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Comparison of solving systems of linear equations with two variables with classic methods and application software

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ABSTRACT

The study and teaching of mathematics, particularly in middle school, is one of the most challenging subjects and often a source of student frustration. Teachers continuously strive to make this field more engaging and easier to understand. Middle school mathematics teachers, dedicated to creating a more favourable learning environment, face ongoing challenges, especially when teaching systems of linear equations with two variables and solving word problems. Their concern stems from unsatisfactory student performance, despite their persistent efforts to improve outcomes. This research aims to address these challenges by offering a new approach to teaching systems of linear equations with two variables and word problems in mathematics. Through innovative strategies and advancements in teaching methods, the goal is to enhance students' understanding and interest in the subject. In doing so, we hope to help middle school teachers have a more positive impact on students' learning progress and foster a deeper appreciation for mathematics. The study focuses on 9th-grade students at "Pjeter Bogdani" middle school in Pristina, where 103 students participated. The students were divided into three groups, each taught the same topic in different ways. The first group used GeoGebra, the second group used Mathway, and the third group was taught using traditional methods without any software. Based on the results, we concluded that the group using GeoGebra achieved the best outcomes. GeoGebra proved to be the most effective tool for explaining the topic and should be considered for future use in classrooms. In this way, the study guides teachers on choosing a more effective approach among the three methods examined, while acknowledging that other methods and tools may lead to different results.

Keywords: mathematics, students, Geogebra, mathway, systems of linear equations with two variables

INTRODUCTION

Mathematics is a powerful tool that requires understanding, respect, and application. In a world where life's challenges often demand complex solutions, mathematics provides us with logic and reasoning that can guide us in our daily lives. However, despite its importance, mathematics is often seen as one of the most challenging subjects for students.

One of the main issues faced by students is the lack of understanding of fundamental concepts and the difficulty in solving mathematical problems. This challenge is not new; it has persisted throughout the history of mathematics education. While earlier periods lacked the technology to improve learning and teaching, today, we are presented with numerous opportunities to change our approach to this subject.

One promising solution is the integration of technology, particularly given that students are regular users of electronic devices. Phones, laptops, and televisions are integral parts of their lives. By incorporating these tools into the learning process, we can create a more innovative and engaging approach to teaching mathematics.

This paper focuses on the topic of solving systems of linear equations with two variables, examining how technology can enhance this learning experience. Through the use of various software applications on mobile and computer devices, we aim to develop a more attractive and effective teaching methodology for students. Along this journey, we anticipate both the challenges and benefits that come with integrating technology into the mathematics classroom.

The purpose of the paper is to compare the effectiveness of three teaching methods in solving systems of linear equations with two variables, using different software and the traditional teaching method. This study aims to determine which of these approaches is most helpful in improving student's understanding and performance in this mathematics topic. In this way, the study guides teachers on choosing the most effective approach for teaching systems of linear equations in two variables.

This paper also aims to help teachers select the most appropriate tools to improve students' results and increase their interest in mathematics, considering the positive impact that technology can have on teaching.

LITERATURE REVIEW

Mobile and online learning applications are becoming increasingly widespread and are now used by millions of students and educators worldwide. Wireless mobile devices such as smartphones, PDAs, and tablets can benefit students' learning both inside and outside the classroom (Drigas & Pappas, 2015). With online learning, students can access content anytime and anywhere, as often as needed. On days when students cannot attend school, they can still learn the topic independently. E-learning ensures that students do not fall behind due to missed lessons and helps them align with modern learning practices (Anwar et al., 2020).

A particularly challenging area for students is solving mathematical word problems. Recent advances in natural language processing, especially transformer-based models, have been explored to support students in tackling such problems (Zong & Krishnamachar, 2023). Another line of research examined how ethnomathematics can support students' understanding of linear systems of equations in two variables, using design-based research that involved three phases:

1) Preliminary design,
2) Teaching experiments, and
3) Retrospective analysis (Nursyahidah et al., 2018).

The increasing availability of smartphones and tablets with higher processing power has also enabled broader use of participatory and social technologies. Today's students, as digital natives, are immersed in video games and social networks, which highlights the gap between their experiences and the traditional education system designed decades ago (Figueiredo et al., 2016).

Mobile applications also show promise in teaching mathematics at the university level. For instance, one study proposed integrating mobile applications into the teaching of geometry and found that digital tools and virtual manipulatives can help students overcome difficulties in learning and understanding mathematical concepts (Alkhateeb & Al-Duwairi, 2019).

Pulungan and Suhendra (2019) identified two main obstacles students face when solving problems: linguistic difficulties and limitations in conceptual understanding. Their case study of four eighth-grade students in Bandung used Newman's Error Analysis to identify sources of errors. While students were able to read the problems, they often failed to grasp their meaning, which led to mistakes in constructing and solving equations. These findings highlight that difficulties often stem not from reading itself, but from a lack of understanding of the material on systems of linear equations in two variables.

Similarly, Rahmadiani et al. (2024) found that students' conceptual understanding of systems of linear equations in two variables remains low. According to their results, students with high mathematical ability were able to master all indicators of understanding, such as restating concepts, identifying examples and non-examples, and classifying objects by their properties. However, students with medium and low mathematical ability struggled to demonstrate these indicators consistently.

Systems of Linear Equations with Two Variables

The students are now familiar with linear equations and the graphical representation of linear equations with two variables. With the knowledge of linear equations with two variables for graphical representation, the authors will start dealing with systems of linear equations with two variables. So, the relationship between two linear equations will be treated, we will look at the solution of linear equations with two variables, but first, let's get acquainted with the meaning of the term systems of linear equations.

Definition 1: If the linear system is given.

$$\begin{aligned} ax + by &= l \\ cx + dy &= e \end{aligned} \tag{1}$$

where a, b, c, d, l and e are constant. The pair of numbers x = x_0 and y = y_0 (where they can also be written in this way (x_0, y_0)), is a solution of the system if the equations are satisfied by the pair. The set of all such ordered pairs is called set solutions for the system.

In (1) the general form of the system of linear equations with two variables is given. In the following, an example will be used to name the parts of the system of linear equations with two variables.

$$\begin{aligned} 2x + 4y &= 3 \quad \leftarrow first\ equation \\ 7x - 3y &= -1 \leftarrow second\ equation \end{aligned} \tag{2}$$

x and y are the variables, 3 and -1 are the constant terms, while 2, 4, 7 and -3 are the coefficients of the system.

Definition 2: A solution of a system of two linear equations with two variables is any ordered pair of real numbers that satisfies both equations simultaneously (Zejnullahu et al., 2019).

This study will deal with three methods for solving systems of linear equations:

- Graphical method
- Substitution method
- Method of elimination

All these methods will be treated in two ways, showing the traditional method of explanation as well as the use of software in solving systems of linear equations with two variables.

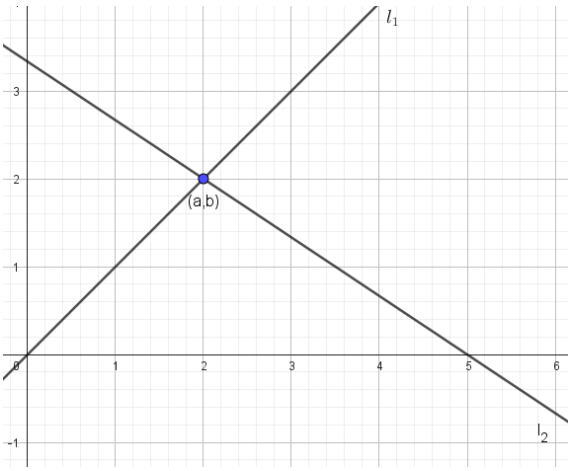

Figure 1. Graphical method-consistent and indendent system (Source: Authors' own elaboration, using GeoGebra)

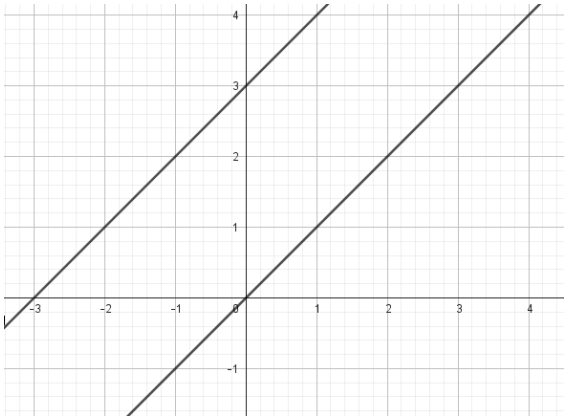

Figure 2. Graphical method-inconsistent system (Source: Authors' own elaboration, using GeoGebra)

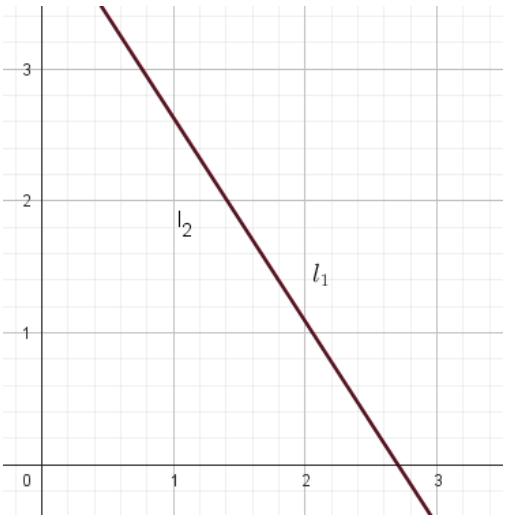

Figure 3. Graphic method – consistent and dependent system (Source: Authors' own elaboration, using GeoGebra)

Solving the System of Linear Equations with Two Variables - Graphical Method

First of all, it is noted that each linear equation with two variables is graphically presented and they represent a straight line in a plane. Therefore, a system of linear equations with two variables represents two linear lines in the plane, which we will mark with l_1 and l_2. The following may be the possible cases:

Lines l_1 and l_2 are expected. The system of equations has only one solution. The solution of the system is the ordered pair (a, b), where the two lines intersect. In this case, the system is consistent and independent (**Figure 1**).

Lines l_1 and l_2 are parallel. The system has no solution. In this case, the system is inconsistent (**Figure 2**).

Lines l_1 and l_2 match. The system has infinitely many solutions. In this case the system is consistent and dependent (**Figure 3**).

Problems of Explanation and Understanding for the Method of Solution of System of Linear Equations with Two Variables

Compared to other areas of mathematics, such as geometry, algebra—particularly the topic of systems of linear equations—is generally more understandable for students. While each area of mathematics presents its own challenges for both teachers and students, in my experience, students tend to find algebra more straightforward and accessible.

However, the topic of systems of linear equations with two variables does come with its own set of challenges for students. When the three different methods for solving these systems are introduced, some students may initially experience fear and anxiety. Despite this, it serves as a stimulating challenge that often sparks interest and encourages deeper understanding. Mastering all three methods is crucial for students, but the graphical method tends to be the one where they struggle the most. As such, it requires more focused attention during instruction.

Another significant challenge for both students and teachers are solving word problems related to systems of equations. Not all tasks present the same level of difficulty, but students often encounter the greatest difficulties when applying the graphical method to solve these problems. Word problems, in particular, are a common stumbling block. Teachers, therefore, must present these problems in a clear and accessible manner, but it's also important to recognize that explaining such tasks can be difficult for teachers themselves, as students often find them complex and hard to grasp.

In order to eliminate and solve these challenges and problems, the teacher should, in addition to explaining and interpreting not only textual but also graphic tasks, use different software and online math problem solvers to solve these tasks. Since every student has technological equipment, the way is to guide the students on how to use this equipment for the development of their knowledge not only in the mathematics part but in every subject that can be used. In cases where technology is integrated in the explanation of mathematics, students are even more interested in learning the lesson and the understanding of the unit is even greater. Problem-solving is the actions and/or methods used by students to understand and solve problem situations. The aim of learning mathematics—particularly problem solving—is to help students build new mathematical knowledge, understand the problem-solving process, and apply various strategies effectively. Problem-solving skills are developed through practice, and this ability is essential not only for mathematics but also for addressing real-life challenges. Therefore, problem-solving must be an integral part of mathematics instruction. According to Mustofa et al. (2020), the problem-solving process consists of four key activities: Understanding the problem, planning a solution, carrying out the plan, and checking the result.

Mathematics teachers who are familiar with informatics can very easily create programs that can be used by students, but for those who do not have this knowledge, we have enough software that allows us to better explain a teaching unit. Among the most popular and useful software is GeoGebra, while among the most used online math solvers are: Mathway, Symbolab, Mathematica, etc. Through its user-friendly interface, GeoGebra facilitates a hands-on approach to learning, offering students a practical and engaging environment to rectify misconceptions and deepen their understanding of mathematical principles (Tuda & Rexhepi, 2024).

Although Symbolab and MathPapa were initially considered, they were excluded from the study due to time limitations and to maintain focus on GeoGebra (as a dynamic visualization tool) and Mathway (as an automated solver) for clearer comparison.

To continue this work, we make use of various software tools and online math problem solvers in order to demonstrate clearly how systems of linear equations with two variables can be solved. We then examine how their use affects students' understanding of the topic. Mobile devices offer significant advantages and provide broad benefits for teaching mathematics, as highlighted by Alkhateeb and Al-Duwairi (2019).

The Most Emphasized Errors of Students During Solutions of Systems of Linear Equations with Two Variables

"Anyone who has never made a mistake has never tried anything new" is a statement by Albert Einstein. Mistakes are a very important part of learning mathematics because they are the key part where we learn things in depth and in a long-term way. Solving a problem in mathematics opens many doors to solving other problems.

When it comes to systems of linear equations with two variables, most students tend to make mistakes. These errors often stem from gaps in their understanding of earlier lessons, which they carry into more advanced topics. Among the three methods taught for solving systems of linear equations, minor errors are common and can be addressed with further explanation. However, the graphical method, in particular, warrants closer attention due to the specific types of errors students encounter.

The difficulties often begin with plotting lines on the coordinate system. It's important to examine how students approach this task and where their misunderstandings arise in order to better address these issues.

Example 1: To solve the system of linear equations with two variables using the graphic method.

$$\begin{aligned} 2x + y &= 4 \\ x - y &= -1 \end{aligned} \tag{3}$$

Solution of the example by the student is presented in **Figure 4**. It can be seen from the picture that the student solved the task incorrectly. First of all, his first mistake was not selecting two points to represent the line. This stems from a lack of knowledge in geometry, but we will clarify this with the GeoGebra software. While tablets have had a significant impact on education, GeoGebra remains a particularly valuable tool for visualizing a wide variety of mathematical concepts in both algebraic and geometric representations (Mollakuqe et al., 2021; Tuda & Rexhepi, 2023).

Then, it can be seen that when any point in the coordinate system is taken, and from it, infinitely many lines are built, as in **Figure 5**.

If only one point is selected, infinitely many lines can pass through it, and we cannot know which one corresponds to the equation given in the task. Therefore, we must determine at least a second point to correctly represent the line. This can be done by assigning

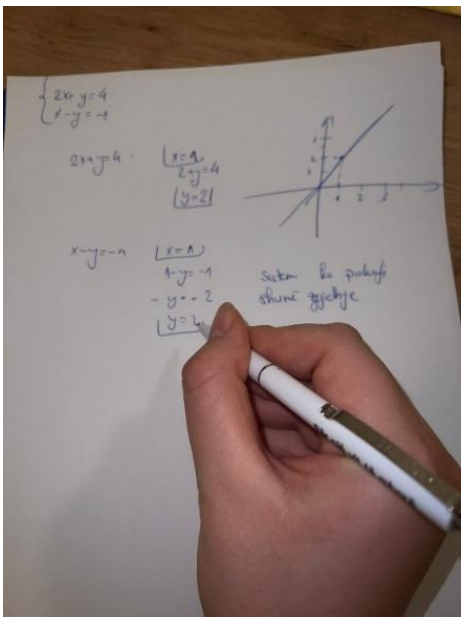

Figure 4. Solution of example 3 solved by a student (Source: Field study)

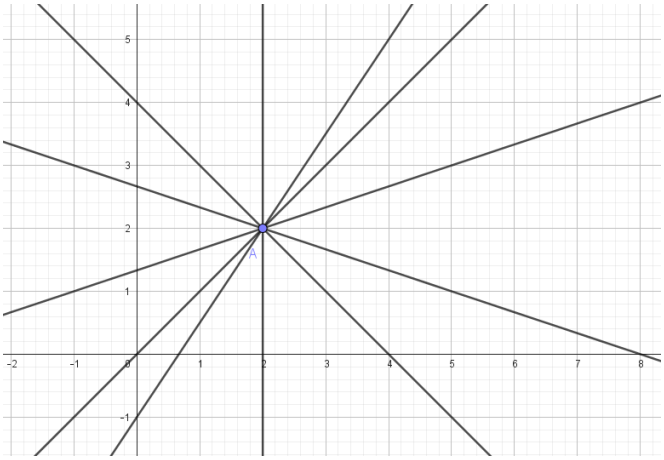

Figure 5. Clarification of example 1 (Source: Authors' own elaboration, using GeoGebra)

convenient values to x or y (for example, 0, 1, or –1) and calculating the corresponding coordinate. Although different values can be chosen, smaller numbers are often preferred because they make the calculations easier and faster.

Problems in Solving Textual Tasks for Systems of Linear Equations with Two Variables

Students often face significant challenges when solving word problems related to systems of linear equations in two variables. These challenges can feel like a long, arduous journey, where the real-world complexities they encounter are presented in the form of mathematical tasks. This is precisely when mathematics should demonstrate its powerful relevance to practical life. However, students frequently struggle to translate real-world scenarios into equations. To make this journey more engaging and manageable, a greater emphasis on solving word problems, accompanied by real-life examples, can help students view mathematics as a valuable tool—a resource that can assist them in navigating both academic and real-life challenges. Problem-solving ability stands for identifying data known and asked, choosing the right problem-solving strategies, solving the mathematical model, and checking the solution that had been obtained (Rakhmawati et al., 2019).

First, let's solve a task and see what steps should be taken to solve a textual task.

Example 2: There are a total of 13 animals in the hangar, including chickens and sheep. There are 40 feet in total. How many chickens and how many sheep are in the shed?

Solution: The best way to solve the problem is to analyse the task piece by piece. First of all, there are 13 animals in a hangar, and those animals are chickens and sheep. So, there are two types of animals, chicken and sheep. In this case, let's have:

x −chickens

y −sheep

Since the chickens and the sheep together are 13 then, it means that their sum is 13. So, the following is obtained:

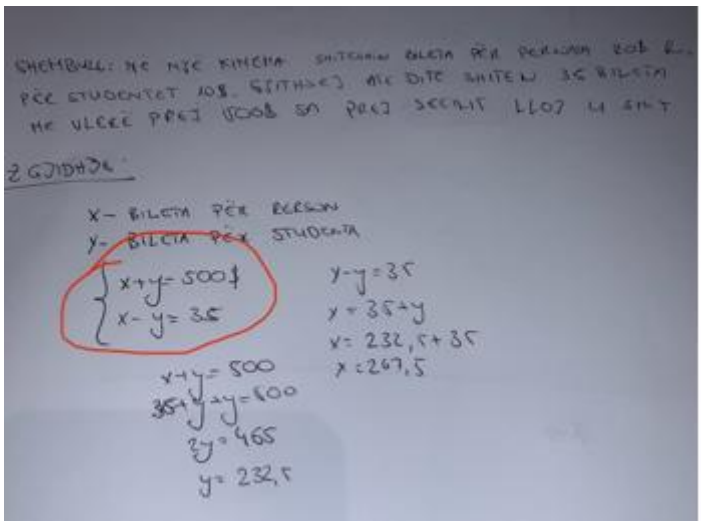

Figure 6. Solution by a student of example 3 (Source: Field study)

$$x + y = 13 \tag{4}$$

Then we have from the task that there are a total of 40 legs. We know that the chicken has 2 legs and the sheep has 4 legs, so the sum of their legs is 40, so:

$$2x + 4y = 40 \tag{5}$$

So, we find the equations, now we build the system:

$$\begin{matrix} x + y = 13 \\ 2x + 4y = 40 \end{matrix} \tag{6}$$

From the above system we have these results:

$$\begin{matrix} y = 7 \\ x = 6 \end{matrix} \tag{7}$$

From the solution of the tasks, we said that with x we marked the chickens and with y we marked the sheep and, in our case, we have 6 chickens and 7 sheep in the hangar.

Next, we will see another example of how the student solved it.

Example 3: In a movie theatre, tickets were sold for $20 per person and $10 for students. The movie theatre sold 35 tickets that day with a value of $500. How many of each type did it sell?

Solution: The student solved it in form presented in **Figure 6**.

The error was made in creating the system. The student very well identified what the unknowns are in our case, but the mistake was in the construction of the system. How the task is solved, we will explain in detail from the teacher's side, but also how we will avoid these problems. Such problems can only be avoided by exercising the students more in this type of tasks, i.e. text tasks, so the more the teacher takes on text-type tasks, the student will learn and be able to solve even more tasks. We will see how the task is solved later.

First, we see that the total number of tickets sold is 35, so the sum of tickets per peson and student will be 35. In this case:

$$x + y = 35 \tag{8}$$

And the money from their sale is $500. Since a ticket per person is $20 and a ticket per student is $10, then we have:

$$20x + 10y = 500 \tag{9}$$

Then the system has the form:

$$\begin{matrix} x + y = 35 \\ 20x + 10y = 500 \end{matrix} \tag{10}$$

We will solve this system using GeoGebra (see **Figure 7**).

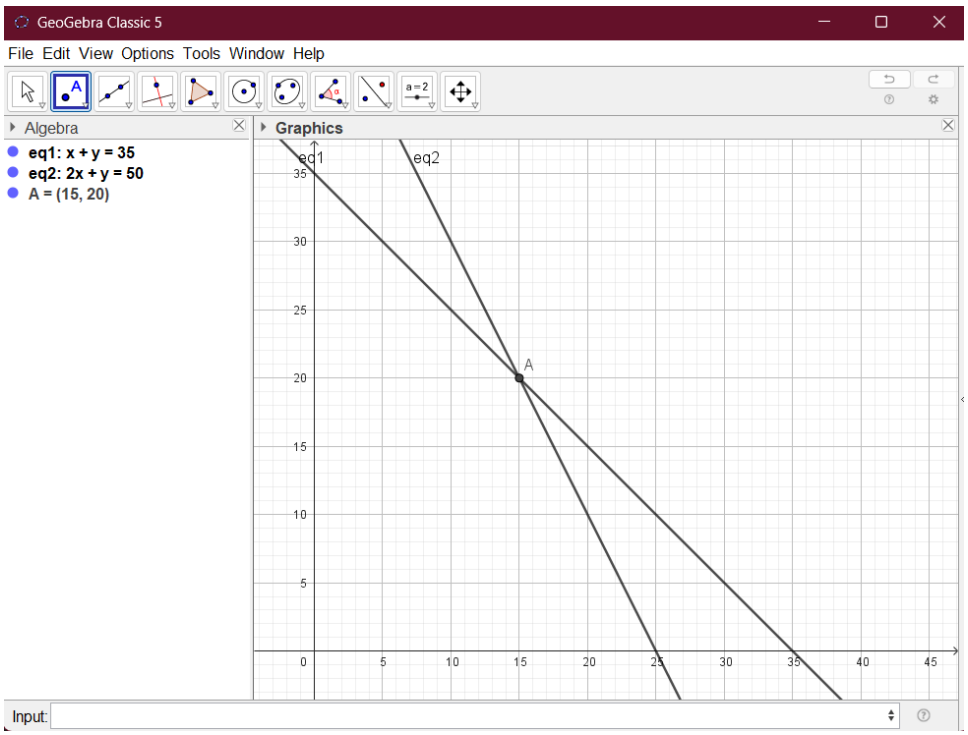

Figure 7. Solving example 3 in graphical way, using GeoGebra during a classwork (Source: Authors' own elaboration, using GeoGebra)

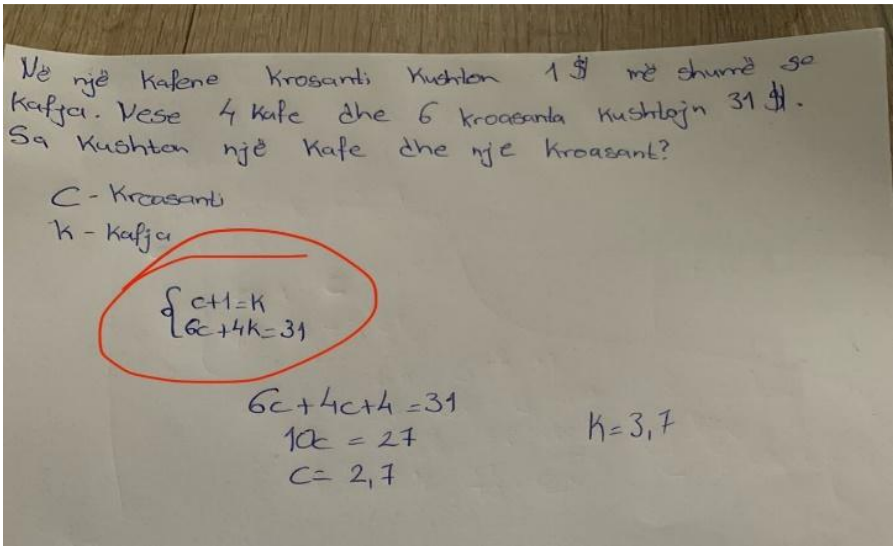

Figure 8. Solution of example 4, by a student in class (Source: Field study)

Since the solution is point A in our case $A = (15, 20)$. Then we understand that 15 tickets were sold per person, while 20 tickets were sold for students.

Example 4: In a cafe, a croissant costs $1 more than a coffee. If 4 coffees and 6 croissants cost $31, how much does a croissant cost and how much does a coffee cost?

Solution: In this case, the solution of the task was made by the student and was solved in form presented in **Figure 8**.

The student made a mistake while solving the task. His mistake was made in the construction of the system. The student very well identified the unknowns and marked them. The task required that the croissant cost $1 more than the coffee, if it is marked with the unknowns of the student, it is in this form:

$$k + 1 = c \tag{11}$$

The student has noted that the coffee costs $1 more than the croissant, which of course gives a different result to the task and in this case the task is wrong. To avoid mistakes of this kind, the only way is to increase the concentration on what is required and how it is clarified, since these mistakes may happen, but with the solution of the tasks these problems will be avoided.

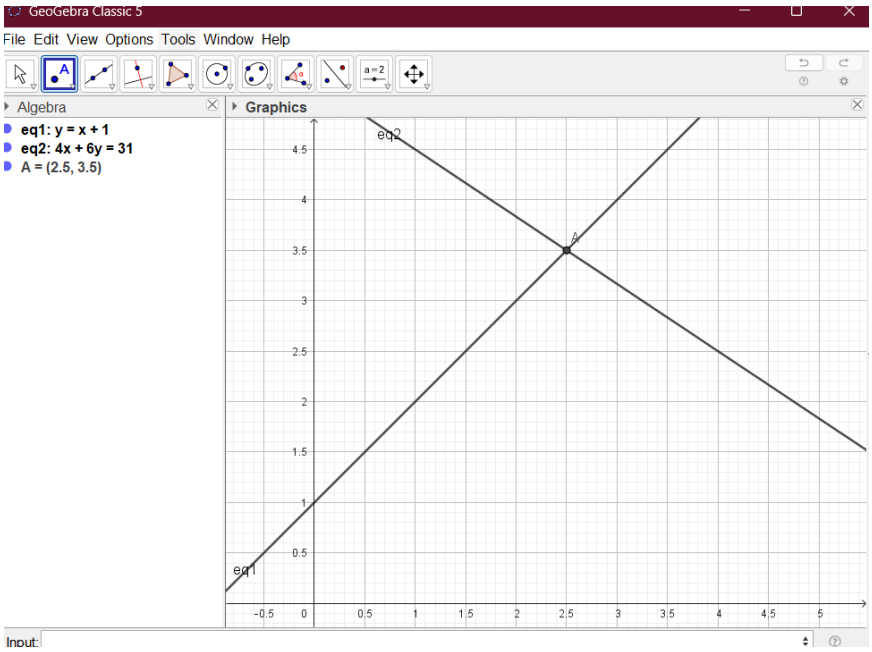

Figure 9. Solving the system y=x+1 and 4x+6y=31, in graphical way, using GeoGebra during a classwork (Source: Authors' own elaboration, using GeoGebra)

The second linear equation of the system is correct, since there are 6 coffees and 4 croissants. Then the solution of the system in this case with the first adjusted linear equation is on **Figure 9**.

So, a coffee costs 2.5 and a croissant costs 3.5.

The Use of Different Software and Online Math Problem Solvers in Solving Examples Related to Systems of Linear Equations with Two Variables

Technology has not always been a part of learning, and this represents a time not too far away when our parents finished their education. Well, in the days when we are educated, the possibilities and ways of learning have also advanced, since the Internet and technology have made our lives even easier. Internet resources, along with other illustrative and demonstration methods, can be used to make mathematics lessons as attractive and engaging as possible (Kamberi et al., 2022).Thus, in mathematics, informatics as well as the development of technology has occupied a very important place, since not only does it receive good information about any specific topic in mathematics, but also the solution of tasks can be done through technology, namely with part of the online math problem solver and different software that are part of the development of technology.

Technology has as many bad sides of use as there good ones since in our days everyone has access and especially children have become among the most users of technological devices, where they have the opportunity to increase their knowledge by using these devices. As teachers, we must give the teaching of mathematics a different direction from the traditional one, and what can be done better than to integrate technology, technological devices, and software in the teaching of mathematics? Mathematics teachers should use these opportunities and increase the attractiveness and desirability of learning mathematics since the interest of students in learning mathematics is decreasing every day. A connection and exchange of traditional learning and the use of software would be the main key to increasing the desire to learn mathematics. School mathematics now integrates a wide range of topics, including set theory, numerical calculations, measurement of length, perimeter, area, and volume of two- and three-dimensional objects, drawing 2D and 3D figures, solving equations and graphing, transformations, trigonometric, exponential, and logarithmic functions, matrix operations and determinants, vectors, and statistics. These concepts are often difficult to teach using only a board and marker; therefore, a variety of digital devices can be used to enhance teaching effectiveness (Joshi, 2016).

Regarding the software and online math problem solver for systems of linear equations with two variables, there are enough options that solve these systems, but each software solves it in different ways. This software will be treated one by one and through examples, we will discuss which software or online math problem solver would be more suitable in solving these systems.

GeoGebra

GeoGebra is one of the most used software by mathematics teachers and students. GeoGebra has arithmetic, algebraic, and geometric possibilities for solving tasks. As one of the easiest software to use, it is also possible to download it to the phone, which is a great advantage for teachers and students, since during the solution of the tasks each student can use GeoGebra and compare the solution of his with that of GeoGebra, thus making a very simple comparison with him. Regarding the solutions of systems of linear equations with two variables above, we have used GeoGebra, and in the solved examples it can be seen that it is a great help, for the understanding and interpretation of the solution of these systems.

Figure 10. Solving the system x+y=3 and 3x-2y=30, using Symbolab (Source: Field study)

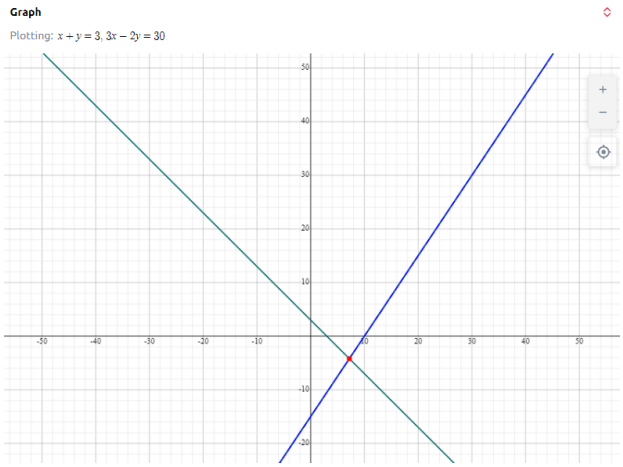

Figure 11. Solving the system x+y=3 and 3x-2y=30, by graphical way, using Symbolab (Source: Field study)

Symbolab

It is an online math problem solver that is also widely used. In addition to giving the solution to the task, in most cases, the procedure for solving the task is also given. So Symbolab in itself contains many more advantages than GeoGebra, but as a teacher, I would have used it only during the explanation of any task, while for the students I would have allowed them to use it only in the first tasks of explaining the systems since, this way of I can influence the students in such a way that their laziness increases, and when they would strengthen their knowledge of the systems, it would not be good for them to use it anymore. Now we will see in what form Symbolab continued to solve the example. Examples as can be seen in **Figure 10** and **Figure 11**.

MathPapa

It is an online math solver or mobile application that solves algebraic problems as well as simpler problems, which in our case is an ideal application to use. The main reason for this preference is that it is a mobile application which can be downloaded on both iPhone and Android devices, the second reason is that it does not have great possibilities, so it does not give much information to the student about the particular solution, since that the student should be able to solve the task by himself, this application gives him the opportunity to see the graph, which in this case helps us in the graphic method of solving systems, as well as we have the solution of the system that at the end the student can see if he has solve the task well. An example solved with this application see in **Figure 12**.

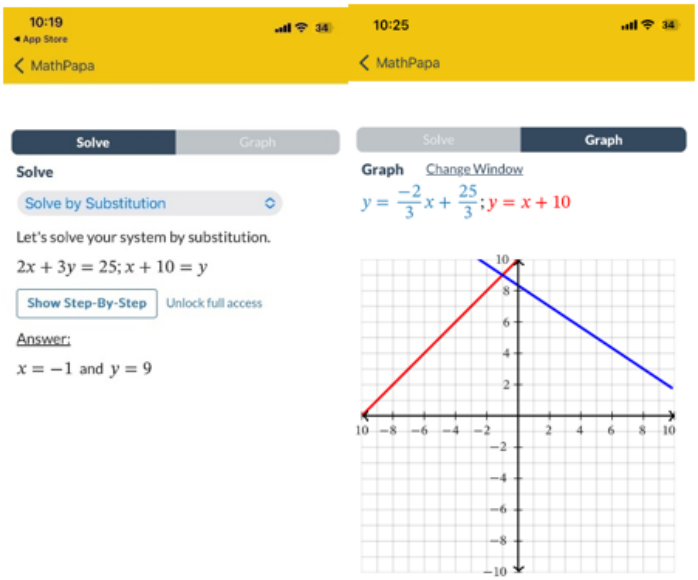

Figure 12. MathPapa problem solving (Source: Field study, using MathPapa)

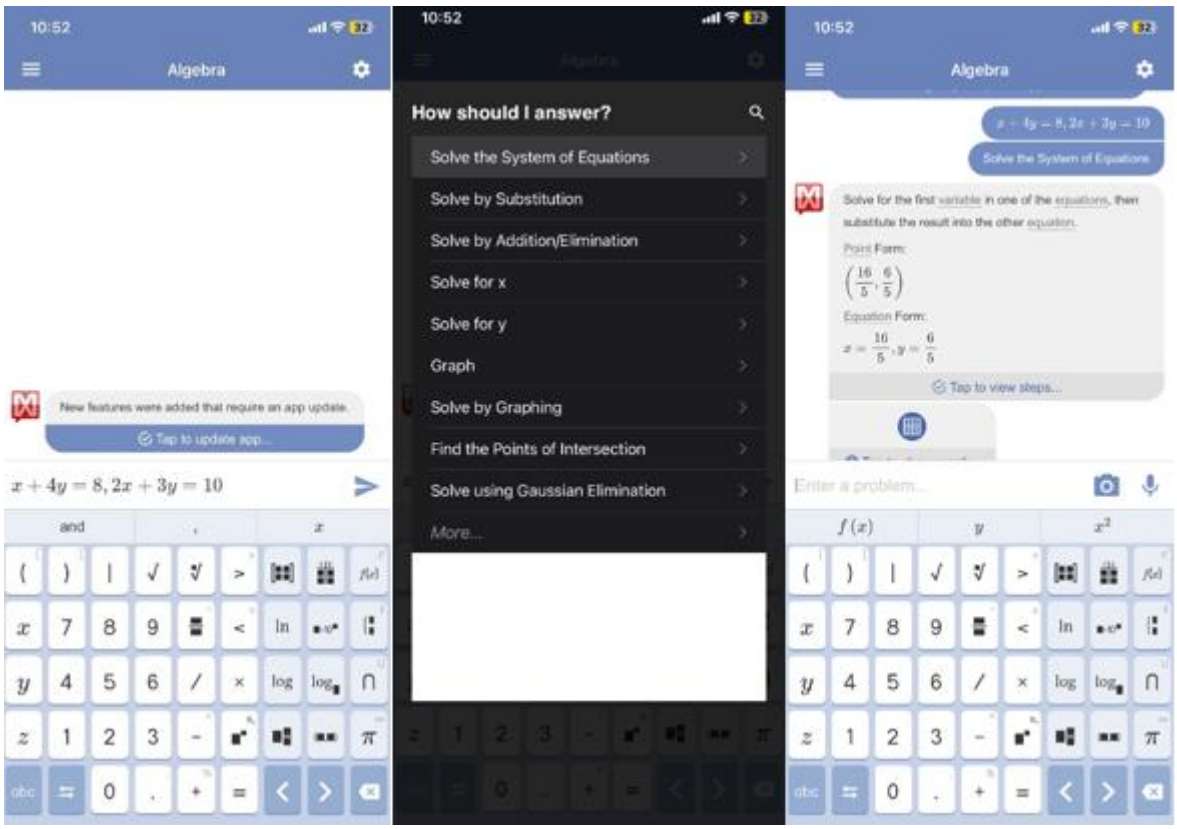

Figure 13. Mathway problem solving (Source: Field study, using MAthway)

Mathway

It is also a widely used online math solver for algebraic solutions, as it can be solved in many ways. So, allow choosing what you want the application to choose. It is an application that can be downloaded both from mobile phones and also on laptops and computers. Mathway is an online math solver that doesn't show you the procedure for solving the task but shows you the result directly. It is also an online math problem solver that should mostly be used by students and especially by teachers for solving tasks and their visual presentation since Mathway also can display graphics. An example solved with Mathay presented on **Figure 13**.

Comparison of the Classical Method with the Modern Method in the Understanding and Explanation of Systems of Linear Equations with Two Variables

Learning mathematics plays a crucial role in the development and advancement of the world, as it offers solutions to problems in nearly every field. Given its importance and the innovations stemming from mathematics, it is essential that we embrace these

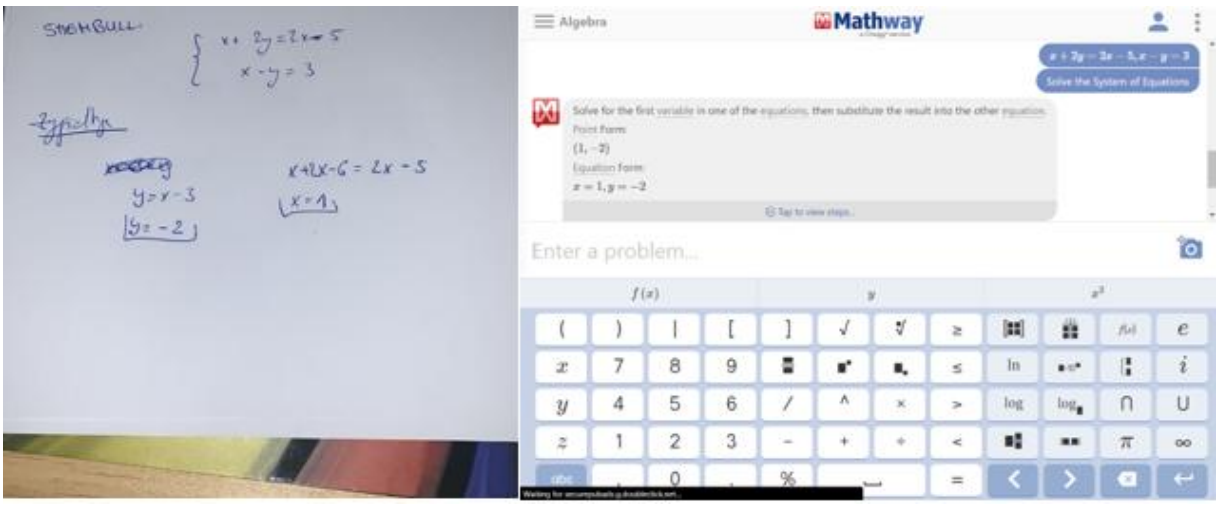

Figure 14. Solution of the system x+2y=2x−5 and x−y=3 using Mathway (Source: Field study)

developments and incorporate them into the teaching and learning of mathematics. Traditional teaching methods, while foundational, are often less effective because they place the responsibility solely on the student after the lesson has been explained by the teacher. This approach, which lacks visualization, real-world connections, and deeper understanding, is no longer sufficient in modern education.

Education has shifted away from the model where the teacher is the sole source of knowledge. Today, students have access to technology and electronic devices that provide quick and accurate answers to their questions. Therefore, as teaching and learning evolve, so too must the methods we use. Integrating technology into the classroom not only makes lessons more engaging but also captures the maximum interest and attention of students, leading to more effective learning outcomes.

The same applies when teaching systems of linear equations in two variables. Traditional methods, which rely mainly on explanation and homework, are often insufficient because they do not actively engage the student during the lesson. By contrast, when students are placed at the center of learning—facing problems directly, asking questions, and working with real-life examples—they achieve a deeper understanding. Discussing these examples with peers further strengthens comprehension and encourages collaborative learning. Besides illustrations important role in understanding the unit is homework, classroom commitment, extra hours, and extra exercises (Aliu et al., 2021).

To explain the systems of linear equations, we would have to use a software or an application that the students can download on their smartphones, and in this case, we will use the software to solve the task. Let's see the Example 5.

Example 5: To solve the system.

$$\begin{aligned} x + 2y &= 2x - 5 \\ x - y &= 3 \end{aligned} \tag{12}$$

Solution: Then if the student has solved the task and he has managed to solve it, but in the end, he does not know if he has a good result or not, a good proof of the result is also the use of the online math problem solver, which in this case I use online math solver MathWay, where now the student is clearly aware that the result is correct and has managed to solve the task well. MathWay as can be seen in **Figure 14**.

So, from the above example we are seeing that the connection of the modern method of explanation with some traditional methods would be the perfect thing in the explanation and understanding of systems of linear equations with two variables. Even the students, if they had any problems during the solutions of the tasks, the applications would be a great help for them.

METHODOLOGY

The experiment or study that was done for this scientific work is based on primary school students, specifically for 9th grade students. The hypotheses that we want to find the solution through this study are: Main hypothesis (H1) and additional hypothesis (H2).

H1: The teaching method through mathematical software (Geogebra and Mathway) is more effective in improving students' performance in solving systems of linear equations with two variables, compared to the traditional method.

H2: The use of mathematical software (such as Geogebra and Mathway) will result in a greater improvement in students' knowledge and skills compared to those using the traditional teaching method.

The study was done so that the students were subjected to a test, where they had questions and tasks of the type of systems of linear equations with two variables. All groups were given the same set of problems, for example:

$$\begin{aligned} 2x + 3y &= 12 \\ x - y &= 4 \end{aligned} \tag{13}$$

The GeoGebra group solved these graphically by finding intersections, the Mathway group used the solver to obtain ordered pairs (with optional steps), while the traditional group solved by substitution or elimination. This ensured comparability and clarified the expected form of solutions. Word problems were also included, e.g., 'There are 13 animals in a shed (chickens and sheep) and 40 legs in total. How many chickens and sheep are there?'

For example, another task stated: 'A cinema sold 35 tickets, some at $20 per adult and some at $10 per student, for a total of $500. How many adult and student tickets were sold?' – In the GeoGebra group, students graphed the equations and identified the intersection point. In the Mathway group, students entered the equations into the solver to obtain the ordered pair (15, 20). In the traditional group, students solved the system using substitution or elimination. This example illustrates how the same task was adapted for each method, ensuring consistency while allowing the tools to shape the solution format. The classification into conceptual knowledge, procedural knowledge, and problem-solving was made based on student responses:

1) Understanding definitions and forms (conceptual),
2) Applying methods step-by-step (procedural), and
3) Translating real-world tasks into equations (problem-solving).

Regarding the study, this paper has a quantitative study, where statistical data were extracted from the answers of students in testing. In this study, two main types of variables have been identified.

Variables

Independent variables

The independent variable in this study is the teaching method used to explain systems of linear equations in two variables. These methods are:

- Use of GeoGebra: Students in the first group used the GeoGebra software to solve the tasks.
- Using Mathway: Students in the second group used Mathway software to solve the tasks.
- Traditional method: Students in the third group solved the tasks without the help of any software, following the traditional teaching method.

Dependent variables

To analyze the students' test results, each response was categorized into one of three dimensions: Conceptual knowledge, procedural knowledge, or problem-solving ability. This classification was based on a rubric developed from the mathematics curriculum and prior research on systems of linear equations.

- Conceptual knowledge referred to the ability to recognize and explain definitions, forms, and properties of linear equations. For example, identifying that two linear equations in two variables represent two intersecting lines in a plane was coded as conceptual knowledge.
- Procedural knowledge referred to the correct application of step-by-step methods, such as substitution or elimination, to solve a given system. For example, correctly applying elimination to solve $2x + 3y = 12$ and $x – y = 4$ was coded as procedural knowledge.
- Problem-solving ability referred to the translation of real-world situations into mathematical models and successfully solving them. For example, correctly formulating and solving equations for the word problem about 13 animals with 40 legs was coded as problem-solving.

Each student's response was reviewed and scored according to these categories. A correct response within a category received a score of 1, while incorrect or incomplete responses received 0. Scores for each student were then summed within each category and entered into SPSS for statistical analysis. To ensure reliability, two mathematics educators independently coded a subset of responses, and disagreements were resolved through discussion. This process ensured consistency in applying the rubric across all responses.

The experimental groups are the Geogebra and Mathway groups, while the control group is the group without the application.

Participants and Data Collection

The use of applications and software in mathematics has now become indispensable as well as necessary to have a successful and understandable class. However, as far as systems of linear equations are concerned, the number of software that can be used is quite large, the problem is up to us which one to choose, or perhaps the use of no application is more efficient for this topic. This was the biggest dilemma and the beginning of our search. To get results, to answer these doubts of ours, we did a study.

The study was conducted on the students of the "Pjeter Bogdani" middle school in Pristina, during the first semester of 2023/2024. The 9th grade students were gathered, and we divided them into three groups, the first group had 35, the second and third groups had 34 students each. They were of the same age and of different genders. The students were divided into groups of first they held extra hours to learn how the applications work, for the Geogebra group as well as for the Mathway group they held 3 extra hours to explain first how the applications work and then what we need more for the topic of linear equations with two variables. Afterwards, a post-test was held so that each group had 30 minutes to solve the tasks, mostly problems that the students had with applications since they have not yet learned how to use applications. Then the final test was held, where each group had 45 minutes of time to solve the test questions. It should also be noted that the children underwent the test with the full knowledge and consent of the parent. The tests done in these groups have contained a variety of questions, starting from questions that include basic knowledge in the concepts of linear equations with two variables, there were also questions of the type for evaluating the procedures for solving a task where those tasks the correctness of the solution of the task was also evaluated, that is, did the student reach the correct result. The whole test contained 7 questions.

This decision to divide the groups in this form is as a result of finding a better and clearer method for explaining this topic, i.e. systems of linear equations with two variables. Based on the results that they have shown, data has been collected and then they are collected in **Table 1**. SPSS software helped us to get this data and process it, in which we entered the data that came out of the tests.

Table 1. ANOVA results

Values			Sum of squares	df	Mean square	F	Sig.
Conceptual knowledge	GeoGebra	Between groups	47.943	5	9.589	49.655	0.000
		Within groups	5.600	29	0.193		
		Total	53.543	34			
	MathWay	Between groups	39.418	5	7.884	68.981	0.000
		Within groups	3.200	28	0.114		
		Total	42.618	33			
	No app	Between groups	55.309	5	11.062	53.866	0.000
		Within groups	5.750	28	0.205		
		Total	61.059	33			
Procedural knowledge	GeoGebra	Between groups	46.200	5	9.240	49.622	0.000
		Within groups	5.400	29	0.186		
		Total	51.600	34			
	MathWay	Between groups	33.959	5	6.792	61.345	0.000
		Within groups	3.100	28	0.111		
		Total	37.059	33			
	No app	Between groups	48.141	5	9.628	36.930	0.000
		Within groups	7.300	28	0.261		
		Total	55.441	33			
Problem solving	GeoGebra	Between groups	53.950	5	10.790	127.718	0.000
		Within groups	2.450	29	0.084		
		Total	56.400	34			
	MathWay	Between groups	37.900	5	7.580	32.158	0.000
		Within groups	6.600	28	0.236		
		Total	44.500	33			
	No app	Between groups	47.532	5	9.506	30.077	0.000
		Within groups	8.850	28	0.316		
		Total	56.382	33			
Overall test	GeoGebra	Between groups	39.300	5	7.860	27.463	0.000
		Within groups	8.300	29	0.286		
		Total	47.600	34			
	MathWay	Between groups	35.641	5	7.128	27.341	0.000
		Within groups	7.300	28	0.261		
		Total	42.941	33			
	No app	Between groups	34.850	5	6.970	37.895	0.000
		Within groups	5.150	28	0.184		
		Total	40.000	33			

ALGEBRAIC CONCEPTS OF THE SUBJECT

The work is formulated entirely using programs and online math solvers such as GeoGebra and Mathway, adhering to the rules given by the highest institutions. This paper focuses more on three basic points.

Understanding Concepts

This aspect of the study examines students' ability to understand and acquire the concepts of systems of linear equations in two variables. It includes theoretical knowledge and interpretation of various forms of equations and their solutions.

Procedural Knowledge

This segment focuses on students' ability to follow procedural steps in solving problems with systems of linear equations. This aspect involves the use of different methods and techniques to arrive at accurate solutions.

Solving Problems

This part of the study analyzes students' ability to apply knowledge and procedures to different situations and practical problems involving systems of linear equations. It measures how well students are able to use learned methodologies to solve new and different problems.

In each of these three points, a study was done and they were divided into groups for the GeoGebra group, Mathway, and the group without any application. General tests and student results are also part of the study and provide important insight into the effectiveness of different teaching methods. This information helps to evaluate the impact of different software and methods on improving students' performance in solving systems of linear equations.

Table 2. Ms and SDs of the three groups

Variable	Group	N	Mean	SD
Conceptual knowledge	Geogebra	35	7.8857	1.25491
	Mathway	34	7.7353	1.13642
	No App	34	6.7059	1.36025
Procedural knowledge	Geogebra	35	7.8000	1.23193
	Mathway	34	7.7059	1.05971
	No App	34	6.6765	1.29616
Problem solving	Geogebra	35	7.6000	1.28795
	Mathway	34	7.5000	1.16124
	No App	34	6.5588	1.30712
Overall test	Geogebra	35	8.2000	1.18322
	Mathway	34	8.1765	1.14072
	No App	34	8.0000	1.10096

Study Application Mechanism

The tests have been created so that they are tests of the caliber of 9th grade students, so with the help of school materials, a test has been created with tasks that they can solve and understand better. Both the tasks and the conceptual questions. The statistics that will be extracted for this scientific work will be worked with SPSS software (Statistical Package for the Social Sciences) and these statistics will be part of the work where we will be based to see the results of our research.

Learning through the application

Students are first taught how to download the programs or access the relevant websites using their phones, iPads, or laptops. They also have continuous access to the internet. Once prepared, they are shown how to use the online math problem solvers, ensuring they do not face difficulties during the lesson explanations or while completing their assigned tasks. Additionally, all students received supplementary materials to guide them in learning how to use the applications. Those using the applications were later supported by other mathematics teachers for deeper learning, receiving help whenever they encountered issues. For the students who did not use the applications, the lesson explanations, tasks, and testing followed the traditional teaching methods.

Problem-solving tasks

Students were then assigned tasks related to solving systems of linear equations with two variables. Those using the online math problem solver occasionally faced challenges in writing the equations and navigating the software, as they were not yet familiar with the applications. On the other hand, students who did not use GeoGebra or the online math problem solver encountered different types of difficulties related to solving the problems using traditional methods.

General test results

At the end of the study, all students were given a general test. Each group's results were recorded, and it was observed that the students who were taught using the applications, despite dealing with new technologies, achieved the highest scores. The detailed results for each group will be presented in **Table 2**.

RESULTS

Based on the data we have collected and based on the post-test analysis, these results were achieved. There were differences between the averages of students using GeoGebra, Mathway, and students without the application. The group that learned using Geogebra had the following results: Conceptual knowledge was respectively (7.9), procedural knowledge was with the result (7.8), knowledge in solving tasks was (7.6) and in the GeoGebra test group was (8.2). The students who used Mathway had these results: Conceptual knowledge was respectively (7.7), procedural knowledge was with the result (7.7), knowledge in solving tasks was (7.5) as well as in test group I Mathway was (8.1). The group that did not use any software had these results: Conceptual knowledge was respectively (6.7), procedural knowledge was with a result of (6.6), knowledge in solving tasks was (6.5) and in the test this group was (8.0) (see **Table 2**). To do an even better search to be more confident about the results obtained, we also performed an ANOVA analysis (**Table 1**) where we obtained results.

To identify the significance of the differences, we use advanced software with results from student tests. From the results, we have that the group that used the GeoGebra software was the one that had the best results in the test, followed by those students who used Mathway as well, and in the end with moderately good results were those students who have not used any application.

Narrative of the Results

Conceptual knowledge

The ANOVA results indicate that the use of GeoGebra led to significantly higher scores in conceptual knowledge ($F(5, 29) = 49.655$, $p < 0.001$) compared to Mathway and no application. This suggests that GeoGebra is more effective in enhancing conceptual understanding.

Procedural knowledge

Similarly, GeoGebra users demonstrated significantly better procedural knowledge ($F(5, 29) = 49.622$, $p < 0.001$) than users of Mathway and those with no application. This implies GeoGebra has a substantial impact on procedural skills.

Problem solving

The analysis shows that GeoGebra users had higher problem-solving scores ($F(5, 29) = 127.718$, $p < 0.001$) compared to both Mathway users and non-users. This indicates GeoGebra's effectiveness in improving problem-solving abilities.

Overall test performance

GeoGebra users performed significantly better overall ($F(5, 29) = 27.463$, $p < 0.001$) than Mathway users and non-users, highlighting its overall efficacy in test performance.

APA Reporting

Conceptual knowledge

An ANOVA was conducted to compare the effect of GeoGebra, Mathway, and no application on students' conceptual knowledge. The results indicated a significant difference among groups, $F(5, 29) = 49.655$, $p < 0.001$. Post-hoc analyses revealed that students using GeoGebra ($M = 7.88$, $SD = 1.25$) scored significantly higher than those using Mathway ($M = 7.74$, $SD = 1.14$) and those with no application ($M = 6.71$, $SD = 1.36$).

Procedural knowledge

An ANOVA was conducted to examine differences in procedural knowledge across the groups. Results showed a significant difference, $F(5, 29) = 49.622$, $p < 0.001$. Post-hoc comparisons indicated that GeoGebra users ($M = 7.80$, $SD = 1.23$) had significantly higher scores than Mathway users ($M = 7.71$, $SD = 1.06$) and non-users ($M = 6.68$, $SD = 1.30$).

Problem solving

An ANOVA for problem-solving abilities revealed significant differences, $F(5, 29) = 127.718$, $p < 0.001$. Students using GeoGebra ($M = 7.60$, $SD = 1.29$) outperformed both Mathway users ($M = 7.50$, $SD = 1.16$) and non-users ($M = 6.56$, $SD = 1.31$).

Overall test performance

The ANOVA for overall test performance indicated significant differences among groups, $F(5, 29) = 27.463$, $p < 0.001$. GeoGebra users ($M = 8.20$, $SD = 1.18$) achieved higher overall test scores compared to Mathway users ($M = 8.18$, $SD = 1.14$) and non-users ($M = 8.00$, $SD = 1.10$).

The analysis of results from ANOVA showed that the use of GeoGebra and Mathway significantly influenced the performance of students in solving systems of linear equations with two variables compared to the traditional method. The use of mathematical software (GeoGebra and Mathway) resulted in a significant improvement of knowledge and skills. The results of the study support the main hypothesis, showing that the use of mathematical software is more effective in improving students' performance in solving systems of linear equations. The students who used GeoGebra and Mathway achieved higher results in the performance tests compared to those who used the traditional method, highlighting the importance of integrating technology in the teaching of the students compared to the students who used the traditional method of teaching. The results of this study also support the additional hypothesis, demonstrating that mathematical software contributes to a greater improvement of students' knowledge and skills compared to the traditional method. This suggests that the use of technology in teaching can offer great advantages in improving student outcomes.

DISCUSSION

This study explored the impact of different instructional methods—GeoGebra, Mathway, and traditional teaching—on students' understanding and performance in solving systems of linear equations. The findings indicate that GeoGebra, a dynamic geometry software, was particularly effective in enhancing students' conceptual understanding, procedural skills, and problem-solving abilities. The results of this study align with previous research highlighting the benefits of using technological tools in education. GeoGebra's interactive features and visualizations contribute to a better understanding of abstract concepts and improve students' mathematical performance. This suggests that incorporating such tools into the curriculum could enhance educational outcomes.

However, there are some limitations to this study. The relatively small sample size and the varying levels of access to technology among students may have influenced the results. Future research should aim to include a larger and more diverse sample to validate these findings and explore the long-term effects of using GeoGebra in mathematics education.

To build on this study, future research could investigate the impact of GeoGebra on different mathematical topics and across various educational levels. Additionally, examining how different teaching methods integrate with GeoGebra and other educational technologies could provide insights into optimizing instructional strategies for diverse learning environments.

CONCLUSION

This study underscores the importance of integrating technology, such as mathematical software, into the teaching and learning of mathematics, particularly in the context of systems of linear equations with two variables. The findings suggest that the inclusion of tools like GeoGebra and Mathway significantly enhances the learning experience and effectiveness for both teachers and students.

Implications for Teaching

The results indicate that incorporating mathematical software into the curriculum can provide significant benefits. Teachers who utilize these tools can offer more engaging and interactive lessons, which can increase students' interest and understanding of complex mathematical concepts. The ability to visually represent and manipulate equations and systems through software helps students grasp the fundamentals more effectively than traditional methods alone.

Student Understanding and Problem-Solving

Students exposed to GeoGebra and Mathway demonstrated a deeper understanding of linear equations and improved performance on related tasks, including textual problems that often pose challenges. Although software alone cannot solve problems for students, it offers valuable support and alternative perspectives that aid in problem comprehension and solution strategies.

Limitations of Traditional Methods

Traditional teaching methods, which focus heavily on theoretical explanations and rote learning, may not be sufficient for fostering a comprehensive understanding of mathematical concepts. The study suggests that a more dynamic and interactive approach, leveraging technology, is necessary to make mathematics more engaging and logically accessible.

Future Directions

The positive impact observed with GeoGebra, Mathway, and similar programs highlights the need for further exploration and adoption of these tools in mathematics education. Future studies should aim to include larger sample sizes and diverse educational settings to confirm these findings and explore the long-term effects of technology-enhanced teaching.

Encouraging Exploration

The study also reveals that students who used these applications are more motivated to explore and delve into advanced mathematical topics. This enthusiasm for further exploration underscores the potential of technological tools to not only improve immediate learning outcomes but also foster a lasting interest in mathematics.

In conclusion, integrating mathematical software into teaching practices can greatly enhance the educational experience, making mathematics more accessible, engaging, and effective for students. Teachers are encouraged to embrace these tools to support and enrich their instructional methods, ultimately benefiting student learning and achievement in mathematics.

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AI statement: The authors stated that they did not use generative AI or AI-based tools during the study.

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