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Challenges of sonification in the teaching and learning of functions

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ABSTRACT

Sonification, the systematic use of non-speech sound to represent data, has increasingly been investigated as a means of widening access to mathematical representations, particularly for students with visual impairments. This conceptual paper examines the challenges involved in using sonification to support the teaching and learning of functions in inclusive secondary mathematics. Guided by a Universal Design for Learning-informed framework for inclusive mathematical technology design, we integrate research on students' difficulties with function graphs, evidence from auditory-graph studies, and design requirements for accessible learning environments. We then analyse Audiofunctions+ as a current, multimodal web platform that addresses some of these requirements. The analysis distinguishes perceptual access to graph features from epistemic access to functional relationships and from opportunities for participation and learner agency. It also shows that sonification does not, by itself, resolve conceptual difficulties: its educational value depends on the auditory mappings, the tasks, the coordination with other representations, and classroom orchestration. We conclude by identifying priorities for tool development, empirical research, and teacher professional development.

Keywords: visual impairment, accessible mathematics, functions, graphs, sonification

INTRODUCTION

Currently, the number of blind individuals reaching higher education is low, and even fewer choose to pursue sciences, particularly natural or exact sciences. This reflects a lack of effective implementation of existing inclusive education plans, especially in the production, translation, and conversion of mathematical notation (Ahmetovic et al., 2021; Llamazares & Arias, 2022). However, solving accessibility issues with mathematical notation represents only the first step of a broader challenge. Indeed,

> [w]orking effectively with mathematical notation demands more than having the ability to read it. One must also be able to 'do mathematics'— that is, to understand, manipulate, and write symbolic expressions in order to solve a mathematical problem or to develop a mathematical proof (White, 2020, p. 3).

Studies in mathematics education for blind learners have documented significant limitations in instructional arrangements that rely on sighted assistants to read, transcribe, or manipulate mathematical expressions on behalf of the learner (e.g., Spinczyk et al., 2019). Such arrangements can restrict timely access to notation, slow down interaction with mathematical objects, and reduce opportunities for independent study. These barriers do not arise from blindness itself, but from the lack of accessibility in resources and in certain teaching practices. Accordingly,

> [t]here is a public need in the field of computer-aided teaching of mathematics by visually impaired students. Computer methods through the effective use of an alternative description of structural information, presented in sound [or tactile] form, can help the blind person to become more independent in the process of learning mathematics (Maćkowski et al., 2020, p. 213).

Among the digital learning tools that support inclusive participation in the mathematics classroom, this article focuses on sonification. Sonification allows the representation of (mathematical) data with the use of non-speech sound, as a technique of rendering sonic information in response to data and interactions (Hermann et al., 2011; Kramer, 1994). Sonification gradually emerged as a potential vehicle for learning (Flowers, 2005; Metatla et al., 2016; Nees, 2018; Vines et al., 2019), particularly in supporting students with visual impairments (Ahmed & Chao, 2018; Davison, 2012; Nees & Berry, 2013).

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Previous studies indicate that sonification can support access to and interpretation of some mathematical and scientific representations (Vines et al., 2019), although evidence of sustained learning effects remains limited and context-dependent. This paper focuses on its application to functions, a fundamental but persistently challenging concept across mathematics curricula (Niss, 2020; OECD, 2024). Analysing the educational potential of sonification for blind and visually impaired students requires attention to two related bodies of research: barriers in mathematics learning and communication for students with visual impairments (Bell & Silverman, 2019; Gerofsky & Zebehazy, 2022), and students' conceptual difficulties with graphs, covariation, monotonicity, limits, continuity, and differentiability. Only a small number of studies explicitly connect these two strands of research (Maffia et al., 2025).

Research on how learners with visual impairments engage with functions and their graphical representations remains scarce. Existing studies identify limitations in the precision, production, and readability of tactile graphs made with embossed paper, elastic bands, or pins (e.g., Tanti, 2007). Because tactile exploration is sequential, learners may require explicit strategies and sufficient time to coordinate local features with the global structure of a graph (Zebehazy & Wilton, 2014). Gerofsky and Zebehazy (2022) also show the value of gesture, sound, movement, and metaphor in interpreting and communicating graphical features. Materials that learners can manipulate independently may support attention to mathematically relevant features such as roots, slope, curvature, maxima, and minima. Dynamic multimodal artefacts that include sonification are therefore worth investigating, but their educational contribution cannot be inferred from accessibility alone.

In this context, the aim of this paper is to identify the main challenges of using sonification in the teaching and learning of functions for visually impaired students. Adopting a framework guided by Universal Design for Learning for the design of inclusive environments for teaching and learning functions leads us to examine the aspects that characterise it: access to representations, access to mathematical concepts and relationships, and the promotion of participation and agency. Given the paucity of studies that specifically examine the difficulties faced by individuals with visual impairments in learning functions, we examined in parallel the challenges commonly experienced by sighted individuals, with a focus on the tasks proposed in research for their diagnosis and characterisation. We focus on the specific elements that constitute the concept of function (such as domain, codomain, variation, monotonicity, limits or continuity) that students, regardless of visual status, typically find challenging, and how mathematics education research suggests these difficulties may be addressed through function graphs. Subsequently, we examine how sonification can enhance the teaching and learning of function graphs while addressing the above-mentioned difficulties for all students. In particular, we present *Audiofunctions+*, an interactive digital platform that employs multimodal and dynamic approaches to make function graphs accessible via sound representation. This analysis helps to clarify both the possibilities and the current constraints of sonification technologies, as well as identify areas that require further development to support the understanding of the concept of function for all learners, particularly those with visual impairments.

CONCEPTUAL FRAMEWORK

This study adopts a framework guided by Universal Design for Learning (UDL) for the inclusive design of mathematical technologies. As Sewell et al. (2022) explain, UDL is built around three psycho-neurological networks –affective, recognition, and strategic– and its central theoretical commitment is that educational experiences should be designed as inclusive for all learners from the outset, rather than through *retrofitting*, that is, adapting a design conceived for a presumed "norm" only after barriers have already emerged (Meyer et al., 2014). From this perspective, inclusion does not merely involve making subsequent adaptations for specific students; rather, it consists of designing from the outset environments, tasks, resources, and forms of interaction that are sufficiently flexible to reduce barriers and expand opportunities for participation for all students, including those with special educational needs.

In the field of mathematics education, this approach is particularly relevant due to the inherent complexity of mathematical learning, which requires the coordinated use of multiple forms of representation –symbolic, graphical, verbal, manipulative, visual, or auditory– along with different modes of reasoning and abstraction processes that can create barriers to access and understanding. Consequently, UDL proposes offering multiple means of engagement, representation, and action and expression, with the aim of fostering full participation and promoting student agency, avoiding reliance on a single form of communication or access to mathematical knowledge (CAST, 2024). Recent evidence from mathematics classrooms supports this orientation: in a qualitative study of how Australian primary and secondary teachers enact UDL, Porta et al. (2026) found that, although teachers rarely referred to UDL guidelines explicitly, their practices substantively aligned with its three principles and functioned as an underlying design logic shaping participation, emotional safety, and learner agency, rather than as a checklist of isolated accommodations applied after the fact. This reinforces UDL's status as a generative orientation for inclusive practice, rather than a compliance exercise.

These general principles become particularly significant in the design of digital technologies for mathematics education. In this regard, Drijvers (2012) argues that technological design cannot be understood in isolation but must be coordinated with task design, classroom activities, and teaching practices. Similarly, Freiman (2020) situates this coordination within the broader field of technology design in mathematics education. Likewise, Healy and Kynigos (2010) complement this perspective by proposing criteria such as deep structural access, body and ego syntonicity, and the multiplicity of representations and forms of communication, specifying the conditions necessary to ensure meaningful access to mathematical activity.

The UDL–oriented framework is traditionally articulated through three core principles that guide inclusive mathematics teaching. The first, multiple means of engagement, focuses on the “why” of learning and aims to promote student interest, motivation, and participation through meaningful problems, authentic contexts, varying levels of challenge, and opportunities for

collaboration. The second, multiple means of representation, addresses the “what” of learning and emphasises the need to present mathematical objects through diverse registers and formats, supporting connections across representations and ensuring access to mathematical meaning regardless of students’ perceptual or cognitive characteristics. Finally, the principle of multiple means of action and expression refers to the “how” of learning, promoting different ways of interacting with mathematics, solving problems, communicating reasoning, and demonstrating understanding through verbal, graphical, symbolic, manipulative, or technological resources.

Building on these principles, the UDL–oriented framework is operationalised through three analytical dimensions that allow the inclusive potential of mathematical technologies for students with visual impairments to be examined. The first dimension, representational access, analyses which properties of mathematical representations –for example, features of graphs, values, or references– become perceptible through sound and its coordination with tactile, symbolic, tabular, or visual representations, extending to non-visual modalities Healy and Kynigos's (2010) concern with body and ego syntonicity. The second, epistemic access, examines the extent to which technological interaction enables reasoning about relevant mathematical relationships –such as covariation, domain and range, continuity, or rate of change– rather than being limited solely to the perceptual identification of features, asking whether non-visual interaction supports genuine engagement with the relationships that define the concept of function, in the spirit of Healy and Kynigos's (2010) deep structural access. Finally, the dimension of participation and agency considers whether students are able to explore, make decisions, formulate and test conjectures, explain reasoning, and collaborate in ordinary classroom mathematical activities, consistent with research on how agency and accountability are distributed within the activity systems of mathematics classrooms (Gresalfi et al., 2009).

Taken together, these dimensions shift the focus from individual limitations toward an analysis of conditions for access, participation, and the construction of mathematical meaning, and they guide the rest of the paper: the synthesis of learning difficulties, the interpretation of auditory-graph studies, the design requirements presented in **Table 2**, and the analysis of Audiofunctions+. The paper offers a conceptual and integrative synthesis rather than a systematic review; the literature was brought together from four intersecting areas: function learning, mathematics education for learners with visual impairments, auditory graphs and sonification, and inclusive technology design. The tables are therefore analytic syntheses used to connect documented learning needs with design requirements; they should not be interpreted as an exhaustive catalogue of the literature or as evidence that the proposed requirements are educationally effective.

ASSISTIVE TECHNOLOGIES IN THE TEACHING AND LEARNING OF MATHEMATICS FOR VISUALLY IMPAIRED STUDENTS: THE ROLE OF SONIFICATION

Technological resources can mediate access to mathematical content in inclusive classrooms, particularly when they make notation, diagrams, and operations available for independent exploration. Specifically, the use of technological resources allows mathematical content developed for students with visual impairments to be described in rich detail during explanations, avoiding not only the misinterpretation of what is intended for them to learn, but also the use of deixis and distortions of “spoken mathematics” (Mendes et al., 2021). Since mathematical technologies provide access to previously inaccessible information, these resources serve as mediators in teaching-learning processes (Piroi et al., 2023; Spinczyk et al., 2019).

Starting from these reflections, it is necessary to question whether and how it might be possible to implement these tools in a classroom context, thus reconsidering them not only as related to their direct intervention with students with visual impairments, but to their transversal use. For this purpose, the design of mathematical technologies - and by using the term ‘design’ we refer to the meaning given by Drijvers (2012), who does not interpret it univocally as "the design of digital technology involved but also the design of corresponding tasks and activities and the design of lessons and teaching, in general" (Freiman, 2020, p. 853) - must allow for the development of a learning environment that is stimulating for students and enables them to achieve meaningful objectives, while simultaneously making mathematical concepts accessible and comprehensible through communicative modalities that foster connection with the content (Confrey et al., 2009).

Mathematical digital technologies, particularly in contexts involving sensory diversity, should meet three key criteria (Healy & Kynigos, 2010):

1. *Deep structural access*, which refers to students’ ability to engage not only with surface-level representations but also with the “underlying structures and generative rules” of mathematical objects. This is especially critical when visual access is limited or absent.
2. *Fostering body and ego syntonicity*, meaning the design of environments in which students can experience mathematical ideas in ways that are “physically and sensorially meaningful”, aligned with their own embodied perspectives.
3. *Providing multiple representations and forms of communication*, supporting diverse ways of expressing, exploring, and negotiating mathematical meaning.

Research on assistive technologies in mathematics identifies computers, speech output, and audio or tactile tools as potentially valuable resources for learners with visual impairments. Positive engagement has been reported when materials permit meaningful and increasingly independent mathematical activity, although such outcomes are not guaranteed by the technology itself (Spinczyk et al., 2019). Ketema Dabi and Negassa Golga (2024) noted three broad issues concerning the role of assistive technologies in supporting the engagement of students with visual impairments:

1. The availability of assistive technologies to support learning mathematics. The degree of accessibility in the classroom and the extent to which teachers integrate assistive technologies into their instruction, together with their cost, significantly influence the availability of such tools for facilitating mathematics education.
2. The usability of assistive technologies in mathematics classes. The use of these technologies in the classroom depends on their ease of manipulation, the appropriateness of the tools for instructional purposes, and the level of user satisfaction.
3. The role of assistive technologies in the inclusion of visually impaired students. This dimension encompasses the engagement of students with visual impairments with assistive technology and their sense of empowerment when solving mathematical problems or performing operations with its support.

Ketema Dabi and Negassa Golga (2024) believe that few studies have analysed the barriers to accessing assistive technologies for visually impaired students in terms of cost, teachers' failure to integrate assistive technologies as part of their teaching activities, technology's constant evolution and incompatibility with existing media. They identified inaccessibility, complexity, erroneous prescription, price, and a lack of professional follow-up as obstacles to the effective use of assistive technologies in mathematics instruction. Moreover, learnability, efficiency, memorability, errors, satisfaction, and ease of use are used to characterise and assess assistive technologies' usability in teaching activities.

Recent work has provided a more differentiated account of sonification and its possible roles in mathematics education. Sonification, understood as the process of transforming data into sound, involves the use of non-speech audio to convey information or make data perceptible. Specifically, it can be defined as the data-dependent generation of sound in a way that reflects the objective properties of the input data (Hermann, 2008). In this context, Nees and Walker (2008) refer to an "auditory graph" as a sonified representation that uses non-speech sound to explore quantitative data.

Different sonification methods produce various auditory graphs based on factors such as data characteristics, audio types, sound mapping strategies, and play order (Dubus & Bresin, 2013; Fitzpatrick & Neff, 2023; Hermann et al., 2011). Discrete event-based sonification provides immediate auditory feedback for specific data points or interactions, making it particularly useful for stepwise exploration and event detection. This approach is characterised by point-wise sounds triggered by particular interactions or individual data values. Common examples include simple tones, text-to-speech, earcons, or beeps, with sonic mappings involving pitch variation, stereo positioning, or spatial comparison. Representative methods include the use of earcons to indicate boundaries and discrete sonification techniques for guiding stepwise interactions, facilitating precise and interactive auditory exploration of data (Dobosz & Hanak, 2024). Continuous sonification represents evolving data trends through uninterrupted sound streams, allowing real-time monitoring of dynamic datasets. It is characterised by continuous auditory streams that reflect ongoing changes in data, with typical audio examples including oscillator waveforms, synthesizers, and spatialised sounds. Representative methods include continuous oscillators and frequency mapping along the y-axis, providing a smooth and ongoing auditory representation of data dynamics (Kim et al., 2019).

Hybrid sonification integrates point-based and continuous methods, enabling richer and more nuanced multivariate auditory representations (Grond et al., 2010; Reinsch & Hermann, 2023). It has also become possible to generate spatial sonification, which enhances spatial awareness by positioning sounds within stereo or 3D space to convey data relationships. This approach relies on position-based auditory cues that guide perception of data structure and distribution. Typical examples include spatialised sound effects, natural sounds, or text-to-speech, which are mapped through techniques such as stereo or 3D panning and volume modulation according to position. Representative methods in the literature include direction-based sonification, spatial comparison through stereo audio, and 3D virtual microphone mapping, all of which contribute to creating richer and more intuitive auditory representations of data (Heuten et al., 2006; Hoque et al., 2023).

Thanks to these advances in technological research, it has also been possible to observe in the educational field how sound can be considered one of the most suitable tools to enable individuals with visual disabilities to learn and discern elements that typically require extensive use of sight. Sound plays an essential role for people with visual impairments by providing auditory cues for navigation, environmental awareness, and information comprehension (Manolino et al., 2025). In this context, sonification brings the possibility of inclusion and a change in the social interaction of students with visual impairments. In addition to favouring their autonomy, it offers a shared medium for studying a lesson, allowing for collaborative work (Tomlinson et al., 2016). These possibilities concern representational access and participation; whether they lead to epistemic access depends on task design and instruction.

DIFFICULTIES IN UNDERSTANDING FUNCTIONS THROUGH THEIR GRAPHICAL REPRESENTATIONS

There is broad consensus that a deep understanding of functions is essential for the study of calculus and for making meaningful connections between calculus concepts and real-world phenomena (Bardini et al., 2014; Parhizgar et al., 2022). However, research indicates that learning the concept of function is a complex process.

To evaluate possible educational uses of sonification techniques for teaching and learning functions, we first identify the mathematical notions, skills and tasks that the technology would need to support. The analysis is organised by linking the tasks to curricular objectives at the secondary education level, as well as to the main difficulties identified in the literature in this area, particularly in interpreting and coordinating graphical, symbolic, tabular, and verbal representations of functions.

Table 1. Tasks, learning objectives, and students' difficulties in learning functions

Tasks	Related difficulties
Learning objectives. Recognising general characteristics of a function through its graph (domain and range, growth, maximum and minimum, continuity, symmetry, sign, trend, curvature and inflection point).	
Recognising which graph does not correspond to a function (and which one does).	• Understanding the role of the variable in a function graph. • Connecting the concept of function with the table of values and its graphical representation.
Interpreting function graphs to identify properties of the functions.	• Relating the points on a function's graph to their description through the table of values. • Realising how domain constraints also affect the range.
Learning objectives. Transposition between different representations (verbal, tabular, analytical, graphical).	
Constructing the graphical representation of a function from its verbal description, table of values or analytical expression.	• Translating verbal descriptions into mathematical relationships. • Understanding that the points of a function's graphical representation are equivalent to the pairs of values which constitute the function. • Realising that $f(x) = y$.
Associating an analytical expression with a graphical representation.	• Understanding what a function represents. • Understanding what the role of a variable in a function is. • Switching between the algebraic expression and the graphical representation of a function.
Graphing a function that meets certain conditions (regarding domain, range, growth, limits, continuity, etc.).	• Realising how domain constraints also affect the range.
Sketching/Graphically representing a function from a real/contextualised situation.	• Using functions for the modelling of a phenomenon or a real situation.
Learning objectives. Operations with functions and their properties. Composition and inversion of functions.	
Operating with or composing functions through their graphical representations.	• Graphically interpreting the meaning of operations with functions. • Performing operations with functions through their graphs. • Finding the graph of the inverse function. • Interpreting inverse functions in context.
Learning objectives. Interpretation of limits and asymptotes through their graphical representation.	
Studying the behaviour and calculating limits at specific points or at infinity for functions that do not have a given analytical expression.	• Understanding of the concept of limit. • Graphically interpreting the terms "tends to" or "approaches" in the context of limit calculation. • Justifying calculations of limits.
Learning objectives. Continuity of a function and its properties. Types of discontinuities.	
Determining whether a function is continuous or discontinuous by analysing its graph.	• Graphically interpreting the terms "tends to" or "approaches" in the context of limit calculation.
Identifying types and points of discontinuity by graphically examining the one-sided limits at those points.	• Identifying types and points of discontinuity by graphically examining the one-sided limits at those points.
Learning objectives. Differentiability of a function and its graphical interpretation.	
Determining whether a function is increasing/decreasing and its maxima and minima by analysing the graph of its derivative.	• Understanding of the concept of derivative. • Interpreting the derivative as the slope of a tangent line at a point.
Sketching/Graphically representing the derivative graph of a function from the graph of that function and vice versa.	• Recognising the covariation between a function's graph and its derivative graph.

Table 1 connects the learning objectives that secondary school students are expected to attain by the end of this stage (Ministerio de Educación y Formación Profesional [MEFP], 2022; Ministero dell'Istruzione, dell'Università e della Ricerca [MIUR], 2012a, 2012b; Ministerstvo školství, mládeže a tělovýchovy České republiky [MŠMT], 2021)[1], with the difficulties highlighted in research on the study of functions through their graphical representations. These difficulties refer to the identification of function properties through their graphs (Abdullah, 2010; Akkoç & Tall, 2003; Cansiz et al., 2011; Dreyfus & Eisenberg, 1982; Hatisaru & Erbas, 2017; Hitt, 1998; O'Shea et al., 2016; Vinner & Dreyfus, 1989); the translation between the analytical, tabular, and graphical representations (Bardini et al., 2014; Hatisaru & Erbas, 2017; Panaoura et al., 2015); the operations with functions in their graphical representation (Abdullah, 2010; Clement, 2001; O'Shea et al., 2016; Paoletti et al., 2018); the study of limits (Szydlik, 2000) and the continuity and differentiability of a function (Munyaruhengeri et al., 2024; Thompson & Harel, 2021).

Trujillo et al.'s (2023) review highlights several recurrent difficulties:

- Giving a clear and mathematically precise definition of a function without relying on a graph.
- Distinguishing graphs that represent functions from those that do not (**Figure 1**) and using functions to model phenomena or real situations.
- Understanding what a function represents, interpreting the role of variables, and coordinating different algebraic forms of the same function.

[1] The authors are currently involved in the European project "SONAIRGRAPH - SONification for Accessible and Inclusive Representation of GRAPHs in Education" (https://sonairgraph.unito.it/), which sees them collaborating particularly with teachers and researchers from Italy, Spain and the Czech Republic, hence the focus on the ministerial curricula of these European countries. The topics cited reflect mathematics curricula that are widespread across various parts of the world, as documented in international comparative studies (Shimizu & Vithal, 2023).

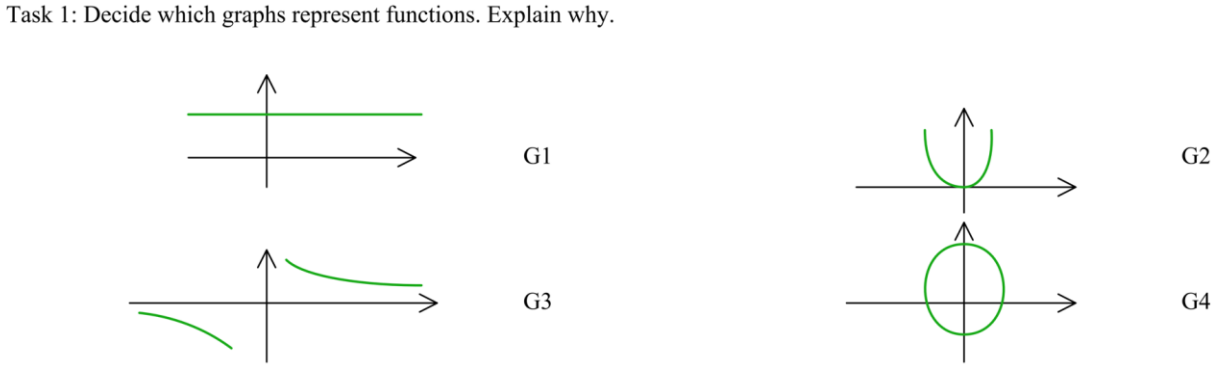


Figure 1. Recognising which graph does (not) correspond to a function (Abdullah, 2010, p. 287. Redrawn by the authors).

- Switching between algebraic, graphical, and tabular representations, including interpreting graph points as ordered pairs of corresponding values.
- Determining whether an expression is a function or not, using function properties and operations, and reasoning about how constraints on the domain affect the range.

Although the terms domain and range are commonly used in teaching functions, students often struggle to fully grasp these concepts, leading them to form an understanding of functions that may differ from the formal definition. A solid understanding of relations, variables, and set theory is among the essential prerequisites for students to comprehend the concept of function (Parhizgar et al., 2022).

Furthermore, a significant part of research focuses on the importance of covariational reasoning (Thompson & Carlson, 2017). Understanding the relationship of covariation in functions (as an invariant relationship between the values of two quantities - one dependent and one independent - as they vary simultaneously) is a fundamental step in learning calculus, though not without difficulties (Fuad et al., 2019; Krüger, 2019). If students develop a productive understanding of variation, they can also approach covariation in a productive way. However, covariational thinking cannot be directly derived from variational thinking nor constructed in isolation, which is why it is important to emphasise the covariation of quantity values from primary education onward (Thompson & Harel, 2021). Moreover, for authors like Paoletti (2020), the covariational meaning of the function can support the development of meanings for a set-theoretic function definition.

Sonification may mediate covariational reasoning by making the coordinated change of two quantities perceptible as an unfolding auditory event. In an auditory graph such as Audiofunctions, movement through x is enacted over time and through keyboard or pointer action, while pitch represents the corresponding y-value (Manolino et al., 2025). A rising, falling, or reversing pitch trajectory can therefore direct attention to how y changes as x changes. Hetzler and Tardiff (2007), for example, found that students could use auditory graphs to identify intervals of increase and decrease, although curvature was more difficult to recognise. Perception also depends on the structure of the auditory presentation: data density and the number of trend reversals affect the identification of trends (Nees & Walker, 2008). These perceptual affordances should not be equated with covariational reasoning itself. We argue that listening remains pointwise if students attend only to successive pitches or isolated coordinates; it becomes potentially covariational when tasks require them to coordinate the progression of x with the direction and rate of change in y across an interval, and to articulate that relationship. Repeated and bidirectional exploration and comparison with visual, tabular, or tactile representations can support this coordination. Fan et al. (2025) provide initial evidence for a further design possibility: varying tempo rather than pitch interval improved participants' perception of slope and acceleration. This result is promising for representing rates of change, but it concerns a specific experimental mapping and should not be generalised to auditory graphs that do not implement variable-tempo sampling.

Some researchers have noted difficulties related to the interpretation and execution of operations between functions through their graphical representations. Very low success rates were reported in tasks requiring the identification of the graphical representation corresponding to a given operation between functions (Clement, 2001; O'Shea et al., 2016) and the conceptualisation of function composition (Abdullah, 2010).

Other studies find that the graphical meanings that students assign to inverse functions are often limited to performing specific actions in different representations. Swapping variables and solving analytically does not produce mathematically equivalent results to reflecting the function over the line $y = x$ (Paoletti et al., 2018). In other cases, students associate the inverse function with the process of 'undoing' the original function without understanding the formal mathematical properties of inverse functions (Teuscher et al., 2018). Providing students with opportunities to compare the results of their procedures across both representations may help them identify inconsistencies, allowing them to develop their understanding of inverse function determination - both graphically and analytically - toward higher conceptual levels (Paoletti, 2020).

Various studies have highlighted the challenges in learning functions, particularly in grasping the notion of infinity and transitioning from intuitive to formal understanding of the concept itself (Kidron & Tall, 2015), understanding the concepts of limit and continuity (Tall, 2019), and comprehending the tangent to a curve and the derivative function (Hitt & Dufour, 2021). Although the concept of limit is "perhaps the most fundamental idea in the standard calculus" (Szydlik, 2000, p. 259), many students develop an incomplete or inaccurate understanding of limits and function continuity (Edwards et al., 2005). Even after receiving targeted instruction intended to clarify the concept, many continue to hold partial or alternative conceptions (Szydlik, 2000).

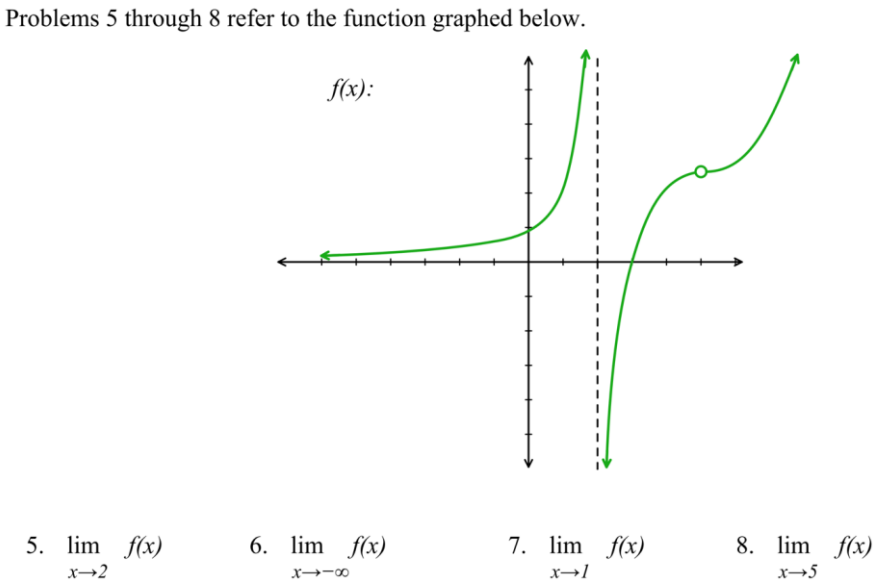


Figure 2. Study the behaviour and calculate limits at specific points or at infinity for functions that do not have a given analytical expression (Szydlik, 2000, p. 263. Redrawn by the authors). [Reprinted with permission from Journal for Research in Mathematics Education © 2000, by the National Council of Teachers of Mathematics. All rights reserved. This figure is excluded from the Creative Commons licence of this article.]

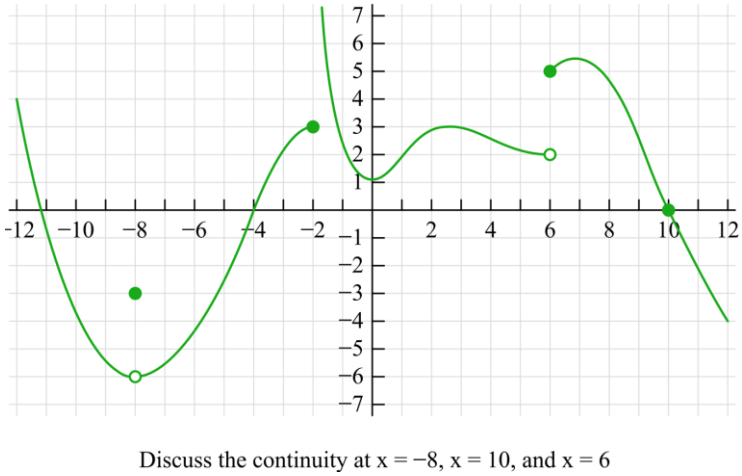


Figure 3. Identify types and points of discontinuity for a function that does not have a given analytical expression (Munyaruhengeri et al., 2024, p. 474. Redrawn by the authors). [The source material is used with permission from Conscientia Beam.]

Munyaruhengeri et al. (2024) found that students have an "operational conception" of limit, and they often confuse how to determine the limit of a function by looking at its graph. On a sketched graph, students have difficulties recognising infinite limits or limits at infinity (**Figure 2** - Szydlik, 2000), distinguishing between the image of a point and the limit of the function at that point, and connecting these ideas with continuity or discontinuity at a point (**Figure 3** - Munyaruhengeri et al., 2024). Moreover, students struggle with the terms "tends to" or "approaches", hindering their understanding of the limits of functions. Finally, regarding the continuity of a function, it is observed that students could not figure out whether a real-world problem could be represented by a continuous or discontinuous function.

Thompson and Harel (2021) summarise some essential aspects found in the literature related to the learning difficulties students face with the differentiability of functions. First, while students are able to carry out different procedures to differentiate functions, they do not understand the quantitative meaning of the derivative in terms of average and instantaneous rate of change. Second, they have issues with understanding the derivative of a function, both symbolically as a limit at a point and graphically as the slope of a tangent line at a point. The difficulties with the concept of both rate of change and slope could have their origin in a poor understanding of the concept of ratio as a multiplicative relation. Finally, the weak understanding of the underlying quantitative meaning of the derivative could be responsible for a rigid focus on common visual characteristics of the graph of a function and the graph of its derivative, not paying attention to the quantitative relations as covariations between these two graphs. The specific research suggests that a) quantitative reasoning with functions expressed dynamically, analytically, and graphically, b) ratio as a multiplicative relation and c) coordination among covariations, provide the basis for meaningful understanding of derivative.

Research with school-age students with visual impairments often reports barriers associated with available materials and visually dependent classroom communication. These barriers can make it difficult to distinguish conceptual difficulties from those arising from the representational environment. Moreover, much of the literature focuses on the introduction and exploration of

elementary graphs; therefore, particularly in light of the curricular contents described above, we remain far from achieving the objectives set at the end of secondary education.

Part of this challenge stems from the limited number of studies exploring more complex functional concepts. This scarcity becomes evident in recent literature reviews. Minholi and Rocha (2024) conducted a systematic review to identify accessible mathematics materials for visually impaired students, discovering merely six studies that addressed functional content and graphical representations - primarily using tools like the Multiplane for quadratic polynomial or exponential functions. Similarly, while Mendes et al. (2021) identified 18 function-related studies in the Brazilian context, nearly all relied exclusively on tactile materials.

On the other hand, Sedaghatjou (2018) illustrates how mathematical communication generally relies heavily on *deixis*. Words like “this”, “that”, “here”, and “there” require visual context such as pointing or gestures to be fully understood. For visually impaired students, interpreting these contextual references presents a significant challenge during functions lectures, as many deictic expressions in mathematical discourse depend on accompanying visual cues, particularly when referencing specific elements of graphs.

Research involving adult users with visual impairments –particularly university students– has addressed somewhat more advanced topics through sonification, although the empirical basis remains limited. Its findings provide a starting point for the a priori identification of potential difficulties related to these topics, the design of tasks and their subsequent implementation –and evaluation– in the school environment.

SONIFICATION IN THE TEACHING AND LEARNING OF FUNCTIONS

Being explicitly visual, a function graph requires adaptation to be accessible for people with visual impairments. A student with visual impairments may rely on tactile perception to access function graphs, through static representations such as embossed diagrams or swell paper graphics. These materials offer intuitive spatial access but are typically limited to fixed content and require time-consuming production. More advanced solutions include dynamic tactile displays, which allow real-time rendering of graphical content via arrays of movable pins. While they enable more flexible and interactive exploration, such devices are often expensive and not widely available (Zhou et al., 2011). Alternatively, auditory access through sonification offers a highly accessible and low-cost solution, transforming graphical information into sound patterns. Unlike tactile formats, sonification supports autonomous exploration and can be used with standard digital devices, making it an auspicious approach for inclusive and independent learning.

Some researchers have begun to investigate the potential and effectiveness of sonification in the study of functions with sighted and non-sighted participants. Specifically, they have investigated the following content:

- Matching monotonicity or curvature properties with visual graph characteristics –working with sighted students (Hetzler & Tardiff, 2006, 2007).
- Estimating the locations of zeros and extrema –working with sighted students (Hetzler & Tardiff, 2006, 2007).
- Investigating limits of continuous functions and derivatives –working with sighted students (Hetzler & Tardiff, 2006, 2007).
- Identifying the position of a graph relative to the x-axis and points of non-differentiability – working with blind and visually impaired participants (Kim et al., 2019).
- Identifying types of discontinuity –working with blind and visually impaired participants (Ohshiro et al., 2021).

In an experiment conducted with sighted students, Hetzler and Tardiff (2006, 2007) analysed students’ responses to various tasks involving the use of auditory graphs. To this end, they developed two sonification tools: a web page with an mp3 file of the graph, and a Visual Basic tool embedded in Excel that sonified a graph entered by the user. On one hand, they observed that students were successful in solving ‘training tasks’ such as estimating numerical values within a given interval based on sonification or associating auditory graphs with visual graphs. Results improved when the sonification incorporated random noise for negative y-values (Hetzler & Tardiff, 2006, 2007).

On the other hand, Hetzler and Tardiff (2007) also observed that students were able to identify subintervals where a given function had local maxima or x-intercepts by listening to the sonified graph. In their experience, once the concept of monotonicity (respectively, curvature) has been introduced, students are presented with the graph in three formats: auditory graph, table of values, and visual graph. They are then asked to identify whether the function is increasing or decreasing (respectively, concave or convex), to indicate how they perceive it in the visual graph, how they identify it in the table of values, and how they hear it in the auditory graph. Most students successfully identified whether a function was increasing or decreasing over a given interval based on its sonification, with a success rate like that of visual identification and higher than identification based on a table of values. Although recognising the concavity or convexity of a function was more difficult for students, the results of their research also showed that the auditory graph was more effective than the table of numerical values for understanding curvature and could be as effective as a visual graph (Hetzler & Tardiff, 2007).

Ohshiro et al. (2021) investigated how participants with visual impairments identified different types of discontinuity using auditory graphs. They developed a study-specific tool with the p5.js library and used distinct auditory behaviours for removable, jump, and infinite discontinuities. Participants detected most discontinuities; infinite discontinuities were identified most readily, whereas jump and removable discontinuities were more easily confused.

In their research involving blind and visually impaired individuals, Kim et al. (2019) focused on identifying optimal sonification methods that support the recognition of relative positions of graphs, quadrant transitions, and differentiability. Participants were

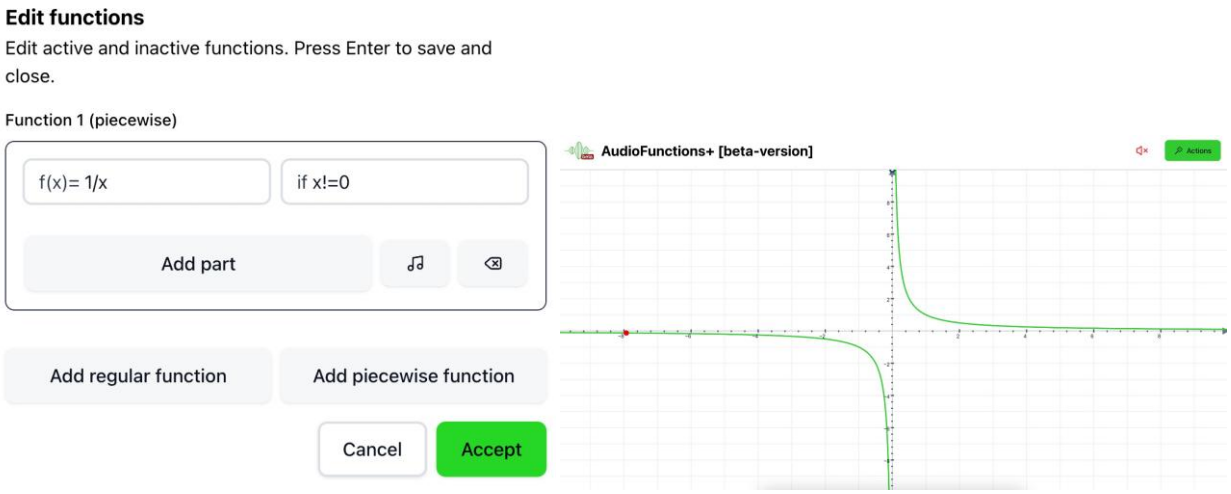
asked to draw the graphical representation of different functions after listening to their sonifications. Results showed that participants produced graphs more similar to the original ones when a sound was provided for the function's value and another for the x-axis simultaneously, and when quadrant changes or discontinuities were marked by distinct sounds or speech-based cues, respectively (Kim et al., 2019).

In contrast to the research presented in the previous section, these studies do not aim to identify students' limitations in learning functions, but rather to test and evaluate the potential of different ways of sonifying function graphs. Indeed, although these works revealed difficulties in recognising characteristics of functions (e.g., monotonicity, continuity, and types of discontinuity), their main objective was to connect the sonification of a function with its graph and its main properties, in order to identify which sonification features are key for detecting those properties. Despite recent advances, further research is needed to determine how sonification can support increasingly sophisticated understanding of functions and their graphical properties. For example, Fan et al. (2025) investigated how different sampling strategies in pitch-based sonification influence the perception of slope and acceleration, while Gatto et al. (2024) combined sonification and tactile tools for linear and quadratic functions.

Table 2 integrates the findings identified in the literature with the synthesis of learning objectives, tasks that support their achievement, and potential difficulties in this process, summarised in **Table 1**. Thus, for each of the main topics in the study of functions, **Table 2** articulates students' learning needs and possible limitations (based on **Table 1** and previous research results) with the requirements that a sonification tool should meet in order to address them.

Table 2. Topics in the study of functions, student needs or difficulties, and requirements for sonification tools

Main Topics	Students' needs or limitations	Requirements
Essential characteristics of a function		
What is a function? What is not a function?	Students have difficulty distinguishing graphical representations which correspond to functions from those which do not.	Include auditory representations of relations that are not functions and tasks that make the single-valued condition explicit.
Relationship of covariation.	Students may attend to isolated input-output pairs without coordinating how two quantities vary together across an interval.	Use sonification to enable dynamic, temporally sequential exploration that supports the perception of covarying quantities.
Identifying the domain and range of a function. Recognising when a function is injective or surjective.	Students struggle to realise how domain constraints also affect the range.	Explore the possibility of sonification along the *y*-axis. Provide explicit cues for the coordinate window.
Operations with functions		
Operating with functions and their graphical interpretation. $f(x) + g(x)$, $f(x) + k$, $k \cdot f(x)$.	Students have difficulties graphically representing an operation of two functions given by their graphs, identifying the graphical representation corresponding to a given operation between functions, and conceptualising the composition of functions through their graphs. The meanings students assign to inverse functions are limited to performing specific actions in different representations.	Display and sonify two functions simultaneously or quickly switch between them to observe and hear the effects of transformations such as translation, composition, scalar multiplication, or inversion.
Monotonicity		
Characterising or recognising increasing/decreasing functions. Determining intervals of increase/decrease, relative extrema and absolute extrema.	Comprehending increasing and decreasing behaviour, as well as the determination of local and absolute extrema, is fundamental for applying functions to real-world problems.	Display two functions simultaneously or quickly switch between them (eventually, fixing the x-value) to compare increasing/decreasing functions, and examples of increasing/decreasing, and relative/absolute extrema.
Curvature		
Characterising or recognising concave and convex functions. Determining intervals of concavity/convexity, and inflection points.	Students have difficulties interpreting the curvature of a function, even when sonification is used.	Display two functions simultaneously or quickly switch between them (eventually, fixing the x-value) to compare concave and convex functions, as well as examples and counterexamples of concavity/convexity intervals and inflection points.
Continuity		
Characterising continuous functions. Identifying types of discontinuities.	Students could not figure out whether a real-world problem represented a continuous or discontinuous function. Students struggle when identifying types and points of discontinuity for a function for which no analytical expression is given.	Represent piecewise-defined functions suitable for working with different types of discontinuities. Sonify different types of discontinuities in distinct ways.
Differentiability		
Recognising when a function is not differentiable at a point.	Students face difficulties with understanding the derivative of a function both symbolically as limit at a point, and graphically as the slope of a tangent line at a point.	Coordinate the graph of a function with a tangent line or derivative graph, using distinguishable mappings and reference cues.
Understanding the quantitative meaning of the derivative in terms of average and instantaneous rate of change.	Students have issues with the quantitative meaning of the derivative in terms of average and instantaneous rate of change.	Adjust the sound speed of the function relative to the slope of the tangent line at a given point.



(a) (b)

Figure 4. Use of Audiofunctions+ to identify vertical asymptotes in a function: (a) definition of the function; (b) resulting graph (Source: Authors' screenshots of Audiofunctions+)

Audiofunctions+: A SONIFICATION TOOL FOR FUNCTION GRAPHS

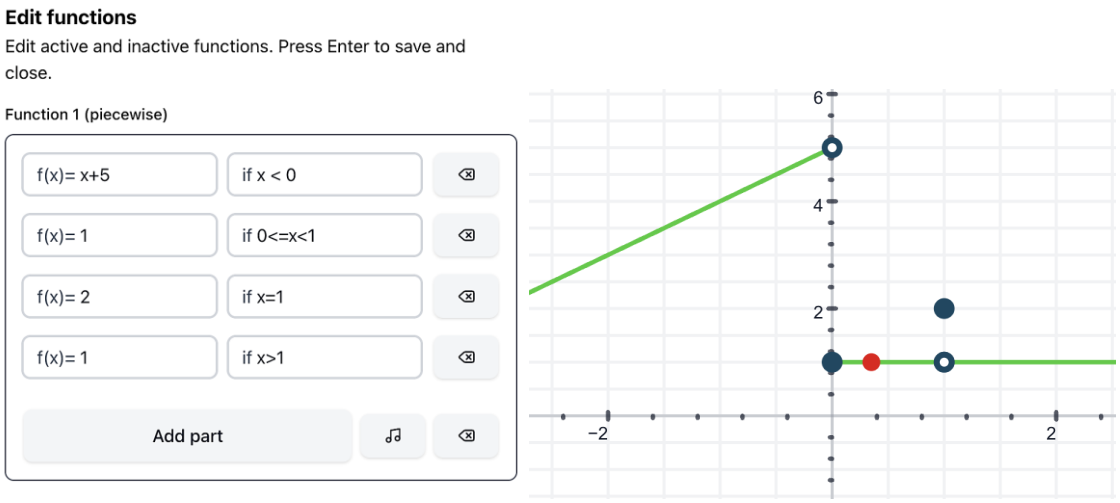
In this section, we provide an example of how some of the requirements presented in **Table 2** can be addressed with a specific sonification tool. *Audiofunctions* was developed in 2014 at the University of Milan as an iOS app for exploring mathematical function graphs on touch screens. Building on research into the educational needs of visually impaired students, the University of Turin's Polin Laboratory, in collaboration with the EveryWare Lab in Milan, extended it into *Audiofunctions.web*, a browser-based tool for classroom use (Ahmetovic et al., 2019) available at https://ewserver.di.unimi.it/audiofunctions. Feedback from research and users later led to *Audiofunctions.web 2.0*, accessible at https://audiofunctions.netlify.app. Within the Erasmus+ Key Action 2 project *SONAIRGRAPH*, coordinated by the University of Turin with several European partners, the tool has been further updated into *Audiofunctions+*, currently available at https://audiofunctions-plus.netlify.app/.

Audiofunctions+ is open-source, free to use, and can support the teaching and learning of functions, with particular attention to non-visual access. Users enter algebraic expressions and adjust the displayed coordinate window. Horizontal position is mapped to stereo panning, while the function value is mapped monotonically to pitch. Mouse or keyboard movement triggers sound according to the current *x*-position and corresponding function value. The application provides continuous and discrete modes, using clarinet and guitar timbres respectively. As described by Manolino et al. (2025), the modes differ in pitch density and threshold-passing behaviour. In the discrete mode, a new sound is produced when the exploration crosses into a different pitch interval; unchanged intervals are represented by silence. Users can store several functions and switch rapidly between their separate graphs with the corresponding number keys. Simultaneous activation, as used for the example in **Figure 7**, requires editing and re-importing the exported session file. These mappings support different forms of exploration but do not provide a one-to-one audible code for every numerical value.

In addition to sonifying the function itself, earcons have been incorporated to facilitate graph exploration. Specifically, an earcon is provided to signal the visualised chart bounds, one to indicate crossing the y-axis (when x = 0), and one to signal when the function is not visible within the set *y*-range or is undefined for an *x*-interval. If the value of the function at a certain *x*-coordinate is negative, then a noise is added to the sound, helping to indicate possible points of intersection with the *x*-axis. The noise disappears when moving back to positive values. Finally, the application offers the possibility to read aloud the coordinates of the current point in the graph of the function.

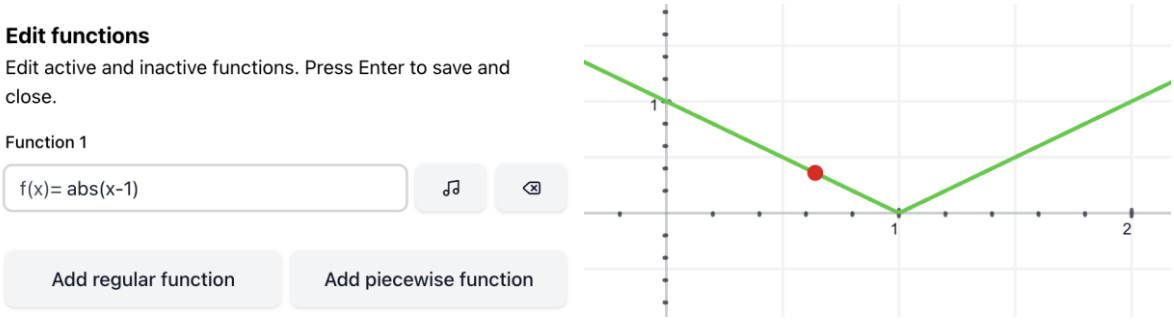
Audiofunctions+ was designed to fulfil some of the detected requirements a sonification tool should have to address students' difficulties with functions (**Table 2**). In order to study the injectivity of a function, the user can scan the function in the current view box to determine if the pitch at two different *x*-coordinates is the same. Because pitch is quantised, hearing the same pitch at two positions can flag a candidate violation of injectivity, but it does not by itself establish that the *y*-values are equal; coordinate readout or a finer scale is needed for confirmation. Growth and decrease are indicated by a change in the pitch of the sound. If the function is increasing, the pitch rises, and if it is decreasing, the pitch falls. Local maxima and minima can be identified by listening for points where the pitch changes direction. Audiofunctions+ also provides cues that may support investigation of vertical and horizontal asymptotes. For instance, when approaching a vertical asymptote from the left side (e.g., $f(x) = \frac{1}{x}$), an earcon signals that the point $(x, f(x))$ lies outside the current view box, while the pitch abruptly shifts from low to high (**Figure 4**).

The zoom level can be adjusted to better explore a specific interval of interest. This can be achieved through zooming in for detailed local analysis or zooming out for a broader global interpretation. Regarding horizontal asymptotes, in continuous sonification the user hears a similar pitch while moving across the function, while discrete sonification produces extended silence



(a) (b)

Figure 5. Representation and sonification of a piecewise-defined function: (a) definition of its parts and conditions; (b) resulting graph (Source: Authors' screenshots of Audiofunctions+)



(a) (b)

Figure 6. Representation and sonification of the shifted absolute value function f(x) = |x - 1|: (a) algebraic input; (b) resulting graph (Source: Authors' screenshots of Audiofunctions+)

in that region. In this case, the zoom level can be adjusted to verify if the same behaviour persists, confirming whether a horizontal asymptote is present.

Audiofunctions+ includes the ability to represent piecewise-defined functions (**Figure 5**). Piecewise functions are described via algebraic expressions and inequalities specifying where these expressions apply. These functions are particularly relevant for investigating removable and finite jump discontinuities. Finite jumps are indicated by a sudden change in pitch. If the function contains in its definition a condition of the form $x = a$, then the point $(a, f(a))$ becomes a point of interest. When the user passes near this point using continuous navigation or with the mouse, it acts as a small magnet producing a snapping effect toward it. Snapping to the point of interest produces an abrupt change in pitch. If the movement is discrete, and the user misses a point of interest, an earcon is played to warn the user.

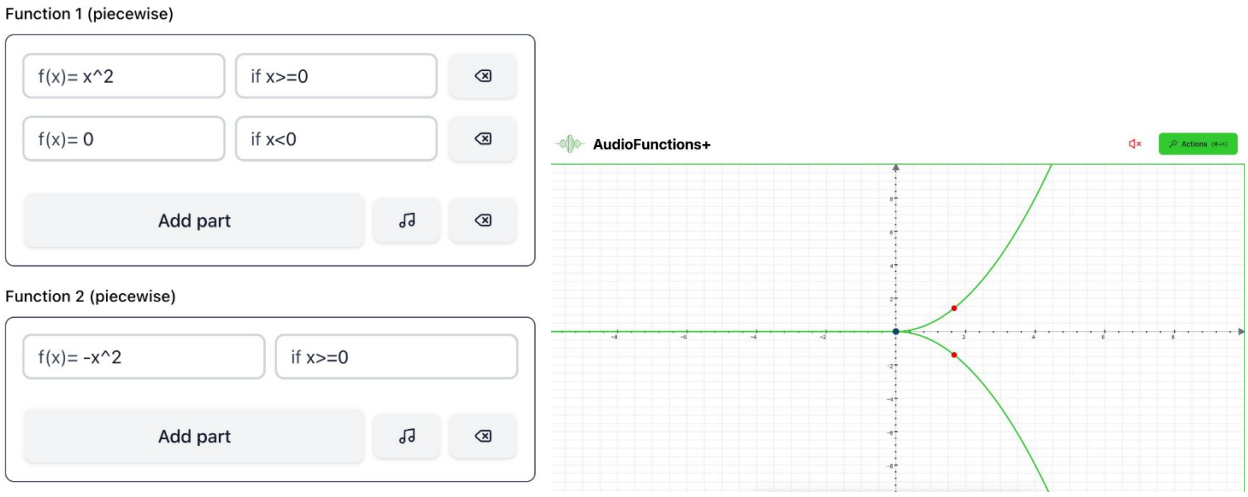
A point of non-differentiability can be recognised by a sudden change in pitch without a smooth transition. For example, the absolute value function in **Figure 6** has a corner at x = 1 that can generate such a cue. Zooming can help determine whether the audible change persists locally, but non-differentiability must ultimately be justified mathematically rather than inferred from sound alone.

An important contribution of Audiofunctions+ is the possibility of representing and sonifying graphs that do not correspond to functions (**Figure 7**).

To help students distinguish functions from non-functions, teachers must manually edit the 'visibility' of the functions[2]. In the example shown in **Figure 7**, once the interval x > 0 is reached, the user perceives two sounds being played simultaneously, which contradicts the single-valued nature required by the mathematical definition of a function.

The following scenario illustrates a possible classroom use; it is a design proposal rather than an empirically evaluated sequence. In a task on monotonicity, local extrema, and covariation, students first predict a function's behaviour and then explore coordinated auditory and visual representations, with a tactile representation available when useful. They scan the graph in both directions, describe intervals in which pitch rises or falls, treat reversals as candidate extrema, and use coordinate readout and

[2] In order to do this, teachers define the two piecewise-defined functions. Then, they export the session to a file (JSON) and edit it so that they make both functions active by setting the attribute "isActive" to true in both functions. Finally, they import the modified file into Audiofunctions+.



(a) (b)

Figure 7. Representation and sonification of a graph that does not represent a function: (a) definitions of the two expressions; (b) simultaneous representation (Source: Authors' screenshots of Audiofunctions+)

zoom to test their conjectures. Comparing the auditory graph with a visual graph and a table of values can then support discussion of what each representation makes available and how the quantities vary together.

The sequence can move from guided familiarisation with mappings and earcons towards more autonomous exploration. Where the interface is used with a screen reader, speech and non-speech audio should be sequenced so that they do not mask one another. Headphones may support individual exploration, while shared audio may support collective discussion. In keeping with UDL, sonification should be available alongside visual, symbolic, tabular, and tactile resources rather than positioned as a universal replacement. This coordination addresses representational access; prompts that require conjecture, justification, and comparison are needed to pursue epistemic access and learner agency.

DISCUSSION

The central challenge in the teaching and learning of functions is not simply to make a visual graph audible, but to design an inclusive learning environment in which auditory interaction supports mathematical activity. Through the conceptual framework adopted here, this requires representational access to relevant features, epistemic access to the relationships that constitute the function concept, and opportunities for participation and agency. Sonification can contribute to all three dimensions: it can make graphical information perceivable without vision, provide a dynamic means of investigating variation, and create a representation that blind, visually impaired, and sighted students can discuss together. Nevertheless, access to sound alone is not equivalent to access to mathematical meaning.

Tables 1 and **2** serve complementary roles in this argument. **Table 1** is a diagnostic and curricular synthesis: it links function-graph tasks with difficulties documented in mathematics education research. **Table 2** is a design-oriented translation of that synthesis: it asks what auditory and interactive resources would be needed to investigate or address those difficulties. The recurring topics across the tables are therefore deliberate rather than redundant. They show how a difficulty such as coordinating variation, interpreting discontinuity, or relating a function and its derivative becomes a requirement for tool and task design.

The evidence is uneven across mathematical properties. Hetzler and Tardiff (2006, 2007) found that auditory graphs supported the identification of monotonicity and extrema, whereas curvature was more difficult. Kim et al. (2019) found that reference sounds and speech cues affected participants' recognition of a graph's position, quadrant transitions, and points of non-differentiability. Ohshiro et al. (2021) reported that infinite discontinuities were easier to identify than jump and removable discontinuities. These contrasts show that no auditory mapping is intrinsically transparent. Outcomes depend on perceptual discriminability, exploration direction and timing, reference cues, prior knowledge, and the task posed. Covariational reasoning, in particular, requires students to interpret coordinated change across intervals rather than merely recognise isolated sounds.

Audiofunctions+ addresses several representational requirements through keyboard-controlled exploration, pitch and stereo mappings, continuous and discrete modes, earcons, coordinate readout, zoom, and rapid switching between stored functions. Priorities for development and empirical evaluation include pedagogically structured uses of multiple-function comparison, operations and inverse functions, explicit representations of tangent lines and derivatives, and temporal mappings for slope and curvature. However, Audiofunctions+ should be regarded as a developing tool rather than a finished product that has resolved all documented learning difficulties.

CONCLUSIONS

Despite the emergence of successful inclusive teaching practices in mathematics education in recent years (Del Zozzo & Santi, 2023), such approaches remain isolated examples rather than institutionalised standards (Dias & Pinto, 2025). Graph-generating technologies are often costly, which can limit their accessibility and use. Therefore, it is essential to develop appropriate materials at the most affordable cost possible for blind individuals (Llamazares & Arias, 2022). Additionally, the role of teachers is crucial, and they must receive training in using these resources to ensure truly inclusive and effective mathematics education for blind students (Piroi et al., 2023; Piñeiro et al., 2025).

The integration of sonification with visual, tactile, tabular, and symbolic representations may contribute to a multimodal approach to mathematics teaching; however, it is necessary to evaluate its specific correspondence and its contribution to curricular learning objectives (Guiotto Nai Fovino et al., 2024). Multiple means of perception and expression may reduce barriers, but they do not eliminate the need for individual accommodations or specialised support. This position is consistent with UDL, which frames access, meaningful participation, and learner agency as design responsibilities (CAST, 2024). Recent evidence from inclusive mathematics classrooms more broadly suggests that these responsibilities are also shaped by contextual conditions – class size, assessment structures, and access to resources– that constrain how far any UDL-informed design can be enacted in practice (Porta et al., 2026).

Research documents a broad range of difficulties in reasoning with function graphs, whereas direct evidence on sonification remains limited, heterogeneous, and concentrated in short studies with adults or university students. The resulting research agenda is therefore to investigate how auditory graphs can support curriculum-based mathematical activity in inclusive classrooms, which combinations of representations are productive for different learners and tasks, and what knowledge teachers need to orchestrate their use.

As a result of our study, we propose five areas of reflection to guide the design and use of sonification-based tools to support the learning of functions among all students, especially those with visual impairments:

1. Delving into the elements of the mathematical structure of functions that students (in general) find difficult and examining what mathematics education research suggests about how to address them.
2. Distinguishing conceptual difficulties from barriers produced by inaccessible materials, visually dependent language, and classroom practices for blind and visually impaired students.
3. Exploring which specific aspects of functions can be addressed with existing sonification tools and which cannot, in order to determine how current applications may help and what developments or improvements are needed to fill additional gaps.
4. Analysing how these tools could be integrated into the mathematics classroom (considering the role of mathematical content, the teacher, and the student).
5. Designing teacher training and professional development initiatives to effectively integrate sonification into the teaching of functions.

As mathematics education has long been recognised as a field where inequities persist due to systemic barriers (Hodge & Cobb, 2019), we believe that inclusion-driven research in this field should advocate for practices that acknowledge and mitigate disparities, seeking an intermediate solution that overcomes the dichotomy between individualisation and equality (Hunt, 2020). These are not mutually exclusive goals but dynamically coexisting principles. Inclusion is neither a fixed endpoint nor a uniform method but an ongoing, situated exercise of researchers' and teachers' professional responsibility.

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