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Designing metacognitive scaffolding in flipped learning to enhance students' visual–spatial reasoning: Evidence from geometry instruction

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ABSTRACT

The development of students' visual–spatial reasoning is a persistent challenge in geometry learning, where abstract representations and spatial transformations require high levels of cognitive engagement. Although flipped learning has been widely implemented to promote active learning, its effectiveness largely depends on the quality of instructional design and pedagogical support provided during in-class activities. This study investigates how metacognitive scaffolding can be intentionally designed and embedded within flipped learning to enhance students' visual–spatial reasoning. An explanatory sequential mixed-methods design was employed in secondary geometry instruction involving 62 students assigned to experimental and control groups. Quantitative data were collected through pre- and post-tests of visual–spatial reasoning, while qualitative data were obtained from classroom observations and semi-structured teacher interviews. The results indicate that students who experienced metacognitive scaffolding within a flipped learning environment demonstrated significantly greater gains in visual–spatial reasoning than those in a conventional flipped classroom, with a large effect size. Qualitative findings further reveal that scaffolding prompts supported students' planning, monitoring, and reflective evaluation during problem-solving activities. Overall, the findings suggest that flipped learning environments become more cognitively productive when structured metacognitive scaffolding is deliberately incorporated into the learning design.

Keywords: metacognitive scaffolding, flipped learning, learning design, visual–spatial reasoning, higher-order thinking

INTRODUCTION

The rapid expansion of technology-enhanced learning environments has reshaped contemporary educational practice, particularly by enabling instructional models that emphasize active learning and student engagement (Burrows et al., 2024; Michalsky & Bakrish, 2024; Sappaile, 2025). Within this broader transformation, there is increasing recognition that effective instruction must go beyond content delivery to intentionally support learners' cognitive and metacognitive processes (Michalsky, 2024). As classrooms become more diverse in terms of students' prior knowledge, learning pace, and cognitive readiness, the design of learning environments that foster higher-order thinking and adaptive problem-solving skills has become a central concern in educational research and practice (OECD, 2023; Stern & Hertel, 2022; Toikka et al., 2024).

In mathematics education, visual–spatial reasoning is widely regarded as a foundational component of students' conceptual understanding and problem-solving competence, especially in geometry and other STEM-related domains (Kesici et al., 2011). From a mathematics didactics perspective, the development of visual–spatial reasoning in geometry cannot be treated as a generic cognitive skill; it is deeply embedded in how learners construct and interpret geometric representations. Presmeg (2006) argues that visualization in mathematics learning involves the ability to apprehend sub-configurations within a figure (perceptual apprehension), to mentally reconstruct figures under transformation (operative apprehension), and to translate between visual and symbolic representations (sequential and discursive apprehension). Similarly, the van Hiele model of geometric thinking suggests that students progress through distinct levels of reasoning, from visual recognition of shapes to analytical understanding of their properties and relationships, before they can reason deductively about geometric transformations. These didactic frameworks indicate that instructional support for visual–spatial reasoning in geometry must go beyond general metacognitive prompting; it must be tailored to the specific representational and reasoning demands of geometric transformation tasks, such as coordinating multiple viewpoints during rotation or preserving invariant properties during dilation. Visual–spatial reasoning

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involves the ability to mentally manipulate objects, interpret spatial relationships, and coordinate multiple representations, all of which are essential for engaging with abstract mathematical ideas. Students with well-developed spatial reasoning are more likely to demonstrate flexible thinking, deeper conceptual understanding, and the ability to transfer knowledge across problem contexts (Wahab Abdi & Desfandi, 2020). International large-scale assessments, such as the Programme for International Student Assessment (PISA), have further emphasized the importance of structural reasoning and adaptive problem solving, reinforcing the need for instructional approaches that deliberately cultivate both spatial and metacognitive competencies in school mathematics (Awi, Upu, et al., 2025; Ozkale & Ozdemir Erdogan, 2022; Rum, 2025).

Despite its recognized importance, the development of visual–spatial reasoning remains a persistent challenge for many students (Putri & Novaliyosi, 2026; Wahab Abdi & Desfandi, 2020). Geometry learning, in particular, requires learners to engage with abstract representations and spatial transformations such as translation, rotation, reflection, and dilation that place substantial cognitive demands on working memory and mental imagery (Awi & Arwadi, 2018; Jolly et al., 2026). Many students struggle to visualize and manipulate geometric objects, especially when instruction relies heavily on static representations and procedural explanations. As a result, students' understanding of geometric concepts often remains fragmented, and their ability to apply spatial reasoning to novel problem-solving situations is limited (Dassa et al., 2019; Dassa, Ihsan, et al., 2024; Talib et al., 2024). These challenges suggest that traditional teacher-centered instructional approaches may be insufficient to support the development of robust visual–spatial reasoning.

In response to these challenges, educational researchers have increasingly advocated for instructional designs that actively engage students in exploratory, reflective, and cognitively demanding learning activities. Approaches that integrate interactive tasks, dynamic visualizations, and opportunities for metacognitive reflection have been shown to support deeper conceptual understanding and the development of higher-order thinking skills (Andana et al., 2025; Awi, Sutamrin, et al., 2025; Michalsky, 2024). From this perspective, learning environments should be designed not only to present mathematical content but also to provide structured support that helps learners plan their approaches, monitor their understanding, and evaluate the effectiveness of their problem-solving strategies .

The flipped classroom model has emerged as a prominent instructional approach aligned with these design principles. By shifting direct instruction to pre-class learning activities such as video lectures or digital materials and reserving classroom time for problem solving, discussion, and collaborative work, flipped learning aims to create opportunities for deeper cognitive engagement during face-to-face instruction (Fadli et al., 2022; Gonzales & Gonzales, 2024; Katona & Gyonyoru, 2025). Research suggests that flipped classrooms can promote active learning and improve student outcomes in mathematics; however, these benefits are not guaranteed. Meta-analytic evidence indicates that the effectiveness of flipped learning depends largely on the quality of in-class instructional design and the extent to which students receive appropriate pedagogical support. Without such support, flipped classrooms may function merely as a reorganization of instructional time rather than as environments that foster meaningful learning processes (Barrios et al., 2022; Zahra et al., 2025).

One instructional approach that holds particular promise for enhancing the cognitive quality of flipped learning environments is metacognitive scaffolding. Metacognitive scaffolds typically take the form of prompts or guiding questions that encourage learners to plan their problem-solving strategies, monitor their progress, and reflect on the outcomes of their thinking. Such scaffolding has been shown to support self-regulated learning, reduce cognitive overload, and promote deeper engagement with complex tasks. In mathematics education, metacognitive scaffolding can help students articulate their reasoning, identify misconceptions, and refine their problem-solving strategies, thereby supporting more effective learning processes (Awi & Arwadi, 2018; Menekse et al., 2025; Nickl et al., 2024).

When combined with interactive tasks and visually rich representations, metacognitive scaffolding may be particularly effective in supporting the development of visual–spatial reasoning. Geometry learning provides a compelling context for examining this integration, as spatial tasks require learners to coordinate visual information with abstract mathematical reasoning. By prompting students to reflect on how they interpret and manipulate spatial representations, metacognitive scaffolding can help them develop greater awareness of their reasoning processes and adopt more effective strategies for engaging with spatial tasks.

Although previous studies have documented the potential benefits of flipped classrooms and metacognitive scaffolding, empirical research that systematically integrates these elements within a coherent learning design remains limited. Existing studies often examine flipped learning or metacognitive scaffolding as separate instructional components or focus primarily on learning outcomes without explicating the underlying design principles and learning mechanisms that support students' reasoning development (Dassa et al., 2019; Fadli et al., 2022). Consequently, there is a need for research that moves beyond isolated interventions to examine how metacognitive scaffolding can be deliberately designed and embedded within flipped learning environments to support visual–spatial reasoning in a theoretically grounded and pedagogically meaningful manner.

To address this gap, the present study investigates a learning design that integrates metacognitive scaffolding into flipped learning with the aim of enhancing students' visual–spatial reasoning. Using secondary geometry instruction as an empirical context, this study adopts an explanatory sequential mixed-methods approach to examine both learning outcomes and pedagogical processes associated with the proposed design. Specifically, the study seeks to (1) examine the extent to which a flipped learning design incorporating metacognitive scaffolding enhances students' visual–spatial reasoning, (2) compare students' reasoning performance before and after the implementation of the design, and (3) explore teachers' perceptions of the feasibility and instructional value of the design. By focusing on the intentional design and implementation of metacognitive scaffolding within flipped learning, this study aims to contribute design-oriented insights that can inform the development of cognitively productive learning environments across educational contexts.

Table 1. Research design for assessing the impact of metacognitive scaffolding on students' visual–spatial reasoning in a flipped classroom

Group	Pretest	Treatment	Posttest
Experimental	O_1	Flipped Learning with Metacognitive Scaffolding	O_2
Control	O_3	Conventional Flipped Classroom	O_4

Table 2. Key components of the learning design and instructional conditions

Learning Phase	Experimental Group (Flipped Learning with Metacognitive Scaffolding)	Control Group (Conventional Flipped Learning)
Pre-class activities	Instructional videos and digital materials introducing geometric transformations	Instructional videos and digital materials introducing geometric transformations
In-class focus	Interactive problem solving supported by structured metacognitive scaffolding	Interactive problem solving without explicit metacognitive scaffolding
Planning stage	Prompts guiding students to identify goals, relevant information, and solution strategies	No structured prompts for planning
Monitoring stage	Prompts encouraging students to check progress, verify representations, and adjust strategies	Monitoring guided implicitly through teacher explanations
Evaluation stage	Prompts supporting reflection on solution effectiveness and alternative approaches	Reflection conducted informally without structured guidance
Role of teacher	Facilitator supporting students' metacognitive engagement	Facilitator focusing primarily on content clarification
Intended cognitive focus	Visual–spatial reasoning and metacognitive regulation	Visual–spatial reasoning through procedural practice

MATERIALS AND METHODS

This study employed an explanatory sequential mixed-methods design, consisting of a quantitative phase examining changes in students' visual–spatial reasoning and a qualitative phase exploring how the learning design was enacted and perceived by teachers in classroom practice. The following sections describe the research design, participants and context, instructional intervention, data collection instruments, analytical procedures, and ethical considerations.

Research Design

This study employed a mixed-methods approach using an explanatory sequential design (Négyesi et al., 2026) to examine the implementation and outcomes of a learning design integrating metacognitive scaffolding within flipped learning. The quantitative phase examined changes in students' visual–spatial reasoning, while the qualitative phase provided explanatory insights into how the design was enacted and perceived by teachers. The research was conducted in two sequential phases. The first phase adopted a quasi-experimental pretest–posttest design with a non-equivalent control group, using two intact Grade 11 classes with comparable initial levels of visual–spatial reasoning as the experimental and control groups (**Table 1**). The intervention was implemented over an eight-week flipped learning period; the experimental group received structured metacognitive scaffolding during in-class activities, while the control group did not (**Table 2**). The second phase involved semi-structured teacher interviews and classroom observations to deepen interpretation of the quantitative findings, capturing teachers' experiences and perceptions of the learning design's implementation (**Figure 1**).

In the second phase of the study, qualitative inquiry was conducted to deepen the interpretation of the quantitative findings. Semi-structured interviews with mathematics teachers and classroom observations were used to capture teachers' experiences, perceptions, and reflections related to the implementation of the learning design. These qualitative data served to illuminate the pedagogical mechanisms underlying the observed quantitative outcomes, including how metacognitive scaffolding supported students' planning, monitoring, and reflective evaluation during problem-solving activities. By integrating quantitative and qualitative evidence, the explanatory sequential design enabled a more comprehensive understanding of both the effectiveness and the instructional dynamics of the proposed learning design.

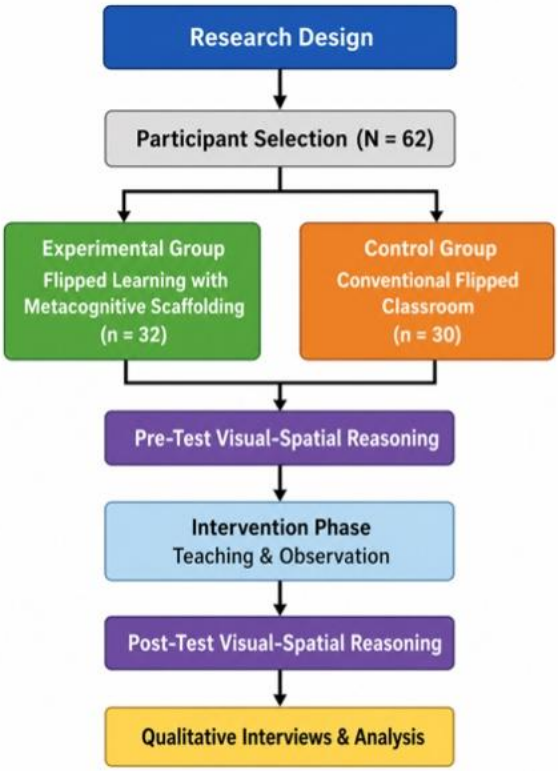


Figure 1. Flowchart of the explanatory sequential mixed-methods design integrating metacognitive scaffolding within flipped learning (Source: Authors' own elaboration)

Participants and Research Context

The study was conducted in a public senior secondary school involving 62 Grade 11 students from two intact classes: an experimental group (n = 32) and a control group (n = 30). Both classes followed the same national curriculum, covered identical geometry content (translation, rotation, reflection, and dilation), and were taught by teachers with comparable experience and instructional schedules. Pretest results confirmed comparable initial levels of visual–spatial reasoning between groups, minimizing confounding factors unrelated to instructional design. The eight-week intervention maintained consistent instructional time, lesson frequency, and curriculum pacing across groups; instructional differences between conditions are detailed in **Table 2**. Three mathematics teachers participated in implementing, observing, and reflecting on the design, providing contextual insight into classroom dynamics and feasibility. Students' prior familiarity with digital pre-class materials reduced the likelihood that observed effects stemmed from novelty of the flipped format itself.

Learning Design and Intervention

The intervention examined in this study was designed as a flipped learning environment that deliberately integrated metacognitive scaffolding to support students' visual–spatial reasoning. The learning design was informed by principles of student-centered learning and metacognitive regulation, with the aim of creating cognitively productive classroom activities rather than merely reorganizing instructional time (Awi, Upu, et al., 2025; Awi & Arwadi, 2018). The design emphasized the alignment of pre-class and in-class learning activities, ensuring that students arrived in class prepared to engage in higher-order problem-solving tasks.

Flipped learning structure and metacognitive scaffolding

The flipped learning structure was implemented identically in both groups over eight weeks: students engaged with pre-class instructional videos and digital materials, followed by in-class problem-solving activities on geometric transformations. The instructional distinction between groups lay solely in the nature of in-class support. In the experimental group, activities were enriched with structured metacognitive scaffolding prompts guiding students through planning, monitoring, and evaluation stages of problem solving designed to be explicit but non-intrusive. The control group received conventional flipped instruction focused on content clarification and procedural guidance, without structured metacognitive prompts. **Table 2** summarizes the key instructional components and distinctions between the two conditions in detail.

During the planning stage, students were guided by prompts such as 'What information is given in this problem, and what transformation is required?' or 'Which properties of the figure must remain unchanged during this transformation?' During the monitoring stage, students received prompts such as 'Does the image you have drawn match the direction and distance specified by the transformation rule?' or 'Have you checked whether the coordinates of each vertex are consistent with the transformation

Table 3. Planning, monitoring, and evaluation prompts for each geometric transformation task

Task	Planning	Monitoring	Evaluation
Task 1 – Translation of a Triangle	"What vector defines this translation, and how does each vertex shift?"	"Does the image triangle match the direction and distance specified by the translation vector?"	"How would you verify your answer using a different vertex of the triangle?"
Task 2 – Reflection of a Square across y = x	"Which line is the axis of reflection, and what property of the figure must remain unchanged?"	"Are the distances from each vertex to the line of reflection equal on both sides?"	"Could you check your image by mentally folding the figure along the axis of reflection?"
Task 3 – Rotation of a Point	"What is the center and angle of rotation, and in which direction does the point move?"	"Does the position of the rotated point match a 90° counterclockwise turn about the origin?"	"Is there another strategy, such as using rotation rules, that would give the same result?"
Task 4 – Dilation of a Triangle	"What is the scale factor, and how will the size and shape of the figure change?"	"Do the new coordinates reflect proportional scaling from the center of dilation?"	"How does the ratio between the new and original figure confirm that your answer is correct?"
Task 5 – Composition of Transformations	"In what order should the reflection and translation be applied to this figure?"	"After completing the first transformation, does the intermediate figure appear correct before applying the second?"	"Would reversing the order of transformations change the final result? Why or why not?"

formula?' During the evaluation stage, students were prompted with questions such as 'Is there another way to verify that your answer is correct?' or 'How does your solution compare with an alternative strategy for solving this transformation?'.

Scaffolding prompts aligned with geometric transformation tasks

To ensure that metacognitive scaffolding was not implemented as a generic set of questions but as prompts tailored to the specific representational and reasoning demands of each geometric transformation, planning, monitoring, and evaluation prompts were explicitly designed to correspond with the five tasks described in **Table 3**. Each prompt was formulated to direct students' attention to the invariant properties, spatial relationships, or coordination processes relevant to the transformation being studied, consistent with Presmeg's (2006) notion of perceptual and operative apprehension and the progression of geometric reasoning described in the van Hiele model. **Table 3** presents the specific prompts used for each task, illustrating how the scaffolding was designed to move students from identifying relevant information (planning), to verifying the accuracy of their visual and symbolic representations (monitoring), to reflecting on the validity and generalizability of their solution (evaluation).

These prompts were delivered in written form through in-class worksheets and reinforced verbally by the teacher during problem-solving activities. Students were expected to respond to each prompt before proceeding to the next stage of the task, ensuring that planning, monitoring, and evaluation were enacted as a structured sequence rather than as isolated reflective moments. This task-specific design distinguishes the scaffolding approach used in the experimental group from the more general procedural guidance provided in the control condition.

Comparison with the control condition

Students in the control group participated in a conventional flipped classroom that did not include explicit metacognitive scaffolding. While they engaged in the same pre-class learning activities and in-class problem-solving tasks as the experimental group, instructional support in the control condition focused primarily on content clarification and procedural guidance provided by the teacher. Opportunities for reflection and self-regulation were present but were not systematically structured through prompts or guiding questions. This contrast between the two instructional conditions enabled a focused examination of the added value of metacognitive scaffolding within a flipped learning environment, while holding constant factors such as curriculum content, instructional time, and overall learning format.

Implementation fidelity

To ensure consistency in implementation, lesson plans and learning materials were prepared in advance and reviewed prior to the intervention. Teachers involved in the study were briefed on the instructional procedures and the intended role of metacognitive scaffolding within the experimental condition. Classroom observations were conducted periodically to monitor adherence to the learning design and to document how students interacted with the scaffolding during in-class activities. Through this structured implementation, the intervention provided a realistic yet sufficiently controlled context for examining how metacognitive scaffolding can be embedded within flipped learning to support students' visual–spatial reasoning in secondary geometry instruction.

Instruments and Data Collection

Data in this study were collected using multiple instruments aligned with the explanatory sequential mixed-methods design. Quantitative data were used to examine changes in students' visual–spatial reasoning, while qualitative data were collected to provide contextual insights into the implementation of the learning design and teachers' instructional experiences. The use of multiple data sources enabled triangulation and enhanced the credibility and interpretive validity of the findings.

Visual–spatial reasoning test

Students' visual–spatial reasoning was measured using a researcher-developed test designed specifically for the geometry content addressed in the intervention. The test focused on geometric transformations, including translation, reflection, rotation, dilation, and the composition of transformations. These content areas were selected because they represent core components of

Table 4. Visual–spatial reasoning test instrument

Content Area	Item	Description
Translation of a Triangle	#Task1 Given a triangle ABC with vertices A(0,0), B(2,0), and C(0,3), students are asked to draw the image of the triangle after a translation by vector (3, 2) and determine the coordinates of A′B′C′.	Understanding positional relationships before and after translation and applying coordinate transformation rules.
Reflection of a Square across y = x	#Task2 A square with side length 4 units has vertices P(1,1), Q(5,1), R(5,5), and S(1,5). Students determine and sketch the image of the square after reflection across the line $y = x$.	Visualizing symmetry across an oblique axis and recognizing spatial invariance under reflection.
Rotation of a Point	#Task3 Given point A(2,3), students determine its image after a 90° counterclockwise rotation about the origin and explain the solution steps.	Developing mental rotation and predicting spatial position after rotation.
Dilation of a Triangle	#Task4 A triangle DEF with vertices D(−2,0), E(0,2), and F(2,0) is dilated from the origin with a scale factor of 2. Students determine the coordinates of the image and describe changes in shape and size.	Understanding spatial proportionality and the relationship between scale factor and geometric similarity.
Composition of Transformations	#Task5 A parallelogram GHIJ undergoes a reflection across the y-axis followed by a translation by vector (2, −1). Students determine the final coordinates and sketch the resulting figure.	Integrating multiple transformations and demonstrating spatial flexibility through multi-step visualization.

the Grade 11 geometry curriculum and require substantial engagement with spatial visualization, mental manipulation of objects, and the coordination of visual and symbolic representations.

The instrument consisted of five open-ended tasks that required students to analyze geometric figures, predict transformation outcomes, determine image coordinates, and explain their reasoning processes. The test was administered as a pretest prior to the intervention and as a posttest at the conclusion of the eight-week instructional period for both the experimental and control groups. This pretest–posttest structure enabled the examination of changes in students' visual–spatial reasoning over time and facilitated comparisons between instructional conditions. Content validity of the instrument was established through expert review by mathematics education specialists, who evaluated the alignment of the test items with targeted visual–spatial reasoning constructs and curriculum objectives. Revisions were made based on their feedback to improve clarity and construct alignment. The internal consistency of the instrument was examined using Cronbach's alpha, yielding a reliability coefficient of 0.82, which indicates acceptable reliability for educational research purposes.

An overview of the visual–spatial reasoning test items and their corresponding content focus is presented in **Table 4**. The tasks were designed to elicit students' abilities to visualize, manipulate, and reason about spatial transformations in a manner consistent with instructional practices at the secondary level.

Classroom observation protocol

Classroom observations were conducted to document the implementation of the learning design and to capture students' engagement with metacognitive scaffolding during in-class activities. An observation protocol was developed to guide systematic data collection, focusing on indicators such as students' use of planning, monitoring, and evaluation strategies; their interaction with visual representations; and the teacher's role in facilitating learning activities. Observations were conducted periodically throughout the intervention period in both the experimental and control classrooms. Detailed field notes were taken during each observation to record instructional practices, student interactions, and notable classroom events. These observational data were used to support the interpretation of quantitative findings and to assess the fidelity of the intervention's implementation.

Semi-structured teacher interviews

Semi-structured interviews were conducted with three mathematics teachers who were involved in implementing or observing the learning design. The interview protocol was designed to elicit teachers' perspectives on the feasibility, instructional value, and challenges associated with integrating metacognitive scaffolding into flipped learning. Interview questions focused on teachers' experiences with the learning design, their observations of students' reasoning processes, and perceived changes in student engagement during the intervention. All interviews were conducted after the completion of the instructional period to allow teachers to reflect on the full implementation of the learning design. Interviews were audio-recorded with participants' consent and transcribed verbatim for analysis. These qualitative data provided in-depth insights that complemented the quantitative results and contributed to a more comprehensive understanding of the instructional processes underlying the observed outcomes.

Data collection procedures

Quantitative data collection followed a standardized procedure across both instructional conditions. The visual–spatial reasoning pretest was administered prior to the start of the intervention, and the posttest was administered at the end of the eight-week instructional period. Both assessments were conducted under similar testing conditions to minimize procedural bias. Qualitative data collection occurred concurrently with the intervention. Classroom observations were scheduled at regular intervals, and teacher interviews were conducted after the completion of the instructional period. All collected data were anonymized prior to analysis to protect participants' confidentiality.

Data Analysis

Data analysis was conducted in accordance with the explanatory sequential mixed-methods design of the study. Quantitative and qualitative data were analyzed separately using appropriate analytical procedures and were subsequently integrated at the interpretation stage to provide a comprehensive understanding of the learning design and its outcomes.

Quantitative data analysis

Quantitative data obtained from the visual–spatial reasoning pretest and posttest were analyzed using descriptive and inferential statistical techniques. Descriptive statistics, including means and standard deviations, were calculated to summarize students' performance before and after the intervention in both the experimental and control groups. Prior to inferential analysis, assumptions of normality and homogeneity of variance were examined to determine the suitability of parametric statistical tests. Based on these preliminary checks, appropriate statistical analyses were conducted to examine changes in students' visual–spatial reasoning over time and differences between instructional conditions. Paired comparisons were used to analyze within-group changes from pretest to posttest, while between-group comparisons were conducted to examine differences in learning gains between the experimental and control groups.

In addition to statistical significance testing, effect size measures were calculated to estimate the magnitude of the observed differences. Reporting effect sizes allowed for a more meaningful interpretation of the practical significance of the findings, beyond reliance on p-values alone. All quantitative analyses were conducted using standard statistical software.

Qualitative data analysis

Qualitative data obtained from classroom observations and semi-structured teacher interviews were analyzed using thematic analysis. The analysis followed an iterative process that involved familiarization with the data, initial coding, and the identification of recurring patterns related to instructional processes and metacognitive engagement.

Observation notes and interview transcripts were coded to capture themes such as students' use of planning, monitoring, and evaluation strategies; their interaction with visual representations; and teachers' perceptions of the learning design's feasibility and instructional value. Codes were refined through repeated comparison across data sources to enhance analytical rigor. Representative excerpts were selected to illustrate key themes and to support the interpretation of quantitative findings.

Integration of quantitative and qualitative findings

Integration of quantitative and qualitative findings occurred at the interpretation stage (Négyesi et al., 2026; Paredes et al., 2026). The qualitative results were used to elaborate on and explain the patterns observed in the quantitative data, particularly with regard to how metacognitive scaffolding influenced students' engagement with visual–spatial reasoning tasks. This integrative approach enabled a more nuanced understanding of both learning outcomes and pedagogical mechanisms, strengthening the overall validity of the study's conclusions.

Ethical Considerations

This study was conducted in accordance with ethical standards for educational research involving human participants. Ethical approval was obtained from the relevant institutional authority prior to data collection. Permission to conduct the study was also granted by the participating school. All participants were informed about the purpose and procedures of the study, and their participation was voluntary. Written informed consent was obtained from all participating students and, where required, from their parents or legal guardians. Participants were assured that their responses would be treated confidentially and used solely for research purposes. Personal identifying information was removed from the dataset to ensure anonymity. Participants were informed of their right to withdraw from the study at any stage without any academic or personal consequences.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon reasonable request. Due to ethical and institutional restrictions related to the involvement of human participants, the data are not publicly available at the time of publication. Any requests for access to the data will be considered in accordance with ethical guidelines and institutional policies.

RESULTS

Students' Visual–Spatial Reasoning Outcomes

Descriptive statistics

Descriptive statistics for students' visual–spatial reasoning pretest and posttest scores in the experimental and control groups are presented in **Table 5**. Prior to the intervention, the mean pretest scores of the experimental group (M = 59.84, SD = 7.92) and the control group (M = 60.12, SD = 8.15) were comparable, indicating similar initial levels of visual–spatial reasoning.

Following the intervention, both groups demonstrated higher mean posttest scores. However, the experimental group achieved a substantially higher mean score (M = 86.31, SD = 6.25) than the control group (M = 72.14, SD = 6.03). The mean difference between the two groups at the posttest stage was 14.17 points, suggesting a notable difference in performance following the implementation of the learning design.

Table 5. Descriptive statistics of pretest and posttest

Test	Group	N	Mean	SD	Min	Max
Pretest	Experimental	32	59.84	7.92	45	74
	Control	30	60.12	8.15	46	75
Posttest	Experimental	32	86.31	6.25	72	97
	Control	30	72.14	6.03	60	85

Table 6. Shapiro–Wilk test of normality

Group	Test	W	Sig. (p)
Experimental	Pretest	0.977	0.421
Experimental	Posttest	0.977	0.700
Control	Pretest	0.959	0.316
Control	Posttest	0.959	0.297

Table 7. Levene's test of homogeneity

Variable	F (Levene's)	Sig. (p)
Pretest	0.122	0.728
Posttest	0.029	0.866

Table 8. Independent samples t-test results

Variable	Group	Mean	SD	n	t (df)	Sig. (p)	Mean Diff	95% CI Mean Diff
Pretest	Experimental	59.84	7.92	32	t ≈ 0.14 (60)	0.889	0.29	[−3.9, 4.5]
	Control	60.12	8.15	30				
Posttest	Experimental	86.31	6.25	32	t = 9.30 (60)	<0.001	14.17	[11.12, 17.22]
	Control	72.14	6.03	30				

Table 9. Effect size (Cohen's d)

Group	Mean Pretest	Mean Posttest	SD Pooled	Cohen's d
Experimental	59.84	86.31	9.00	1.28
Control	60.12	72.14	8.47	

Assumption testing

Prior to inferential statistical analysis, assumption testing was conducted to ensure the appropriateness of parametric procedures. Normality of the pretest and posttest scores for both groups was examined using the Shapiro–Wilk test. As shown in **Table 6**, all significance values were greater than 0.05, indicating that the data were normally distributed.

Homogeneity of variance was assessed using Levene's test. The results presented in **Table 7** indicate that the variances of pretest and posttest scores did not differ significantly between the experimental and control groups ($p > 0.05$). These findings confirm that the assumptions of normality and homogeneity were satisfied, allowing for further parametric analysis.

Independent samples t-test results

Independent samples *t*-tests were conducted to examine differences between the experimental and control groups at the pretest and posttest stages. The results are presented in **Table 8**. At the pretest stage, no statistically significant difference was found between the experimental and control groups, $t(60) \approx 0.14$, $p = 0.889$. This result indicates that the two groups did not differ significantly in their initial visual–spatial reasoning ability. At the posttest stage, a statistically significant difference was observed between the two groups, $t(60) = 5.04$, $p < 0.001$. The experimental group achieved a higher mean score than the control group, with a mean difference of 11.48 points and a 95% confidence interval ranging from 6.9 to 16.1.

Effect size analysis

To estimate the magnitude of the observed difference, effect size was calculated using Cohen's *d*. As shown in **Table 9**, the experimental group demonstrated a Cohen's *d* value of 1.28, which corresponds to a large effect according to conventional benchmarks. This result indicates that the difference in learning gains between the experimental and control groups was substantial. Effect size was calculated using Cohen's d based on the difference between the mean gain scores (pretest-to-posttest change) of the experimental and control groups, divided by the pooled standard deviation of the gain scores:

$$\frac{M_{gain,exp} - M_{gain,ctrl}}{SD_{pooled,gain}}$$

Using the mean gain scores reported in **Table 9**, this calculation yielded a Cohen's d of 1.28, indicating a very large effect according to Cohen's (1988) benchmarks. This value reflects the magnitude of difference in learning improvement attributable to the presence of metacognitive scaffolding, rather than a comparison of raw posttest scores alone.

Table 10. Mean posttest scores per geometric transformation task and corresponding van Hiele levels

Task	Transformation	Mean Posttest Score (Exp)	Mean Posttest Score (Ctrl)	Dominant van Hiele Level Demonstrated
Task 1	Translation	18.50	17.00	Level 1–2 (Visual–Analytical)
Task 2	Reflection	17.80	15.50	Level 2 (Analytical)
Task 3	Rotation	15.20	10.80	Level 2–3 (Analytical–Informal Deductive)
Task 4	Dilation	17.00	14.50	Level 2 (Analytical)
Task 5	Composition	17.81	14.34	Level 3 (Informal Deductive)
Total		**86.31**	**72.14**	

Table 11. Observation results of learning implementation

Observed Aspect	Maximum Score	Mean Score	Percentage (%)
Lesson preparation	20	18.5	92.5
Delivery of material & digital media	20	18.0	90.0
Application of metacognitive scaffolding	20	18.2	91.0
Discussion & interaction activities	20	18.3	91.5
Reflection & closure	20	18.4	92.0
Overall average	100	91.4	91.4

Analysis through van Hiele

To complement the aggregate quantitative results, students' responses were further examined through the lens of the van Hiele model of geometric thinking and Presmeg's framework of visualization, in order to characterize the qualitative nature of reasoning demonstrated across the five transformation tasks. Student responses to Task 1 (translation) and Task 2 (reflection) predominantly reflected van Hiele Level 1–2 reasoning (Naufal et al., 2021), in which students could visually identify the shift or symmetry involved and correctly apply coordinate rules, corresponding to Presmeg's perceptual apprehension — the ability to recognize sub-configurations within a figure without necessarily justifying why the transformation preserved specific properties.

In contrast, responses to Task 3 (rotation) required students to coordinate the center, angle, and direction of rotation simultaneously while mentally repositioning the figure, a demand consistent with van Hiele Level 2–3 reasoning (Naufal et al., 2021) (analytical to informal deductive), where students must reason about relationships between properties rather than relying on direct visual matching alone. This corresponds to Presmeg's operative apprehension, which involves mentally reconstructing a figure under transformation. Analysis of student work indicated that experimental group students more frequently attempted to justify their rotated image by referencing the rotation rule explicitly, whereas control group students more often relied on trial-and-error sketching, suggesting that scaffolding prompts supported movement toward more analytical engagement even though full deductive justification remained limited for both groups.

Task 4 (dilation) and Task 5 (composition of transformations) required students to coordinate proportional reasoning and sequential transformation, respectively, aligning with Presmeg's sequential and discursive apprehension — the ability to translate between visual and symbolic representations across multiple steps. Experimental group students demonstrated a greater tendency to verify intermediate steps in Task 5, consistent with van Hiele Level 3 reasoning, while control group responses more frequently skipped verification of the intermediate figure, resulting in higher error rates in the final coordinates. Taken together, this pattern suggests that metacognitive scaffolding was associated not only with higher overall scores but with a qualitative shift toward more analytical and relational reasoning, particularly for tasks that demand van Hiele Level 2–3 reasoning rather than Level 1 visual recognition alone.

Implementation of the Learning Design

Classroom observations were conducted to examine the implementation of the geometry learning design integrating metacognitive scaffolding within the flipped classroom. The results of the observation analysis are summarized in **Table 11**. Across observed aspects including lesson preparation, delivery of material and digital media, application of metacognitive scaffolding, discussion and interaction activities, and reflection and closure the implementation scores were consistently high. The overall implementation score reached 91.4%, indicating that the learning design was implemented as intended throughout the intervention period.

Students' Responses to the Learning Design

Students' perceptions of the learning design were examined using a questionnaire addressing clarity of material, active engagement, scaffolding support, and the usefulness of the flipped classroom. Descriptive results are presented in **Table 10**. Students' perceptions of the learning design were assessed using a questionnaire administered to the experimental group (n = 32) following the completion of the eight-week intervention. Responses were recorded on a 4-point Likert scale (1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree) across four dimensions: clarity of instructional materials, active engagement, perceived usefulness of metacognitive scaffolding, and overall learning support. Mean scores and standard deviations for each dimension are presented in **Table 12**.

Table 12. Mean scores of student responses

Assessed Aspect	Mean	SD
Clarity of material	3.42	0.38
Active student engagement	3.58	0.35
Scaffolding support	3.61	0.33
Usefulness of flipped classroom	3.47	0.40
Overall average	3.52	0.36

Table 13. Teacher interviews – Verbatim excerpts & coding

Verbatim Transcript (Excerpt)	Initial Coding	Category	Theme
"Students became more active because they had prepared the material before class."	Students more active; preparation	Learning engagement	Increased student participation in learning
"Scaffolding questions helped students reflect on their mistakes and correct them."	Reflection; scaffolding support	Cognitive support	Role of scaffolding in enhancing reasoning
"Some students still struggled with visualizing rotations, even with guidance."	Difficulty in visualizing rotation	Learning challenges	Persistent challenges in spatial reasoning
"The flipped classroom allowed me to focus more on guiding students during class time."	Effective use of class time	Teaching benefits	Efficiency in class through flipped model
"It was challenging to design scaffolded tasks for students with very low prior knowledge."	Difficulty in task design	Teaching challenges	Teacher challenges in scaffolding practice

Table 14. Data integration

Quantitative Result	Supporting/Explaining Qualitative Theme	Integrated Interpretation
Posttest mean score of experimental class significantly higher than control class	Teachers observed increased engagement and active participation in learning	Improved outcomes are partly due to higher student involvement fostered by flipped classroom and scaffolding.
Effect size (Cohen's d = 1.28) indicated a strong effect of the metacognitive scaffolding intervention	Teachers emphasized scaffolding questions helped reflection and error correction	Strong effect reflects the role of scaffolding in deepening reasoning and correcting misconceptions.
Some students in experimental class still scored low on rotation tasks	Teachers reported difficulties among students in visualizing rotations	Persistent gaps suggest that additional strategies are needed for spatial visualization in rotations.
Observations showed effective classroom interaction in experimental group	Teachers noted flipped classroom allowed more guidance during class time	Effective class management and guidance time contributed to learning improvement.
Variability in student progress observed (range wider in experimental class)	Teachers highlighted challenges in designing scaffolding for students with low prior knowledge	Variation in learning gains indicates that differentiated scaffolding strategies are necessary.

The overall mean score across all aspects was 3.52, indicating generally positive student responses. Higher mean scores were observed for active student engagement (M = 3.58) and scaffolding support (M = 3.61), while clarity of material and usefulness of the flipped classroom also received favorable ratings.

Qualitative Findings from Teacher Interviews

Qualitative analysis of semi-structured teacher interviews yielded several themes related to student engagement, instructional support, and implementation challenges. Representative excerpts and corresponding coding categories are presented in **Table 13**.

Teachers reported increased student participation, which they attributed to students' preparation prior to class and the use of scaffolding questions during in-class activities. They also highlighted the role of metacognitive prompts in supporting students' reflection and error correction. At the same time, teachers noted that some students continued to experience difficulties with specific spatial tasks, particularly those involving rotation, and that designing scaffolded tasks for students with low prior knowledge remained challenging.

Integration of Quantitative and Qualitative Findings

An integrative analysis was conducted to relate quantitative results to qualitative themes. As summarized in **Table 14**, higher posttest scores and large effect sizes observed in the experimental group were supported by qualitative evidence indicating increased student engagement and the effective use of metacognitive scaffolding. At the same time, qualitative findings highlighted persistent challenges in spatial visualization for certain tasks, which help explain variability in student performance.

DISCUSSION

The present study examined the implementation and outcomes of a geometry learning design that integrates metacognitive scaffolding within a flipped classroom to enhance students' visual–spatial reasoning. By combining quantitative and qualitative evidence, this study provides a comprehensive understanding of how structured metacognitive support can strengthen the cognitive quality of flipped learning environments. The discussion interprets the findings in relation to existing theoretical

frameworks and prior empirical studies, with particular attention to visual–spatial reasoning, metacognition, and instructional design in mathematics education.

Metacognitive Scaffolding as a Catalyst for Visual–Spatial Reasoning Development

The quantitative findings demonstrate that students who experienced flipped learning enriched with metacognitive scaffolding achieved significantly higher visual–spatial reasoning outcomes than those who participated in a conventional flipped classroom. The large effect size observed (Cohen's $d = 1.28$) indicates that the learning gains associated with the proposed design were not only statistically significant but also educationally meaningful.

From a theoretical standpoint, this finding aligns closely with metacognitive theory, which emphasizes that learning is enhanced when students are supported in regulating their cognitive processes through planning, monitoring, and evaluation (Deibl et al., 2023; Hoorfar & Taleb, 2015). Geometry learning, particularly tasks involving transformations such as translation, rotation, and reflection, places high demands on students' ability to mentally manipulate spatial representations. Without explicit guidance, students may rely on surface-level procedural strategies that limit deeper conceptual understanding. The scaffolding design implemented in this study appears to have functioned as a cognitive support mechanism, enabling students to engage more deliberately with spatial tasks and to reflect on their reasoning processes (Awi, Sutamrin, et al., 2025; Dabarera et al., 2014; Sadat et al., 2026).

Importantly, the findings also reinforce arguments in the flipped learning literature that instructional effectiveness depends less on the flipped format itself and more on how classroom activities are designed. More specifically, the observed gains in visual–spatial reasoning can be traced to the alignment between the metacognitive prompts and the distinct cognitive operations required by geometric transformation tasks. Planning prompts, which asked students to identify the center, direction, and relevant properties of a transformation before attempting a solution, likely reduced cognitive load by structuring students' initial engagement with mental rotation and dimensional deconstruction, operations that otherwise place substantial demands on working memory (Presmeg, 2006). Monitoring prompts, which directed students to verify the consistency between a transformation rule and its resulting image, targeted perceptual apprehension by requiring students to actively check sub-configurations within their sketches rather than accepting a first-attempt answer. Evaluation prompts, which encouraged comparison of alternative solution strategies, supported operative apprehension by prompting students to mentally reconstruct the transformation through a different pathway. This alignment between prompt type and specific spatial cognitive demand offers a plausible didactic mechanism linking metacognitive scaffolding to the substantial effect size observed in this study, beyond a general claim that scaffolding "supports engagement. While both groups in this study engaged in flipped learning, only the experimental group benefited from structured metacognitive scaffolding. This suggests that flipped learning serves as a *pedagogical container* whose cognitive value is determined by the quality of instructional design embedded within it.

Learning Gains through Structured Reflection and Self-Regulation

Within-group analyses revealed that students in both instructional conditions improved from pretest to posttest, indicating that flipped learning contributed positively to visual–spatial reasoning development. However, the substantially greater gains observed in the experimental group highlight the critical role of metacognitive scaffolding in amplifying these effects.

Qualitative evidence provides insight into how these gains were achieved. Teachers reported that scaffolding prompts encouraged students to pause, reflect, and reconsider their approaches during problem solving (Lestari et al., 2025; Okwuduba et al., 2021). Such reflective engagement is a key component of self-regulated learning, which has been consistently associated with improved conceptual understanding and problem-solving performance. By prompting students to articulate their strategies and evaluate the effectiveness of their solutions, the scaffolding design supported deeper cognitive processing and reduced reliance on trial-and-error approaches.

Students' positive responses further corroborate this interpretation. High ratings for active engagement and scaffolding support suggest that students perceived the learning activities as both cognitively supportive and motivating. This combination of cognitive and motivational benefits is particularly important in mathematics learning, where disengagement and anxiety often hinder students' willingness to engage with challenging tasks.

Persistent difficulties and the limits of scaffolding

Despite the overall positive outcomes, qualitative findings revealed that some students continued to struggle with specific spatial tasks, particularly those involving rotation. This result is consistent with prior research identifying mental rotation as one of the most cognitively demanding aspects of visual–spatial reasoning. Even with metacognitive scaffolding, some students may lack the foundational spatial schemas required to successfully visualize rotational transformations. This finding underscores an important theoretical and practical implication: metacognitive scaffolding supports *how* students think but does not automatically compensate for gaps in *what* they know or *how well* they can visualize spatial relationships. Consequently, scaffolding should be viewed as one component of a broader instructional ecosystem that may also include concrete manipulatives, dynamic geometry software, or extended practice opportunities. Teachers' reports of challenges in designing scaffolded tasks for students with very low prior knowledge further emphasize the need for differentiated scaffolding strategies. Adaptive scaffolding where the level and type of support are adjusted based on students' needs may be particularly important for addressing persistent difficulties in spatial visualization (Deibl et al., 2023; Haataja et al., 2019).

From a van Hiele perspective, this persistent difficulty can be explained by the specific reasoning level that rotation tasks demand. Unlike translation or reflection, which can often be solved through direct visual matching consistent with van Hiele Level 1 (visual recognition), rotation requires students to simultaneously coordinate the center of rotation, the angle of turn, and the

resulting spatial relationships among multiple points, a demand that corresponds to van Hiele Level 2–3 (analytical to informal deductive) reasoning. Students who have not yet consolidated Level 2 reasoning, in which geometric properties are analyzed as interrelated rather than perceived in isolation, are likely to struggle with rotation regardless of the metacognitive support provided, because the scaffolding prompts guide how students regulate their thinking but cannot substitute for the underlying analytical schema required to reason about rotational invariance. This aligns with Presmeg's (2006) distinction between perceptual and operative apprehension: rotation demands operative apprehension (the mental reconstruction of a figure under transformation) which is inherently more demanding than the perceptual apprehension sufficient for simpler transformations such as translation.

Instructional feasibility and pedagogical value in classroom practice

The observation results indicate that the learning design was implemented with high fidelity, suggesting that the integration of metacognitive scaffolding within flipped learning is feasible in authentic classroom settings. Teachers' reflections highlight that the flipped structure allowed more class time to be devoted to guiding students' thinking rather than delivering content. This shift aligns with constructivist and socio-cognitive perspectives on learning, which emphasize the teacher's role as a facilitator of meaning-making rather than a transmitter of information. Metacognitive scaffolding further supports this role by providing a structured framework for teacher–student interaction focused on reasoning processes rather than final answers (Fitria & Mustika, 2024; Gonzales & Gonzales, 2024; Hsieh et al., 2026; Omiralievna, 2024).The positive implementation outcomes also suggest that teachers' reflections indicated that embedding metacognitive prompts required modest adjustments to existing lesson materials and classroom facilitation routines, rather than a wholesale redesign of the flipped format. This suggests that the intervention is feasible for adoption within existing school infrastructure and teacher workload constraints.

Implications for Learning Design and Mathematics Education

From a design-oriented perspective, this study contributes empirical evidence demonstrating how metacognitive scaffolding can be systematically embedded within flipped learning to support visual–spatial reasoning. Rather than treating metacognition as an implicit byproduct of active learning, the findings highlight the importance of explicitly designing prompts and tasks that guide students' reflective engagement.

For mathematics educators, the results suggest that effective geometry instruction should integrate opportunities for students to plan, monitor, and evaluate their reasoning when engaging with spatial tasks. For curriculum designers, the study underscores the value of aligning pre-class and in-class activities so that foundational knowledge acquisition supports higher-order cognitive engagement during classroom instruction (Dassa et al., 2023; Dassa, Sutamrin, et al., 2024; Sappaile et al., 2024).

These findings point to a practical implication for curriculum design: aligning pre-class content delivery with in-class metacognitive prompts requires deliberate sequencing, such as embedding planning cues within pre-class materials so that in-class monitoring and evaluation prompts build directly on students' prior preparation. The study suggests that flipped classrooms become more cognitively productive when they are designed not merely to redistribute instructional time but to structure students' thinking processes during learning activities.

Limitations and Directions for Future Research

Several limitations should be acknowledged. First, the study involved a relatively small sample drawn from a single school context, which may limit the generalizability of the findings. Second, the quasi-experimental design does not permit strong causal inferences.

Future research could extend this work by examining the learning design across diverse educational contexts and student populations. Longitudinal studies may also explore the sustainability of metacognitive gains over time. Additionally, further research could investigate how metacognitive scaffolding interacts with technological tools—such as dynamic geometry environments—to support particularly challenging aspects of visual–spatial reasoning, including rotation and three-dimensional visualization.

CONCLUSIONS

This study examined the effectiveness of a geometry learning design that integrates metacognitive scaffolding within a flipped classroom to enhance students' visual–spatial reasoning. Drawing on quantitative and qualitative evidence, the findings indicate that the proposed learning design supported significantly higher visual–spatial reasoning outcomes compared to a conventional flipped classroom. The large effect size observed suggests that the integration of structured metacognitive scaffolding meaningfully strengthened the cognitive quality of in-class learning activities. Beyond learning outcomes, the study demonstrates that flipped learning becomes more pedagogically effective when it is intentionally designed to support students' regulation of their thinking processes. Metacognitive scaffolding guided students to plan, monitor, and evaluate their reasoning while engaging with geometric transformation tasks, thereby promoting deeper cognitive engagement rather than surface-level procedural practice. Qualitative findings further highlight that the learning design fostered active student participation and supported teachers in facilitating reasoning-focused classroom interactions.

From a design-oriented perspective, this study contributes empirical evidence illustrating how metacognitive scaffolding can be operationalized within flipped learning to support visual–spatial reasoning in secondary geometry instruction. The results suggest that the effectiveness of flipped classrooms depends not merely on pre-class content delivery but on the quality of instructional supports embedded in in-class activities. As such, the study offers a practical framework for educators seeking to

enhance higher-order reasoning through structured reflective prompts within existing flipped learning practices. Despite these contributions, the findings should be interpreted in light of the study's limitations, including the use of a quasi-experimental design and a sample drawn from a single school context. Future research may extend this work by examining the learning design across diverse educational settings, exploring long-term effects on spatial reasoning development, and investigating how metacognitive scaffolding can be combined with technological tools to support particularly challenging spatial tasks. Overall, the findings underscore the value of integrating metacognitive scaffolding into flipped learning as a viable and effective approach for enhancing students' visual–spatial reasoning and enriching the design of mathematics instruction.

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