

# ChatGPT and mathematical modeling of motion for teaching linear functions in secondary education

Rosa Isela González-Polo <sup>1</sup> , Apolo Castaneda <sup>2\*</sup> 

<sup>1</sup> Universidad Autónoma del Estado de México Av. Filiberto Gómez s/n, Guadalupe, Barrio de Tlacopa 50010 Toluca de Lerdo, Estado de México, MEXICO

<sup>2</sup> Instituto Politécnico Nacional, Legaria 694, Col. Irrigación, Miguel Hidalgo, 11500 Ciudad de México, CDMX, MEXICO

\*Corresponding Author: [acastane@ipn.mx](mailto:acastane@ipn.mx)

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## ABSTRACT

This exploratory study analyzes the pedagogical mediation conditions under which ChatGPT's intervention in mathematical modeling amplifies coordination among semiotic registers and favors functional thinking. Twelve second-year secondary school students participated in four teams that modeled the rectilinear motion of a person captured on video and constructed piecewise linear functions. From the articulation between the theory of semiotic representations and Realistic Mathematics Education, the study documented how the teams coordinated the four registers when interacting with ChatGPT. The analysis identifies four conditions that made that mediation productive: the selection of a contextualized situation, the iterative refinement of the prompt, the open stance of the teacher, and the availability of multiple representations as an object of contrast. Given its limited scope, the results offer a first approach to how generative artificial intelligence redistributes cognitive work toward conceptual analysis, and require replication in other groups.

**Keywords:** ChatGPT, mathematical modeling, linear functions, semiotic registers, secondary education

## INTRODUCTION

In basic education, many students approach the concept of a function from a calculation-oriented perspective, in which the prevailing idea is that a function transforms input values into output values. This operational conception, common in early schooling, treats the function as a sequence of steps for obtaining results, without yet constituting a mathematical object on which to reason. Doorman et al. (2012) document that this bias persists even in advanced schooling; at these levels, the tendency to identify functions with procedures, formulas, or patterns persists, hindering the transition toward more structured forms of thinking.

An additional obstacle is the restricted use of representations. Bagossi et al. (2022) document that many students fail to coordinate tables, graphs, and algebraic expressions as different expressions of the same functional relationship, instead treating them as isolated entities. According to the theory of semiotic representations (Duval, 2006), this fragmentation is particularly problematic. Mathematical understanding requires recognizing the same object (in this case, the function) across multiple registers. The transition toward a structural view requires more than exposure to multiple formats; it requires deliberate opportunities for students to analyze how quantities co-vary, how those variations are expressed in different registers, and how those expressions can be coordinated. This form of reasoning, grounded in co-variation, correspondence, and the explicitation of invariant relationships, entails an epistemological shift: the function must move from being an instrument of calculation to being an object that can be interpreted, modeled, and employed in new mathematical contexts.

However, a dimension remains underexplored: how recent digital technology modifies this learning scenario. Generative language models such as ChatGPT introduce an unprecedented capacity to automatically generate multiple representations (tables, algebraic expressions, graphs, natural-language explanations) simultaneously and instantly. While Duval (2006) points out that coordination among registers is central to mathematical understanding, theories of cognitive load (Paas & Sweller, 2012) note that the laborious manual production of representations consumes cognitive resources that could be devoted to coordination and justification. Generative artificial intelligence (AI) thus introduces a pedagogical opportunity: could the automatic generation of representations release the cognitive load of production, allowing students to focus on the coordination among registers that is central to functional thinking? However, this potential depends critically on how AI is integrated, not as a substitute for student reasoning, but as a semiotic mediator that expands the space of mathematical dialogue oriented toward coordination (Pepin et al., 2025; Schorch et al., 2025).

Beyond the fragmentation of registers, there are specific conceptual difficulties that hinder the understanding of functions. Breidenbach et al. (1992) document that many students confuse the univocality condition (one input produces one output) with properties such as injectivity, leading them to discard valid functional relationships that do not fit incorrectly assumed criteria. This conceptual ambiguity shows that the problem does not lie solely in the multiplicity of representations, but in the absence of opportunities for students to build meaning of functions from observable phenomena.

The incorporation of digital technologies expanded the didactic possibilities for studying functions. Environments such as GeoGebra and Tracker represent dynamic phenomena and graphically explore the relationships among variables, but they share a limit: they visualize the phenomenon without constructing the conceptual model that explains it. Without a pedagogical intention, students tend to read the function as a dependency among visual elements rather than as a structural object (Batiibwe, 2024), and the automatic generation of representations may remain at a superficial level. The transition from observed data to the expressions that capture the invariant relationships is a task of teaching, not of technology, and depends on how the teacher organizes its use within a coherent sequence (Clark-Wilson et al., 2022; Hoyles, 2018). Generative artificial intelligence opens the possibility of reorganizing that mediation within the didactic sequence.

Generative artificial intelligence presents itself as a recent technological mediation in mathematics education. Since its appearance in late 2022, tools such as ChatGPT have been integrated experimentally into classrooms for their capacity to generate step-by-step explanations in natural language together with symbolic reasoning processes (Pepin et al., 2025; Schorcht et al., 2025). Unlike Tracker and GeoGebra, these models articulate descriptive analysis with theoretical construction, since verbal exploration, the formulation of conjectures, and the gradual elaboration of algebraic expressions advance together with the generation of tables, graphs, and diagrams within a single space of dialogue. The first documented uses show a complex panorama; the tool's performance is consistent in initial exploration, the reformulation of statements, and the recognition of patterns (Pepin et al., 2025), but it diminishes with tasks of structural reasoning, graphical analysis, or nonlinear functions (Almarashdi et al., 2024). This limitation responds to the nature of mathematical work rather than to the tool, given that meaning construction depends on how it is integrated into the didactic sequence and how the teacher organizes its use (Dasari et al., 2024). Among the documented risks are dependence on the responses without critical analysis and the non-reflective reproduction of procedures.

When ChatGPT is positioned as an interlocutor and not as a solver, its potential changes. Instead of delegating the resolution, the tool accompanies the student in testing formulations, reviewing strategies, and verbalizing relationships among quantities. In the teaching of linear functions, this takes on meaning, given that when expressing how two quantities covary, when exploring ways of representing that dependency, or when justifying why an algebraic expression captures the observed relationship, the tool expands the space of coordination among registers that Duval (2006) places at the center of functional understanding. The decisive condition is that the teacher orient the interaction toward conceptual justification and not toward obtaining answers. This study takes up that question in a particular context: second-year secondary school students who construct piecewise linear functions from the analysis of a rectilinear motion captured on video, with ChatGPT as a mediator of algebraic reasoning and the coordination of representations. The central question is under what conditions of pedagogical mediation the intervention of ChatGPT in that modeling amplifies coordination among the registers (observed data, graphical representations, and algebraic expressions) and favors the construction of functional thinking instead of the reproduction of procedures.

## THEORETICAL FRAMEWORK

In mathematics, access to objects is not direct, given that all thought relies on systems of signs that represent them and not on the objects themselves (Duval, 2006). That semiotic mediation is constitutive of mathematical knowledge. Understanding a concept entails recognizing that a single structure is expressed in several registers, graphical, algebraic, tabular, and verbal, each with a distinct aspect, and coordinating them with one another as articulations that give access to the totality of the object, beyond a mechanical translation from one to another. Learning occurs when the student identifies the invariants that remain constant whatever the representation; without that conscious coordination, knowledge fragments and each register remains an isolated activity. In modeling contexts such as that of this study, those transitions go from the data observed in the video to the tables, from these to the graphs and to the algebraic expressions that capture the functional relationship.

The cognitive difficulty underlying that fragmentation consists in dissociating the meaning, the function as a relationship between variables, from the representation that expresses it, whether the graph, the table, or the formula. When the student does not maintain that distinction, they confuse the representation with the object and become trapped in a particular register (Duval, 2006). In linear functions, coordination requires recognizing that the slope of the graph, the coefficient of the formula, and the rate of change of the table express the same constant relationship. This articulation is difficult, especially in transitions that involve formulas (Ceuppens et al., 2018), but it is important for the appropriation of the concept of function. The pedagogical question is how to create conditions so that the student attains that coordination deliberately and recognizes the invariants that make the linear function a coherent unit beyond the register that expresses it.

The theory of semiotic representations attends to the operations of coordination, but not to how objects originate or what situations make them comprehensible. Realistic Mathematics Education offers a complementary path by situating learning in contexts that the student recognizes as real, through their connection with everyday phenomena or with prior cognitive structures (Gravemeijer, 2007). From this perspective, mathematical objects emerge as progressive constructions from the mathematization of meaningful situations, and not as already-given abstract entities (Freudenthal, 1991). This stance does not contradict Duval, given that the object is neither in reality nor in the representation, and is constructed through processes of interpretation,

**Table 1.** Phases of the didactic activity

| Phases of the activity                        | Theoretical foundation                                                                                                                                                                                                                                                              |
|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| a) Choice and exploration of the phenomenon   | The choice of a contextualized phenomenon activates the principle of reality of Realistic Mathematics Education and allows students to recognize the situation as meaningful (Freudenthal, 1991).                                                                                   |
| b) Preliminary work of visual analysis        | The identification of reference points and the estimation of units allow the articulation of visual language with the construction of magnitudes, mobilizing the graphical register in a phase prior to formalization (Duval, 2006).                                                |
| c) Graphical anticipation                     | The initial sketch functions as an anticipatory semiotic representation and establishes a basis for future comparison; it is linked to the predictive thinking that drives modeling (Gravemeijer, 2007).                                                                            |
| d) Analysis of variables involved             | The discussion about the variables allows the transition from everyday language to functional language, reinforcing the need to formulate models and to establish correspondences among registers (Kaiser & Sriraman, 2006).                                                        |
| e) Interaction with digital technologies      | The use of ChatGPT to help organize information and to generate piecewise-defined algebraic expressions, reinforcing the articulation among registers and modeling phases (Almarashdi et al., 2024).                                                                                |
| f) Interaction with ChatGPT to resolve doubts | ChatGPT acts as an interlocutor that allows doubts and expand the analysis students to review expressions, contrast ideas, and extend functional analysis through explanations in natural language, supporting the reorganization of mathematical thinking (Schorcht et al., 2025). |
| g) Group confrontation and reflective closure | The collective analysis favors the reorganization of the model, the comparison of initial and final representations, and consolidates functional understanding through the contrast between symbolic and graphical expressions (Blum & Leiß, 2007).                                 |

generalization, and coordination. This process privileges the transition from the concrete to the formal, through experiences that connect the variations of quantities with their representations. In linear functions, the graph acts as a bridge between the observed context and the algebraic expression, and gives rise to contrasting functions, identifying invariants such as the slope, and seeing how that invariant is expressed in various forms (Freudenthal, 1991).

The articulation between the two perspectives gives the study a way to assess what happens with ChatGPT. The first referent, with Duval, attends to whether the intervention amplifies or reduces the coordination among registers. A second, with Realistic Mathematics Education, attends to whether that coordination arises from real mathematization work or whether students limit themselves to accepting what the tool produces.

## MATERIALS AND METHODS

The methodology for the intervention rests on mathematical modeling, which understands knowledge as something that is constructed when the student translates a phenomenon into a system of quantifiable relationships that they can interpret, represent, and communicate (Blum & Leiß, 2007). Modeling advances through different phases: understanding the phenomenon, structuring the relationships, mathematizing the variables, analyzing the model, interpreting the results, and validating them in the original context. This process goes beyond applying formulas. Realistic Mathematics Education complements modeling by selecting meaningful contexts, given that learning occurs in processes of progressive mathematization that start from situations that are real for the student and construct formal structures (Freudenthal, 1991; Gravemeijer, 2007). Neither of the two proposes presenting the concepts as already-given entities; both construct them in a sequence that articulates observation, representation, and formalization. These frameworks make it possible to justify a situation such as that of the rectilinear motion of a person captured on video, a situation from which the students construct piecewise linear functions that relate time and position.

The study is qualitative and interpretive in nature, interested in the processes by which students construct functional meaning when working with the video and with ChatGPT as a mediator (Stake, 2010). In this design, ChatGPT intervenes at specific moments as a generator of representations; it produces tables, graphs, and algebraic expressions from the data that students collect, thereby shifting the students' work from producing representations by hand to validating and coordinating them. The analysis follows the transition between empirical experience and symbolic elaboration, and documents how that mediation expands the opportunities to link registers and reorganize interpretations. The design articulates three components: the real situation of the motion in video that provides the empirical data, the mediation of ChatGPT that produces multiple representations, and the student's work that coordinates them to construct the functional meaning.

**Table 1** presents the phases of the modeling activity and the articulation of the theoretical frameworks that sustain it. Each phase specifies the type of task to be developed, the knowledge mobilized, and the representations that emerge. Through this organization, it becomes possible to visualize how progressive modeling interacts with the principle of reality of RME, the coordination of semiotic registers (Duval), and the automatic generation of representations by ChatGPT to build the concept of function.

The intervention took place in an urban public school with 12 second-year secondary school students (13-14 years old), organized in teams of three. The students had prior experience with proportionality, basic graphical representations, and initial algebraic notation, but they had not formally dealt with the concept of function. The activity was implemented in two fifty-minute sessions within the unit on relationships between variables, where variation and covariation were introduced from contextualized situations prior to the formal definition. The choice of the group responded to two criteria: the teacher's willingness to collaborate in an innovation experience and the opportunity of that curricular moment. The work took place in the usual classroom, with the furniture rearranged and moments of collective discussion led by the teacher; the sessions were recorded in audio and video.

Two complementary resources were used. With a mobile phone, the rectilinear motion of a person running in a straight line for about twelve seconds was recorded; from the video, the students extracted approximate distance and time data and

**Table 2.** Coding system of the analysis

| Family                          | Code                                     | What it records                                                                                                                               | Example (excerpt from the interactions)                                                                                                                                   |
|---------------------------------|------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Validity of ChatGPT's responses | 1. Acceptance by technological authority | The validity of the result is justified by its origin or its appearance                                                                       | "This answer is better because it has more numbers considered, that is, more data."                                                                                       |
|                                 | 2. Empirical contrast                    | The video is used to review a representation                                                                                                  | "But in the video, at the end it already goes faster, and this line keeps rising the same."                                                                               |
|                                 | 3. Tabular contrast                      | Values or increments from the tables are compared to decide about the model                                                                   | "We have 5.8 meters and here it says 6.04; we have to check which one matches the video better."                                                                          |
| Coordination among registers    | 4. Graphical-algebraic coordination      | Graphical meaning is attributed to elements of the expression                                                                                 | "This line is more inclined because in the equation the slope is 1.42 and in the other it is 0.80."                                                                       |
|                                 | 5. Slope-velocity coordination           | The same invariant is recognized in the algebraic, tabular, graphical, and empirical registers                                                | "In the second part it increases 1.42 meters each second, that is why the line is more inclined and you can see it goes faster."                                          |
|                                 | 6. Interpretation of parameters          | The coefficients stop being just numbers and come to name magnitudes of the phenomenon                                                        | "The 3.20 is not another velocity; it is the distance it had already traveled when it reached four seconds."                                                              |
| Modeling decisions              | 7. Problematicization of the breakpoint  | The change point is analyzed as a decision supported by the table and the video                                                               | "We put the change at 4 seconds because from there the increments in the table are larger and in the video it begins to go faster."                                       |
|                                 | 8. Mathematical refinement of the prompt | The formulation of the request conditions the structure of the generated model                                                                | "We have to tell it to check whether there are two velocities and to make a piecewise function, not a single line with the average."                                      |
| Cross-cutting                   | 9. Relevant non-verbal participation     | Gestures and actions that sustain reasoning without words; it avoids reading the lower verbalization of two students as an absence of thought | The student points with their finger to the separation between the average line and the last points of the graph; then they mark the place where the inclination changes. |

Note. The examples are excerpts taken from the teams' interactions during the sessions, translated from Spanish.

constructed an initial table. In the second session, ChatGPT was integrated as a generator of representations, and with the data already collected, it produced organized tables, graphs, and candidate algebraic expressions for the model. The students contrasted those expressions with the pattern of variation observed in the video and in their own tables, a validation practice that was the axis of the activity. The teacher assumed a deliberately non-directive role; that is, instead of anticipating answers, they formulated questions and focused their intervention on three aspects: the reading and interpretation of the graphs, the precision of mathematical language, and the joint reflection on the coherence among the video, what ChatGPT generated, and the model that the teams constructed.

The activity was implemented in late November 2025 with the version of ChatGPT available at that time, GPT-5.1, released on November 12, 2025 (OpenAI, 2025). The students worked with the free version, and the teacher had the paid version, which gives access to the more complete variants of the model. The interaction included sending an image of the video with trajectory marks, so that the model's image-processing functions were used. No parameters were adjusted beyond the standard conversation interface.

To document the process of building the functional model, four types of records were collected, capturing different dimensions of the activity. First, the written and graphical productions that students elaborated manually on paper, including data tables, diagrams of relationships, and graphs built from the data extracted from the video. Second, screenshots of each interaction with ChatGPT, documenting both the prompts students sent and the responses generated by the tool. Third, photographs of collaborative teamwork, showing the dynamics of organization and discussion. Fourth, audio and video recordings of the sessions in their entirety, capturing students' conversations, arguments, and verbal reflections. These records documented how students moved among different forms of representation throughout the motion modeling process, from the initial data extracted from the video to the algebraic expressions ChatGPT generated.

The collected material was analyzed as a single exploratory case study, with four embedded units corresponding to the four teams that developed the sequence (Stake, 2010; Yin, 2018). The unit of analysis was the teams, since the coordination of registers occurred through each team's interaction with ChatGPT and with the teacher. The analysis was developed on two planes. In the first, of a deductive nature, the major categories were established from the theoretical referents, the coordination among semiotic registers (Duval, 2006), and the levels of progressive mathematization of Realistic Mathematics Education (Gravemeijer, 2007). In the second, an inductive approach, a set of codes was constructed from the students' formulations through thematic analysis (Braun & Clarke, 2006). The articulation between prior categories and emergent codes corresponds to an analysis of abductive orientation, in which theory orients the reading but does not limit what the material can contribute (Timmermans & Tavory, 2012).

The analysis of the four teams' productions led to nine codes, organized into three families plus one cross-cutting code (Table 2). The three families correspond to the validity criteria of ChatGPT's responses, coordination among registers, and modeling decisions; the cross-cutting code captures non-verbal participation, which was important for the teams with little oral exchange.

The audio and video of the sessions were transcribed to follow the line of reasoning, and the photographs of the plenary sessions served to reconstruct the moments of collective discussion. These records were organized by team.

The coding was carried out in successive phases. First, each member of the author team read the material of the four teams separately, identified the relevant episodes, and assigned codes to them. The researchers then contrasted their readings,

**Table 3.** Route from the evidence to the four conditions

| Condition                                                     | Codes (Table 2) | Teams and moment                                                                                                                                                                                              | What the evidence shows                                                                                                                                                                                                                                                                                                                |
|---------------------------------------------------------------|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Meaningful and complex situation, with two velocity phases | 2, 5            | Before measuring, three of the four teams drew two segments with different inclination, the fourth moved from a single line to two after contrasting with the video                                           | The motion had two phases, an initial slower one and a final faster one, which the teams could distinguish when observing the video, although recognizing them entailed interpretive work and was not immediate for everyone. That is why the situation gave rise to different readings from the beginning, and not to a single answer |
| 2. Iterative refinement of the prompt                         | 8               | Faced with two different responses from ChatGPT, a piecewise model and an average line, the teams reformulated the prompt so that the tool would check whether the velocity changes before choosing the model | The way of formulating the prompt determined what ChatGPT returned, a single line or a piecewise function. That is why adjusting the prompt was a mathematical task, about which feature of the motion to consider                                                                                                                     |
| 3. Open stance of the teacher                                 | 2, 7            | Faced with the model $y = 1.43x$ , the teacher neither approved nor rejected it; with questions they directed attention to the separation between the line and the points                                     | By not giving the answer and returning the analysis with questions, the teacher left in the teams' hands the decision to accept or reject the model, instead of telling them whether it was right or wrong                                                                                                                             |
| 4. Multiple representations for contrast                      | 2, 3, 4, 9      | The teams contrasted their manual table, ChatGPT's table and graph, and the video, for example when comparing 5.8 with 6.04 meters and returning to the video                                                 | The teams stopped accepting a response from ChatGPT and began to judge it by comparing among themselves the video, the tables, the graph, and the equation; validity came to depend on those sources coinciding                                                                                                                        |

confirmed the coincidences, and discussed the disagreements until reaching a shared interpretation. With the episodes already coded, a summary chart was constructed (**Table 3**) that brought together the sections of interest of each team, elaborated again separately and confirmed in a joint session (Miles et al., 2014).

To give consistency to the analysis, two triangulations were considered (Stake, 2010). The first, of researchers, consisted of contrasting the independent readings in identifying the episodes and relevant themes; the second, of sources, in cross-checking the different records collected from each team. This was supplemented by dialogue with the teacher throughout the intervention, particularly in the closing conversation, where they presented their assessment of the experience, the difficulties observed, and their point of view on the technology used.

**Table 3** brings together the route that goes from the coded episodes to the four conditions, with the codes that support each one, the teams and the moment where it is documented, and what the evidence shows.

The four pedagogical conditions come from two different origins, in correspondence with the abductive orientation of the analysis. Three were defined as design criteria derived from the theoretical framework, the selection of a meaningful and complex situation according to the reality principle of Realistic Mathematics Education (Freudenthal, 1991; Gravemeijer, 2007), the open stance of the teacher in line with the research on mediation with digital technology (Clark-Wilson et al., 2022; Hoyles, 2018), and the availability of multiple representations to compare and validate in accordance with the coordination among registers of Duval (2006). The analysis showed that the three criteria functioned during the activity, and **Table 3** gathers that evidence. The iterative refinement of the prompt followed a different path, since it was not anticipated in the design and appeared during the sequence, when the comparison of the two responses from ChatGPT made it clear that the request had to be reformulated. During the analysis, it was recognized as a fourth condition that emerged from the data.

## RESULTS

As part of the intervention design, the researchers proposed several examples of motion that could serve as situations for modeling functions. The teacher selected the motion of a person moving in a straight line for two reasons: it was an observable and quantifiable situation that students could relate directly to their own physical experience, and it allowed the generation of two phases of constant but distinct velocity, which would enable the identification of a piecewise linear function. For this purpose, a student was recorded running 15 meters in a straight line on a soccer field for approximately 12 seconds.

In the first session, students viewed the video and identified two phases of motion with distinct velocities. The first showed displacement at a low and constant speed. The second showed a constant increase in velocity. To quantify these changes, students used two calibration references: the length of the path (15 meters) and the student's height (1.50 m). With this scale, they analyzed specific frames. Two strategies emerged in the data extraction. One team measured directly on the screen, recording approximate positions. Another counted frames to achieve greater precision. Both built time-distance tables that differed in accuracy but shared the same structure.

In this initial phase, without ChatGPT's intervention, students intuitively coordinated what they observed in the video with their numerical records. The teacher guided the analysis by emphasizing that changes in the variation of position were the central mathematical property to be modeled. In this way, what students identified visually in the video became the starting point for the functional construction (See **Figure 1**).



**Figure 1.** Frame of the video sent to ChatGPT, with trajectory marks (Source: Authors' own recording)

Note: The video can be viewed at: <https://www.youtube.com/watch?v=CRdgE2AT-XQ>

### Anticipatory Production and Initial Graphical Interpretation

After viewing the video, but before proceeding with systematic data extraction, students were asked to produce an anticipatory graph of how distance would vary over time. The instruction was to construct that relationship based solely on visual perception, without measurements or technical tools. This initial production documented how students established correspondences between what was observed and what was represented, without formalized language.

The productions showed common features. Three of the four teams traced the trajectory as two straight segments of different inclination, and the fourth represented it with a single ascending line. In two teams, the marks on the axes showed equal intervals, and in the other two, the separation was irregular, so that the graphs varied in precision. Even with those differences, all of them placed the start of the motion at the origin of the plane and understood the phenomenon as a displacement that grows continuously. In Team 1, the discussion prior to the drawing showed the identification of the two rhythms, "first it goes slowly, and then it runs faster, yes there is a change, that is, it was going at one speed, and then it increased." Another member put it as "a line like this... and then more upright." Faced with the teacher's question about what it meant that the second segment was more inclined, a student answered "that in the same time it covers more distance," an incipient formulation of covariation.

### Data Recording from the Video: Systematization and Exploration of Patterns

With the anticipatory graph as a starting point, students began extracting quantitative information from the video. This phase represented a clear transition in the activity, shifting from visual perception toward the need to record measurable magnitudes. From a still image of the video and the total distance covered (15 meters), the teams defined visual reference points and discussed strategies to estimate the person's position at different times. Without specialized tools, they relied on approximate time counts and stable visual references to determine relative distances.

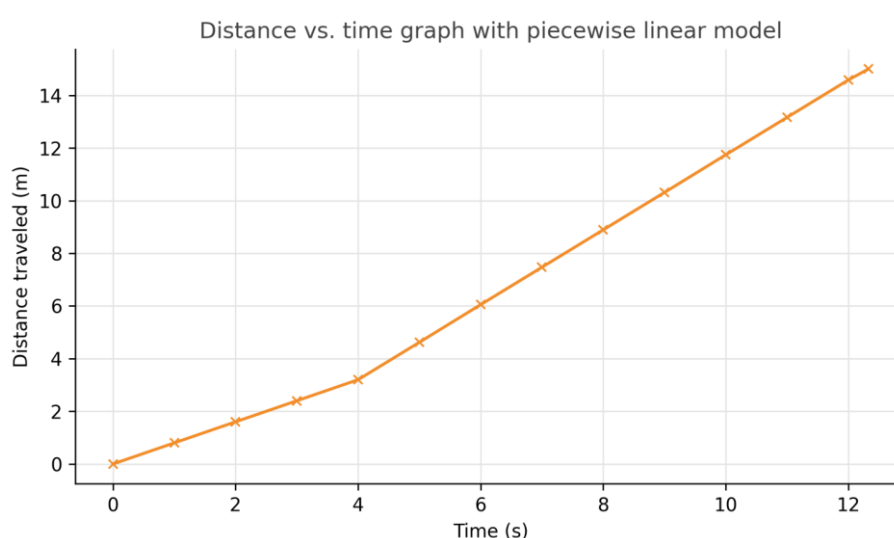
The systematization occurred through the construction of tables that related elapsed time to estimated distance. The four teams manually elaborated these initial tables based on direct observation of specific segments of the video and comparisons of relative positions. In general, all of them maintained a coherent structure in their records, with regular time intervals of 2 to 5 seconds, depending on each team's strategy, and indicating approximate distances. This initial table, elaborated without technological mediation, would later serve as a reference point for the more precise tables ChatGPT would generate in the second session.

### Interaction with ChatGPT: Data Extraction and Model Adjustment

With the preliminary data systematized, students worked with ChatGPT in the second session. The prompt sent to the tool integrated the elements needed for the model to process the motion as a quantifiable experiment. It included an image of the video with trajectory marks, calibration references (height 1.50 m, total distance 15 m), the estimated frame rate, and visual reference points. The request was that ChatGPT estimate positions in meters, generate a time-distance table, adjust a piecewise linear model, and graph the results.

**Table 4.** Time-distance table generated by ChatGPT for one of the teams (Source: ChatGPT output, GPT-5.1, obtained during the second session)

| $t(s)$ | $d(m)$ |
|--------|--------|
| 0      | 0      |
| 1      | 0.80   |
| 2      | 1.60   |
| 3      | 2.40   |
| 4      | 3.20   |
| 5      | 4.62   |
| 6      | 6.05   |
| 7      | 7.47   |
| 8      | 8.89   |
| 9      | 10.31  |
| 10     | 11.74  |
| 11     | 13.16  |
| 12     | 14.58  |
| 12.33  | 15.00  |

**Figure 2.** Distance-time graph with the piecewise linear model generated by ChatGPT for one of the teams (Source: ChatGPT output (GPT-5.1) obtained by the authors during the second session; redrawn by the authors)

ChatGPT's response produced four elements: a table with uniform time increments and accumulated distances rounded to two decimal places; two linear equations that described the motion in two distinct phases, with slopes that reflected the initial low velocity and the subsequent acceleration; and a graph that showed the data points along with the fitting lines. These elements were presented simultaneously, allowing immediate comparison among different representations.

During the design of the intervention, the researchers communicated a strategic orientation to the teacher on how to present these results. It was agreed that the results generated by ChatGPT should not be assumed as final answers to be validated afterward. Instead, the teacher would maintain a deliberately open stance, avoiding closing the dialogues around correct and incorrect answers. The emphasis would lie on critical review and on the exploration of multiple perspectives on what the numbers, graphs, and equations communicated. In this way, when ChatGPT was incorporated into the activity, the teacher maintained the agreed-upon open stance (Table 4 and Figure 2).

The table generated by ChatGPT led to an interpretation of the data. The teams contrasted it with their previous records and found small differences, on the order of 0.04 to 0.28 meters depending on the interval. In Team 1, a student observed, "at six, it puts 6.04; we have 5.8. It is not much, but there is a difference." At eight seconds, the difference reached 0.28 meters. At first, a criterion of authority predominated, "ChatGPT has many more numbers... so it is better," until the teacher asked whether having more numbers necessarily made it better. The teams then returned to the video and reviewed their counts, and the same student concluded "ChatGPT's table is more regular and may be more precise, but it is not accepted just because ChatGPT made it, we have to check some points with the video, we do not know whether it is taking the information correctly." Team 2 formulated an equivalent criterion, "there is not one that is automatically the correct one, if they approximately coincide, it gives more confidence," and proposed "using one to check the other."

The comparison between the graph generated by ChatGPT and the anticipatory graphs that students had initially built proved particularly productive. Some students recognized that their anticipatory graphs had represented the motion as a single

trajectory, without identifying the difference in slopes between the two segments. Others identified that the piecewise linear structure offered a mathematical description of what they had intuitively observed as "acceleration" in the video.

The teams read the slope of each equation and related it to velocity, a concept they had worked on in their physics course and that the teacher related to the activity. In Team 1, the table showed that the distance increased 0.80 meters per second in the first segment. A student said, "then this means the velocity, how fast it goes," and another added, "yes, but it is rather the first velocity... there are two velocities," because the second segment advanced 1.42 meters per second. Team 3 arrived at the same thing during the sharing session. Faced with the question of what a greater slope meant in the video, a member answered, "more velocity; if it goes faster, you see the line more inclined," and another added that the slope is not reduced to the inclination, "it is also how much the distance changes when the time changes by one second." With the support of ChatGPT's representations, the relationship between the coefficient and the velocity, which in the previous phase had remained unclear, made it possible to relate the algebraic register to the motion situation.

The coordination of the four registers appeared at specific moments, but it was not observed in the whole group. In Team 1, it became evident when a student brought together the equation, the table, and the graph in a single sentence, "yes, we have to see that... the 1.42 in the formula, the increments of 1.42 in the table, and the more inclined line say that it goes faster... but it is the same information." Another classmate added what is observable in the video, "... and in the video you can see that it runs more... that is, you can see how it is a different format of the same thing." In that exchange, the equation, the table, the graph, and the motion in the video entered into a single reading. The mediation of the tool expanded the space of that coordination and kept the interpretive work in the team's hands.

The prompt used with ChatGPT emerged from an iterative collaborative process among teachers and students. The first version was conversational and less technical. It included a description of the video, calibration information (height of person 1.50 m, total distance 15 m), a visual reference point (the orange torso), and the didactic purpose of studying functions with secondary school students. In response to this request, ChatGPT generated a piecewise linear model with two distinct equations, a table of values per second, and a graph that showed both phases of the motion. This first result made it possible to begin the didactic analysis using representations useful to the activity's objective.

In a second iteration, students requested greater temporal precision in the data table, with intervals of every 0.5 seconds. In response, ChatGPT explicitly recognized that the observed velocity was not constant and proposed a simplified linear model ( $y = 1.43x$ ) as an approximation to an average velocity. The resulting graph showed the model's line clearly separated from the observed points, especially in the final segments where the acceleration was greater. This second response, although technically correct, proved less useful for the didactic purpose of visualizing the change of velocity between phases.

The comparison between the two results opened a space for formative analysis in the group. Students observed how changes in the prompt formulation (requesting greater precision) produced structurally distinct responses (a single model versus a piecewise model). This analysis led to collaborative work on what elements a prompt should contain to obtain didactically useful results. Students identified technical aspects, such as the video's frame rate and visual reference points; mathematical aspects, such as piecewise fitting and the constant-slope criterion; and contextual aspects, such as the explicitation of the didactic purpose. The final version of the prompt, built collectively, integrated these elements into a structure that subsequently served as a template for future motion analysis activities.

### **Prompt for Motion Analysis**

#### ***Motion analysis for the teaching of functions***

I attach:

1. A video (file: {VIDEO}) showing the object in motion.
2. A JPG image (file: {IMAGE}) with the trajectory marks.

#### ***Calibration data***

- Actual height of the object: {OBJECT\_HEIGHT} m
- Total distance covered: {TOTAL\_DISTANCE} m
- Frame rate: {FPS} fps
- Reference point: {REFERENCE\_POINT}

#### ***Didactic context***

This analysis will be used with secondary school students to explore linear functions. Students need to identify changes in velocity as changes in slope, and to coordinate tabular, graphical, and algebraic representations.

#### ***Request***

1. Calculate the pixels  $\rightarrow$  meters ratio using the calibration marks.
2. Estimate the position of the reference point at regular time intervals and convert it to meters.
3. Generate a table with columns  $t$  (s) and  $d$  (m), rounded to two decimal places.
4. Analyze whether the motion presents one or several phases of constant velocity. If there is more than one phase, identify the points of change and justify that identification.

5. Adjust the corresponding mathematical model (simple linear or piecewise), presenting the equation of each segment with its slope.
6. Graph distance vs. time, showing the estimated points and the fitting lines.
7. Indicate the limitations of the analysis, the estimated margin of error, and which factors could affect the precision of the model.

### Confrontation Between the Anticipatory Graph and the Constructed Table

During the comparison among representations, students had access to two distinct responses generated by ChatGPT across successive iterations of the prompt. The first response offered a piecewise model with two distinct linear equations. The second proposed a simplified single-equation model with an average velocity of 1.43 m/s. This difference between the two responses opened a productive space for discussion about which model better represented the observed motion.

This moment illustrated the risk of uncritical reproduction that can arise with ChatGPT, and at the same time how to reverse it. At first, some teams took the simplified line as good because the tool presented it as correct and it was simpler, "the chat says it is the average velocity... yes, that is the answer, because what is obvious is that it is a line." The teacher intervened without resolving, "I am not going to tell you whether it is right or wrong; look at the graph and tell me what relationship there is between the line and the points." The teams then read the separation, "the line is above the points," and a student explained that "it is saying that it traveled more meters than it actually traveled," while another observed that "the formula takes everything as if it were going at the same velocity." The conclusion distinguished the planes, "it may be correct as an average, but it is not useful to show how the motion was," "it erases the change," "it makes it even." With that contrast, the teams adopted the piecewise model as the representation closest to the phenomenon. The episode showed the difference between accepting a result because the tool generates it and validating it by considering the phenomenon.

### Model Development

Working with the model generated by ChatGPT, students analyzed the structure of the piecewise function. The first segment, corresponding to the interval  $0 \leq t \leq 4$  seconds, was expressed by the equation  $d(t) = 0.80t$ , with a constant slope of 0.80 m/s. Students interpreted this value as the initial velocity of the displacement, consistent with the phase of motion in which the advance was more gradual. The second segment, corresponding to the interval  $4 < t \leq 12.33$  seconds, was expressed as  $d(t) = 3.20 + 1.42(t-4)$ . In this expression, the value 3.20 represents the distance accumulated over 4 seconds, and the slope of 1.42 m/s was interpreted by students as the increase in velocity during the second phase. This reading introduced the notion of change in velocity in a situated manner, without requiring the previous formalization of complex kinematic concepts.

The table of time and distance values served to verify the consistency of the piecewise model. When calculating the differences between consecutive values, the teams verified that in the first segment the increments were 0.80 meters per second and in the second, 1.42. In Team 1, faced with the teacher's question about where to verify the 0.80, a student answered, "in the table, from zero to one it rises 0.80, from one to two it rises 0.80." This calculation supported the reading of the slopes as velocities in each segment. From this work with the data, the teams reorganized their initial ideas and accepted that the piecewise function was a pertinent representation for phenomena with changes of rhythm.

The algebraic expression generated by ChatGPT became an object of analysis. The students questioned the meaning of the coefficients and the location of the breakpoint at  $t = 4$ . In Team 4, the question "why does it start with 3.20 if the other formula starts with zero?" led to interpreting that value as the distance already traveled, "when the second part starts it had already covered 3.20 meters, it is a new beginning," and to recognizing the term  $t - 4$  as "the time that passed after the fourth second." About the breakpoint, a member of Team 1 asked, "why did it choose four, it could be five; that is not clear." The group verified it in the table, and another student pointed out "before four it increases about 0.80 per second, after it increases about 1.42," so that placing the change at five would leave that datum wrong. Team 4 added the nuance regarding the approximation, "it does not mean that exactly when the clock says four it changed abruptly," and proposed treating the value as "an approximate point where the rhythm changes." In these exchanges, ChatGPT functioned as an interlocutor that answered questions and offered alternatives, with which the discussion shifted from calculation toward the interpretation of the model and the justification of the constructed expression.

The analysis of the intervention identified four pedagogical conditions that proved decisive in enabling ChatGPT's mediation to amplify the coordination among semiotic registers. The first was the selection of a meaningful and complex situation to activate interpretive work, the motion with two phases of distinct velocity. The second was the iterative refinement of the prompt as collaborative work among teachers and students, which revealed how decisions about the formulation have direct conceptual implications. The third was the deliberately open stance of the teacher, previously agreed upon with the researchers, which kept interpretive work at the central axis of the activity. The fourth was the availability of multiple representations (video, manual tables, ChatGPT-generated results, and anticipatory graphs) as a basis for contrast and validation. The analysis identified how these four conditions emerged in an articulated manner during the intervention. None of them, on its own, would have sustained the intervention's formative value.

## DISCUSSION

The findings highlight how ChatGPT's intervention expanded coordination among semiotic registers during the modeling work. The analysis documented the simultaneous articulation of four registers: the motion video, the tables built manually by

students, the representations generated by ChatGPT, and the initial anticipatory graphs. Drawing on Duval's (2006) framework, mathematical understanding demands conversion among registers, a process that goes beyond mechanical translation from one symbolic system to another. Coordination occurs when the student recognizes the same mathematical object in different representations and articulates the invariant features preserved in the conversion.

The episode in which two teams interpreted the slope of the equation as the velocity of the motion illustrates that conversion. The articulation was achieved with greater clarity than in the previous phase without ChatGPT, where the coordination was limited to comparing the video with the manual records. In this phase of work, the empirical register of the video was decisive since it offered the referent to contrast the other registers.

Recent literature on ChatGPT in mathematics classrooms converges on the idea that the tool's usefulness depends less on its intrinsic capabilities and more on the pedagogical conditions that frame its use. Studies on collaborative argumentation document that the tool primarily builds verbal and algebraic representations, with variable performance in conversions among registers (Urhan et al., 2025), while reviews of the field note that its conceptual contribution depends on pedagogical integration and warn of overreliance (Pepin et al., 2025).

The mediation of ChatGPT in the coordination of registers has a documented limit, since the tool makes systematic errors as complexity increases and can offer technically incorrect responses that an inexperienced user would accept (Frieder et al., 2023). In this study, that risk became visible with the simplified model  $y = 1.43x$ . This work contributes evidence that the availability of multiple representations, articulated with the teacher's mediation, turns that risk into a formative opportunity. When contrasting the line, the graph, and the table, that inconsistency became evident, so that coordination with generative AI requires, in addition to having the representations available, a scenario of contrast and a mediation that accompanies the comparison and the validation.

A second aspect of the analysis concerns how progressive mathematization takes shape when students work with AI-generated representations in realistic modeling contexts. The findings document three distinct moments in this transition. The initial anticipatory graph corresponds to the situational level, in which students mobilized semiotic resources from direct experience with the video, without recourse to technical procedures (Gravemeijer, 2007). The manual construction of tables with regular intervals corresponds to the referential level, where students organized numerical data and detected patterns in distance differences. Formulating the piecewise linear model with specific equations ( $d(t) = 0.80t$  and  $d(t) = 3.20 + 1.42(t-4)$ ) corresponds to the general level, where the mathematical structure synthesizes the regularities of the observed phenomenon.

This sequence draws on the principle of reality of Realistic Mathematics Education, where mathematical objects emerge progressively from meaningful situations (Van den Heuvel-Panhuizen, 2020). The motion video functioned as a phenomenologically rich context, since it offered a quantifiable situation that students could link with their direct physical experience. The specific complexity of the phenomenon, with two phases of distinct velocity, opened the way for mathematization, advancing from intuitive description to a piecewise algebraic expression. The study provides evidence that progressive mathematization, originally formulated for contexts without technological mediation, preserves its analytical relevance when generative AI tools are incorporated into the didactic design.

Recent research on RME and digital technologies shows that integrating digital tools strengthens the development of higher-order thinking skills when it preserves the central principles of the realistic approach (Listyaningrum et al., 2025). The warnings about the limits of this integration point in two converging directions: the "black box" risk, in which students obtain results without understanding the underlying mathematical processes (Cevikbas et al., 2023), and the overreliance that can limit the critical evaluation and validation of models (Kandemir & Eryilmaz, 2025). The field thus identifies a necessary condition: digital tools expand pedagogical possibilities as long as the didactic design preserves the interpretive work of the student.

This research contributes a distinction about how progressive mathematization is reorganized when ChatGPT intervenes in the transition among levels. The situational and referential levels were developed before the tool's intervention, during the work with the video and the manual construction of tables. ChatGPT intervened in the transition toward the general level, contributing the piecewise algebraic model. This sequence documents a didactic design in which the tool provides resources for formalization without replacing the initial levels of mathematization, making it possible to interpret and validate the equation generated by ChatGPT rather than accepting it as a finished product.

This organization of the didactic design directly addresses the "black box" risk noted in the literature as characteristic of interventions with digital technologies (Cevikbas et al., 2023). When students have previously moved through the situational and referential levels, the algebraic formalization provided by the tool finds interpretive referents in prior mathematizing experience. The model  $d(t) = 0.80t$  does not appear as an imposed formula, but as an expression that recovers the regularity that students had already identified in their manual tables. This finding connects with the next dimension: the specific role of prompt formulation as a situated mathematical practice in contexts of modeling with generative AI.

The first version of the prompt led to a piecewise function with two equations. The second was more detailed in requesting finer intervals, but it generated a single average line that ignored changes in velocity; it was a less useful model despite its greater precision. This shows that prompt formulation is a mathematical decision, not a technical adjustment, because it defines which feature of the phenomenon the model preserves or loses.

Recent literature recognizes prompt engineering as a mediating competence between the tool and mathematical tasks. In this scenario, two predominant approaches are documented: a technical approach focused on specific learning objectives and a procedural approach focused on fostering thinking with the tool (Qian, 2025). This literature recognizes the prompt as having a central formative value that goes beyond its status as a secondary technical element.

The study contributes a specific distinction to this conversation for contexts of mathematical modeling. The three planes that students articulated when refining the prompt, technical, mathematical, and contextual, differ from those characterizing prompts oriented to solving closed mathematical problems, where the formulation is limited to specifying the statement and requesting the solution. In modeling contexts, formulating the prompt involves deciding which physical phenomenon is represented, the requested precision, the expected mathematical model, and the level of conceptualization appropriate to the educational context.

Kock et al. (2025), in a study of engineering students who developed schemes of use for ChatGPT, documented that students established criteria for the tool's limitations and developed their schemes over time. Our findings extend this observation to the secondary level and to the context of functional modeling. The students who participated in the iterative refinement of the prompt developed two articulated forms of learning: technical interaction with the tool and the construction of mathematical criteria for what constitutes an adequate response to the modeled phenomenon. The collective identification that the model  $y = 1.43x$  oversimplified the phenomenon required mobilizing mathematical knowledge about the distinction between single- and piecewise-fitting. This development of criteria constitutes an additional learning that the design of the intervention made possible.

The fourth dimension of this discussion refers to the teacher's open stance, a condition foreseen in the design, and to how it became present during the activity. The deliberately open stance of the teacher, agreed upon with the researchers during the intervention's design, avoided closing the dialogues around correct and incorrect answers. The teacher guided students to critically examine the tool's results, compare representations, and justify their interpretations. This stance became evident in the episode on the simplified model  $y = 1.43x$ , where the teacher asked the teams to observe the separation between the model line and the observed points, rather than directly validating or rejecting the tool's response.

Recent literature recognizes that the teacher's role in classrooms mediated by generative AI is being reconfigured toward functions of critical mediation rather than content transmission. This reconfiguration responds to a central tension in the field: while the tool accelerates the obtention of results, students may become dependent on it and delegate skills of analysis and self-regulation that learning seeks to develop (Qian, 2025).

The teacher's interventions, recorded throughout the sequence, illustrate the form of mediation that accompanied the work with the tool. The teacher's open stance constituted an active form of mediation that fostered students' interpretive work. His interventions occurred at specific moments: during the elaboration of the anticipatory graph, the comparison between ChatGPT's table and the manual records, and the episode of the simplified model, to direct attention toward the contrast among representations. These targeted interventions, rather than direct validation, kept interpretive work at the center of the activity.

The coordination of registers documented arose from the interaction among the tool, the teacher, and the modeling activity, not from ChatGPT by itself. There was no attempt to separate the contribution of each element; the study inquired into the conditions that make that articulation productive. Although the analysis focused on the work of the teams and not on teacher mediation as an object in itself, it was possible to observe that the teacher's open stance was a condition that helped organize the experience.

## CONCLUSIONS

This research aimed to analyze how ChatGPT's mediation in modeling activities can amplify the coordination among semiotic registers in constructing the function concept. The central question that guided the study was the following: under what pedagogical mediation conditions does the intervention of this tool, in a context of motion analysis captured on video, expand the coordination among semiotic registers and favor the construction of functional thinking rather than the reproduction of procedures? The design of the intervention articulated three dimensions in response to this question: the selection of a real motion situation, the mediation of ChatGPT as a tool that generates multiple representations, and the teacher's pedagogical mediation in sustaining students' interpretive work.

In this study, the answer to that question rests on four articulated pedagogical conditions, the selection of a meaningful and complex situation that activates interpretive work, the iterative refinement of the prompt as joint work between teacher and students, the open stance of the teacher, which kept interpretive work at the center of the activity, and the availability of multiple representations as an object of contrast and validation. None of them, on its own, would have sustained the formative value of the intervention; their articulation made ChatGPT's mediation amplify the coordination among registers and favor the construction of functional thinking instead of the reproduction of procedures. As it is a limited experience, it is important to read these conditions as findings that require being contrasted in other groups and contexts before assuming them as general guidelines.

The limitations of this study point to directions for future research. The work with 12 students and 2 sessions offers an initial approach that requires replication with larger groups and longer sequences. The design does not separate the contribution of ChatGPT from the teacher's mediation or from the modeling activity, given that the sequence articulated the three elements simultaneously. In particular, teacher mediation did not constitute an analytical dimension in itself; the claims about the teacher's role rest on the recorded interventions and on the closing conversation, and must be read with that reservation.

Further research should examine how this form of mediation operates with other functions (quadratic, exponential, trigonometric), with more complex motion phenomena, and with students at different educational levels. Future studies should also investigate how students transfer the competencies they developed to situations in which they must design their own prompts without the teacher's collaboration. Research in this direction can contribute to outlining a framework for the conditions that make the integration of artificial intelligence tools productive in mathematics teaching, especially at a time when these technologies are being rapidly incorporated into the classroom without sufficient didactical reflection.

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**Ethical statement:** The authors stated that the study was conducted in accordance with the ethical principles for research involving human subjects established in the Declaration of Helsinki. Approval from an institutional ethics committee was not required, since the study consisted of a regular classroom activity within the official mathematics curriculum, carried out by the group's teacher during ordinary class time, involved minimal risk, and collected no sensitive personal data; neither the authors' institutions nor the participating school require ethics committee review for this type of classroom-based educational research. The authors further stated that the study was authorized by the principal of the participating public secondary school. Written informed consent was obtained from the parents or legal guardians of all participating students, who were minors, and the students gave their assent to take part and to be recorded. Audio and video recordings, photographs, and screenshots of the ChatGPT interactions are stored on password-protected devices accessible only to the authors. Participant identities were anonymized in the transcripts and throughout the manuscript, and no participant can be identified in the published figures.

**AI statement:** The authors stated that ChatGPT (GPT-5.1, OpenAI) was the object of study and was used by the students and the teacher during the classroom sessions, as described in the Materials and Methods section; the outputs reproduced in Table 4 and Figure 2 were generated by ChatGPT during those sessions. In preparing the manuscript, the authors used Claude (Anthropic) to support the stylistic editing, and Grammarly to check grammar and spelling. The coding, analysis, and interpretation of the data were carried out by the authors. The authors reviewed and edited all AI-assisted text and take full responsibility for the content of the article.

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