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Connections between achievement emotions and covariational reasoning: The case of Valeria

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ABSTRACT

This research explores the link between achievement emotions and covariational reasoning, a type of mathematical reasoning involving two variables. The study employs a case study approach, focusing on a high school calculus student named Valeria, and develops a theoretical framework based on the control-value theory and levels of covariational reasoning. The results reveal that Valeria's ability to coordinate variables and her perceived importance of solving mathematical problems influence her experience of achievement emotions, including enjoyment and frustration. The case study enables the creation of a hypothetical model that explains how students feel achievement emotions when tackling mathematical tasks that require covariational reasoning. The study highlights the significance of comprehending the interplay between emotions and mathematical reasoning to cultivate advanced cognitive and emotional abilities. This research is essential in bridging the gap between emotions and mathematical reasoning, a topic that has been previously overlooked in research on the affective domain.

Keywords: achievement emotions, covariational reasoning, networking of theories, control-value theory, coordination of variables, mathematics education

INTRODUCTION

In recent years, the interest in the study of emotions in school subjects has grown and they have been termed *academic emotions* (Frenzel et al., 2021; Pekrun, 2006; Pekrun et al., 2017). In the particular case of mathematics, the study of students' academic emotions began with analyzes related to mathematical problem-solving (e.g., Middleton et al., 2018). Later on, it was found that there were other activities and situations in the classroom that also triggered emotions–mostly negative–, such as exams, lack of mathematical understanding, passing mathematics courses, etc. (Lewis, 2013; Martínez-Sierra & García-González, 2016; Pérez-Tyteca et al., 2013). Among the most frequent students' academic emotions reported in the literature, mathematical anxiety stands out, followed by a wide group of negative emotions, such as fear, stress, anguish, guilt and dislike.

According to Panero et al. (2020) the influence of emotions on individual mathematical learning and performance is one of the so-called "solid findings" in mathematics education research (Education Committee of the European Mathematics Society, 2011). However, this influence is not related to merely mathematical processes, but rather to class goals (Martínez-Sierra & García-González, 2014, 2016). This may be one of the reasons why authors such as Di Martino et al. (2017) make a call to investigate the influence of emotions in particular mathematical activities, in order to uncover the potential relationship between the type of mathematical thinking that is demanded for such activities and the emotions that are triggered.

Students' emotions and covariational reasoning have been studied but separately (Bell & Javier, 1981; Castillo-Garsow, 2012; Frank & Thompson, 2021; Mainhard et al., 2018; Moore & Carlson, 2012; Muis et al., 2018; Pekrun et al., 2017), this is, without considering how the students' covariational reasoning influences the students' emotions when addressing a mathematical activity or task. This is perhaps one of the reasons for the scarce research that relates student affect and cognition in academic contexts, such as the mathematics classroom (e.g., Sonnert et al., 2015; Tapia & Moldavan, 2007; Wade et al., 2016). In this regard Nava Guzmán et al. (2021) suggest a theoretical coordination of frameworks to address the study of affect and cognition, focusing particularly on students' emotions and covariational reasoning. Such a theoretical coordination is elaborated and illustrated in this paper.

In order to study students' emotions when carrying out a mathematical activity related to covariational reasoning, the control-value theory of achievement emotions (Pekrun, 2006) in coordination with the covariational reasoning levels (Thompson &

Carlson, 2018) are used. As we will argue later, this coordination of theoretical frameworks allows us to categorize the kind of academic emotions experienced by students when they are involved in processes of solving mathematical tasks related to covariational reasoning, which is fundamental for mathematical subjects such as calculus. Although both theoretical approaches have been conceptualized from different fields of knowledge, a possible meeting point between the two is their application in the analysis of the emotions experienced by students when facing mathematical tasks that require covariational reasoning. Thus, using a theory-networking strategy (Prediger et al., 2008) in this study the following research question is addressed:

What are the academic emotions experienced by a high school student when facing tasks that require the use of covariational reasoning?

In the next section of this article are introduced notions from control-value theory and covariational reasoning that form the theoretical basis for the theory-networking strategy implemented in the study reported.

The research aims to examine the emotions experienced by students during an activity. The control-value theory was employed to understand how the students' emotions are triggered by the actions they take, with particular focus on their attention towards the activity. The covariational reasoning framework is used to identify the actions that students need to take in order to coordinate two different quantities. It is precisely these actions that serve as a meeting point between these two perspectives from two different domains.

CONTROL-VALUE THEORY OF ACHIEVEMENT EMOTIONS

Pekrun (2006) considers academic emotions as set of emotions that are triggered in academic contexts, classifying them into:

(1) social emotions,

(2) topic emotions,

(3) epistemic emotions, and

(4) achievement emotions.

In this article special emphasis is placed on achievement emotions that students experience when they are engaged in mathematical activities that demand covariational reasoning. Achievement emotions are those experienced by students when their academic activity produces positive or negative effects on their learning and school achievement (Pekrun, 2006). The control-value theory provides an integrative framework for analyzing the antecedents and effects of achievement emotions experienced in academic settings (Pekrun, 2006). This theory assumes that achievement emotions arise as a result of an appraisal that students make of the academic activity that they face, in terms of

(1) the *control* that they have over the activity and

(2) the *value* that they assign to the activity.

The control is related to the student's knowledge required to solve the academic activity and related skills. The value refers to the importance that the student attaches to the activity in terms of potential benefits, such as the acquisition of new knowledge, or obtaining good grades.

Consider the following illustrative case. There is a student who knows the algorithm to add quantities of two-digits and greater. Her teacher assigns her the task of solving 10 sums with 5-digit quantities and tells her that this task will give her extra points for her final grade for the course. When solving the task the student will have high control of the activity, since she has the mathematical knowledge necessary to solve the sums. In addition, the student tries to correctly solve the 10 sums, because he knows that this will help her improve her final grade–that is, she assigns a positive value on this activity. Applying the control-value theory, it could be interpreted that this student experiences enjoyment during the development of this activity, due to her high control and positive value assigned to this activity.

The students' *perception* of control and value of an academic activity is considered a key precursor for the type of emotion experienced. This is to say, the appraisal that students make of an academic activity produces in them the idea of "feel in control over, or out of control of achievement activities and outcomes that are subjectively important to them" (Lichtenfeld et al., 2022, p. 8). This implies that students' appraisals of control and value of an academic activity are relevant for the achievement emotions that they experience.

Thus, link between students' appraisals and their achievement emotions becomes important, since it allows to infer what achievement emotions students may experience according to their appraisal of an academic activity or task. **Table 1** shows the types of achievement emotions that, according to control-value theory, could be experienced on combinations of appraisals in terms of value and activity control. More instances of connections between appraisals and emotions can be found in Pekrun (2006).

Table 1. Illustration of types of achievement emotions that could be triggered depending on appraisals that a student makes of an academic activity

Object focus	Appraisals		
	Value	**Control**	**Achievement emotions**
Academic activity	Positive	High	Enjoyment
	Negative	High	Anger
	Positive/negative	Low	Frustration
	None	High/low	Boredom

COVARIATIONAL REASONING

Thompson and Carlson (2018) conceptualize *covariational reasoning* as a special type of mathematical reasoning, which has been fundamental for the development of mathematical notions such as function. For instance, this kind of reasoning has been present in Newton's (1736) mathematical developments when he was trying to find the rate of change of the product of two changing quantities (see for example Newton, 1736). Covariational reasoning can be defined as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson et al., 2002, p. 354).

Due to its crucial role in mathematical thinking, research in mathematics education has attempted to identify the ways in which students develop their covariational reasoning. For instance, it is known that some students might perceive that the change of a particular variable is continuous, while a different group of students conceive it as a discrete change. Thus, different *levels of covariational reasoning* have been identified (Thompson & Carlson, 2018, p. 435), which are used to recognize students' ability to coordinate and envision the increases and decreases of two variables that are modified simultaneously. The levels of covariational reasoning that have been identified are the following:

Smooth continuous covariation: At this level the person perceives the variation (increase or decrease) of the value of a quantity or variable as happening simultaneously with changes in another variable's value. The person perceives the change in the two variables as smooth and continuous.

Chunky continuous covariation: At this level the person thinks of the variation of the value of a variable as changing simultaneously with changes in another variable's value; however, the individual might also think of the variation of the variables as changing by continuous intervals or chunky continuous variation.

Coordination of values: In this case the person manages to coordinate the values of the two variables x and y with the anticipation of creating a discrete collection of pairs (x, y).

Gross coordination of values: Here the person perceives that the value of the variables changes but cannot envision that individual values of quantities change together. The person only discerns loose overall changes in two quantities' values.

Pre-coordination of values: In this situation the person notices that the values of the variables change but cannot distinguish that this happens simultaneously.

No coordination: Here the person does not perceive that the two variables change. The person focuses only on the change of one of the variables.

The identification of how a student coordinates the simultaneous changes of two variables is important, since according to the degree of coordination that the student achieves, it is possible to classify her covariational reasoning according to the levels previously described. Thompson and Carlson (2018) consider that the levels of covariational reasoning should not be used as instructional aims to ensure students' learning progress, but rather as a means to identify and describe how students think and relate quantities simultaneously. It is in this sense that these levels are used in this paper.

NETWORKING BETWEEN THE CONTROL-VALUE THEORY AND THE COVARIATIONAL REASONING LEVELS

As mentioned before, the conceptual framework used in this study has been constituted through a networking of theories. Prediger et al. (2008) define the networking of theories as a process to establish relationships between components of theoretical approaches while respecting the plurality and autonomy of each theoretical approach. These authors consider that the plurality of theories can be productive when different approaches and traditions enter into a dialogue, which allows establishing types of networking that range from disregarding or neglecting theoretical approaches, to being globally unified. These are two extremes of a continuous landscape, where it is possible to locate different degrees of integration between theoretical approximations, for example contrasting, comparing, synthetizing theories, among others (Prediger et al., 2008, p. 170).

In the study reported here, the networking strategy of *combining* was implemented. According to Prediger et al. (2008) this networking strategy is appropriate to build conceptual frameworks that provide analytical tools for the sake of a practical problem or the analysis of a concrete empirical phenomenon–in this case, the study of academic emotions that may emerge when solving activities that require the use of covariational reasoning. This strategy allows us to observe an empirical phenomenon from different theoretical perspectives.

The combination of theoretical approaches does not require a complete compatibility of the theories. Even theories with contradictory basic assumptions could be combined to look at the same empirical phenomenon from different perspectives and try to understand it. This networking strategy is illustrated later.

METHODS

This section describes the empirical method implemented in the research study. The description is divided into the following four sections:

(1) the case study under consideration,

(2) the data collection instruments,

(3) the procedure, and

(4) the data processing.

The Case Study Under Consideration

An instrumental case study (Stake, 1998) was used in this investigation. In particular, the mathematical work of a high-school student whose pseudonym is "Valeria" was considered. Through the analysis of the achievement emotions and the levels of covariational reasoning expressed by Valeria, we seek to reveal relationships between her emotions and her reasoning levels, with the expectation of formulating generalizations that go beyond this particular case. According to Blatter (2018), a case study favors the understanding of the phenomenon under investigation by identifying specific mechanisms and cause and effect routes–in this case, between emotions and covariational reasoning. The identification of such mechanisms and routes allow us to build descriptions and interpretations of the studied phenomenon.

There was a selection process for the person who would be the case study. First, the written productions of three groups of students in the fifth semester of a Mexican public high school were analyzed. These written productions were generated by the students when trying to solve mathematical tasks that involve covariational reasoning. Subsequently, those written productions in which there was no evidence of the presence of covariational reasoning were discarded. Valeria was a student who did manifest covariational reasoning. Once identified, she was invited to participate in this study. Valeria agreed to participate anonymously in the study (she gave written informed consent). Thus, it was decided to work with Valeria as she was one of the students who had more manifestations of covariational reasoning, but also because she is an expressive and participative person always willing to give her opinion and participate in class.

Data Collection Instruments

To collect the empirical data on which this research study is based, two types of instruments were used: a set of two mathematical tasks that require covariational reasoning, and two semi-structured interviews focused on the student's appraisals regarding the covariational mathematical tasks. Each instrument is described in more detail below.

Covariational Mathematical Tasks

Two covariational mathematical tasks were used to identify the covariational reasoning manifested by Valeria. Although the analysis focused only on Valeria's solutions, these mathematical tasks were implemented to the entire calculus class to which Valeria belonged during two regular lessons. The students in the calculus class knew that their solutions of the tasks would be evaluated and counted as part of their final grade for the course.

The first covariational task is called "the Ferris wheel" and the second task is called "the paper box". They are adaptations of the tasks used by Moore and Carlson (2012) in which manipulative materials are used; that is, "artifacts used in mathematics education: they are handled by students in order to explore, acquire, or investigate mathematical concepts or processes and to perform problem-solving activities drawing on perceptual (visual, tactile, or, more generally, sensory) evidence" (Bartolini & Martignone, 2020, p. 487).

Two manipulatives were used in the adapted tasks: a piece of yarn (for the Ferris wheel task) and sheets of paper (for the paper box task). The main reason why it was decided to use manipulative materials in the covariational tasks was to try to trigger interest and engage the student in the tasks, but also to offer her a concrete sensory experience. As Kontas (2016) points out, manipulative materials can be an auxiliary means to concretize mathematical notions, in this case the simultaneous change of quantities.

The Ferris wheel task (task 1)

The Ferris wheel is a mathematical task adapted from Moore and Carlson (2012) designed to identify the reasoning involved in relating quantities that have continuously changing values. The adapted text of the task is the following:

> Consider a Ferris wheel with a radius of five meters that takes 1.2 minutes to complete a full rotation. April boards the Ferris wheel and begins a continuous ride on the Ferris wheel. If the platform to board the Ferris wheel is three meters off of the ground at the bottom of the Ferris wheel, sketch a graph that relates the total distance traveled by April and her vertical distance from the ground (Moore & Carlson, 2012, p. 55).

The version of the Ferris wheel task used in this study has three modifications with respect to the original task. First, in the text of the original task the distances are expressed in feet, but in the adapted task they are expressed in meters because Valeria uses the metric system. Second, as mentioned above, the adapted task includes the use of a piece of yarn. A purpose of using the piece of yarn was to allow the student to use it as an instrument for measuring line segments and curve segments. It was expected that this manipulative would facilitate addressing and solving the task. Second, the original task asks to draw a graph relating the distance traveled by the passenger April on the Ferris wheel and her vertical distance from the ground. In the adapted version of the task the student was suggested to use centimeters instead of meters for the graphical representation.

Additionally, she was asked to answer a set of questions related to the activity (e.g., how does the height and distance traveled by April change from the moment she begins her ride on the Ferris wheel until she reaches ¼ of a rotation? How do the height and distance traveled by April change between half a rotation and ¾ of a rotation?), and to complete a table (**Table 2**) containing the values of height and distance traveled at different moments of April's ride. These additional activities were included to favor the manifestation and identification of instances of covariational reasoning in the student.

Table 2. The task that had to be filled with values of distance & height traveled by April during her ride

Ferris wheel rotation	Distance traveled	Height
¼ rotation		
½ rotation		
¾ rotation		
Full rotation		

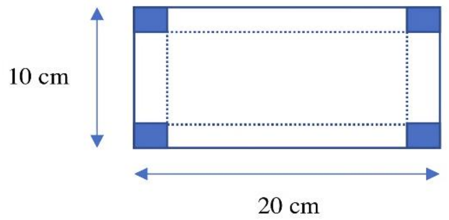


Figure 1. Supporting illustration shown to Valeria as part of the first mathematical task (Source: Authors' own elaboration)

Table 3. The task that had to be filled with values of constructed boxes

Length of the cut squares	Box height	Box width	Box length	Box volume
1.0 cm	1.0 cm			
2.0 cm	2.0 cm			
2.5 cm	2.5 cm			

Table 4. The task that had to be filled with numerical values corresponding to dimensions of different boxes

Length of the cut squares	Box height	Box width	Box length	Box volume
0.5 cm	0.5 cm			
1.0 cm	1.0 cm			
1.5 cm	1.5 cm			
2.0 cm	2.0 cm			
2.5 cm	2.5 cm			
3.0 cm	3.0 cm			
3.5 cm	3.5 cm			
4.0 cm	4.0 cm			
4.5 cm	4.5 cm			

The paper box task (task 2)

The adapted formulation of "the paper box" task is as follows (adapted from Moore and Carlson, 2012, p. 51):

> Starting with a 20 cm × 10 cm sheet of paper, a box is formed by cutting equal-sized squares from each corner of the paper and folding the sides up. For this construction you are provided with sheets of paper and the following illustration as support (**Figure 1**). Cut squares in each corner with a side of one cm, and measure the height, length, width and volume of the box that is formed. Calculate the same dimensions for the boxes that are formed by cutting out squares with sides of two cm and 2.5 cm.

There are three adaptations made to the original task as presented by Moore and Carlson (2012). First, the use of inches was substituted for centimeters as it is the measurement system Valeria is most familiar with. Second, the actual use of sheets of paper as a manipulative was introduced; particularly to build three boxes of different sizes that favor the visualization of the change in the volume of the boxes as the size of the cutout squares increases. Third, instead of requesting the formula that predicts the volume of the box–which is requested in the original task–, the student was asked to fill in a numerical table that contains the dimensions of each constructed box (**Table 3**).

She was also asked to answer a set of questions about how the size of the cutout of the sheets of paper affects the dimensions of the boxes (e.g., How does the height of the box change when the size of the cut square increases? How does the volume of the paper boxes change when the size of the cutout changes from 0.5 cm to two cm?); and to fill out a numerical table with the values of height, width and length of different boxes.

Finally, Valeria was requested to trace a graph using the values of **Table 4**, where the size of the cutouts of the sheets of paper and the volume of the boxes are related. These additional activities have the purpose of favoring the manifestation and identification of instances of covariational reasoning.

Semi-Structured Interviews

Two semi-structured interviews were conducted with Valeria. Each interview took place immediately after finishing the lesson in which Valeria had solved a mathematical task. The guide for the interviews is constituted by two groups of questions: the first group focuses on inquiring about the control that Valeria had over the task, while the second group focuses on inquiring about the value assigned by her to the task. **Table 5** shows the guiding questions for the interviews.

Table 5. Guiding questions for semi-structured interviews conducted with Valeria

No	Questions related to control over mathematical task
1	How capable did you feel of solving the task when you started it? Why?
2	How did you feel during the activity? Why?
3	How did you feel after solving (or not solving) the task? Why?
4	*In case she solved the mathematical task: What allowed you to solve the task? Why?
5	*In the event that her answer to the previous question refers to a particular skill: How do you feel about having that skill? Why?
6	Do you think your solution is correct? Why?
7	Do you think that the task allows you to develop some kind of skill? Why?
8	*In case that her answer to the previous question is positive: How do you think the task helped you develop that skill?
9	*In case that her answer to question 7 is positive: How do you feel about the task by allowing you to develop skills? Why?
10	How important is it for you to develop a skill through the resolution of a mathematical task? Why?
11	How did you feel when you finished the task? Why?
12	How do you feel when the teacher acknowledges your effort after finishing the task? Why?
13	The task has a score assigned that is part of your calculus course final grade, how did this fact make you feel when trying to solve the task?
14	Task has a score assigned that is part of your calculus final grade, how does this make you feel in relation to your calculus goals? Why?

The first group of questions related to control over the task are focused on investigating Valeria's perception of her own ability to solve the task during different moments of the solution process; her perception of the reliability of her solution; as well as the emotions experienced during the process of solving the task. The second group of questions focuses on investigating the value that Valeria assigns to the task, particularly, her valuation of the task as a means to develop skills; and her valuation of the task as a means of recognition and academic progress within her calculus class.

Procedure

The study reported in this paper was developed at the beginning of the COVID-19 pandemic, for this reason different technological platforms were used to implement the mathematical tasks and collect empirical data.

The empirical data on which this study is based are Valeria's written answers (the numerical tables she filled out and the questions she answered as part of the mathematical tasks), in addition to her responses to the semi-structured interviews. The first author of this study was responsible for implementing the mathematical tasks and conducting the interviews. Google Classroom was used to share the mathematical tasks and to record the answers and procedures of Valeria and the rest of her classmates. Google Meet was used to develop the regular lessons in which the mathematical tasks were solved, and to clarify any doubts that the students might have during this process. The semi-structured interviews were conducted privately with Valeria through Zoom. These interviews were recorded for later analysis.

Data Analysis

The analysis of the empirical data was carried out in two stages. In the first stage, the emphasis was on identifying in Valeria's written production her level of covariational reasoning. The written responses were initially coded by the first author of the study using the levels of covariational reasoning. Subsequently, an investigator triangulation (Rothbauer, 2008) was implemented in which the initial coding was discussed with the rest of the authors to agree on whether the empirical evidence had been adequately classified according to the levels of covariational reasoning. When Valeria's written answers were not clear enough to make inferences–for instance, short answers or non-detailed mathematical procedures–, and when seeking to confirm particular interpretations of the written data, these instances were identified to later try to clarify them through the semi-structured interviews with Valeria. Such interviews were transcribed for further analysis.

In the second stage, the emphasis of the analysis was on inferring Valeria's emotions. For this, the control and value appraisals that Valeria associated with each mathematical task were identified. Here it is important to underline two connections: a connection between Valeria's covariational reasoning and her control over the mathematical task; and another connection between the importance that the task has for Valeria and her value appraisals. Possessing a certain level of covariational reasoning necessary to address the mathematical task favors Valeria to perceive control over such task; for instance, being able to coordinate the variations in height and distance traveled in the Ferris wheel task. On the other hand, the importance that Valeria assigns to the task influences her value appraisals; an example would be that the grade assigned to the task is important to Valeria, thus influencing her value appraisals.

To illustrate the data analysis, an example of the inference of an emotion through control and value appraisals follows. The example is made up of Valeria's written productions and interview transcripts.

Control and value appraisals when Valeria coordinates values

In the Ferris wheel task, Valeria is asked to complete a table with the values corresponding to the distances and heights of April's position (the passenger on the Ferris wheel) at different times of her ride. Valeria's answers to this part of the task are shown in **Figure 2**. A pictorial representation of the variables involved in the covariational situation that Valeria used as a reference to tackle the mathematical task is shown in **Figure 3**.

Figure 2 shows that Valeria was able to determine the values of the distances and heights of different positions of the passenger April. Although the values of the distances written by Valeria are not as numerically precise as those of the heights, the way she filled in the table suggests that she had the ability to coordinate the values of the two variables. That is, Valeria was capable of anticipating the change of the variables according to the position in which the passenger is located. Thus, she seems

1/4 de vuelta	7.5 m	8 m
1/2 vuelta	15.5 m	13 m
3/4 de vuelta	23.5 m	8 m
Vuelta completa	31.5 m	3 m

Figure 2. Table filled by Valeria with values expressed in meters corresponding to distance (middle column) and height (right column) of position of passenger of Ferris wheel at different moments of her ride (left column represents such moments and reads as from top to bottom: quarter turn, half turn, three quarter turn, and full turn) (Source: Elaboration by Valeria, the study subject)

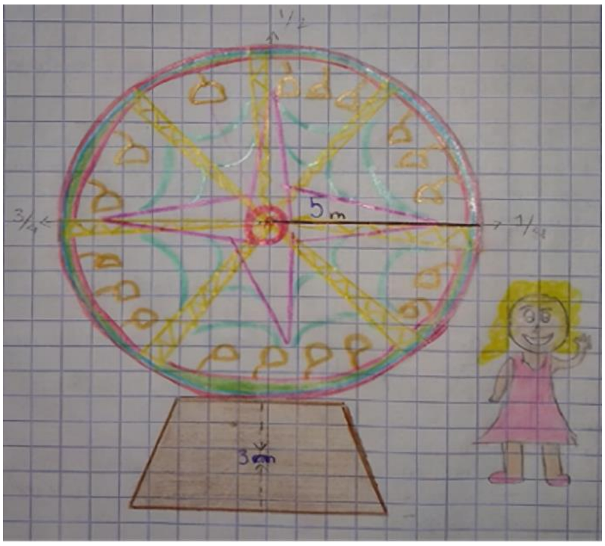


Figure 3. Representation of Ferris wheel task made by Valeria and used as a reference to address task (Source: Elaboration by Valeria, the study subject)

to manifest covariational reasoning at the *coordination of values* level. On the other hand, the numerical information presented in **Figure 2**, as well as the representation of variables in **Figure 3**, are interpreted as evidence that Valeria had a high control of the Ferris wheel task.

To corroborate these interpretations about Valeria's level of covariational reasoning and her potential control appraisal, she was interviewed. The interview confirmed that Valeria exhibited covariational reasoning at the coordination of values level. This is illustrated with the following transcript of an excerpt from the interview in which Valeria makes explicit the process that she followed to coordinate the distance and height values.

The following code is used in the transcript: **key phrases suggesting evidence of covariational reasoning are highlighted in bold**; underlining is used to highlight control and value appraisals; square brackets [ ] are used to indicate the level of covariational reasoning and the types of control and value appraisals manifested. To make a clarification, parentheses ( ) are used.

> 01 Interviewer: What increases or what decreases during one turn of the Ferris wheel?
>
> 02 Valeria: **The distance traveled increases and the height changes, as the passenger rides the mechanical game** (she coordinates the distance and height values associated with the passenger's position) [here the coordination of values is corroborated].
>
> 03 Interviewer: How would you describe the change in values in the Ferris wheel situation?
>
> 04 Valeria: **At the beginning the distance and the height increase**; then **they continue to increase** because she is positioned at the highest part of the game or in the middle of the turn. **Passing this the distance increases and the height decreases** [coordination of values] because she returns to the same position as in a quarter turn (Valeria uses the representation shown in **Figure 3** as a reference to describe the change in quantities).

The above excerpt confirms (utterances 02 and 04) that Valeria had the ability to coordinate the distance values with the height values and anticipates the change of these variables according to the position of the passenger. In addition, her representation of the Ferris wheel as a circle divided into four sections (see **Figure 3**) was a reference for her to locate the position of the passenger and determine the pairs of distance and height values recorded in the table (**Figure 2**). All this suggests that Valeria has a high control of the activity, since she possesses the mathematical knowledge that allowed her to solve the task. In this case, the

Table 6. Appraisals of control and value that Valeria manifests in a moment of the first covariational task in which she manages to coordinate values

Covariational reasoning	Appraisals		Achievement emotions
Coordination of values	**High control**	**Positive value**	
A1. Valeria is able to describe changes in distance and height in position of passenger during a complete turn of Ferris wheel	Ca1. Valeria shows ability to solve task without major difficulties	Va1. Valeria seems to care about task, and for that reason she tries to confirm that her solution to task is correct	Enjoyment

interview was also used to corroborate this interpretation; in particular she was asked how she felt during the mathematical activity. She answered the following:

> 05 Valeria: I felt satisfied because I finished the activity … I felt very good for answering the questions, doing my drawing, and completing the table to finish the activity on time and correctly [high control appraisal].

In her response, a high control appraisal is perceived. Valeria acknowledges that she was able to solve the activity and hand it in on time. In addition, this situation generated a feeling of well-being that she expressed with the phrases "I felt satisfied", "I felt very good".

To investigate the value appraisal of the task, Valeria was asked her opinion about the development of skills through the task. In this regard, the interviewer asked her:

> 06 Interviewer: Do you consider that the task allows you to develop skills?
>
> 07 Valeria: Yes, because I reasoned to answer the task (she acknowledges that the task allows her to exercise reasoning).
>
> 08 Interviewer: How do you feel about the task in relation to the fact that it did allow you to apply reasoning?
>
> 09 Valeria: It makes me feel satisfied because I answered according to what I reasoned (apparently reasoning to answer the task triggers satisfaction in Valeria). In the case of the graph, I had doubts about the way it was drawn. To be sure that it was well drawn, I looked for a similar activity on the internet, and the curve (referring to a graph found on the internet) resembled the one I drew, so I assumed it was fine.
>
> 10 Interviewer: What led you to carry out this search on the internet?
>
> 11 Valeria: Since the activity has a value for the final grade, then I did not want to fail in my answer, and I wanted to make sure that it was correct.

When Valeria was asked if the activity allowed her to develop skills, her answer was affirmative (utterance 07) and suggests that she recognizes the application and development of her (covariational) reasoning. Likewise, the application of her reasoning makes her feel satisfied as shown in utterance 09. Utterance 11 leads us to interpret that Valeria formulates a positive value appraisal of the task, related to her successful solution of the task, and the value that this task had for her final grade.

The identification of Valeria's high control and positive value appraisals suggest that an emotion that Valeria experienced during the resolution of the mathematical task was *enjoyment* (see **Table 1**).

Table 6 summarizes these findings and illustrates the networking strategy of *combining* between the control-value theory and the levels of covariational reasoning. The combining strategy consists of analyzing Valeria's solution process and the associated interview side by side, using the two theoretical approaches. This allows generating a meeting point between the theoretical approaches. The codes A1 (action 1), Ca1 (control appraisal 1), and Va1 (value appraisal 1) are used in **Table 6**.

RESULTS

The data analysis allows to identify two other achievement emotions experienced by Valeria when facing task 1 and task 2. These identified emotions are reported synthetically through the following two episodes.

Episode 1–Enjoyment

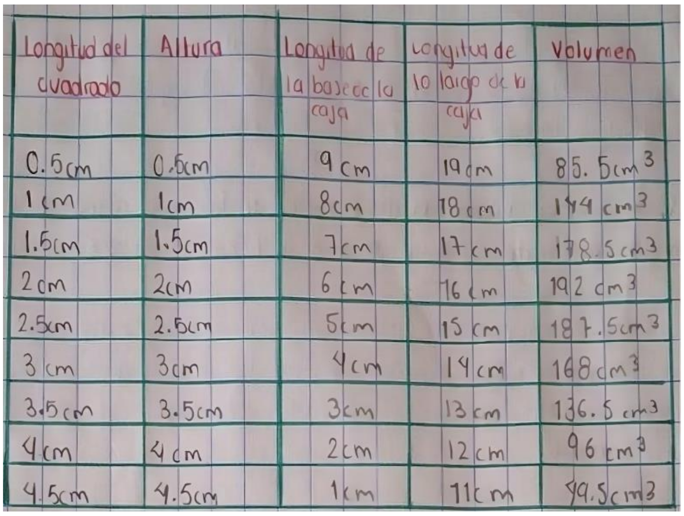
Table 7 presents a summary of the analysis of episode 1, where it is concluded that Valeria reasoned at a level of coordination of values. This conclusion is based on her construction and use of the illustration of the Ferris wheel (**Figure 3**), on her capacity to complete a table with values of the distance and height associated with April's ride on the Ferris wheel (**Figure 2**) and confirmed through the interview with Valeria. Taking this evidence together, it is reasonable to affirm that Valeria exhibits an ability to describe and coordinate two variables.

Valeria's ability to describe (A1) and coordinate (A2) the change in distance and height as the passenger's position changed, allowed her to solve the task without complications by identifying the values of the variables at different moments of the ride. This supports her high control appraisal (Ca1 and Ca2). Valeria seems to care about the task because of its value to her final grade, so she does an internet search to confirm that her solution to the task is correct. This is interpreted as evidence of Valeria's positive value appraisal (Va1) of the task.

Table 7. Summary of analysis of episode 1, where it is concluded that Valeria shows ability to coordinate values of two variables(it is inferred that she experienced enjoyment)

Covariational reasoning	Appraisals		Achievement emotions
Coordination of values	**High control**	**Positive value**	
A1. Valeria is able to describe changes in distance and height in position of passenger during a complete turn of Ferris wheel	Ca1. Valeria shows ability to solve task without major difficulties	Va1. Valeria seems to care about task, and for that reason she tries to confirm that her solution to task is correct	Enjoyment
A2. Valeria is able to coordinate values of height and distance in a complete turn of Ferris wheel	Ca2. Valeria identifies values of height and distance at different moments during a turn of Ferris wheel		

Figure 4. Paper boxes built by Valeria (this is a photograph that she sent as evidence of her solution to task) (Source: Photograph taken by Valeria, the study subject)



Longitud del cuadrado	Altura	Longitud de la base de la caja	Longitud de lo largo de la caja	Volumen
0.5cm	0.5cm	9 cm	19 cm	85.5 cm3
1 cm	1cm	8cm	18 cm	144 cm3
1.5cm	1.5cm	7cm	17cm	178.5cm3
2 cm	2cm	6 cm	16 cm	192 cm3
2.5cm	2.5cm	5cm	15 cm	187.5cm3
3 cm	3cm	4cm	14cm	168 cm3
3.5 cm	3.5cm	3cm	13cm	136.5 cm3
4cm	4 cm	2cm	12cm	96 cm3
4.5cm	4.5cm	1cm	11cm	49.5cm3

Figure 5. Table elaborated by Valeria, where she wrote values of height, width, length, and volume of boxes that are formed by cutting out squares of different sizes (Source: Elaboration by Valeria, the study subject)

Finally, considering Valeria's high control and positive value appraisals as a whole, it can be inferred that Valeria experienced an emotion of enjoyment during the resolution of a mathematical task in which she managed to coordinate the values of two variables.

Episode 2–Frustration

Episode 2 refers to Valeria's solution process of task 2 (the paper box task). In this case, Valeria was able to construct three paper boxes that met the initial conditions of the problem (**Figure 4**). She used these boxes as a reference to fill in the first requested table (the one shown in **Table 3**), and she was also able to fill in the second requested table (**Figure 5**).

The activity of building the paper boxes was positively appraised by Valeria, as illustrated in the following excerpt from the second interview:

> 12 Interviewer: How did you feel building the three boxes?

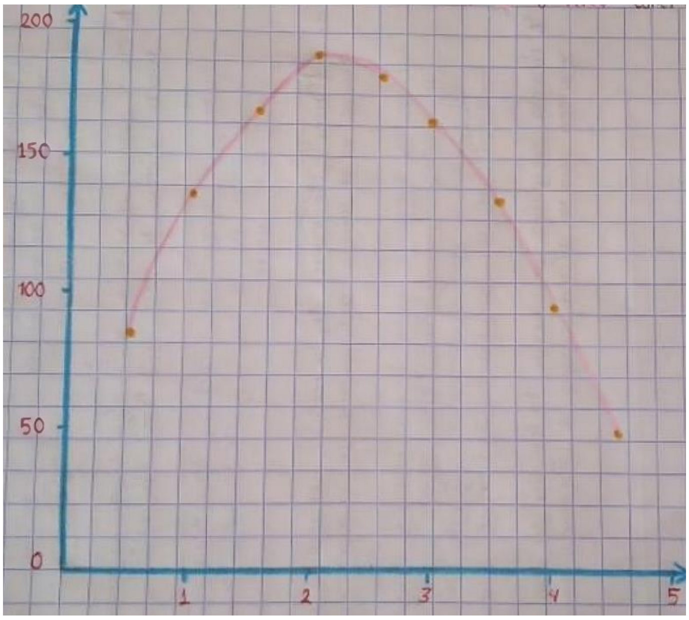


Figure 6. Graph traced by Valeria after searching on the Internet for activities similar to paper box task (size of cutouts of sheets of paper and volume of boxes are related) (Source: Elaboration by Valeria, the study subject)

> 13 Valeria: It had been a long time since I did this type of activity and doing it again made me feel good.
>
> 14 Interviewer: Why did it make you feel good?
>
> 15 Valeria: I already have some time without being in class (face-to-face lessons) and right now with this reinforcement, I went back to doing what I did at school. Not just homework and stuff.

However, Valeria had difficulties drawing the graph that was requested as part of task 2. During the interview Valeria was questioned about her solving process of the task. There she states that at some point during her solution process, she was able to locate the positions of the points with coordinates (x, y)=(*size of the cutout square*, *volume of the box*) on a Cartesian plane, but she was not sure what the shape of the graph should look like. The following excerpt from the interview illustrates how Valeria felt about this situation:

> 16 Interviewer: What allowed you to solve the task?
>
> 17 Valeria: **To observe the boxes that I built, and notice how when the box height increases, the width and length of the box also change, affecting thus the volume**.
>
> 18 Interviewer: How did you feel about not completing the task because of the graph?
>
> 19 Valeria: Frustrated, I did not know how to do it.

Valeria knew that this task also had value for her final grade of the course. For this reason, she turns to the internet to look for a task similar to the one with the paper box, and thus finds out the shape of the graph. With this information she manages to draw the graph correctly (see **Figure 6**):

> 20 Interviewer: How did you make the graph?
>
> 21 Valeria: I investigated what the graph of the construction of a paper box looked like and I saw that it was curved. So I thought the box graph was like that.

Searching for mathematical information on the internet allowed Valeria to complete the task and hand it in. However, the interview reveals that she was still somewhat restless because she did not fully understood how to completely solve the task:

> 22 Interviewer: How did you feel when you turned in the task?
>
> 23 Valeria: Calm for having handed it in, but I did not feel completely satisfied because I did not understand how to make the graph.

In this episode, Valeria manifests covariational reasoning at a gross coordination of values level. The paper boxes that Valeria built seem to have been instrumental in perceiving the change in variables (see utterance 17); however, her understanding of how

Table 8. Summary of analysis of episode 2, where it is concluded that Valeria reasoned at a level of gross coordination of values (it is inferred that she experienced frustration during solving process of task)

Covariational reasoning	Appraisals		Achievement emotions
Gross coordination of values	Low control	Positive/negative value	
M3. Valeria acknowledges change of length and volume in paper boxes	Ca3. Valeria is able to fill in numerical tables requested in task, but has difficulty drawing graph	Va2. Valeria refers positively to task (she says that it makes her feel good) Va3. Although she manages to complete task, she says she feels dissatisfied with it	Frustration

Table 9. Hypothetical model of achievement emotions experienced by students when facing a mathematical task that requires covariational reasoning

Covariational reasoning level	Appraisals		Achievement emotions
	Control	Value	
(6) Smooth continuous covariation	High	Positive	Enjoyment
(5) Chunky continuous covariation			
(4) Coordination of values		Negative	Anger
(3) Gross coordination of values	Low	Positive/negative	Frustration
(2) Pre-coordination of values			
(1) No coordination of values			
(1), (2), (3), (4), (5), & (6)	High/low	None	Boredom

these variables change together does not appear to be robust. This is manifested in the difficulty she has in drawing the graph of the function that relates both variables. Her explicit acknowledgment of this difficulty is considered a low control appraisal.

To overcome such difficulty, Valeria searches for mathematical help on the internet focusing on determining the shape of the graph that is requested in the paper box task. This type of behavior is identified by Pekrun (2006) as a *preventive action*, which has the purpose of reducing failure and increasing the certainty of achievement.

Table 8 presents a summary of the analysis of episode 2. Here it is concluded that Valeria reasoned at a level of gross coordination of values. It is also inferred that due to the low control appraisal–which was not modified with the preventive action implemented–, in combination with the change from positive to negative value appraisal, it produced a feeling of frustration in Valeria.

HYPOTHETICAL MODEL OF ACHIEVEMENT EMOTIONS AND COVARIATIONAL REASONING

According to Stake (1998), instrumental case studies have the aim of reaching an understanding of a general phenomenon that involves the case studied as one of its elements or links. In this regard, Valeria's case allows us to understand the relationship between covariational reasoning and academic emotions, particularly achievement emotions. It seems that at a higher level of covariational reasoning there will be greater control of the task, and this may enable a positive value assessment of the task–as long as the person perceives the task as important or relevant. This finding makes it possible to establish a hypothetical relationship between the different levels of covariational reasoning and achievement emotions (see **Table 9**).

In the first column of **Table 9**, the levels of covariational reasoning are presented, and they are related to the second column of control appraisals. The reasoning levels (4), (5), and (6) are associated with a high control in the resolution of the task, while the reasoning levels (1), (2), and (3) are associated with a low control in the resolution of the task. In addition, the value appraisals depend on the importance or appreciation that the student associates with the task, for instance with respect to her own learning or her grades. In this way, the aggregation of control and value appraisals of the task will condition the achievement emotion experienced by the student. For instance, in episode 1 Valeria displayed covariational reasoning at the coordination of values level, which allowed her to have a high control of the activity. Additionally, Valeria expressed a positive value appraisal of the task–apparently because of the importance the task had for her final grade–, so it is inferred that she experienced enjoyment. In the case of episode 2, Valeria showed low control of the task, which is associated with a low level of covariational reasoning (a gross coordination of values). In addition, during the resolution of the task Valeria transitions from a positive value appraisal to a negative value appraisal from which it is inferred that she experienced frustration when trying to solve it.

It is possible to expect that when students show more complex levels of covariational reasoning (levels 4, 5, and 6) the control value remains high; while at low levels of covariational reasoning (levels 1, 2, and 3) low control values are expected. It is the combination with high or low value appraisals–or the transition from one to the other–what determines the emergence of negative or positive achievement emotions. In the case of boredom, this is an achievement emotion that can be manifested in the student at any level of covariational reasoning. Regardless of whether her control appraisal of the task is high or low, the student could experience boredom as long as the task lacks importance for her, that is, when there is no value appraisal of the task.

DISCUSSION

In this study, the academic emotions experienced by a high school student when facing tasks that require the use of covariational reasoning were determined. Considering the student Valeria as a case study, it was determined that she experienced two achievement emotions (enjoyment and frustration), while manifesting different levels of covariational reasoning: coordination of values and gross coordination of values. This answers the research question posed.

However, the case of Valeria help us to understand the achievement emotions students could experience when trying to solve this type of mathematical activities. The achievement emotions (Pekrun, 2006) that students may experience when faced with mathematical tasks that require covariational reasoning depend on their ability to reason covariationally (which is associated with their control appraisal), and the importance they give to the mathematical task (associated with their value appraisal).

Based on these results, a hypothetical model was produced, which suggests that according to the level of covariational reasoning (Thompson & Carlson, 2018) that a student manifests, they will formulate high or low control appraisals (see **Table 9**). For the levels smooth continuous covariation (6), chunky continuous covariation (5), and coordination of values (4), the control appraisal is expected to be high because the student's covariational reasoning at these levels requires abilities such as coordination and prediction of the variables' behavior as they change simultaneously. In the case of the levels gross coordination of values (3), precoordination of values (2), and no coordination of values (1), a low control appraisal is expected because the student could not adequately synchronize the changes of the variables involved. Once the student formulates a control appraisal according to how they perceive their ability to solve a covariational task, it is the value appraisal that defines the achievement emotion that the student could experience. This value appraisal could be related to the importance of the task for the student.

The hypothetical model synthesized in **Table 9** is a theoretical window that gives us a first approximation to the connections between covariational reasoning and students' achievement emotions. It is also an example of how two theoretical approaches–one from the field of educational psychology and the other from the field of mathematics education–can be amalgamated through the networking strategy of combining, to study an empirical phenomenon. This research illustrates how a theory-networking strategy makes it possible to study an empirical phenomenon for which there are no tailor-made theoretical tools, facilitating the construction of a pragmatic theoretical bricolage to understand the complex emotional reality, in this case the achievement emotions of a high school student when solving activities that require covariational reasoning.

One of the contributions of this study is to illustrate the functioning of the networking strategy "combining" using two theoretical approaches from different research areas. This kind of theoretical strategy is useful for the study of multifaceted phenomena that cannot be described or studied using a single theoretical approach. Also, this research responds to the need pointed out by Middleton et al. (2018) to study the affective dimensions associated with the mathematical activity, by identifying a meeting point between the achievement emotions and the covariational reasoning of students.

One limitation of this study is that only achievement emotions were inferred because the focus of the student's attention was on the academic activity (Pekrun, 2006). To identify the four academic emotions, a more comprehensive analysis of the emotional dimension related to not just the mathematical activity, but other academic activities would be necessary. For instance, if the goal of the research was to examine epistemic emotions, it would be essential to identify cognitive appraisals regarding the relationship between new information and existing knowledge or beliefs (Muis et al., 2018) as these appraisals play a significant role in determining the types of emotions students may experience in school.

A characteristic of this research is that it was conducted during the pandemic and relied on technology such as Zoom to conduct interviews. While this allowed us to reach the students and understand their covariational reasoning and control value appraisals, it raises the question of how the findings would have been different if face-to-face interviews had been conducted.

Achievement emotions are emotions that can be inferred because of the control and value appraisals that students make when their attention is on the activity (see **Table 1**). Other academic emotions like social, epistemic, and topic emotions, cannot be inferred the way it was proceeded in this research because they are emotions that do not happen exclusively in academic contexts and the focus of interest is not on the activity (Pekrun, 2006).

CONCLUSIONS

In this study the academic emotions experienced by a high school student when solving tasks that require the use of covariational reasoning were investigated. To study the connections between academic emotions and covariational reasoning, theoretical notions from the control-value theory (Pekrun, 2006) and covariational reasoning (Thompson & Carlson, 2018) were combined. It is concluded that it is the level of coordination of variables that triggers high or low control appraisals, which in combination with the student's value appraisals of the task, allow to infer the type of emotion that the student experiences when trying to solve such kind of mathematical tasks. The findings of this study allow to establish a hypothetical relationship between the different levels of covariational reasoning and achievement emotions (see **Table 9**). In particular, between the level of variable coordination and the control appraisal over the mathematical task.

Although the study is limited to a single individual, the hypothetical model established in this research offers valuable insights that could be generalized to other students who engage in mathematical tasks requiring covariational reasoning. It is important to recognize that the relationship between achievement emotions and covariational reasoning is influenced by the students' control and value appraisals, which are directly linked to their ability to reason covariationally and the importance they assign to the mathematical task.

The conclusion of this study asserts that there is a hypothetical relationship between different levels of covariational reasoning and achievement emotions. To better understand this connection, it is essential to consider the key factors that contribute to this relationship: control appraisals and value appraisals.

Control appraisals are influenced by the level of coordination of variables that a student exhibits during a mathematical task involving covariational reasoning. When students demonstrate higher levels of covariational reasoning, they are more likely to perceive a higher degree of control over the task, which in turn can lead to positive achievement emotions such as enjoyment and pride. Conversely, when students struggle with coordinating variables and exhibit lower levels of covariational reasoning, they may perceive a lower degree of control over the task, potentially resulting in negative achievement emotions like frustration and anxiety.

Value appraisals, on the other hand, refer to the importance that students assign to the mathematical task. When students perceive the task as valuable or significant, they are more likely to experience positive emotions, as they see the task as meaningful and worth their time and effort. In contrast, when students assign low value to a task, they may experience negative emotions due to the perceived lack of relevance or importance of the activity.

In summary, the study's findings suggest that the relationship between different levels of covariational reasoning and achievement emotions is dependent on the students' control and value appraisals. As students demonstrate higher levels of covariational reasoning, they are more likely to perceive greater control over the task and assign higher value to it, resulting in positive achievement emotions. The hypothetical model presented in this study (**Table 9**) provides a framework for understanding this complex relationship, offering valuable insights for educators and researchers alike. By examining this relationship further, it is possible to develop targeted strategies to support students in experiencing positive emotions while engaging in mathematical tasks requiring covariational reasoning.

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