSTRACT

Recently, some mathematics education researchers have challenged the notion that recommended beliefs and practices are "washed out" or ignored by beginning secondary mathematics teachers. Yet, the focus of these studies tends to be how coursework informed beginning mathematics teachers' practices and no other components of teacher education such as field placements or more specifically community-based field placements. Having documented moments of interdependence (nos/otrx) or grappling with multiple conceptions of mathematics by pre-service secondary mathematics teachers as they work with Black and Latinx students during an after-school mathematics club, it is possible that beginning secondary mathematics teachers could also enact moments of interdependence during their first year of teaching. As previous research has documented that teacher education program coursework and student teaching impacts the practices of beginning teachers, then community-based field placements could also impact practices of first-year teachers. This qualitative interview study juxtaposes moments of interdependence by three White women as volunteers at an after-school mathematics club with moments of interdependence during their first year of teaching. Although moments of interdependence did occur during their first year of teaching, these moments were uncommon. This study offers implications for research and teaching.

Keywords: nos/otrx, interdependence, beginning secondary mathematics teacher

INTRODUCTION

Mathematics education programs are designed for preservice mathematics teachers to deepen their content knowledge (Bloom, 2004; Burton et al., 2008; Conference Board of the Mathematics Sciences, 2012), develop pedagogical content knowledge (Hill et al., 2008; Shulman, 1986), as well as developing the dispositions and stances to teach equity and social justice to an increasingly diverse student population in U.S. public schools (Association of Mathematics Teacher Educators, 2017; Poole-Parrilla, 2023). Although researchers have noted that recommended stances and practices from coursework seem to get "washed out" (Zeichner & Tabachnick, 1981, p. 7) during the first years of their teaching, other mathematics teacher education researchers have challenged the notion that recommended stances and practices of mathematics teacher education programs are disregarded by beginning secondary mathematics teachers (Corven et al., 2022; Ensor, 2001; Goos, 2005; Kinser-Traut & Turner, 2020; Kirwan & Edwards, 2023; König et al., 2024; Lloyd, 2013; Peressini et al., 2004). These researchers acknowledge the affordances and constraints beginning secondary teachers endure, such as the pressure to conform to colleagues' traditional teaching approaches, maintain fidelity to pacing plans, as well as difficulties with classroom management (Ensor, 2001; Gregg, 1995; Lloyd, 2013; Peressini et al., 2004; Prescott & Cavanagh, 2008). But some researchers argue that by the third year, advanced novice teachers develop more nuance and flexibility with their pedagogy and classroom practices (Kirwan & Edwards, 2023). So, it's not that beginning teachers disregard recommended stances and practices, but that they are in the process of learning to teach from their experience in the classroom (Brown, 2023; Makar, 2024). However, most of these studies tended to focus on coursework (except for Goos, 2005; Kinser-Traut & Turner, 2020). Mathematics teacher education programs are comprised of not only courses but also field placements. Could these field placements experiences also have a lasting impact on the stances and practices of beginning secondary mathematics teachers?

Researchers have documented the promise placements have for informing stances and practices of preservice teachers (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Kinser-Traut & Turner, 2020; Lloyd, 2005). For example, positioning bilingual students as experts was not only a practice recommended in methods courses but was also supported and enacted during student

teaching (Kinser-Traut & Turner, 2020). Perhaps when coursework is supplemented by field placements that align with recommended stances and practices from coursework, these recommendations might be less likely to be "washed out" (Zeichner & Tabachnick, 1981. p. 7).

Similarly, community-based field placements could also be sites for enacting recommended practices (Gerardo, under review; Bussert-Webb, 2011; Douglass, 2023; Kirtman, 2008; Leonard & Evans, 2008; Vomvoridi-Ivanović, 2012). For example, a community garden could be an opportunity to explore geometry and measurement concepts, and teacher candidates could gain insight into the mathematical experiences (positive and negative) of bilingual students (Bussert-Webb, 2011). At an after-school mathematics clubs, preservice secondary mathematics teachers had the opportunity to engage in moments of interdependence with students (Gerardo, under review). This teacher-student dynamic involves working alongside each other and grappling with each other's mathematical ideas and problem-solving approaches. The low stakes setting of community field placements provide opportunities to take risks and experiment with planning lessons and how they work with students. Therefore, if some researchers argue that recommended stances and practices from coursework continue into the first year of teaching, could the stances and practices from community-based field-based placements also be enacted by beginning secondary mathematics teachers?

This study seeks to address this gap in literature by juxtaposing the stances and practices of beginning secondary mathematics teachers in relation to their experiences at a community-based field experience. This qualitative interview study documented the experiences of three White women beginning secondary mathematics teachers as pre-service teachers and then as first-year teachers. How they approached working interdependently with students as volunteers at an after-school mathematics club is described. Then examples of how the stance and practice of interdependence were enacted during their first year of teaching are provided. Working interdependently with students could help beginning teachers reject the ascribed role of mathematics teachers as the expert and students as the novice in mathematics classroom. In turn, this could result in increasing the participation and engagement of Black and Latinx students in secondary mathematics classrooms because their mathematical sense-making could be valued and leveraged by the mathematics teacher.

LITERATURE REVIEW

Mathematics teacher education researchers have documented a variety of experiences for supporting the professional development of preservice secondary mathematics teachers to teach reform-based lessons that include rich mathematical tasks and engaging in mathematical discussions (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Hamilton, 2023; Lloyd, 2005). When referencing these experiences in terms of vision for high quality teaching, stances and practices of beginning secondary mathematics, except for Goos (2005), Hamilton (2023), and Kinser-Traut and Turner (2020), the focus of this research has tended to be coursework and no other components of teacher education programs. Because the focus of this study is field-placements, specifically community-field placements, the following brief overview will focus on the experiences of preservice secondary mathematics teachers during a school and community-based field placements. Afterward, I provide a brief overview of the possible transfer of recommended stances and practices into the first year of mathematics teaching and some challenges experienced by beginning secondary mathematics teachers.

Negotiating Mathematical Authority During School-Based Field Placements

Researchers, investigating classroom field experiences of secondary mathematics student teachers, study a variety of student teacher dynamics. Some researchers focused specifically on the impact mentor teachers have on experiences of student teachers (Leatham & Peterson, 2010; Peterson & Williams, 2008; Rhoads et al., 2013) and others on how university supervisors mentored student teachers (Blanton et al., 2001a; Fernandez & Erbilgin, 2009; Rhoads et al., 2011). The following studies were selected because secondary mathematics student teachers negotiated authority with students during their school field placements (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Lloyd, 2005). These are not examples of interdependence, but they do describe how the student teachers negotiated authority with students in the classroom.

The argument generally posed by some researchers was that student teaching was not just an opportunity to teach but also a learning experience for how to teach mathematics (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Lloyd, 2005). In one case study of a middle school student teacher, his teaching practices were contrasted to fictional narratives he wrote in response to two prompts asking him to discuss the teacher he strives to be and the kind of teaching he hopes to avoid (Lloyd, 2005). He valued having students explain their work but at first rarely probed student thinking, although he facilitated mathematical discussions. After many students complained about his teaching and their grades on a recent exam, the student teacher reflected on his practice and realized he might not be deepening their conceptual understanding. He began to probe their thinking more often, prompted students to ask questions of groups presenting their work while he played the part of a student who would ask clarifying questions. As a result of this student-teacher conflict, the student teacher's practice shifted to better align with the goal of students engaging in mathematical discussions to develop conceptual understanding.

The following two studies were more explicit in terms of sharing authority with students (Blanton et al., 2001b; Brendefur & Frykholm, 2000). The researchers of one study proposed four constructs for mathematics communication, the constructs ranged from "univocal" teaching (Brendefur & Frykholm, 2000, p. 149), where the teacher mostly lectured students, to "instructive" teaching (Brendefur & Frykholm, 2000, p. 149), where students and the teacher engage in a mathematical discussion to deepen their understanding and shape subsequent instruction. The unit of analysis was the experiences of two high school student teachers, one man and one woman. Both student teachers initially expressed their inclination toward engaging the students in mathematics discussions: his practice did not reflect this, but hers did. He seemed concerned with maintaining the pace of his cooperating teacher and did not find asking open-ended questions an efficient use of time. In contrast, she overcame her concerns

about off-topic conversations during small group discussions and persevered. With the support of the university supervisor, the two developed questions to promote more mathematical student interaction and she became more comfortable with increasing student participation during whole group discussions and ultimately expressed discomfort with taking on the role as the "knowing authority" (Brendefur & Frykholm, 2000, p. 137). It was notable this approach was not the cooperating teacher's but that the student teacher was allowed to enact stances and practices that aligned with her beliefs about teaching mathematics.

Similar to the previous study, these other researchers categorized classroom discourse as "univocal" or "dialogic" (Blanton et al., 2001b, p. 230). Univocal discourse describes a transmission model of teaching (i.e., teacher lecturing), whereas dialogic discourse describes classroom discussions when students interpret, question, validate, or even reject what classmates share. The student teacher, a woman, taught in a seventh-grade general mathematics and pre-algebra class in an urban school setting. Early in her student teaching, as the "mathematical authority" (Blanton et al., 2001b, p. 239), she was usually the "filter of discourse" (Blanton et al., 2001b, p. 22) and directed student exchanges. Her university supervisor prompted her to teach a lesson by presenting the problem first and having students justify their solution. The student teacher was uncomfortable, and students initially resisted, but she persisted, and a shift occurred in her teaching. Instead of emphasizing the correct answer, she began to focus on students' strategies. By experiencing the "power of students' thinking" (Blanton et al., 2001b, p. 240), her teaching shifted from focusing on correct answers to building upon students' unsuccessful attempts, and she used questions to understand her students' mathematical sense-making. As a result of the participation of students in these mathematics discussions, the classroom discussions became a positive, mediated experience for the student teacher to consider more opportunities for students to engage in mathematical discussions during class. The researchers of this sociocultural participant observation study contend that mathematics classroom discourse can mediate student teachers' learning about teaching mathematics. Although student teachers guide the classroom discourse, they are also "subject to its mediated effects" (Blanton et al., 2001b, p. 229).

A more recent study focused on the beliefs of pre-service teachers regarding mathematical authority in relation to the classroom during their teacher education program (Hamilton, 2023). A qualitative case study of 4 pre-service mathematics teachers from a large Southeast public university. Generally, all student teachers described an emphasis on wanting students to discover mathematical concepts through reform-based approaches and yet shared that as the teacher that they were the ultimate mathematical authority in the classroom. One student teacher noted that individual learning was necessary to support students' conceptual understanding of mathematics, taught in more traditional manner but did incorporate time for individual selfreflection before engaging in group discussions. In contrast, another student teacher developed a more confident approach in her questioning of students to privilege students' mathematical voices. Unlike the previous studies where there were some reflection and willingness for positioning students as mathematical authorities by relinquishing their own position in the classroom (Blanton et al., 2001b; Kinser-Traut & Turner, 2020), the student teachers in this study did not. The first student teacher discussed above acknowledged that students were capable of mathematical authority with the condition that students' mathematics was correct whereas the second student teacher referred to above was more willing to acknowledge that student's mathematical insights were positive contributions during class. Student teaching did not seem to change their perception of mathematical authority but reinforced their beliefs. What did occur for each student teacher experience was the development of "personalized pedagogy" (Hamilton, 2023, p. 96) or practical approaches for them, to provide certain opportunities for students to engage with mathematical concepts and share their reasoning and solutions.

Working With Students During Community-Based Field Placements

If student teaching is an opportunity to learn about teaching mathematics, could community-based field placements also be an experience to develop a "personalized pedagogy" (Hamilton, 2023, p. 96) and experience the "power of student's thinking" (Blanton et al., 2001b, p. 240)? One recent study that took place at a Makerspace Lab, four pre-service teachers gained experience exploring a makerspace lab e.g., 3-D printing, laser cutting, and woodcutting) to consider its potential to explore STEM topics with students (Douglass, 2023). Generally, they were excited, curious and developed a belief that these experiences would be beneficial for students and expressed excitement for integrating makerspace opportunities for their students. In other community-based field placements, mathematics preservice teachers worked with students different from themselves (Kirtman, 2008; Leonard & Evans, 2008), engaged with students in Spanish and attempted to bridge home and community knowledge with the mathematical content they worked with (Bussert-Webb, 2011; Vomvoridi-Ivanović, 2012), and other pre-service teachers were even challenged to develop reform-based culturally relevant mathematics lessons (Leonard & Evans, 2008). At another after-school mathematics club, preservice secondary mathematics teachers engaged in moments of interdependence (Gerardo, under review). They worked alongside the students and grappled with multiple mathematical perspectives. One volunteer, an Asian-American man, reflected on his and a Black student's conceptions of geometry. As a mathematics major, he did not dismiss the student's understanding but considered it almost on an equal status to the mathematics he is learning in his undergraduate courses. Another volunteer grappled with his own problem-solving process for a computer puzzle game he saw a girl play. He did not impose his solutions but compared and contrasted his strategies with those she used. With these opportunities to experience the "power of student's thinking" (Blanton et al., 2001b, p. 240) during community-based field experiences, is it possible that, to some degree, interdependence might also occur during the first year of teaching mathematics?

From School-Based Field Placement to the First Years of Teaching

There is evidence from the broader perspective of mathematics teacher education that practices from field experience, like those from coursework, do continue into the first years of teaching. Some of these studies have documented the degree with which knowledge for teaching is maintained post-graduation (Corven et al., 2022; Kirwan & Edwards, 2023; König et al., 2024) as well as high quality vision for teaching mathematics (Hayes, 2023). Some researchers documented that content taught for longer period in courses was maintained by beginning teachers when they were assessed by completing mathematical tasks, regardless

of additional professional development in content (Corven et al., 2022). Other researchers (König et al., 2024) noted that general pedagogical knowledge also seemed to transfer and was maintained by in-service teachers (Germany and Austria) emphasizing the importance of these concepts in coursework in teacher education program. Regarding high quality vision for teaching mathematics, it seems that novice teachers, other researchers theorize that if early career teachers do not seem to be enacting recommended practices early in their career that they may begin doing so during their third year of teaching (Kirwan & Edwards, 2023). By this time early career teachers are considered advanced beginners (Berliner, 2004) and that early concerns regarding control of the classroom, both from classroom management and unpredictability of students' responses, that they've gained enough experience and confidence to enact more complex and nuanced practices that enable them to enact reform-based teaching practices.

Similarly, documenting the impact of field placements and practices in their early years of teaching, one study focusing on the integration of technology into mathematics lessons, Goos's (2005) case study noted to what degree this transfer of practice for integrating technology into mathematics lessons occurred during student teaching and the first-year teaching in Australia. This transfer of practice occurred as a result of being an "active agent" (Goos, 2005, p. 55). The mathematics teacher's experiences in different social contexts helped the student teacher deepen his understanding of how to integrate technology, and he was able to reinterpret recommended practices and apply them in light of his beliefs and practices. Another example is that of an Asian-American elementary mathematics teacher who earned her licensure and ESL endorsement in a program supporting an ambitious mathematics teaching approach centering children's mathematical thinking and community funds of knowledge (Kinser-Traut & Turner, 2020). In her coursework and student teaching, the emphasis was to "share authority ... when teachers attend to students' ideas and traditional teacher/student power dynamics are disrupted" (Kinser-Traut & Turner, 2020, p. 9) by having students contribute ideas, make connections, and determine correct answers and procedures. During student teaching, the woman tended to "hand over authority" (Kinser-Traut & Turner, 2020, p. 15) and minimally participated. Similarly, during her first years of teaching, she elicited students' thinking but more often engaged in mathematical reasoning with the students. This was notable because the curriculum at the school was a teacher-centered curriculum. Recognizing that she needed to scaffold sharing authority with students, she increasingly engaged in this practice and was able to maintain it throughout the academic year.

Mathematics teacher education has recommended stances and practices that do not seem to be "washed out" (Zeichner & Tabachnick, 1981, p. 7) as documented by the stances and practices enacted of some beginning secondary mathematics teachers (Brown, 2023; Ensor, 2001; Hayes, 2023; Kirwan & Edwards, 2023; Lloyd, 2013; Peressini et al., 2004; Poole-Parrilla, 2023). Therefore, could developing a stance for working interdependently, working alongside, and learning with students during community-based field placements also continue as a stance and practice into the first year of teaching? By broadening the focus of transferable practices related not only to pedagogy and content but also to teacher-student dynamics (Kinser, 2025; Poole-Parrilla, 2023), teacher education researchers might also identify moments when mathematics teachers and students reject traditional classroom roles (i.e., teacher as expert-authority and student as novice-subject). When working with working with minoritized students who can be perceived as nonmathematical (Martin, 2009), could moments of interdependence increase the likelihood of positioning them as knowers and doers of mathematics? By building upon the results of a previous study in an after-school mathematics program (Gerardo, under review) and working with a different group of participants, how could the practice of moments of interdependence by beginning secondary mathematics teachers open opportunities to acknowledge students' mathematical sense-making during an after-school mathematics club? Then as first year teachers, do moments of interdependence occur that disrupt the traditional teacher as expert and -student as novice binaries?

Except for Goos (2005) and Kinser-Traut and Turner (2020), few studies have attempted to document how field placements impact the practices of first-year mathematics teachers. No studies have considered to what degree community-based field experiences inform the practices of beginning secondary mathematics teachers. The focus of research has been coursework (Ensor, 2001; Kirwan & Edwards, 2023; Lloyd, 2013; Peressini et al., 2004; Prescott & Cavanagh, 2008). This study sought to address these gaps. To what degree does the stance and practice of interdependence, that was enacted at a community-based field experience, continue into the first year of teaching? This research examined the experiences of three White beginning secondary mathematics teachers. This study juxtaposed their experiences of interdependence during an after-school mathematics club with moments of interdependence as first-year teachers. Stances and practices from a community-based field experience seemed to continue into the first year of teaching but were enacted inconsistently. This study is guided by the following research questions:

- **RQ1** How did volunteering at an after-school mathematic club inform the stance of preservice secondary mathematics teachers to engage in moments of interdependence with youth?
- **RQ2** How did the stance that was developed or learned while working in an after-school mathematics club transfer to the teaching context as beginning secondary mathematics teachers?

Theoretical Framework

Researchers who have investigated how teachers and students negotiate the roles of authority and expert in mathematics classrooms (Blanton et al., 2001b; González & DeJarnette, 2012; Kinser-Traut & Turner, 2020) tend not to challenge the teacher-student binary. This research appears to take for granted ascribed teacher and student roles (i.e., novice and experts). Even the frameworks used (e.g., positioning theory as per van Langenhove & Harré, 1999) have assumed asymmetrical relationships between teachers and students. Mathematics teachers are expected to "teach" and the student to "learn," but teacher-student dynamics are complex, dynamic, and constantly (re)negotiated (Gonzalez & DeJarnette, 2012, 2015).

What might occur if teachers and students solved problems together and discussed each other's ideas? These actions might result in mathematics teachers and students rejecting assumed teacher and student roles. In two previous studies (Author, 2021), interdependence was documented during an antiracist mathematics teacher seminar among preservice teachers and facilitators

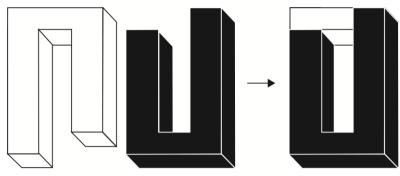


Figure 1. The puzzle piece representing nosotrx (dependence) (Source: Author's own elaboration)

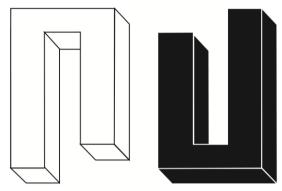


Figure 2. The puzzle piece representing nosotrx (independence) (Source: Author's own elaboration)

(Gerardo, submitted) and during an after-school mathematics club between preservice teachers and students (Gerardo, under review). Therefore, it might be possible that interdependence may also occur in secondary mathematics classrooms. This study builds upon previous applications of interdependence by considering the classwork experience of three beginning secondary mathematics teachers. What follows is a brief description of nos/otrx (i.e., interdependence).

Nos/otrx (interdependence) is a Chicanx feminist theoretical concept. It is currently written with an "x" as a rejection of the gender binary, although it was originally proposed as nos/otras (Anzaldúa, 2002; Keating, 2000, 2005) to center a feminist perspective and reject the masculine plural form in Spanish. Thus, nos/otrx is a rejection of binaries, social categories, and labels that perpetuate an "us" (nos) and "them" (otras) divide. Expanding on the application of nos/otrx as applied in previous studies, the dynamic considered in this study is how beginning secondary mathematics teachers and students work alongside each other and consider multiple mathematical perspectives (e.g., sense-making and problem solving). What if mathematics teachers and students re-envisioned working together that considered the sensibility of being "in each other's world... affected by the other, and... dependent on the other" (Keating, 2000, p. 215). What does a rejection of categories and "we're in each other's worlds... [and] dependent on the other" have to do with mathematics education?

Traditional teacher-student dynamics in mathematics classrooms can be described as *nosotrx* (independence or dependence). For example, in the school mathematics tradition (Cobb et al., 1992; Gregg, 1995), there are assumed and highly regimented teacher-student dynamics. These can be described as nosotrx (dependence), where the teacher is the authority figure or content expert, and the student is the subject and novice (see **Figure 1**). Traditional approaches to mathematics teaching include teacher "chalks and talks" (Ensor, 2001, p. 311), students "copying and memorizing," and a repetition of these steps in a series of assigned practice problems. The "chalk and talk" dynamic is an example of dependence because the teacher transmits a specific mathematical approach upon students (see **Figure 1**) as opposed to the teacher and student engaging in a mathematical discussion. In **Figure 1**, the puzzle piece of the teacher forms the base and the students' puzzle piece is held up and in place, thus symbolizing the dependency of the student to the teacher. The inverse of this is also an example of nosotrx or dependence.

Well intentioned reform-based approaches recommend the teacher not be the "arbiter of mathematical knowledge" (Stein et al., 2008, p. 315) but instead orchestrators of mathematical discussions. Within this dynamic, the tendency is to position students as competent (Cohen et al., 1995). Instead of the teacher retaining the expert role, the students take on this role. All other students, and perhaps the teacher, are now dependent on the student to express their expertise. Ideally, there is dialogue and the opportunity to listen and challenge each other's conjectures. Yet, if students are positioned as experts, these are moments of nosotrx. This example of positioning students as the expert is also represented by **Figure 1**. This time, the puzzle piece that represents the base is not the mathematics teacher but the student that holds up the other puzzle piece that, in this example, represents the classmates and the teacher as dependent on a particular student who shares their expertise.

Lastly, the teacher may or may not be dependent on the student's mathematical expertise. Instead, they may be independent and maintain their distance by not engaging in mathematical discussions with students. This lack of engagement or distance is also an example of nosotrx (see **Figure 2**). In **Figure 2**, the puzzle pieces are shown separately from the other puzzle piece representing the teacher not engaging with students or also moments when students choose to not answer or response to

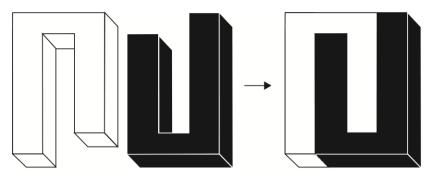


Figure 3. The puzzle piece representing Nos/otrx (interdependence) (Source: Author's own elaboration)

teacher's prompts. Therefore, disrupting traditional classroom roles or categories may result in moments in interdependence (nos/otrx) among the teacher and students.

What would nos/otrx (interdependence) look like in mathematics classrooms? **Figure 3** is a visual representation of two interlocked puzzle pieces. Each puzzle piece is not dependent on (or subsumed by) the other. The pieces are alongside one another, interdependently creating a new shape while maintaining their respective shape (i.e., sense-making or mathematical understanding). Thus, **Figure 3** visually represents how mathematics teachers and students can be "dependent on each other" (Keating, 2000, p. 215) as well as "in each other's world" (Keating, 2000, p. 215).

If previous studies are any indication (Gerardo, submitted, under review), perhaps students suggest working with the teacher on a mathematical problem they came across on social media (e.g., X or TikTok). Perhaps a recently discussed mathematical concept (i.e., fractals) spurred additional thoughts and the student sought to engage the teacher in additional research and conversation about the topic. During mathematical discussions, perhaps the mathematics teacher could take part, not just as the orchestrator of the discussion but also to engage, be questioned, and justify their own claims similar to and along with the students. All parties can grapple with each other's mathematical ideas. Granted, there are responsibilities a teacher cannot relinquish (e.g., planning, grading, attendance) and students have expectations of teachers (e.g., teaching, being fair and consistent, providing a welcoming classroom). Nevertheless, the teacher and students might be able to engage in moments where they reject traditional mathematics teacher-student dynamics and work alongside and grapple with each other's mathematical ideas. Thus, they are "in each other's worlds ... and dependent on one another" (Keating, 2000, p. 215) as they do and discuss mathematics in the classroom. Given that this teacher-student dynamic occurred during a mathematics teacher education seminar (Gerardo, submitted) and at an after-school mathematics club (Gerardo, under review), could this also occur in secondary mathematics classrooms? After all, researchers have suggested recommended stances and practices from courses and school-based field placements are enacted by beginning secondary mathematics teachers (Brown, 2023; Ensor, 2001; Goos, 2005; Kinser-Traut & Turner, 2020; Kirwan & Edwards, 2023; Lloyd, 2013; Peressini et al., 2004; Poole-Parrilla, 2023).

METHODOLOGY

This was a qualitative interview study (Kvale & Brinkmann, 2009). The following describes the methodology, the context of the study and participants, data collection, data analysis, and limitations for this study.

First, nos/otrx (interdependence) was not only the theoretical lens of this study, but also framework also informed this study methodologically. As a Chicanx researcher with a strong political stance advocating for and working with Black and Latinx youth and families, I could not impose upon or reject the beginning mathematics teacher's actions or sense-making for working with students. Nos/otrx reminded me "we're in each other's world... and [that] we're all dependent on the other" (Keating, 2000, p. 215), so as the interviewer, it is important to be mindful of my own stances while also attempting to acknowledge and respect those of the participants. Second, the interview "journey" (Kvale & Brinkmann, 2009, p. 49) with the participants and engaging in interdependence complemented "establishing a good interviewing partnership" (Weiss, 1994, p. 61) with the mathematics teachers in this study. Although I was a mentor and friend of these mathematics teachers, the interviews posed new dynamics between us that I could not take for granted. I began each interview with small talk and after each interview, off the record, continued our initial conversation. Other times, if I sensed they felt overwhelmed or the need to vent, I would stop the interview to let them collect themselves. Therefore, during the interview, I attempted to negotiate a nos/otrx interview partnership and journey along with the beginning secondary mathematics teachers as they reflected upon their first year of teaching. With this approach, I hoped to engage in interdependence of "being in each other's world ... dependent on the other" (Keating, 2000, p. 215) with the three mathematics teachers.

Context

As discussed in previous studies, this study is part of a larger longitudinal project (Gerardo, submitted, under review; Gutiérrez et al., 2013; Gutiérrez et al., 2017; Irving, 2014) documenting the professional development of pre-service secondary mathematics teachers as they participate in an antiracist mathematics teacher education program. By anti-racist, we mean centering the needs and values of students of color who have been historically marginalized through the school system as well as preparing teachers to advocate for historically marginalized students. Marginalized students are defined here as students who are Latinx, Black, Indigenous, and from low-income neighborhoods. These preservice teachers were engaged in moments of interdependence in

whole group discussions during seminar (Gerardo, submitted) and as volunteers with Black and Latinx youth at an after-school mathematics club (Gerardo, under review). This study extends prior studies by juxtaposing moments of interdependence of these three teachers at the after-school mathematics club with their experiences as first-year teachers. Although this analysis does not focus on the racial identities of the beginning mathematics teachers and their students, it is worth noting that the teachers are White, and the students were primarily Black and Latinx and working class (with the exception of Annie's classroom). I acknowledge that in the high-stakes content area of mathematics, racial identity can have an impact on how historically marginalized students are perceived by mathematics teachers (Martin, 2009). Thereby investigating asymmetrical instructor-student dynamics is a necessary contribution to our understanding of K-12 mathematics classrooms. So, I chose to investigate teacher-student dynamics to understand the degree that mathematics teachers challenge assumed unequal status of students in formal and informal mathematics settings when working with minoritized students: the mathematics classroom (current study) and an after-school mathematics club (current study and Gerardo, under review).

The three beginning secondary mathematics teachers in this study are Taylor, Annie, and Rose (pseudonyms), White women, who were scholars of the last cohort of the R1 Scholars Program (RSP) (N = 6). During their first year of RSP as undergraduate juniors, they overlapped with Wendy and Cecilia of the previous two studies (Gerardo, submitted, under review). All were mathematics majors who were earning a secondary education certification. As first-year teachers, Rose and Taylor taught at different charter public schools in a major Midwest metropolitan city. Rose taught ninth-grade algebra, and Taylor taught seventh-grade pre-algebra. Both schools were Title I schools with a majority of Black and Latinx students, whereas Annie taught at a suburban high school in a different Midwest state where most students were White and middle class. Though her school was predominantly White, Annie remained purposeful in her approach for working with the Black students in her ninth-grade algebra and tenth-grade geometry classes.

All three teachers had a degree of freedom regarding curriculum and pedagogy. Rose taught from a problem-centered curriculum with a partner teacher, while Taylor was the only seventh-grade pre-algebra teacher at her school. She had access to a reform-based textbook but was not required to teach from it and used a variety of resources. In contrast, Annie was part of a large mathematics department. A teacher-developed district-wide algebra curriculum was provided, but she had flexibility regarding pedagogy (e.g., modifying lessons to be more student-centered, integrating cooperative learning strategies). This was not the case for geometry, where she was not provided with a curriculum but received guidance from her colleagues. Annie had a degree of freedom to choose both the content and pedagogical approaches she thought appropriate.

Data Collection

Three beginning secondary mathematics teachers were interviewed five times over a period of a year. The first interview occurred in the summer after their graduation. The subject of this interview was to discuss their experiences volunteering at IDM (see **Appendix**, Phase I). Participants were asked about specific interactions with youth, how their identity as White women may have impacted how they worked with Black and Latinx youth, and ways this experience may inform their work with students in their first year of teaching. Additionally, field notes were used to develop specific scholar questions regarding specific interactions with youth observed during their time at IDM.

The next four interviews (see **Appendix**, Phase II questions) were conducted during their first year of teaching. The purpose of the first interview of Phase II in October was to capture their experiences during their first months of teaching. The second interview occurred in late January to document the end of the first semester of teaching and how the prospect of standardized exams seemed to impact their teaching. The third interview occurred in April to capture their experience regarding standardized exams and their goals and expectations for the last few weeks or months of the academic year. The final interview that occurred in June or July, depending on availability, was an opportunity for member checking (Lincoln & Guba, 1985), asking clarifying questions, and asking them to reflect on the challenges and successes during their first year of teaching along with their goals and expectations for their second year of teaching.

Data Analysis

A "bricolage" approach (Kvale & Brinkman, 2009, p. 233) was used for the data analysis. The approach I used was a combination of coding (Gibbs, 2007), matrices (Miles et al., 2014), and diagrams (Bliss et al., 1983) to engage with the data, identify patterns, and develop themes. All audio recordings were transcribed and read to develop an overall sense of each of the mathematics teacher's experiences at IDM and during their first year of teaching. After rereading each transcript, they were organized and coded using Excel spreadsheets. Early readings identified contexts (e.g., planning, working with students, working with colleagues). Additional readings helped to further distil these transcripts into examples that reflected theoretical codes (e.g., interdependence, difficulties with classroom management), codes informed by the literature, or data-driven codes (e.g., grappling with tracking, teacher or student-centered lessons, or traditional versus reform-based teaching), codes that were identified informed by the data collected (Gibbs, 2007). Once a general sense of the beginning secondary teacher's experiences was developed, the data was culled by identifying moments of *nos/otrx* (interdependence) or nosotrx (dependence) with students. This process was repeated for each transcript, and then research memos (Miles et al., 2014) were written to help record patterns and possible explanations. Then diagrams were sketched (Bliss et al., 1983) to help map and compare experiences for each participant and then across the participants. These memos and sketches, in tandem with the Excel spreadsheet, were used to identify the most salient examples of nos/otrx and helped identify possible explanations for their experiences at IDM and during their first year of teaching.

FINDINGS

The following excerpts from the three beginning secondary mathematics teachers highlight moments of nos/otrx (interdependence). The first three examples are related to their experiences and sense-making while at IDM (to address **RQ1** for this study), while the last three examples are those that occurred during their first year of teaching (to address **RQ2** for this study). And as a result of the complexity of teacher-student interactions, moments of nosotrx (dependence) were also identified as they recalled their experience at IDM and in their classroom.

Moments of Nos/otrx During I Do Mathematics

During the first interview, the beginning secondary mathematics teachers were asked to recall their experiences at IDM. The following excerpts were from their first interview the summer before teaching. In this first excerpt, Taylor recalled a dynamic similar to a strategy previously mentioned, "not knowing... [then] you're figuring it out together" (Gerardo, under review) and she shared her goal for not being the "math authority" as a classroom teacher.

"There wasn't one authority"

I was new to the games the students are new to the games ... I honestly didn't know... [at] IDM it was always something new you had to learn about these students ... [and] about what they knew about the game. The students knew that there were different ways of winning the game or different strategies and I think math is just a bunch of strategies. And at IDM there wasn't one authority, "Oh this person knows how to play every game and they're good at every single game." It wasn't like that. It was much easier to see the different strengths of the students. [At] IDM I [was] seen more as a helper but in student teaching [I was] seen as the math authority in the classroom. I might know the most math but it's not the only math. And I don't want my students to think that . . . it's difficult to show my students that my way of doing math isn't the only way.

In this excerpt, Taylor described moments of nos/otrx (interdependence). First, there "wasn't one authority" at IDM, not even scholars. After all, Taylor admitted she was "new to the games . . . and didn't know [them.]" So, this became an opportunity to "learn" about the students and what they "knew" and to recognize their "different strengths." Taylor's insight paralleled findings by another pre-service teacher from a previous study who shared that "not knowing" (Gerardo, under review) as an opportunity for the students and teacher to be "figuring it [the puzzles] out together," or, in terms of nos/otrx, "be in each other's worlds" (Keating, 2000, p. 215) at IDM. This approach contrasts with the traditional roles of preservice teachers in community-based and school-based field placements as the knower of mathematical content (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Bussert-Webb, 2011; Kinser-Traut & Turner, 2020; Leonard & Evans, 2008; Lloyd, 2005). Some preservice teachers attempted to "share" authority with students (Blanton al., 2001b; Brendefur & Frykholm, 2000; Hamilton, 2023; Kinser-Traut & Turner, 2020; Lloyd, 2005) but were never relinquishing their role as content expert and authority. Furthermore, these pre-service teachers, in both formal and informal field placements, were not learning content or deepening teachers' mathematical understanding alongside the students. Not only did Taylor relinquish her role as an authority at IDM, but she was willing to learn from and with the students. Nos/otrx occurs when mathematics teachers work alongside students and engage with different mathematical or problem-solving perspectives. At IDM, Taylor had the opportunity to engage with "different strategies" students knew for the different games and puzzles.

Not only was Taylor describing a particular approach for working with students at IDM, but she also seemed to express her stance of a personalized pedagogy (Hamilton, 2023) regarding her role as a classroom teacher. She expressed rejecting the role as the "math authority" in the classroom to emphasize to her students that she may "know the most math but not the only math." As she stated earlier, "math is a bunch of strategies," so if students are familiar with and can use their own strategies, it seems she could accept these student strategies equal those of the teacher. She wanted to emphasize "[her] way of doing math isn't the only way." By rejecting her role as the authority in the classroom, this norm may provide the opportunity for many "ways of doing math" by students. This stance contrasts with pre-service teachers who described themselves as the "ultimate authority" in the classroom although expressed varying degrees that students could also be mathematical authorities, so long as their mathematics was correct (Hamilton, 2023). Whereas Taylor seems to be more considerate of students' mathematical contributions in her classroom, which aligns with more ambitious teaching or student-centered approaches for teaching mathematics. By validating students' approaches to do mathematics, this approach may result in moments of nos/otrx in her classroom because she and the students could be in "each other's [mathematical] worlds" (Keating, 2000, p. 215).

Although Taylor seemed to engage in moments of nos/otrx with students, she also described moments of nosotrx (dependence). When working with students, it was unclear to what degree did Taylor exchange ideas with the students at IDM. She acknowledged "learn[ing] what they knew," but the degree of dialogic communication (Blanton et al., 2001b) she engaged in with students was unclear. Similarly, she described herself as a "helper" at IDM. As a mathematics teacher, "a helper" is a good role to take on when assisting students, but this role maintains an asymmetrical role as the expert. It is notable that Taylor assisted and guided students as they worked on puzzles and games at IDM, but this possibly limited her engaging with student's mathematical sense-making.

Overall, Taylor described rejecting the role of authority at IDM and acknowledged the opportunities she had to learn from and with students. It also seemed she had similar expectations as a teacher where she would seek to reject the role of authority. However, teacher-student dynamics are complicated, and moments of nosotrx would also occur as a first-year teacher.

"We were helping each other out"

In this second excerpt, Annie described a different dynamic when working with students at IDM, one where she emphasized "learning from each other" (i.e., the students):

Grading kind of gets in the way of having good relationships because students know that I am the person who gives them their grade. So, if I'm the one giving them their grade it's hard for them to have fun and be goofy with me. [Whereas at IDM] I wasn't their teacher, I wasn't grading them ... it was more of a like a buddy-buddy kind of thing I was able to just kind of participate with them, they were helping me I was learning from them they were learning from me we were helping each other out with these activities ... I learned a lot from them like the different kinds of games and strategies they were doing and that lends itself to being able to have that back-and-forth kind of thing ... Well sometimes, they didn't want to tell their strategies. I got really competitive, so sometimes they wouldn't want to share [when] they were playing against me.

In this excerpt, Annie described moments of nos/otrx. At IDM, she was able to reject the role as the teacher (the one who grades them) and take on a "buddy-buddy" dynamic with the students. However, not only did she describe working alongside students, but she mentioned a kind of "back and forth" dynamic where she "was learning from them and they were learning from me, we were learning from each other." This recalls the dialogic discussion some student teachers have attempted with their students (Blanton et al., 2001b; Brendefur & Frykholm, 2000). Specifically, questioning techniques can elicit student thinking but Taylor makes no mention of her questioning technique but seems to allude to engaging in conversations with students at IDM where she was "buddy-buddy" with them and that resulted in them "learning from each other." This seems to reflect a more dialogic (Blanton et al., 2001b) and instructive teaching (Brendefur & Frykholm, 2000) approach for engaging with student's mathematical thinking.

For Annie, this "back and forth" dynamic seemed to be an integral aspect of working alongside of students at IDM. For moments of nos/otrx to occur, it is insufficient to merely work alongside one another; it is also necessary to be "affected by the other ... dependent on the other." (Keating, 2000, p. 215) IDM was an opportunity for Annie and the students to share their sensemaking of the games and puzzles. Unlike Taylor, who was explicit about what her role would be in the classroom, Annie expressed her concern for replicating these "buddy-buddy" dynamics with her students. After all, her responsibility will be to "give grades," which positions her as the authority. This role, as well as expectations from parents and students, may create a chasm too challenging to overcome and work interdependently with her future students.

Because teacher-student interactions are complex, Annie also described a moment of nosotrx (independence) that was unique to her interaction with students: competition. None of the scholar described themselves as competitive or seeking to be competitive against the students at IDM (Gerardo, under review). As a result of being competitive, Annie possibly contradicted her own "buddy-buddy" dynamic because what resulted was an interruption to "learning from each other," and instead, students "sometimes wouldn't want to tell" their strategies with her. After all, why would they share a possible winning strategy with the opponent? Competition likely motivated students to play against (and win against) Annie, but it did so at the cost of an opportunity for "learning from each other" that she initially described. Although, we could interpret competition as nos/otrx because you must counter opponent's moves or they to yours moves, thereby having each other's strategy in mind. But Annie states that students did not want to "tell" their strategies and therefore when playing against an opponent, these were moments of nosotrx because there was less of the "back and forth" conversation with the students.

"What I think I hear you saying is...."

In the next excerpt, Rose described grappling with a tension regarding her role at IDM. She was not as explicit as either Annie or Taylor regarding working alongside students at IDM (though she did), but she did express her willingness to hear what students had to say which is a necessary aspect of nos/otrx.

To me it's always a bit of a challenge because we want to help students. My traditional sense of I'm going to help you by explaining why this isn't working for you, you know, that's not helpful (chuckles). [At IDM] I was trying to redefine my definition of what's going to be helpful and getting students to take charge and explain to me and get that extra level of understanding of being able to put their thoughts into words. It was difficult and still is and I think that's something I see in my teaching practice of not wanting to take over the extra the explanation but instead, "Oh, what I think I hear you're saying is" and then just saying whatever it is they say. It's a challenge... It's something that I'm definitely going to use in my teaching practice as far as being able to let students explain to me what they're doing and where they're struggling and why they're struggling.

The moment of nos/otrx Rose described was related to "hearing" what students explain to her. She recognized the challenge of "redefining" a "helpful" approach for working with students. Feeling compelled to "take over the explanation," she had realized this was not helpful for students to develop that "extra level of understanding," So, she worked to be more of a listener which is an example of "we're in each other's world" (Keating, 2000, p. 215). Her struggle to shift from a "helper" to a "listener" role is similar to the challenges that pre-service mathematics teachers endured during their field placements (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Hamilton, 2023). Some really worked to be more dialogic and privilege the voiced of students whereas others spoke in terms of reform-based teaching, but their practice did not reflect this. Pre-service teacher's experiences during field placement as well as their own beliefs and visions regarding the teaching and learning of mathematics can support or not a change to one's pedagogy and interaction with students. Rose desired to shift her role, but it was not an easy process. And similar to Taylor and Annie, Rose also described moments of nosotrx when recalling her role at IDM.

As a result of Rose grappling with the tension of redefining herself as a listener and not the one explaining, she described aspects of nosotrx. For example, it seemed she felt most comfortable explaining to students. This tendency echoes univocal communication (Brendefur & Frykholm, 2000) approaches, part of the traditional role of the teacher as the one explaining. However, Rose meant well because she was "helping [students] by explaining" errors, but she realized this was not the most productive approach for promoting student's mathematical understanding. Her realization was similar to various student teachers who recognized that not probing student thinking or providing enough opportunities for students to justify their work seemed to limit the development of conceptual understanding (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Hamilton, 2023; Lloyd, 2005). Yet, the inverse—positioning students as the expert—is also an example of nosotrx. By positioning students to "take charge and explain," this is not nos/otrx or an example of being in "in each other's world" (Keating, 2000, p. 215) but instead role reversal where univocal communication is not enacted by the teacher but by the student. Rose's change in her role at IDM also echoed efforts by preservice teachers as they attempted to share authority (Blanton et al., 2001b; Kinser-Traut & Turner, 2020). But unlike Annie, the teacher-student interaction that Rose describes may not have been an example of the "back and forth" dynamic or engaging in "learning from each other." Perhaps, Rose and the students took turns as the expert, which is not precisely an example of interdependence. Instead, interdependence occurs when Rose and the student engage with multiple conceptions of mathematics (each other's mathematical sense-making).

These previous excerpts described various moments of nos/otrx. Annie described how at IDM, there was no authority, and "not knowing" was an opportunity to learn from students. In the classroom, her goal was to emphasize "her way is not the only way to do math," so students could potentially suggest their strategies for doing mathematics. Annie described a "back and forth" dynamic where she and the students at IDM learned from each other. Clearly, she highlighted the bidirectionality of working interdependently. Rose, though grappling with the tension of redefining the teacher's role, expressed a desire to hear what a student had to say regarding their mathematical understandings and struggles which is fundamental aspect of nos/otrx. Yet, working with youth presents complex dynamics, so all three also described moments of nosotrx. Let us consider how these three women's goals played out in their first year of teaching and if what they had learned in their teacher education program was "washed out" (Zeichner & Tabachnick, 1981, p. 7).

Moments of Nos/otrx During the First Year of Teaching

The following excerpts are three examples from each of the beginning student teachers describing moments of nos/otrx and nosotrx. The context of each particular beginning secondary mathematics teacher seemed to have afforded and constrained the possibility for interdependence to occur.

"It really opened my eyes to how students think . . . and interpret"

Upon being offered her first full-time teaching position, Taylor was told teachers can offer a colloquium, an elective course for students. She was so excited about the opportunity she offered to lead an origami colloquium. It was accepted, and she led this election during her first semester of teaching.

[M]y colloquium is origami. And it is the most relaxing, chill teaching experience. They come in and start by finding out what we're going to be making that day and then we spend the rest of the 100 minutes making origami. I get to talk to students about movies and what they did that weekend. I don't feel like I'm teaching the kids, I feel like we're just hanging out. It is [also] very much like IDM because once I see that a student has got it, I take them and say, "Hey, can you help out so-and-so over there? They're struggling." And they do! And it's awesome.

[T]here's some days where I'm like, "Students, we're going to see what you know. Take a Chromebook, explore the websites that we always use, find something that you want to make and then try and make it. And I will come help people who feel like they need help." And then a lot of the students go find something together and be like, "Oh let's try making this one." And then they can help each other make it.

I really hope I'll get to teach the origami class because it really opened my eyes to like how my students think and how they interpret what I ask them to do.

As Taylor described the origami colloquium during her first interview, she described some aspects of nos/otrx. First, the colloquium allowed her to "hang out" with the student. Recalling what she hoped, to avoid being the authority in her classroom, it seemed she was able to reject this status during this elective course. Depending on what Taylor's expectations were for the students, there was a high degree of freedom for how and what origami constructions students would do in class. By allowing students to choose how and what they did during the colloquium, this was an opportunity for "opening [Taylor's] eyes" to how they think and interpret what she asked them to do. Again, Taylor seemed to recall how at IDM, there was no one authority, and it was an opportunity to "learn about what students knew." That Taylor was able to propose (and have accepted) a course akin to IDM is a clear example of the kind of recontextualization (Ensor, 2001; Kinser-Traut & Turner, 2020), a "link-to-practice" (Lloyd, 2013, p. 105) and enacting a personalized pedagogy (Hamilton, 2023, p. 96) that an experience such as an after-school mathematics club can inform the decisions and practices of a first-year teacher.

Yet, as Taylor described aspects of nos/otrx, she also described moments of nosotrx. First, Taylor's description of the colloquium as "hanging out" with students might have been missing an essential element from Annie's description of "buddy-buddy" interactions with students at IDM: "learning from each other." Taylor may have rejected her role as the authority, but it is unclear how much dialogic communication (Blanton et al., 2001b) occurred with students to be in "each other's [mathematical] world" (Keating, 2000, p. 215). It seemed she may have taken a more passive approach with students' free exploration, whereas

she could have probed for mathematical thinking and process during colloquium. This approach is reminiscent of the early practices of a student teacher who took an off-stage, passive approach to probing student thinking (Kinser-Traut & Turner, 2020). Closely related, it seems the common and direct transfer from IDM was positioning students as competent (Cohen et al., 1995), which is an example of nosotrx. She stated that "very much like IDM . . . once I see that a student has got it, I take them and say, 'Hey, can you help out so-and-so over there?'" which is an important teacher-move to make classrooms more student-centered but not necessarily enacting interdependence with students. As was noted by researchers, making particular stances and practices explicit (Ensor, 2001; Kinser-Traut & Turner, 2020; Lloyd, 2013) could increase the likelihood of these moves occurring in mathematics classrooms of first-year teachers, which in Taylor's case, she was able to enact during the origami colloquium.

Perhaps moments of nos/otrx during IDM were recommended implicitly to some cohorts, whereas the practice of positioning students as experts was stated explicitly to Taylor's cohort of volunteers. Even during days when Taylor declared it was a day "to find out what you [students] know," the students were again positioned as experts of origami construction. Even though Taylor wanted to relinquish her role as the math authority, she described her role as "coming to help" students when necessary. Her role was similar to that at IDM, a helper. As well intentioned as a helper role is as a mathematics teacher, a degree of authority and expertise was maintained by enacting this role during the origami colloquium. Taylor seemed to have the best intentions in replicating IDM as a course elective, but moments nosotrx (dependence) seemed to have occurred more often than moments of nos/ortx (interdependence). Yet it is noteworthy of her transference of IDM into the classrooms during her first year of teaching.

"Bouncing ideas off of each other"

During the following excerpt, Annie described a lesson involving congruent triangles. She expressed the importance of opportunities for students to practice concepts that would lead to understanding, and underlying this approach was her desire for students to collaborate. By providing opportunities for practice, moments of nos/otrx among students may have been possible.

I like to give students a chance to practice the material, with people in their groups because they can discuss it with each other ... work through problems or exercises together to bounce ideas off of each other ... [this] allows them to kind of find their own misconception without me having to tell them. Which I think is a lot more beneficial than them doing a problem wrong and me finding out what's wrong with it and then telling them. I think giving them time to figure out what they did wrong is really important because they'll figure out what their own mistake was.

So, within her first year of teaching, Annie may not have engaged in moments of nos/otrx, but she did attempt to provide the opportunity for interdependence among her students. The purpose of having students collaborate was to have them work and discuss with each other and, ultimately, to "bounce ideas off of each other" and "find their own misconceptions." This dynamic was similar to her description of IDM, when she engaged in a "back and forth" with students that resulted in them "helping each other." It seemed Annie witnessed this dynamic among students when working in groups. Pedagogically, she felt it beneficial for students to identify their own mistakes and misconceptions rather than her telling them. This approach seemed to parallel the "contribute" and perhaps "instructive" communication (Brendefur & Frykholm, 2000, p. 128) approach some preservice teachers attempted to facilitate to help students deepen their mathematical understanding as well as placing the onus on them to deepen their understanding of the content (Hamilton, 2023). This approach seems to reflect the personalized pedagogy (Hamilton, 2023, p. 96) that was informed by her experience volunteering at IDM and was able to enact in her classroom. While Taylor created a class to challenge the notion of "no one math authority," Annie attempted to provide opportunities for her students during lessons to share their mathematical sense-making in her geometry class. Yet, there were aspects of nosotrx (independence) that Annie also described

There are two aspects of nosotrx (independence) I noticed in what Annie recalled. First, it seemed students engaged in moments of being "in each other's [mathematical] world" (Keating, 2000, p. 215) but not with Annie. It is unclear to what degree she engaged with the students in the "back and forth" dynamic she described at IDM. When facilitating geometry inquiry lessons, interdependence might be challenging to enact, because when students seek the teacher's assistance, they tend to position the teacher as the primary knower or expert (González & Dejarnette, 2015). This is not to say interdependence is impossible for the teacher and students to engage in the classroom but perhaps challenging considering the ascribed role teachers and students take on.

Second, it is unclear to what degree the students were able to maintain moments of nos/otrx. Group dynamics are fraught with negotiating authority and expertise for many reasons such as gender, personality, or perceived competency (Esmonde et al., 2010). The students were collaborating, but it is unclear to what degree they were "dependent on each other" and "in each other's mathematical world" (Keating, 2000, p. 215). Pedagogically, it is a sound approach to have students work in groups, and notable that Annie implemented a "personalized pedagogy" (Hamilton, 2023, p. 96) for promoting group work as a first-year teacher, but perhaps promoting a culture of nos/otrx, might increase the likelihood for more equitable collaborations and participation in the pursuit of co-constructing mathematical knowing together, with each other, and the teacher.

"Catching myself being more of a student"

In this last excerpt, Rose described an instance where she momentarily rejected her role as the mathematics teacher. She described this as part of her hope of promoting to her students that mathematics could be fun and a "puzzle to be solved":

I want my students to enjoy working with math, I think that math is just fun to do and to figure out and is a puzzle to be solved but they don't see it that way. They see it very much as something they have to do. I've gotten a few kids to kind of get on board of, you know, "Let's challenge ourselves!" and they'll humor me every now and then. A couple of times some

of them will ask me about a question that I haven't solved, and it'll be, you know, quirky enough that I'll have to sit and think about it for a second before I can actually work through it. And, um, every now and again I'll catch myself being more of a student working through a problem than a teacher trying to help my students' learning. And they'll see me figure a problem out and be like, "Oh my gosh, yeah! Yes! This one, guys look, it's this. Isn't that cool?" And they're like, "What are you talking about?" Like, all right, back in teacher mode you probably need that explained or maybe just more talk to students at a little bit of a slower pace but they all kind of laugh for a second because they see me get so excited about it, "Oh my gosh, guys I figured it out how cool is it that I figured it out?"

Of the three beginning secondary mathematics teachers, Rose was the only one who described a moment of nos/otrx where she took on the role of a student. The circumstances were a little surprising because it was an unsolved homework problem. Generally, she was confident that at a glance she could "figure it out," but not always. As a result of her students successfully challenging her, she momentarily took on the role alongside them as a student. Rose's experience as the mathematics teacher seems similar to Annie's and other preservice teachers at IDM, where "not knowing" became a learning opportunity (Gerardo, under review). After all, Rose "caught herself being more of a student" and "worked through the problem." For Rose, there were also pedagogical implications, because she was able to model having fun and the challenge of doing mathematics during class. This approach seemed to be in contrast to the tendency of sharing authority (Blanton et al., 2001b; Hamilton, 2023; Kinser-Traut & Turner, 2020) as simply positioning students as experts during whole group mathematical discussions as well as of maintaining the role as the "ultimate authority" in the classroom (Hamilton, 2023, p. 96). Yet, it seems this moment of rejecting her role as the math authority was fleeting.

Rose described various aspects of nosotrx, even though she "caught herself being more of a student" while working on an unfamiliar homework problem. While momentarily rejecting her role as the teacher, working interdependently with students seemed limited. First, it is unclear to what degree she was "dependent on others [the students]." It seems she took on the role of the student, worked on the homework problem herself, and not listening or "hearing ... students' struggle" with the homework problem. Then, soon after her moment of clarity, she reverted to teacher mode to explain and "talk at a slower pace." Even though it seemed she was sharing authority (Blanton et al., 2001b; Kinser-Traut & Turner, 2020) with students, it seemed she felt compelled to maintain univocal or unidirectional communication (Blanton et al., 2001b) with the students as well as retain her role as the "ultimate authority" (Hamilton, 2023, p. 96). She admitted to enduring this tension while discussing her experience at IDM related to redefining how to help students. Although an unfamiliar homework problem for Rose and a challenging problem for students seemed ripe for dialogic communication (Blanton et al., 2001b) and gaining insight to "be in each other's world" (Keating, 2000, p. 215), but the inertia of maintaining the teacher mode possibly made interdependence difficult to maintain. This tension parallels research related to the vision of teaching mathematics where what they describe as their approach for teaching reform-based mathematics is not always what they do when working with students (Makar, 2024).

The last three examples discussed by the three beginning secondary mathematics teachers described moments of nos/otrx. Taylor facilitated an origami elective course that was an opportunity for her to "open her eyes" to what students know and interpret related to the origami constructions. And similar to IDM, there seemed to be no single math authority in her classroom, whereas Annie, during her geometry classes, attempted to engage her students in opportunities to "bounce ideas off of each other" which parallel the "back and forth" she experienced with students at IDM. Although Rose may not have engaged in "back and forth" with the students, she momentarily rejected her role as the teacher and took on the role as a student. Working through an unfamiliar homework problem, she, too, seemed to be able "to be [in the students' mathematical] world" (Keating, 2000, p. 215). Yet, it must be noted these moments of interdependence were inconsistently enacted and sustained. After all teaching mathematics is complex, dynamic, challenging for beginning secondary teaching (Corven et al., 2022; Ensor, 2001; Goos, 2005; Kinser-Traut & Turner, 2020; Kirwan & Edwards, 2023; König et al., 2024; Lloyd, 2013; Peressini et al., 2004). In the following section, I offer possible explanations for the occurrence of both nos/otrx and nosotrx.

DISCUSSION

The three beginning secondary mathematics teachers engaged in moments of interdependence with students at a community field placement, an after-school mathematics club, as pre-service teachers. The informal field placement, the use of mathematical puzzle and games seemed to afford opportunities for the pre-service teachers to grapple with multiple mathematical worlds (Keating, 2000, p. 215), their own and that of the students. As a result, as first-year secondary mathematics teachers, they seemed to have applied and considered interdependence, although not as often as I expected, as first year teachers. The following section I address the research questions and possible reasons for the transfer of this stance and practice from an after-school club and into mathematics classrooms.

Opportunities for Personalized Pedagogy

Perhaps community-based field placements are also opportunities to develop personalized pedagogy (practices) (Hamilton, 2023). Whereas classroom field placements are formal teaching settings with a mentor teacher, a curriculum to teach and assessments to conduct, informal field placements allow for more flexibility and opportunities for experimentation because there is no required curriculum to teach and assessments to conduct. Even the relationships and interactions (Gerardo, under review) that pre-service teachers have with students can take on different dynamics in community-based field placements. In other out-of-school field placements, these relationships were salient for pre-service teachers (Vomvoridi-Ivanovic, 2012). Pre-service teachers had the opportunity to talk in Spanish with students, could connect informally and discuss interests and hobbies, and

get to know students holistically (Bussert-Web, 2011; Vomvoridi-Ivanovic, 2012). Additionally, after-school clubs can be spaces where mathematics and STEM tasks can also be the focus of discussion (Flavin et al., 2024; LópezLeiva et al., 2022). The focus of this current study was not the pre-service teacher-student relationships and how these occur but more specifically moments when pre-service secondary mathematics teachers grapple with mathematical content and problem solving (their own and that of the students that they work with). Perhaps the mathematical games and puzzles that the students (and pre-service teachers) worked on, possibly increased the likelihood that pre-service teachers enacted and developed a personal pedagogy to have students share and discuss their approaches for the games and puzzles. In addition, the preservice teachers were often unfamiliar with these same puzzles and games, possibly also opened the opportunities for them to reject their role as the expert and learn, alongside the students, how to play or solve the various challenges at the after-school club. Perhaps these circumstances of the flexibility of what can be done during an after-school club and the unfamiliarity of the activities by the pre-service teachers possibly compelled them to learn from and with the students. In turn, these experiences with students in an after-school math club were opportunities for developing a personalized pedagogy that was not solely about developing relational pedagogy or relationships with students but interdependence when mathematical sense-making and problem solving were the focus of their interactions.

Beliefs and Vision

Researchers have argued that practices that mathematics teacher education programs promote and those that are enacted / embraced by pre-service teachers occur because these recommendations align with prior held beliefs (Goos, 2005; Hamilton, 2023; Kinser-Traut & Turner, 2020; Lloyd, 2013). Yet other research has demonstrated that beliefs and visions can be impacted and can be changed through a variety of experiences during teacher education programs (Brown, 2023; Kirwan & Edwards, 2023; Poole-Parrilla, 2023). In this current study it is possible that enacting interdependence at an after-school mathematics club and then considering enacting aspects of interdependence as beginning secondary mathematics teachers occurred as a result of a combination of aligning to their own beliefs and their experiences as RS1 scholars.

First factor to consider is that the after-school mathematics club was a component of an antiracist secondary mathematics education program. As described previously, this program centered the experience of historically marginalized students in order to prepare secondary mathematics teachers to work with minoritized students. This is not to say that all three-beginning secondary mathematics teacher's beliefs and this vision for working with Black and Latinx students was changed but, minimally, impacted by the participation in a two-year anti-racist program. The majority of the students in the after-school mathematics club were Black and Latinx, so it was a community field placement that was diverse and where the pre-service teacher worked with students ethnically different from themselves. Perhaps the developing stances regarding working with minoritized students increased the likelihood of engaging in moments of interdependence, in particular for working with students. The impact of the overall program cannot be diminished but to what degree the program of the after-school mathematical club impacted the teacher's stances and vision was not quantitatively measured but qualitatively analyzed. Second, field placements seem to impact the practices of pre-service teachers (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Lloyd, 2005) and the act of teaching can also be learning opportunities for pre-service as well as in-service teachers (Blanton et al., 2001b; Brendefur & Frykholm, 2000; Kirwan & Edwards, 2023; Makar, 2024). So, the double-factor of a program that included a community-based field placement that informed and impacted the practices of these beginning secondary mathematics teachers may have increased the likelihood of moments of interdependence occurring in their classrooms.

Active Agency

Finally, as a result of enrolling in an anti-racist mathematics teacher program and participating at a community-based field placement where the opportunity to develop a personal pedagogy for engaging in moments of interdependence, perhaps another explanation is "active agency" (Goos, 2005, p. 55). Through a sociocultural lens, this concept argues experiences in various settings provide opportunities for developing sophisticated practices that align with a mathematics teacher's beliefs. Perhaps experiences such as IDM have the potential for stances and practices such as interdependence to continue into the classroom setting. After all, IDM was an extensive community-based field experience that occurred over a period of three semesters for a total of 15 weeks. Such a prolonged and extensive field experience may help explain why and how a stance and practice of interdependence also occurred during participants' first year of teaching. This parallel results by other researchers documenting the transfer of recommended stances and practices into the first year of teaching (Corven et al., 2022; Ensor, 2001; Goos, 2005; Kinser-Traut & Turner, 2020; Kirwan & Edwards, 2023; König et al., 2024; Lloyd, 2013; Peressini et al., 2004). Specifically, teacher education experiences, such as an after-school mathematics club, that can "link-to-practice" (Lloyd, 2013, p. 105) seem to be a promising approach for increasing the likelihood of maintaining and enacting these practices into the first year of teaching. Therefore, active agency helps elucidate how participating in different contexts that centered student's mathematical thinking as well as enacting moments of interdependence increased the likelihood that beginning teachers consider enacting these moments during their first year of teaching.

LIMITATIONS

This study was a qualitative interview research study. Some field notes were used to inform the personalized questions for each scholar, but the primary data source were interviews. These interviews started one academic semester from their time volunteering at IDM. Their recollections were possibly vague, and I was asking for them to recall specific interactions they had with students. Interviews could have been conducted much closer to their time of volunteering at IDM or video recordings taken during IDM sessions so that the scholars would have an opportunity to respond to the interactions they had with students. I had less

concern regarding their interviews during their first year of teaching because I could probe for specific interactions that occurred prior to conducting the interviews (i.e., the day of, or the day before or the week before). In-classroom observations or recordings could have also been conducted (this was a logistical choice during the time of data collection) to triangulate the findings from the interviews. And, since these were self-reported recollections of their experience, and I was a research assistant on the longitudinal study, there may be concerns regarding them wanting to please me as a member of the research team or censor themselves from certain actions or thoughts that may not have aligned with our anti-racist mathematics teacher program. I acknowledge these possibilities in participants' responses. Last, it is difficult to extricate the RSP program, IDM, the general teacher education program the participants completed as well as the participants' beliefs, vision and practices. All these components of the teacher's professional development are inter-related and informing each other and identifying to what degree each contributed to how the participants interacted with students during their first year of teaching is difficult to ascertain.

Future research can methodologically include other data sources (i.e., artifacts, audio and video recordings) as well as document students' perspective of moments of interdependence with not only the teachers but also with other students in both formal and informal mathematics contexts.

CONCLUSION

Community-based field placements, specifically after-school mathematics clubs, are instructive and complementary to classroom field placements for pre-service teachers. In this study, three beginning secondary mathematics teachers were able to engage in moments of nos/otrx with students. That is, they grappled with multiple mathematical and problem-solving perspectives, their own and that of the students. And as first-year mathematics teachers, to varying degrees, they engaged or provided opportunities to increase the likelihood of nos/otrx to occur in their classrooms. Yet these are complicated teacher-student dynamics and classrooms are constrained by pacing plans, curriculum, and assessments to conduct moments of nos/otrx that explained why these moments did not occur more often. Nevertheless, it is notable that these beginning mathematics teachers did engage in these moments with students. The following are some implications regarding moments of nos/otrx for mathematics teacher education research and programs

Nos/otrx (interdependence) (Keating, 2022) and other Chicanx theoretical frameworks are not common frameworks applied in mathematics education (except for Gerardo, under review; Gutiérrez, 2012, 2015, 2016, 2017; Gutiérrez et al., 2023). Further theorization of nox/otrx in mathematics education is necessary to understand the contribution and knowledge-production possible from frameworks uncommonly applied to education research. Mathematics classrooms are not spaces devoid of the larger social and political factors (e.g., racism, sexism, classism, xenophobia, etc.). Frameworks that are developed as a result of marginalized experiences such as nos/otrx (Keating, 2022) have a place in mathematics education where new insight can result from an intersubjective framework that challenges social categories and binaries that may assist in our efforts to deconstruct the roles and binaries (expert / novice and teacher / student) extant in classrooms.

Perhaps mathematics teacher education programs could institutionalize community-based field placements. In these out-of-school field placements, there is a much wider degree of agency for pre-service teachers to enact / practice particular beliefs / vision for not only pedagogy and lesson planning, but also for how they work with students, specifically historically marginalized students. Not only in terms of relational pedagogy (Kinser, 2025), where the focus is social and personal connection with students but interactions where the mathematical sense-making is centered (nos/otrx) (Gerardo, under review). In this manner, pre-service teachers listen to student's sense-making and problem solving alongside their own understanding of puzzles, games, and even mathematical problems. During these moments of nos/otrx, the pre-service teacher and students are in solidarity and mindful of multiple mathematical perspectives or "in each other's [mathematical] worlds" (Keating, 2000, p. 215). This is not to say that preservice teachers may not develop a lesson or a formal mathematical task but how they engage with students can differ beyond a teacher (expert) and student (novice) binary and instead engage in dialogic (Blanton et al., 2001b) mathematical discussions where the pre-service teachers and students are both engaged in the process of sense-making and problem solving a game, puzzle or mathematical task. As a result, moments of nos/otrx may become a practice that pre-service teachers may consider engaging in more constrained sites such as classroom field placements or their own classroom as in-service teachers.

And when community-field placements are more common, then could additional support continue during the first years of beginner mathematics teachers? Researchers have recommended continued support from teacher education programs during the induction years of beginning teachers (Brown, 2023; Condon, 2024; Hayes, 2023; Kirwan & Edwards, 2023; Makar, 2024). If, as argued that experienced novice teachers (Kirwan & Edwards, 2023) develop more complex and flexible approaches to their teaching that they feel more comfortable enacting more reform-based approaches for teaching of mathematics, could continued support to engage in moments for nos/otrx where centering student thinking and opportunities to learn with students may increase the likelihood that moments of nos/otrx will occur more frequently. Perhaps students, who have traditionally not felt welcomed in mathematics classrooms will feel that they are valued and have mathematical ideas to contribute to the classroom because moments of nos/otrx is the norm their mathematical classroom. And just as important, by engaging in these moments, teachers model the sociocultural aspect of learning mathematics by engaging in mathematical discussions with students and further establishing being in each other's (mathematical) world as a part of the classroom culture. Last, this explicit process of engaging with multiple mathematical ideas might make explicit the process of learning to teach by teaching (Blanton et al., 2001b; Brown, 2023; Makar, 2024) as teachers engage in moments of nos/otrx with students and learn alongside of them and engage in the social construction of mathematical knowledge.

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