

Combining Theoretical Approaches: Socio-Didactic-Mathematical Norms and Perspectives in Pre-service Secondary Mathematics Teachers' Discourse

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ABSTRACT

This study is part of wider research that seeks to investigate the existence (or not) of socio-didactic-mathematical norms in the discourse arises within a group of pre-service secondary mathematics teachers when they are solving a didactic-mathematical task. In addition, we try to analyze whether any of these norms could have relationships with teachers' perspectives that they can adopt in their future practice. The data comes from the transcriptions of the audio recordings of the dialogues among the pre-service teachers when they are solving a didactic-mathematical task in the classroom. Based on our analysis, we have been able to identify five socio-didactic-mathematical norms. Three of them were in some way related to the mathematical content and its learning. The other two are related to teachers' role, providing information about characteristics that the future teachers associate with said role. Furthermore, we have identified features related to teachers' perspectives through the above-mentioned norms.

Keywords: pre-service secondary mathematics teachers' discourse, socio-didactic-mathematical norms, teachers' perspectives

INTRODUCTION

The last few years have seen a growing number of studies related to the different norms that can be identified in discourse. In these studies, the norms have been considered with different nuances. From a sociological perspective, norms or prescriptions have been specified as demands if they are shown in an obvious way or norms if they are hidden assumptions or underlying ideas (Biddle & Thomas, 1996). Authors such as Herbel-Eisenmann (2003) have provided information about different terminologies that have been used with respect to the word "norm", according to the adopted theoretical approach.

From an initial cognitive perspective, "with constructivism as a guiding framework" (Yackel & Cobb, 1996, p. 459), broadened with a sociological perspective, Yackel and Cobb (1996) have focused on socio-mathematical norms, considering these norms as "normative aspects of mathematics discussions specific to students' mathematical activity" (p. 461). They have shown how these norms regulate mathematical argumentation and influence the learning opportunities for the students and the teacher (Yackel & Cobb, 1996). Other authors such as Tatsis and Koleza (2008) have focused on social and socio-mathematical norms, contemplating them in the same sense as the above-mentioned authors. They have studied how these norms are established during the interactions of pre-service teachers when they solve mathematical problems and their effect in the problem-solving process. The construct socio-mathematical norms has been used by Gorgorio and Planas (2005), with a heavier social weight than in Yackel and Cobb's interpretation, focusing on the different re-

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interpretations of the same norms in multiethnic classrooms where the immigrant students felt that they themselves or their contributions were valued negatively. The lack of negotiation caused obstacles to immigrant students' participation in the mathematical conversations. Furthermore, taking into account Sfard's works (2007; 2008), Sánchez and García (2014) have considered the socio-mathematical and mathematical norms that arise in the interaction between pre-service primary teachers when they solve a mathematical task, incorporating the mathematical topic addressed (the mathematical definition) as a new variable.

In general, research related to norms has provided new insights about how they are established during the social interactions generated in different settings (Yackel & Cobb, 1996), its important role in students' learning (McNeal & Simon, 2000; Yackel, Cobb, & Wood, 1991) at different educational levels and, therefore, the role of the teacher to develop the norms in classroom (Dixon, Andreasen, & Stephan, 2009; McClain & Cobb, 1997; Yackel, 2002), showing its influence in mathematics teaching/learning processes.

Precisely the characterization of the teachers' role in the classrooms has been considered by other researches. In particular, from a conceptual framework based on a social constructivist perspective, a coordination of cognitive and social perspectives on knowing and coming to know, Simon and colleagues (Simon, Tzur, Heinz, & Smith, 2000; Tzur, Simon, Heinz, & Kinzel, 2001) have utilized the term perspective "to postulate a broad pedagogical structure composed of a multiple conceptions that collectively organize some aspects of a teacher's practice" (Tzur et al., 2001, p. 228). Taking into account the importance of identification of these perspectives that underlie teacher's practice, in other occasions we have considered the existence of relationships between teachers' perspectives and the relational architecture that is established in classrooms (Escudero & Sánchez, 2008), and the way in which the construction mechanisms of knowledge are modeled by teachers in classrooms (García, Gavilán, & Llinares, 2012). Nevertheless, how some norms that regulate classroom practices could encourage the development of viewpoints or positions related to the adoption of a teacher's determinate perspective have not been given in-depth consideration. The relationships of some norms that are socially shared by future secondary mathematics teachers and the possible perspectives that they can generate become our object of interest in this work.

CONCEPTUAL FRAMEWORK

As we have mentioned in the previous section, different theoretical approaches and perspectives has been combined or coordinated in mathematics education research (Wedegé, 2009). In this study, we agree with Cobb (1994) in the sense that: "The central issue is then not that of adjudicating a dispute between opposing perspectives. Instead, it is to explore ways of coordinating constructivist and sociocultural perspectives in mathematics education" (p. 13).

Taking in mind this comment, in our research, on the one hand, the way of considering norms is based on the point of view of Sfard (2008). In her theoretical approach, this researcher highlights the unity of communicating and thinking (cognition). She suggests the word commognition, emphasizing that in the discourse proposed in thinking, cognitive processes and interpersonal communication processes are different manifestations of the same phenomenon. Sfard (2008) considers norms to be "metadiscursive rules that are widely endorsed and enacted within the discourse community" (p. 300). On the basis of this consideration of norms, without minimizing the importance of other norms, here we focus on socio-didactic-mathematical norms (SDMNs) that arise in the interaction within groups of pre-service secondary teachers when they are solving a task proposed in a course of a teacher education program.

In this study, we try to identify SDMNs in pre-service secondary teachers' discourse on the basis of the commonly accepted constraints, conditions and particularities related to mathematics as a school subject matter, its teaching and its learning. They may arise in the interaction between these pre-service teachers when they are solving a task related to teachers' professional activity in a specific context: teacher education. We assume that "these constraints and conditions are consequence of the context in which mathematical practices of these students have been developed" (Sánchez & García, 2014, p. 308). In addition to these specific norms, we are aware that other norms exist, but they are not addressed in this part of our study.

On the other hand, in our conceptual framework, we include the above-mentioned notion of perspective in the sense of Tzur et al. (2001). In particular, we consider the three different perspectives characterized by these authors: traditional, perception-based and conception-based. They emphasize that these perspectives are their characterization of teachers' practice, and not how the teachers themselves would describe their practices. A traditional perspective is characterized by "a passive stance toward learning coupled with a

“harvesting” stance toward mathematics, it exists outside and independent of the learner (or knower) who needs to obtain it” (Tzur, 2010, p. 56). In this perspective, teaching is characterized by “teachers’ attempts to transmit particular mathematical ideas to students” (Tzur et al., 2001, p. 247). As Tzur points out, in this perspective “teachers feel responsible for logically organizing and clearly presenting the mathematical content” (Tzur, 2010, p. 56). This content, for him, has “crystallized” through millennia. A perception-based perspective considers “the teacher’s primary role is not to directly transmit the intended ideas to students, but to orchestrate conditions that engage students in actively seeing and connecting those ideas” (Tzur et al., 2001, p. 247). In contrast to traditional perspective, a perception-based perspective emphasizes on learning as necessarily an active process; teaching based on this perspective emphasizes “students coming to see mathematical ideas and relationships through their own experience of the mathematics” (Tzur et al., 2001, p. 247). Finally, in the conception-based perspective teachers feel responsible for engaging learners in realistic tasks, orienting learning reflections through a reorganization of previously established schemes (Tzur, 2010). From such a perspective, “mathematics is thought of as a web of conceptions that humans abstract through reflection. Learning is the building up and the continual transformation of one’s conceptions. Teaching, promoting intended conceptual advances” (Tzur et al., 2001, p. 247).

In this article we focus on how socio-didactic-mathematical norms, which arise in the interaction within groups of pre-service secondary mathematics teachers when they are solving a didactic-mathematical task, could be related to the distinct aspects that characterize the different perspectives. The research questions behind this study are:

- Is it possible (or not) to identify socio-didactic-mathematical norms in the discourse of pre-service secondary teachers in the context of solving a didactic-mathematical task?
- Could these norms have any relationship with their future teachers’ perspectives?

METHOD

Taking into account the nature of the research questions posed, this study adopts a qualitative/interpretative approach, since it tries to describe, decode and interpret the meanings of events happening in a particular social context. In the following, we present the different subsections included in our methodological design.

Participants and Context

In the part of the research reported here, participants were future secondary mathematics teachers enrolled in the Master’s Degree in Secondary Education Teacher Training at a large university in Spain and, specifically, in the “Mathematics learning and teaching” course. This Master’s is a postgraduate course of 60 credits ECTS (European Credit Transfer System), recently implemented in Spanish universities. It plays an essential role: features the necessary professional requirements that enable an individual to become a teacher at secondary school level.

The “Mathematics learning and teaching” course (12 credits ECTS) was focused on mathematics education issues. In this course, the university teacher proposed didactic-mathematical tasks in the classroom. These tasks tried to bring the future teachers closer to the professional work of secondary mathematics teachers. For instance, each task could be related to a professional activity, such as interpreting pupils’ mathematical productions, and a mathematical content, for example geometry. For solving these tasks, the university teachers provided some articles from mathematics education literature. Other information was supplied by university teachers’ explanations and the mathematical content knowledge coming from the previous background of these pre-service secondary teachers (a degree with strong mathematical content). The pre-service teachers worked in small groups and the groups were given total autonomy with respect to their way of working. The pre-service teachers had to write a final report featuring the findings of their analysis. They were used by the university teacher as a means of evaluating pre-service teachers’ work.

On the course, there were 28 pre-service secondary teachers enrolled and they worked in seven small groups in two-hour sessions per week. There were two 5-pre-service teacher groups, two 3-pre-service teacher groups, and three 4-pre-service teacher groups. They were informed of our research (characteristics, aim, confidentiality issues, etc.), and they should decide for themselves whether or not they wished to participate in the study.

In particular, participants were 20 of 28 pre-service secondary teachers (6 females and 14 males), corresponding to five of the seven groups (namely G1, G2, etc. on our research), that decided to participate voluntarily in the study. Although they came from different specialties, all of them had a university degree related to mathematics or other scientific specialties. These pre-service teachers have not had formal teaching experiences. Their ages ranged from 23 to 35, and they were ethnically and racially similar.

Data Collection Procedures

Taking into account that in the commognitive framework of Sfard the discourse of students is “the principal object of inquiry” (Sfard, 2008, p. 276), researchers have used different learning environments to access to the discourse (Wille & Boquet, 2009). The data for our study consists mainly of the transcriptions of audio recordings of the dialogues among the pre-service teachers while solving a didactic-mathematical task proposed in the above-mentioned course. It was very different to traditional tasks that are usual in the scientific fields in which these pre-service teachers had obtained their degrees (Sánchez & García, 2009). There was no intervention on the part of the researchers neither in the design nor in the implementation of the tasks of the course. We chose one of the tasks proposed by the university teacher. This task was a didactic-mathematical situation related to a teacher’s professional activity (analyzing school problems taken from secondary school textbooks) and a mathematical content (functions), including two articles (see Appendix). The process of solving this task was the context in which the data were generated.

The choice of this task was precisely conditioned by encompassing professional activities which are very present in the work of teachers, and a mathematical content (functions) that is familiar to the pre-service teachers. We thought this can encourage the communication among them when they solve the proposed didactic-mathematical task, providing an opportunity for a broad exchange of views and favoring our access to pre-service teachers’ discourse.

The recordings were collected during seven sessions of two hours a week per each one of five participating groups.

Data Analysis

Once the dialogues were transcribed into written text, in a first step we analyzed the discourse on the basis of two properties proposed by Sfard (2007; 2008): word use and narratives. In our study, we adapt Sfard’s proposal with respect to a discourse related to mathematics, its teaching and its learning. Mathematical words were replaced by didactic-mathematical words. Examples of these words are: modes of representation, mathematical element, pupils’ understanding and so on. In the case of narratives, we focused on endorsed narratives considered as sets of propositions that are accepted and labeled as true by the pre-service secondary mathematics teachers. Some examples of endorsed narratives are shown in **Table 1** (highlighted in bold). Although the words were presented in isolation at times, they were usually incorporated into the narratives.

The three members of the research team participated in the identification of the two properties. The common identifications were assumed and the problematic identifications were discussed to be accepted or rejected. In this way, the analysis process was validated.

In a second step, from the previous analysis, we began to identify in the pre-service teachers’ endorsed narratives features coming from the way of considering mathematics as a subject matter in the school context.

In a third step, when different narratives shared features related to a same issue, we analyzed its meaning in an attempt to infer norms. In particular, SDMNs were inferred by the researchers on the basis of different features linked to aspects coming from the way of considering mathematics as a subject matter to be taught and learnt.

In **Table 1**, we present a brief example from a group of pre-service teachers (Group 7), to show how we infer SDMNs.

In **Table 1**, the successive inclusion in the discourse of endorsed narratives that emphasized different features related to the teacher’s role lead us to identify the socio-didactic-mathematical norm that we have named “The teacher validates the knowledge and clarifies doubts”.

Finally, in the fourth step, these norms were considered from the different characteristics of the perspectives that have been described in the conceptual framework of this study (Tzur et al., 2001; Tzur, 2010). In our example, we can say the given teacher’s role by these pre-service secondary teachers, expressed in this norm, fits with a traditional perspective.

Table 1. Example of the process followed to infer SDM norms

Examples of endorsed narratives		Identified features	Inferred norm	
<i>(G7, Page 63) [Context: Pre-service teachers discuss the information provided in relation to the translations between modes of representation]</i>				
1619: D:	We are going to call him [referring to the university teacher] to see if the theory is the ...	The teacher confirms results and clarifies doubts		
1620: A:	What?			
1621: D:	That we are going to call him, right?			
1622: A:	Yes , but if this is all clear to us, the question is the most ...			
1623: M:	But we are going to ask him the doubts....			
<i>(G7, Page 67) [Context: Pre-service teachers try to decide the best procedure to solve problem]</i>				
1723: A:	Solving an equation, exactly, okay. I think that's done, right?	The teacher confirms what has been done		
1724: M:	Now we call him [referring to the university teacher] to check. Now when he comes we show him the answer to question 1.			
1725: A:	Ok.			
<i>(G7, Page 75) [Context: Pre-service teachers have finished part of the task]</i>				
1911: A:	[addressing to the university teacher as the representative of the group] We have done three problems if you'd care to take a look...	The teacher validates the response to allow them to continue	"The teacher validates the knowledge and solves the doubts"	
1912: S:	To see what you think ...			
<i>(G7, Pages 81-82) [Context: Pre-service teachers are discussing the interpretation of a graph]</i>				
2058: M:	... the interpretation, ...yes. I've thought about it, it is very important but I don't know if it is an element.	We can put whatever and then confirm with the teacher		
2059: A:	Okay , then put it as a mathematical element and we can ask him [referring to the university teacher] afterwards when he comes...			
2060: M:	I'm going to write it in pencil and then we will know that we have to ask about it.			
<i>(G7, Pages 84-85) [Context: Pre-service teachers discuss whether the problem they are analyzing presents a local or global situation]</i>				
2127: A:	I think that the problem is in ...	The teacher guides about the way forward		
2128: M:	I would ask him [referring to the university teacher]			
2129: A:	I think that the problem is the solution. Is it global or local? ... Because you say, well yes, I know that this is the maximum but because I take a look to all...			
2130: M:	Right			
2131: A:	Exactly, but I think that it is more of the solution, but we are going to ask him.			
2132: M:	Ok.			
2133: A:	I think that's what it is, but well now we can ask him...			

FINDINGS OF THE STUDY

Based on our analysis, we have been able to infer five SDM norms in our study. The number of the norm indicates the order of appearance in the analysis process followed by the researchers. Of course, we do not want to say that all the discourse fit into these five norms. They are exclusively the norms that we have inferred.

Three of them (SDMN 2, SDMN 4, and SDMN 5) were in some way related to the mathematical content and its learning. For instance, from the different endorsed narratives identified in Groups 1 and 6, we were able to infer the SDM norm "To teach the most common or necessary mode of representation is table of values" (SDMN 2). In **Table 2**, we show endorsed narratives that have been selected as representative examples of other narratives that express analogous ideas (column 1) and their identified features (column 2) that leads us to infer the SDMN 2 (column 3).

Table 2. Representative example of all the endorsed narratives, identified features in them and inferred norm (SDMN 2)

Representative examples of endorsed narratives		Identified features	Inferred norm
<i>(G1, Page 3) [Context: Pre-service teachers discuss whether the pupils get the formula directly or they need the table of values]</i>			
63: J:	That they are not able to find the formula directly...		
64: M:	They are giving you values, for you to do... it is that if they do not give you values how you get the formula . Let's see, what you say is true, you can directly make the formula.	The pupils need the table of values to get the formula	
65: L:	Yes, but...		
66: M:	But also you can put the table , you do not have to pass of the text to the formula directly.		
67: L:	I tell you one thing I think, be aware that they are pupils who need the table .		
68: S:	The logical thing is to make the table first...		
69: M:	It is the most logical ...		"To teach the most common or necessary mode of representation is table of values" (SDMN 2)
<i>(G6, Pages 165-167) [Context: Pre-service teachers discuss whether the pupils draw the parable directly or they need the table of values]</i>			
3413: F:	And now in middle would be, through a verbal situation requires numeric values. You take the example you have put, f(x)=x², hence passes to graphical mode. This problem [referring to the provided articles] is done without tables of values.		
3414: P:	Means that tables can also be ...	The pupils need the table of values to draw the parable	
3415: E:	[addressing to the university teacher as the representative of the group] We... it is that... the problem 3 for example, go you to the 3. Whether it is that clear, if we put thus also we can say that, they give you that algebraic but you will have to make to table. That the pupil does not take 3x² and draws it.		
[...]			
3443: R:	But it is that, through a table... But nobody says you to do a table, it says you to draw it. Another thing is that you do that representation through a table		

This SDMN 2 emphasizes the importance that is given in our Spanish educational context to the use of the table as a way to obtain a graphical representation, minimizing the use of certain properties (cutting with axes, vertices) that can allow a better picture of the situation.

Other endorsed narratives identified in Group 4 lead us to the SDM norm "A mathematical result is (or is not) correct depending on situation" (SDMN 4). In **Table 3**, we show some representative examples of these endorsed narratives.

The SDMN 4 indicates how a solid training in scientific content helps to link mathematics results to the posed situation, establishing an explicit relationship between the graphical representation and the verbal situation. Precisely, this solid training can lead to the SDM norm "Explanations in the answers to the tasks are not necessary because time is wasted" (SDMN 5), identified in Group 7 as we can see in the representative example shown in the **Table 4**.

The endorsed narratives from which this example is representative suggest that, for these future teachers, communication is neither considered a relevant mathematical process nor seems to recognize the importance of its role in learning.

The other two SDMNs (SDMN 1 and SDMN 3) were linked to teaching in general and teachers' role in particular. One of them (SDMN 1: "The teacher validates the knowledge and clarifies doubts") was identified in all the groups and has been detailed in the analysis section (see **Table 1**). We can infer that future teachers need a teacher's presence and opinions to validate their work. Two aspects emerge: Doubts are not discussed thoroughly in the groups, and possible alternative answers are not discussed. Future teachers think that accurate solutions and alternatives come from the teacher. We can think that this is normal practice in teaching, and nothing new has been identified. Nevertheless, this show how a determinate way of seeing the mathematical classroom (teacher-centered teaching) is perpetuated.

Table 3. Representative example of all the endorsed narratives, identified features in them and inferred norm (SDMN 4)

Representative examples of endorsed narratives		Identified features	Inferred norm
<i>(G4, Page 109) [Context: Pre-service teachers discuss what representation modes are needed to solve the problem]</i>			
2683: M:	Whether to understand what is the maximum height, use the parable, how long is the flight? Well, you have to interact with the situation because that is not worth us negative values because that would mean that the plane has crashed...		
2684: D:	Exactly		
[...]			
2689: D:	“h” negative would mean that the plane would fly below ground level, which is not credible. Then, how long is the flight?, here we really begin with a non-verbal textual representation, but we continue with a graphical representation to understand what is ...	We have to interact with the situation to know the mathematical result correct	“A mathematical result is (or is not) correct depending on situation” (SDMN 4)
2690: J:	The values allowed of the function ...		
2691: D:	Exactly , because really with the graphical expression, with the mathematical expression, it is not sufficient because the mathematical expression allows any value. However, the graphic expression, the graph itself would not make sense for example draws it below ground		
2692: M:	So you have to interact with the verbal situation		

Table 4. Representative example of all the endorsed narratives, identified features in them and inferred norm (SDMN 5)

Representative example of endorsed narrative		Identified features	Inferred norm
<i>(G7, Page 58) [Context: Pre-service teachers discuss whether they have to explain the answers to the task or not]</i>			
1474: D:	Come you to another [problem] ...		“Explanations in the answers to the tasks are not necessary because time is wasted” (SDMN 5)
1475: A:	But we will have to explain , not?		
1476: D:	You are going to lose all afternoon.		
1477: A:	But whether this is silly. Whether that is done in a second... what we will do.	It is not necessary to explain the answers to the task, because it is a waste of time	
1478: M:	What we will do is nonsense.		
1479: A:	Exactly , whether it is better to discuss this [referring to other problem], right?		

The other (SDMN 3: “To introduce a mathematical content the teacher should always follow an established sequence”) was inferred in two groups (G1 and G4), on the basis of endorsed narratives such as those shown in **Table 5**.

With respect to the SDMN 3, a previous content already established should be taught through a “correct” sequence. A teacher must follow that sequence to transmit the content, being a “person who transmits mathematical knowledge”; this knowledge is perfectly structured both in its way of teaching and learning.

Both norms (SDMN1 and SDMN3) are related to teacher’s role. We can observe in them a distinction between norms such as SDMN1, which exist in the context of pre-service teachers as learners (related to these pre-service teachers’ practices when they are the learners), and norms such as SDMN3, which exist in the context of pre-service teachers as future teachers.

Finally, with respect to the relationship between SDM norms and different teacher’s perspective, our results have shown: on the one hand, a view of the mathematics and its learning that give importance to the table of values as a mode of representations commonly used in the classrooms (SDMN2), and does not give too much importance to communication (SDMN5). On the other hand, to transmit, to validate and to solve are verbs that can describe actions, or conditions that characterize a teacher’s role for these pre-service teachers (we do not try to generalize to other pre-service teachers) (SDMN1), and an emphasis on following a pre-

Table 5. Representative example of all the endorsed narratives, identified features in them and inferred norm (SDMN 3)

Examples of endorsed narratives		Identified features	Inferred norm
<i>(G4, Page 114) [Context: Pre-service teachers discuss the order in which to present the problem to their future pupils]</i>			
2784: D:	Look, I propose the following, first a table... afterwards the representation of one of the variables that we can call height with respect to the other one which is boiling temperature...	Pre-service teachers identify what appears to them to be a correct sequence that is assumed without posing more alternatives	"To introduce a mathematical content the teacher should always follow an established sequence" (SDMN 3)
2785: M:	Okay.		
2786: D:	Then we already have here that we can present... to the pupils... first the data processing to the table and then a graphical representation of the problem ...		
<i>(G4, Page 116) [Context: Pre-service teachers discuss the order in which to present the problem to their future pupils]</i>			
2787: D:	Okay.		
2845: J:	First mathematical element the representation in table of values, and its construction by means of a specification, you know... the process of... We will take advantage of that every thousand down, then the second mathematical element is the graphical representation of the linear function as a relative line in the plane. The following is, rule of proportionality ... Do you agree?	Pre-service teachers identify what appears to them to be a correct sequence that is assumed without posing more alternatives	
2846: D:	Yes		

established sequence on the presentation of mathematical school content (SDMN3). In some way, we could say that all these aspects are related to some characteristics identified in a teacher's traditional perspective. Furthermore, the SDMN 4 leads us infer features related to perception-based perspective. The mathematical background of these teachers can promote the use of relationships between the different modes of representations to give sense to the situation posed.

DISCUSSION AND FURTHER CONSIDERATIONS

Our results extend the work of authors who have dealt with different types of norms incorporating the definition, identification and study of socio-didactic-mathematical norms. Following authors such as Tatsis and Koleza (2008) or Yackel (2001), among others, this research deals with norms that are not established between students and the teacher in the classroom, but that arise in the interaction between the students themselves (pre-service teachers in this study) when a task is proposed, in order to minimize the teacher's influence on the discourse generated in the resolution process of the task (Tatsis, 2007).

In relation to the works in which the participants were pre-service teachers (among others, Dixon, Andreasen, & Stephan, 2009; McNeal & Simon, 2000; Roy, Tobias, Safi, & Dixon 2014; Tatsis & Koleza, 2008; Sánchez & García, 2014), a contribution of this study is the incorporation of didactic-mathematical tasks as a new element that allows to situate these pre-service teachers in their future professional activity. Our study has enabled us to inform researchers and teacher educators about what really happens in the interaction generated among the pre-service teachers when they try to solve this kind of tasks. The process of solving these tasks has allowed us to access future teachers' discourse related to mathematics, its learning and teaching, and the norms that regulate it.

From this discourse, we have been able to identify SDM norms. Some of them could be closely related to Spanish educational context that emphasizes the use of some representation modes over others and over translations between modes; in addition, it does not promote mathematical communication as an important element in students' mathematical education. Other SDMN are related to their way of considering teacher's role, providing information about characteristics that these future teachers associate with this role.

The identification of these norms can provide information to researchers and teacher educators in a double sense. On the one hand, showing how certain norms, which can come from previous school contexts of these pre-service teachers, can influence their future work. On the other hand, highlighting how certain norms may appear in very different school contexts, showing a common background that may have a sociohistorical origin

(Martin, 2000) linked to a particular western culture and shared by these pre-service teachers. If we consider that we have identified some of these norms in different groups of pre-service teachers, an important aspect arises from this result. The SDMN might be considered as something shared. This could indicate a cultural feature of the society in which future teachers find themselves, feature that could be related to the social consideration of teachers' work.

Furthermore, we have identified features of determinate teachers' perspectives through these norms. The relationship between norms and perspectives could be reflected in pre-service teachers' future professional work. We need to continue research on school norms at any level to improve the overall vision of what role norms have in the teaching/learning process. In particular, further research is needed to determine if these perspectives are transferred or not to their practice as future teachers (Roy et al., 2014).

Other questions are related to the task. We wonder whether different tasks might give rise to different SDMN that are related to other characteristics of the perspectives. Furthermore, in our case, the lack of coherence between previous tasks that these pre-service teachers have experienced in their educational path and the fact of having to solve a task situated in a professional activity can favor the emergence of the norms identified here.

To sum up, the sociocultural approach adopted in this study has contributed to provide explanation about the results that have been obtained under other theoretical approaches. In particular, how SDM norms contribute to the generation of teachers' perspectives. If we take into account that future secondary mathematics teachers are key elements in the improvement of mathematics education, norms and perspectives become relevant elements that should be taken into consideration in the teaching/learning processes. The implementation of new approaches and methodologies in mathematics teacher education programs can be minimized by these norms and perspectives.

We want to highlighting that this work has not intended in any case to generalize, but approaching the reality of a few pre-service teachers in a specific context, in order to identify some aspects for further studies.

Additional Information

An earlier version of this paper was presented as a Research Report at the 37th Conference of the International Group for the Psychology of Mathematics Education (PME).

Disclosure statement

No potential conflict of interest was reported by the authors.

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APPENDIX

Didactic-mathematical task: Analyses of school mathematical problems on functional dependence.

You have just incorporated to a new school as mathematics teacher who will teach in ESO' second cycle and Bachelor's degree. The school has adopted a textbook for ESO' the fourth course. One of the contents of this cycle is related to the study of the functional dependence (real functions of real variables). We have collected some mathematical problems on this concrete issue and we purport that you can analyze the above-mentioned problems.

Analyze the following problems trying to identify:

- The textual representation of the problem.
- The nature of the activity that the own context in which the problem is presented, it demands of the student.
- The interpretation of the problem.

Examples of mathematical problems proposed taken from secondary school textbooks:

Problem 1

At sea level water boils at 100 °C. At this temperature is named boiling point. When you climb a mountain the boiling point varies depending on height, so that every 1000 m of climbing goes down approximately one degree of temperature of the boiling point of water.

Express through a formula the temperature t of the boiling point when we are to h meters of altitude. What is the temperature of the boiling point on the top of Mount Everest (8848 m)?

Problem 2

Calculate the variation rate of the function $f(x) = x$ in the intervals indicated and observe the sign of the rates obtained:

a) [2, 3] b) [3, 4]

Problem 3

Represent on the same axes the following functions:

$$y = 3x^2 \quad y = 3x^2 - 1 \quad y = 3x^2 + 4$$

$$y = 2x^2 \quad y = 2x^2 + 3 \quad y = 2x^2 - 5$$

$$y = -3x^2 \quad y = -3x^2 - 2 \quad y = -3x^2 + 3$$

Problem 4

A plane flies between Cádiz and Ceuta. Its flight height is given by the following formula:

$$h(t) = 800t - 30t^2$$

where $h(t)$ is the height of the plane in meters at t minutes after taking off from Cádiz.

Represent the graph to determine:

Height at which the plane begins its descent

How long is the flight?

Articles provided:

García, M., & Llinares, S. (1994). Algunos referentes para analizar tareas matemáticas. [Some benchmarks to analyse mathematical tasks]. *SUMA*, 18, 13–23.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1–64.