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# Analyzing in-service teachers' process of mathematical literacy problem posing

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**ABSTRACT**

Posing mathematical literacy problems is one of the challenging issues in literacy studies. The present study focused on how to develop the ability to pose mathematical literacy problems. Within the context of the training given to middle school mathematics teachers on how to pose mathematical literacy problems, literacy problems were introduced. Each of the problems was discussed and evaluated regarding the structural characteristics and differences from the routine problems. Respectively, the steps of (i) completing an incomplete problem with a context as a mathematical literacy problem; (ii) evaluating the problems in the textbooks in terms of realistic problems and converting them into mathematical literacy problems; and (iii) conducting exercises for writing original mathematical literacy problems. It was concluded that teachers were more successful in creating contexts, that they tended to write operation-oriented problems, that they often used sentences ending with the words "please calculate" and "find" by being influenced by the traditional word-problem sentences contained in the textbooks, and that they had more difficulty in developing problems which should bring forward a mathematical suggestion than other problem types.

**Keywords:** mathematical literacy, in-service mathematics teacher education, problem-posing, professional development, real-world problem

## INTRODUCTION

The isolation of school mathematics from real life continues to be one of the main problems of teaching. The previous studies (Ellerton, 2013; Ellerton & Clarkson, 1996) stated the necessity of establishing a link between real-life and school mathematics in teaching practices to bridge the gap regarding this isolation. Traditional mathematics learning environments, rather than considering these needs, distanced the problem-solving topics from everyday life (Kerekes et al., 2009). Therefore, it is pointed out the difficulties encountered in posing real-life problems, which are the difficulties encountered in establishing a link between real-life problems and the mathematical expressions of their solutions and transferring the problem contexts to real-life situations (Nesher, 1980). Literature suggested to link between mathematics and real-life that the problems dealt with in learning environments should include "real-life problems" integrated with the types of information in real life (Frankenstein, 2009). This increases the importance of "mathematical literacy" (ML), which means the role of mathematics in everyday life and the ability to use mathematics to solve problems encountered in everyday life (McCrone & Dossey, 2007).

The fact that countries have started to refer to the Program for International Student Assessment (PISA) results in the development of educational policies (Breakspear, 2012) has led to an increase in the emphasis on ML and ML problems. In Turkey, the Ministry of National Education (MEB) recently published a mathematics curriculum for elementary and middle school students, which included, as the first objective, "improving students' ML skills and enabling them to use those skills effectively" (MEB, 2017). Despite this effort, Turkey's poor results in the PISA have demonstrated the unfruitful mathematics curriculum to achieve its objective. Also, ML problems were not adequately emphasized in the mathematics teaching process and available resources (e.g., textbooks). In many studies, researchers resulted that there are limited number of ML problems in textbooks (Cakir, 2009; Dede & Yaman, 2005; Iskenderoglu & Baki, 2011). Accordingly, the students are not familiar with PISA problems since they have not encountered such problems in their courses and textbooks. All these results revealed the need for problems–like ML problems–requiring students to use their mathematical knowledge and skills (Demir & Altun, 2018). The main challenge is the appropriate context selection for ML problems and the difficulties in posing these problems. Posing realistic and open-ended problems under ML is more challenging than the problems in traditional mathematics textbooks. Posing realistic problems also requires specific abilities and effort in identifying the level of conformity of a problem to ML, writing a context for the word problems appropriate to ML, or posing an ML problem in a chosen subject. This information advocates that teachers should become more experienced in problem posing to understand better posing real-life problems. We aimed to make a conceptual analysis of the process to explain how middle school mathematics teachers can develop the process of writing ML problems due to the particular importance of problem posing both in the development of problem-solving skills and providing the materials for ML problems. This conceptual analysis was made on ML problems posed by teachers due to an in-service training course conducted using ML problems.

### Problem-Posing Process, Previous Studies, and Their Foci

The problem-posing, inherent in mathematical thinking and accompanies problem-solving, is an essential component of mathematics learning and teaching (Kilpatrick, 1987). The development of mathematics has often resulted from new problems posed by mathematicians and the attempt to solve the problems associated with these problems (Stickles, 2011). In this context, developing the ability to pose mathematical problems is as important as solving problems (Bonotto, 2013). Polya (1957) included problem posing in a four-stage problem-solving process, and later, the study by Brown and Walter (1983) attracted attention to problem posing. However, problem-posing was not given importance until the 1990s. Even though its importance was emphasized many years ago (Einstein & Infeld, 1938), it was understood that problem-posing was a substantial component of mathematics education in the 1980s and 1990s when this phenomenon concentrated upon more extensively (e.g., Brown & Walter, 1983; Ellerton, 1986; Silver & Cai, 1996).

In the literature, problem-posing studies positively affect learning for two main reasons (Cai et al., 2013). Firstly, problem-posing activities are required cognitive processes (Cai & Hwang, 2002). Posing a problem, such as posing a new problem based on a given problem or reproducing an existing problem, etc., requires moving beyond problem-solving procedures. Such activities enhance students' conceptual understanding, advance their ability to reason and communicate mathematically, and keep students' interests alive (NCTM, 1991). Secondly, problem-solving processes require the production and resolution of complementary problems (Polya, 1957). Thus, the ability to pose complex problems allows for higher-order problem-solving skills to develop. Considering this, we concluded an urgent need to develop mathematics teachers' skills to create purposeful and functional problems in the ML context (Chapman, 2012; Osana & Pelczer, 2015).

When the importance of problem-posing activities increased, researchers started to discuss problem-posing processes from various perspectives (e.g., Silver & Cai, 1996). The studies with participants of different ages and experiences (in-service teachers, pre-service teachers, and students at different academic levels, etc.) included problems and tasks of varying complexity and created in particular methods. (Silver, 2013). A significant part of such studies was studying the link between problem posing and solving (Cai & Hwang, 2003). Numerous experimental studies were conducted to explore the potential value of problem posing in helping students to be better problem solvers and to investigate the relationship between problem-solving and posing (Cai & Hwang, 2002; Cai et al., 2013; Ellerton, 1986; Greer et al., 2009; Silver & Cai, 1996; Verschaffel et al., 2009; Xie & Masingila, 2017). In these studies, problem-posing had a positive effect on students' problem-solving skills (Leung & Silver, 1997); that teachers could provide insight into students' understanding of mathematical concepts and processes (English, 1997); and that students' experiences with problem-solving increased their perception and motivation (English, 1998; Silver, 1994), helped them to develop more creative approaches to mathematics (Van Harpen & Presmeg, 2013), reduced their concerns about problem-solving, helped them to adopt a more positive approach to mathematics, and enriched their problem-solving strategies (Brown & Walter, 1990; English, 1998; NCTM, 2000; Silver, 1994).

In the last 20 years, researchers were interested in problem posing in terms of the professional development of teachers (Verschaffel et al., 2000). Some of the studies on problem posing examined the problem-posing processes of mathematics teachers and students based on realistic, real-life situations, as is the case in this study (e.g., Chen et al., 2011; Kerekes et al., 2009; Tichá & Hošpesová, 2012). Chen et al. (2011) researched realistic problem solving and posing in Chinese primary schools and revealed that teachers behaved much more realistically when solving and posing problems. It is stated that when combined with both open-ended inquiry and specific pedagogical constraints, the mathematical perspective exists in real-life can be very productive for developing teachers' problem-posing abilities (Osana & Pelczer, 2015). In the studies carried out in the last few years, as a result of training on the structure and characteristics of ML questions in the context of PISA, the mathematical literacy problem posing skills of teachers and prospective teachers (Canbazoglu & Tarim, 2021; Demir & Altun, 2018; Ozgen, 2019; Sahin & Basgul, 2018) varied. analyzed in terms of variables (type, difficulty level, context, content, and processes). According to the results mathematical literacy awareness levels has been increased and problem posing skills suitable for the nature of PISA developed.

In this study, taking into account the need to nurture mathematically literate students (Gellert, 2004; Jablonka, 2003), the real-life problem-posing process, which aims to bring a realistic dimension within the context of ML, is discussed. Specific features of ML problems were posed and discussed in this study. Besides the inclusion of a context that is experienced or can be experienced, the student's understanding that it may be an event that they may encounter in real life. The elimination of the disconnection between mathematics and real-life will require the use of the mathematical knowledge in real-life which the problems should include implicit information (such as the election system, body mass index, number of heartbeats, etc.) and are worth solving by going beyond mere exercises.

### Analysis of Posed Mathematical Problems

When the studies on mathematical problem posing were examined, various analysis schemes were created and used for the analysis of the problems posed during the study process. In his study, Kulm (1994) considered the problems in four stages: understanding the concept, solving the problem, creativity of the problem, and problem-solving by peers. Silver and Cai (1996) proposed a three-stage analysis for the posed problems: In the first stage, non-mathematical questions, mathematical questions, and three-dimensional evaluation in the form of expressions; in the second stage, the problems identified as mathematical can be solvable and unsolvable; in the third stage, the semantic and linguistic-syntactic suitability of the solvable problems. In another study, Silver and Cai (2005), by making some changes to the first analysis scheme, examined the problems according to quantity, originality, and complexity. This study also includes similar sub-dimensions. In another study, Stickles (2011) examined the problems that were posed primarily in terms of exercises, problems, and non-problems. Those that were accepted as problems were considered in six dimensions: sufficient information, additional information, type of purpose (including general and specific), implicit assumptions, initial conditions, and mathematical structure. Bonotto (2013) divided the problems posed into two as mathematical and non-mathematical problems and then classified the mathematical ones as reasonable and unreasonable. Reasonable problems were examined in two further dimensions, including reasonable problems with sufficient and insufficient knowledge. Xie and Masingila (2017), on the other hand, identified three dimensions of these five categories regarding the problems posed: (1) solvable mathematical problems, (2) unresolved mathematical problems, and (3) non-mathematical problems. These analysis schemes were formed by influencing one another to resemble each other in many ways and support each other.

The complexity of posed problems has been addressed in several dimensions: linguistic-syntactic and semantic structures. The *linguistic-syntactic* structure of mathematical problems focuses on identification, relational and conditional expressions (Mayer et al., 1992; Silver & Cai, 1996). Such as an identification expression is "how many pens do Sevil have?"; a relational expression is "how many more pens does Sevil have than Aylin?"; a conditional expression is "if Sevil buys two more pens, how many pens will she have in total?" Mayer et al. (1992) found that problem-solving difficulties were related to linguistic complexity. Problems with conditional and relational expressions were more difficult for students than those with a term of identification. Therefore, the existence of conditional or relational expressions can be taken as an indicator of problem complexity. In another aspect of complexity, problems were analyzed using the semantic classification scheme for problems developed by Marshall (1995). These semantic structure relations were classified under five categories: change, grouping, comparison, re-expression, and diversification. The case of *change* characterizes a problem in which a particular thing changes permanently in a measurable amount over time (a decrease or increase in quantity). The case of *grouping* occurs when many small groups are meaningfully included in a large group. The case of comparison applies when you want to identify which of two things is larger or smaller. *Re-expression* is present when a particular relationship between two different things is defined in a specific period. The case of *diversification* occurs when a specific relationship that connects two things is generalized on other symptoms of these things. Problems that fall into the category of more than one semantic relationship were more complex than others (Silver & Cai, 1996).

### Mathematical Literacy and Mathematical Literacy Problem Posing

Even though the fact that the problems arising from the world of "pure mathematics" can provide a rich source for mathematical activity, benefiting from mathematics for real-life appears to be associated with "applied mathematical problems" (Chen et al., 2011), which suggest the conception of ML. Applied mathematics problems are essential in contributing to developing competencies in posing and solving mathematical problems in different contexts, analyzing the results effectively, reasoning, and communicating (Jablonka, 2003). Even though problem-solving is a widely studied concept, the advantage of posing problems is one of the issues still under discussion. At this point, the answer to the questions "which references are effective in writing ML problems?", "what are the limitations, opportunities, and challenges?" can provide practical ideas for the problem-posing studies.

ML was initially, depending on NCTM standards, announced as a goal of mathematics teaching in the late 1990s. ML, considered as the degree of reflection of mathematics on human life, as can be seen from the definitions, which are partially different, given in OECD sources (OECD, 1999, 2003, 2006, 2009), is defined as understanding and defining the role of mathematics in real life and making mathematical decisions if needed in life, in constructive, associative and reflective ways, and making mathematics a way of life (Altun & Bozkurt, 2017). The increased importance of ML has attracted attention to PISA, whose central theme is the evaluation of literacy and is organized by the OECD (Breakspear, 2012). The effect of ML at the international level, such as PISA, has led to its reflection in the curricula of countries (e.g., Indonesia, South Africa, and Turkey). Countries took the PISA results as a reference to evaluate the education systems, and the low success levels caused educational executives to be sensitive about the subject over time. The fact that Turkey has failed to achieve the desired level of success in PISA has increased the awareness level of the education stakeholders about this issue. It revealed an urgent need to determine the changes in teaching worldwide and develop skills in the production of ML questions.

As a matter of fact, in the particular purposes of the middle school mathematics education program, developing ML skills is used as the first item. Similarly, the High Schools Entrance Exam after 2018 and PISA mathematics questions were quite similar that showed the idea of training mathematically literate individuals were adopted in teaching. However, the measures taken to date have been limited to expressing literacy is the first aim of the curriculum. They include questions aiming to measure literacy in examinations, and the instructional practices and ML problems that could nurture mathematically literate individuals are not adequately covered in the teaching process.

Since the previous studies made it clear that many students ignored essential aspects of reality in problem-solving and that the mathematical actions they carried out repeatedly were based on a superficial analysis, the problem of establishing a bridge between mathematics and real-life became clearer (Schoenfeld, 1991; Verschaffel et al., 2000, 2009). At this point, some theoretical and experimental studies showed that students' unrealistic perspective on mathematics could be affected by the culture of courses. Schoenfeld (1991) and Verschaffel et al. (1994) claimed that students suspended this perception in getting used to the culture of their mathematics course and problem-solving. More specifically, Chen et al. (2011) stated that students' neglect of their thoughts about real-life was caused by the teaching practices and two aspects of class culture: (1) the stereotypical and unrealistic nature of the word problems used in the textbooks and classroom, and (2) the way these problems were designed and handled by the teachers in mathematics courses (Verschaffel et al., 1999). Similarly, a study conducted by Dede and Yaman (2005) that involved reviewing a textbook in Turkey investigated to what extent the problem-solving and problem-posing activities in PISA were included in the second-grade mathematics and science textbooks and concluded that the number of these activities was not sufficient. A study conducted by Gatabi et al. (2012) examined to what extent the problems in a 9th-grade mathematics textbook in Iran reflected the characteristics existing in the literature. It was found that the new Iranian textbook, which was developed for all students to meet the diverse needs compatible with ML, had an insufficient number of problems and a much poor variety of contexts and did not allow the students to experience the fundamental processes of ML. Considering the importance of harmonizing the problems in textbooks with the desired results in terms of ML in students' learning (Bao, 2004), it is seen that the interventions are not yet sufficient.

Learning in schools is the first step towards students' becoming mathematically literate, and they should have the best experience in the school. From this point of view, it is clear that the quality of ML teaching at school will be invigorated by the teacher's ability to choose and pose ML problems. To support ML in the classroom and develop ML skills, teachers should first understand ML (Mosher, 2015). Due to this importance attributed to ML, the research topic in the present study was identified to improve and analyze the ability of mathematics teachers to develop ML problems. The relevant research questions related to this topic are, as follows:

1. How do in-service teachers interpret the structure of ML problems?
2. What are the opinions regarding their experiences of problem-posing of in-service teachers who are involved in professional development training for problem-posing?
3. What is the conceptual/qualitative structure of the ML problems posed by in-service teachers?
4. We sought answers to these questions within the scope of this study.

## METHOD

Even though this study focuses on problem posing in general, it is about teachers posing real-life problems to increase the ML capacity of students in response to the need felt today. Within the scope of the study, the conceptual analysis of the real-life problems posed by middle school mathematics teachers who have received real-life problem-posing training and their views on the process of posing these problems are presented. The instructional treatment, which was structured for this study in a similar way conducted by Grundmeier (2015), is the provision of professional development of in-service teachers to pose ML problems. The main components of the methodology are the research definitions, participants, instructional approach, data collection, and data analysis used by the researchers of the study.

### Working Definitions

The problem-posing skill is a type of skill that requires the ability to use mental activities and involves revealing new problems by changing the existing problems as well as by posing the problems posed under certain conditions (Silver, 1994; Ticha & Hospesova, 2009). Cai et al. (2013) discussed problem posing in two ways: both writing a similar problem and posing an original problem appropriate to a given context. Silver (1994) stated that problem-posing can be in three stages: before solving a problem (posing problems that represent subgoals for the larger, main problem in planning), while solving (establishing a series of successively more refined problem for complex problem solving) and after solving ("looking back" phase of problem solving discussed by Polya, 1957) it. Stoyanova (2000), on the other hand, discussed problem-posing activities in three categories: (i) unstructured situations, (ii) semi-structured situations, and (iii) structured situations. Within the context of these problem-posing processes, the problem-posing activities in this study is implemented in two ways: (a) reformulating the problems given and (b) posing new problems (Silver, 1994). Accordingly, it is important to explicate what is meant by posing new problems and re-formulating them. Posing new problems is the creation of problems that are appropriate to the structure of ML problems and contain specific contexts. In reformulating the given problems, the aim is to write the questions to a given problem and convert the problems in textbooks to an ML problem.

### Participants

Within the scope of the study, the focus was on a training program aimed at providing mathematics teachers with skills to pose ML problems. In qualitative research to achieve this goal, middle school mathematics teachers were the participants of this study. A total of 28 (11 females and 17 males) middle school mathematics teachers were selected voluntarily, participated in the training. Most of these teachers were 35 years of age or older. They had about 15 years of experience (or more) in middle school mathematics teaching. 11 of these teachers, who taught mathematics to 5th-8th grade middle school students, were male, and the remaining 17 teachers were female.

### Instructional Treatment and Data Collection

Within the framework of the planned training, ten weeks (total of 40 hours) of ML training was given in four-hour sessions each week. The sessions were conducted in a middle school in the city center to not hinder the teachers' ongoing teaching processes at their school. We planned this training to include general information about ML, the structure of ML problems, problem posing and ML problem posing, and to progress in an overlapping order. At the same time, we evaluated the previous training session at the end of each week during the training period. We reorganized the following session in light of these evaluations.

The activities planned in weekly were carried out in a participatory-based problem-solving manner. During each session, an attempt was made to generate an interactive learning environment based on constructivist learning. In the training sessions, interactive applications were presented, and, in this process, a domain expert and the two researchers worked as instructors. The teachers who participated in the study discussed mathematics content taught at the secondary level during the training process and evaluated it in terms of ML competencies. As a result of these discussions, ML problem-solving and problem-posing practices related to different subjects were implemented.

We conducted both a preliminary and actual process for problem-posing activities during in-service training. We called this a sequential process in five stages. These stages were, as follows:

1. Identifying ML problems and classifying them in terms of structural characteristics, process skills, and subject areas (two weeks),
2. Understanding the role and importance of ML problems amongst the problem types (one week),
3. Solving sample ML problems and posing similar problems (three weeks),
4. Discussing the differences of ML problems from traditional mathematical problems and converting traditional mathematical problems into ML problems (three weeks), and
5. Posing original ML problems (one week).

The main objective of this training was for the middle school teachers to develop their ability in problem posing. For this purpose, a series of sequential studies mentioned above was conducted. The preliminary activities given in the first two stages were completed before starting to pose problems. First, to raise awareness regarding ML problems, the sample questions used in the PISA published by the OECD were introduced as teaching material. The different aspects of ML questions from the traditional question structures found in the textbooks were discussed. This discussion was conducted initially in small groups and then throughout the whole class, following the principles of constructivist teaching. In these discussions, subject areas, process skills, competence levels, and the areas of life they were related to were addressed. Forms of questions used in examinations such as multiple-choice, complex multiple-choice, close-ended forms requiring a short answer, and open-ended forms requiring a short answer were introduced. The variety of responses was highlighted. In addition to the outlined structural features of the ML questions, the importance of open-ended questions requiring short or long responses for explaining and defending (putting forward the argument) the individual's unique thinking styles was emphasized. The relationship between the five primary purposes of mathematics indicated by the NCTM–(i) learning to value mathematics, (ii) using mathematics in communication, (iii) mathematical reasoning and proving it, (iv) self-confidence is one's ability to do mathematics, and (v) mathematical problem solving (NCTM, 1989)–and open-ended questions was emphasized.

Problem-posing activities were carried out in certain stages by considering the relevant literature (e.g., Cai et al., 2013; Stoyanova, 2000). The last three of the stages given above were aimed at writing ML problems. In this context, (i) completing an inadequate problem with a given context as an ML problem, (ii) evaluating the problems in the textbooks in terms of real-life and converting them into ML problems, and (iii) writing original ML problems. While the activities in items (i) and (ii) are related to reformulating the given problems suggested by Silver (1994), item (iii) is directed towards posing new problems. Each of the teachers' problem-posing tasks, of which some were performed during the training and others were carried out after the training and sent by mail, was given oral and written feedback. The missing and defective parts of the problems were discussed and fixed by the group.

One of the methods used in problem-posing exercises (the one referred to in point i) is, by tackling traditional questions selected from the current middle school textbooks, to convert these into ML problems by looking for an answer to the question–if appropriate–"why should I make a calculation?" If there is a correct answer to the question posed to the chosen problem, it is already a real-life problem. If it is not, the answers to the question "why should I make a calculation?" create severe opportunities for interventions to be made to the problem to convert it into a real-life ML problem. For instance, consider this problem was taken from the 8$^{th}$-grade textbook: "One of two taps fills an empty pool in 8 hours and the second one fills it in 24 hours. If both taps are on at the same time, how many hours does it take for the two taps to fill the pool?" The teachers were requested to ask the following questions:

1. Why should I fill the pool?
2. Why should I calculate the filling time?

and were expected to revise the problem by taking into account the likely answers to these questions. One of the possible answers suggested from the conversion process was as follows: "Ayfer receives the information as of 17:00 that there will be a water cut for three days. The water tank of the house will be filled as a precaution against the water cut. Based on her past experiences, she knows that one of the two water sources can fill the tank in 8 hours and that the other can fill it in 24 hours. It is 13:00! Does she have enough time to fill the tank by using both water sources?" This context makes the problem meaningful by making it necessary to calculate how long it will take for the two resources together to fill the water tank. Since the converted new form of the problem formed the basis to decide to plan the time, it was considered an ML problem (Altun & Bozkurt, 2017). Thus, when the problem was given a real-life context, the mathematical results obtained in the solution ensured the necessity (functionality) of re-evaluation of the problem within the context. It was observed that this method, of which different examples are given in **Appendix A**, was reasonably functional in posing ML problems.

At the same time, together with this method, tolerance for making a change to the root of the question was allowed. Most of the questions in traditional textbooks could be used as a resource for writing ML problems.

A web environment was designed to provide dynamism and continuity in the ML problem posing-oriented in-service training. The teachers, who accessed this web page http://bursaarge.meb.gov.tr/yasamtemelli/ (**Figure 1**) with their user information, shared the problems they posed. All users could see the problems posted on the system. Another feature of this web environment is that we could present our feedback and suggestions to the teachers simultaneously. Therefore, the problem-posing included a

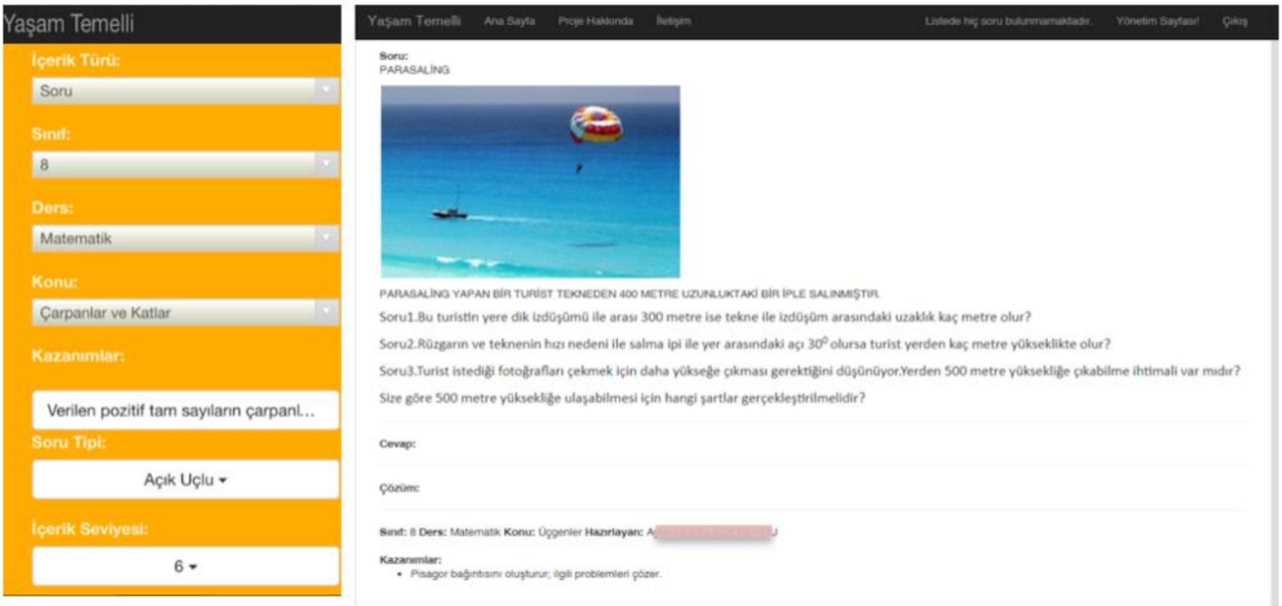



**Figure 1.** Web environment where problems posed are posted online


**Figure 2.** Examples of problems in ML problem identification test

continuous cycle in the form→evaluating→revising→re-posing the problem. A total of 131 problems were posted and assessed in the system. Approximately three weeks after the launch of the in-service training, problem-posing exercises were initiated, and the web environment was introduced to the teachers in this process (**Figure 1**).

Consequently, while the face-to-face training was continuing, the teachers started to upload questions into the system. The most notable advantage of this process for us was to examine the problems posed before the training and make revisions to the training by considering the insufficiencies we noticed in the teachers. At the same time, this procedure made the process proceed more smoothly and without interruption.

In the training process, the teachers first identified ML problems' differences from routine-verbal problems and their type of structure and characteristics. Subsequently, an open-ended ML Problem Identification Test, developed by the researchers consisting of seven questions in which there were problems involving both real-life ML problems and non-ML problems, was applied to the teachers. While four of the questions in this test had ML overtones, the other three were word problems found in regular textbooks. Examples of two of these questions are given in **Figure 2**. The teachers were explicitly required to explain how and on what basis they decided whether the questions were ML-type or not. The purpose of this test was to identify whether the problems had an ML-type rather than solving the problems in the test and explaining the reasons. Thus, we aimed to determine how the teachers interpreted the structure of ML problems.

In this voluntary training, teachers' views on the teaching process and real-life problems were obtained through focus-group interviews and writing. Here, an attempt was made to identify the teachers' opinions about their exercises for posing real-life problems, the difficulties/complications they experienced in these exercises, and the strengths and weaknesses of this training to improve teachers' problem-posing skills. As a result, they were expected to reflect on the problem-posing processes they experienced individually and their views on the training conducted. Finally, teachers who had the chance to pose many ML problems and experienced the process during the problem-posing training sessions (especially the reformulation of given problems) were asked to pose a specific ML problem at the end of the process. These documents, which consisted of the problems posed, constituted another data collection tool of the study.

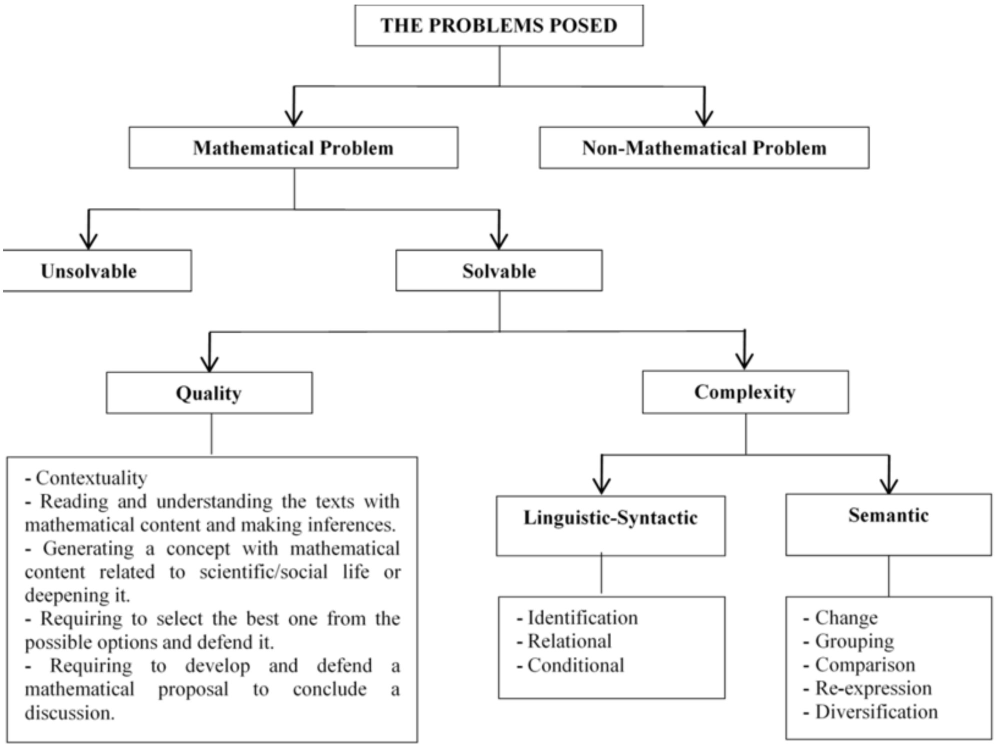



**Figure 3.** Analysis diagram for structural features of ML problems

## Data Analysis

In this context, the procedures followed respectively were to analyze teachers' focus-group and written opinions, the results of the ML problem identification test, and the problems posed by the teachers. The transcripts of the data obtained from the video-recorded focus-group interviews and the black-and-white opinions were evaluated together and analyzed by content analysis. Consequently, a wide variety of data was obtained by combining different sources of data.

The ML problem identification test was applied to identify how the teachers perceived the ML question structures before posing and examining ML problems, was analyzed qualitatively. The test has no equivalent score/total points. Within the scope of this test, teachers' answers to the solutions of the questions were not taken into consideration. For each question, we considered the teachers' responses in the form such as "I think/I do not think that this is an ML problem, because…". In fact, we analyzed the reasons of teachers' decisions on problems they examined as ML problem or not. Thus, teachers' analyses of ML problems were collected under two categories (ML and non-ML problems).

Solvable ML problems posed by teachers were analyzed in two dimensions, namely quality, and complexity (**Figure 3**). In evaluating the quality of the problem, we examined whether the problems possessed the structural features of ML problems. A synthesis of analysis frameworks found in the relevant literature was used to evaluate the complexity of the problems (**Figure 3**). The diagram in **Figure 3** was designed to analyze ML problems posed following the purpose of the study by combining the points emphasized in the classifications made in the relevant literature. In particular, the analysis framework in **Figure 3** was formed by adding "the quality" part to evaluate the compatibility of the problems with ML to the diagram developed by Silver and Cai (1996, 2005) for the analysis of problem-posing practices.

The problems that teachers posed were primarily classified according to whether there was a mathematical problem or not. The problems with a mathematical answer were considered *mathematical problems*. In the second step, mathematical problems were categorized as *solvable* or *unsolvable*. It was decided that the problem could not be solved if the problems posed did not contain sufficient information or if an objective that was not in line with the information provided was posed. The last step in the analysis process involved examining the nature and complexity of solvable problems. *Complexity* is associated with *linguistic-syntactic* and *semantic* structures embedded in the problems posed. For the linguistic-syntactic complexity dimension, the structure proposed by Mayer et al. (1992) and the classification scheme developed by Marshall (1995) in terms of semantics was used. Altun and Bozkurt (2017) evaluated ML problems in terms of *quality* and classified the ML problems according to their structural characteristics: (0) contextuality, (1) producing a concept with mathematical content related to scientific/social life or deepening the existing concept, (2) selecting the best one from the possible options and defending it, (3) developing and defending a mathematical proposal to conclude a discussion, and (4) reading and understanding texts with mathematical content and making inferences. The reason why contextuality is considered as the zero items in the categorization is that ML problems must be put into context. The posed problems were evaluated as ML problems if they had at least one of these characteristics.

Each posed ML problem was evaluated according to the analysis diagram shown in **Figure 3** and subjected to document analysis. All problems posed by the teachers were analyzed separately by the two researchers of the study for inter-coder reliability. Then discussions were held to provide consensus on the categories that differed between the researchers.

**Table 1.** Teachers' opinions on ML problems as a result of ML problem identification test

| Argument | Opinion |
|---|---|
| Yes, it is an ML problem. This is because... | It involves evaluating the data and reaching a conclusion.<br>It requires making a mathematical relationship & identifying the relationship with the change of circumstances.<br>It has an aspect that relates to social life.<br>It requires a formula based on scientific knowledge.<br>The mathematical knowledge is effective in the decisions to be made.<br>It includes chart / table-reading information.<br>A solution to a current problem is offered.<br>It includes generalization and argumentation skills.<br>It requires interpretation by using the information. |
| No, it is not an ML problem. This is because… | It is not aimed at solving a problem or producing information.<br>It contains only the calculation.<br>It measures processing skills.<br>It is far from real life.<br>There is no knowledge generation.<br>It has no routine structure.<br>It is only reinforcement of the subject.<br>It measures information directly.<br>It has the character of a practice question. |

### Ethics Approval

Requirement for consent was waived by the ethics committee for this study. The research has been conducted in accordance with the ethical standards of the American Psychological Association (APA). The utmost care was taken by the researchers to make ensure this. The anonymity and well-being of the participants was respected at all times. No data relating to individual participants are included in the paper.

## RESULTS

The study's findings were presented as teacher inferences about the characteristics of mathematical literacy problems, conceptual examination of ML problems posed by teachers, and teachers' opinions on in-service ML problem-posing experiences.

### Teacher Inferences on Mathematical Literacy Problems

Each ML problem, which was dealt with during the teacher training process, was first solved by the teachers and tackled in such dimensions as the structure of the problem, the mathematical competencies required, the student level, etc. In the ongoing process, on the other hand, the differences between the routine-word problems that the teachers often encountered in textbooks and the ML problems regarded as real-life were discussed. As a result of the classroom observations, it was stated that the textbook-type usual problem structures were considered disconnected from real life by the teachers and that problems requiring mathematical competencies, especially the modeling and argumentation competencies, were rarely encountered.

The ML Problem Identification Test was applied to determine how teachers interpreted the ML problems and evaluated a problem as an ML problem or non-ML problem. When the answers given to the test were examined, it was revealed that some teachers decided on whether there was an ML problem by solving the questions and commented on some others through direct question expressions and structures. As a result, it was revealed which questions the teachers regarded as an ML problem and which questions the teachers considered a non-ML problem, and the results are presented in **Table 1**.

When **Table 1**, which presents the teachers' reasons for classifying the problems as an ML problem or non-ML problem, was examined, it was determined that the situations considered in this open-ended test were issues such as mathematical competencies, the context of the problem, and levels of the cognitive domain. Consequently, these data collected from the teachers were also reflected in the process as a part of the in-service training.

Another issue reflected in the process was the example situations teachers could refer to while searching for a context to pose an ML problem. During the teacher training, the data obtained while discussing ML problem-solving and the problem-posing process led to the consist of the items listed below. These items were determined as examples to be referred to while looking for a context for the ML problem to be posed. The first item guided the problem poser towards the possibility of posing an ML problem on a social movement. For example, we shared elections in countries as an example of a social situation. The following items can also be examined in this context:

1. Explanation of the confusion in social movements and elimination of the chaos (parliamentary elections).
2. Clarifying problems based on mathematical grounds with mathematical foundations.
3. Choosing an option that matches the parameters.
4. Questions describing natural phenomena (Fibonacci sequence).
5. Questions about the development of mathematical knowledge and experiences revealing the original structure (Gauss's sum of numbers from 1 to 100).
6. Mathematically-based questions aimed at solving social or personal conflicts (questions such as "Is… fair?").

**Table 2.** Evaluation of posed problems in terms of ML quality

| Number of structural characteristics of problems | Number of questions (n) |
|---|---|
| One characteristic | 30 |
| Two characteristics | 53 |
| Three characteristics | 26 |
| Four characteristics | 2 |
| Five characteristics | 0 |

**Table 3.** Linguistic-syntactic complexity level of the problems posed

| | Identification | Relational | Conditional |
|---|---|---|---|
| Number of questions (n) | 65 | 34 | 12 |

7. Mathematically-based questions about social life regulations and social rules (scoring system, train and ferry timetables).
8. Mathematically-assisted explanation of scientific events (heartbeat, obesity, and body mass index).
9. Making inferences by utilizing all kinds of graphics, tables, posters, etc. that promote an occasion.
10. Any situation where mathematics is used in the decision-making process.
11. Clarifying conditions that have existed since the beginning of history (wolf-sheep-grass problem).

These themes were the sources of real-life problems, and ML problems could be written according to these sources. Consequently, an infrastructure was provided for the teachers who would structure their ML problems in the training process and their subsequent professional lives.

### Conceptual Analysis of the Mathematical Literacy Problems Posed

Problems posed during the activities in the training and web environment[1] were examined to classify the process of posing an ML problem and the problems posed. In the context of generating new problems, a total of 131 problems were posed by the teachers. Nine of the problems posed were identified as non-mathematical problems. It was determined that authentic information in the context precluded the problem. The problem turned into a problem of reading comprehension rather than a math problem. 11 of the mathematically accepted problems were discussed in the category of unsolvable problems. In this case, some contexts did not contain the information required for the solution or were left incomplete and that there was not enough information for the answer. The remaining 111 problems were analyzed according to the analysis diagram in **Figure 3**. In this process, it was observed that the teachers were able to create original and appropriate contexts and find suitable ways to reach authentic information. It was found that they were successful in posing the problem of establishing a relationship between two sets of data that required the expression of knowledge in different ways. **Table 2** shows the evaluation of the problems in terms of quality to determine the suitability of the problems posed as ML.

When the problems were examined in terms of quality, the striking result was those with two characteristics. Problems with these two characteristics were those which include contextuality as well as one of the other four characteristics:

1. Producing a concept / deepening it.
2. Choosing the appropriate one amongst the possible options.
3. Developing mathematical proposals.
4. Making inferences from texts with mathematical content.

No problem was posed that contained all the ML problem qualities.

Some teachers understood the concept of the ML problem as something requiring long-term calculations and writing problems with a lot of operations. Consequently, they tended to pose such problems. Even though the teachers formed a context suitable for posing an ML problem with this approach, it was seen that they converted their problem with a context into a common word problem and posed problems ending with words such as “please calculate, …” “find...”. They failed to pose open-ended problems that would require an estimate and accept all of the answers given in a given range but often tended to pose problems with one correct answer.

It was observed that the teachers had more challenges in developing mathematical suggestions for the solution of a problem than the other problem types in writing mathematical problems. These problems mostly fall into the formulation (modeling) category have the structure of writing a mathematical statement (inequality or equality) of a change that is described verbally or making a change. The teachers were not successful enough in intervening in real-life through mathematics and posing such problems requiring decision-making. It was thought that teachers had challenges with such problems due to the lack of opportunity in their own experiences.

In the problems the teachers posed, it was revealed that teachers posed more than one problem related to the same context, which was similar to the PISA mathematical question format. The first of the problems posed concerning the same context was usually at the level of linguistic-syntactic identification and were at a weak level in terms of complexity. The second or third problems, if any, were those with higher complexity. As a result of the analyses on these issues, the complexity level of the problems teachers posed in linguistic-syntactic terms was given in **Table 3**.

[1] The web environment is just a tool for teachers to send us their posed problems and our feedback on these problems to them.

**Question 39: Pide (Turkish pizza with meat or cheese) Parlor**

| | | |
|---|---|---|
| Mincemeat: 25TL/kg<br>Filling ingredient: 5TL/kg<br>Flour: 3TL/kg<br>Baking cost: 0,05 TL/unit | Pide with mincemeat topping: 5TL/kg<br>Mincemeat: 150 gr<br>Filling ingredient: 100 gr<br>Flour: 100 gr | Cantık: 2 TL/kg<br>Mincemeat: 50 gr<br>Filling ingredient: 75 gr<br>Flour: 75 gr |

The ingredients and cost table of pide and cantic (pizza-type pide) with mincemeat topping baked by a pide maker is illustrated above.

Based on this:

**Question 39.1: Pide Parlor**

How much does it cost to the pide maker to bake the pide with mincemeat? → Identification

**Question 39.2: Pide Parlor**

Does the pide maker make more profit from 2 pides with mincemeat or from 5 cantıks? → Relational

**Question 39.3: Pide Parlor**

If the price of mincemeat has increased 5TL, what should the price of pide with mincemeat and cantic be for the pide maker to make the same amount of profit? → Conditional

**Question 39.4: Pide Parlor**

If the pide maker wishes to make the same amount of profit from 1 cantık as he does from pide with mincemeat, what kind of changes should he make to the price and ingredients? → Conditional

**Figure 4.** Linguistic-syntactic analyses of problems posed by a teacher-1

**Call Center:** A call center served 57,897 people on May 26, 2015. Average interview duration was 146 seconds. Teams of 12 people were established at the workplace and the success of each team was compared according to the average interview time of the group members. The top 3 fastest employees of the two fastest groups are shown in the table.

| Team A | |
|---|---|
| 3. | 105 sec |
| 2. | 105 sec |
| 1. | 100 sec |

| Team B | |
|---|---|
| 3. | 116 sec |
| 2. | 115 sec |
| 1. | 111 sec |

**Identification** **1.** With 388 people working in this call center, what is the average phone call duration for an employee?

**Relational** **2.** Since the average of team A in this call center is lower than the average of team B, what comments can be made about other employees of these teams?

**Figure 5.** Linguistic-syntactic analyses of problems posed by a teacher-2

**Table 4.** Complexity level of questions from semantic point of view

| Number of relations included | Number (n) |
|---|---|
| 1 | 20 |
| 2 | 49 |
| 3 | 38 |
| 4 | 3 |
| 5 | 1 |

When **Table 3** was examined, it was found that the majority of the problems posed were (n=65) at the level of identification and weak in terms of complexity. For instance, the problems that a teacher posed for the same context and their linguistic-syntactic complexity levels were given in **Figure 4**.

It was seen that in the problems posed by the teacher in **Figure 4**, the complexity levels increased, respectively. Problems posed by another teacher were shown in **Figure 5**. The complexity level of the first problems, which were posed for the same context, was low, while the complexity level increased in the second problem.

**Table 4** presents the classifications related to the level of semantic complexity (change, grouping, re-expression, comparison, and diversification) of the teachers' questions.

**Table 5.** Examples of semantically-analyzed problems and the number of relations included

| Number of relations | Examples |
|---|---|
| 1 | Calculate the body mass index of a person with a height of 183 cm and weight 65. *[Change]* |
| 2 | Since the friend of 10-year-old Serap is going to go to Sanitra Hotel in Antalya with her family, Serap also wants to go there and she tries to convince her family that this hotel is cheaper. Her father thinks that Bodrum is more suited to their budget. Do you think Serap is justified*? [Grouping / Comparison]* |
| 3 | Assuming that Talha told Emre the code 23443, what numbers could Emre have found to find the password of the cabinet? *[Change / Grouping / Re-expression]* |
| 4 | Provided that the items bought should be different, according to the choice that the very first item bought will be given 10% bonus, the second, 15% and the third, 20%, in which order should the items be bought respectively for the 21.5 TL bonus to be uploaded onto the card? *[Change / Grouping / Diversification / Comparison]* |
| 5 | Ahmet and Mehmet will rent a car for a journey of 1000 km. When they go to the car rental company, they learn that hourly diesel car rental is 20 TL and gasoline car rental is 12 TL. Considering that a liter of diesel costs 5 TL and a liter of gasoline costs 4 TL in our country, they try to decide which one to rent. At that time, the owner of the car rental company says that the fuel consumption of the car is as much as the square root of a speed of 100 km / h. Ahmet drives the vehicle slowly, while Mehmet prefers to drive it fast. According to the hourly rental price, who should choose the vehicle and which vehicle should he choose? *[Change / Re-expression / Grouping / Diversification / Comparison]* |

**Table 6.** Analyses of the problems in the category of unsolvable problems

| Situations regarding unsolvable questions | Number of questions (n) |
|---|---|
| Complexity due to unclear relationship | 2 |
| Complexity due to unclear conditionality | 3 |
| Lack of information at the root of the question | 6 |

As a result of examination of the problems from a semantic aspect, the categories the emerged the most are: comparison (n=10), change-comparison (n=19), change-comparison-grouping (n=19), and change-grouping-diversification (n=10). Considering that the complexity of the problems will increase as the number of relations in semantic terms increases, the problems containing four (n=3) and five (n=1) relations are quite a few. For example, **Table 5** illustrates the sample problems related to how many of the relation types were included in the semantically analyzed problems. All problems in **Table 5** are solvable. However, since the texts of the problems in the table are long, only relevant problem statements showing a semantic relationship are included.

Some problems (n=11) that the teachers posed had a mathematical dimension but were considered unsolvable. It was revealed that these problems were unsolvable either because they tried to create problems with high complexity or because there were missing data. Detailed information on this is given in **Table 6**.

When **Table 6** is examined, it is noticeable that regarding the reasons why the problems cannot be solved, the leading one is the case when there is not enough information at the root of the question (n=6). Apart from this, the fact that the expressions were not clear in the process of posing relational or conditional problems that caused the problems to be unsolvable from a linguistic-syntactic aspect. Examples of problems included in the unsolvable category are given in **Figure 6**. The problems included in **Figure 6** contain one example for each category related to unsolvable problems.

### Teachers' Opinions Regarding Their Experiences of Problem Posing

At the end of the training period, focus-group interviews with the teachers to identify their experiences of posing ML problems and their thoughts about the in-service training process. At the same time, their written evaluations were taken. Teachers stated that having been trained with the traditional question structure was inadequate in achieving the objectives of mathematics teaching and that it was a fundamental need to include ML problems in education. The training was found valuable in terms of ML, recognizing ML problems, and posing ML problems.

Teachers stated that problem contexts could not be written only by thinking about them, but that they became aware of the situations (contexts) that might be an ML problem during their daily lives, and they posed the problems in this way. However, they pointed out the difficulty of finding a context based on everyday life and of posing appropriate problems based on this context. They also expressed that their perception that "the measure of the quality of the problem is its level of difficulty; a quality problem is a difficult one" changed with this training. The quality was more concerned with whether it was a real-life situation or not. They emphasized that the ML problems were very different from the usual word problems, and since they only used the questions in the existing textbooks or wrote similar questions rather than posing problems in their teaching experiences, they realized that they needed to concentrate on problem posing. During the interviews, even though the teachers found this problem-posing-oriented study beneficial, they stated that skill development - the problem-posing skill - was a long-term process. To improve themselves in question posing, they emphasized that they needed to get further training, which might continue this training. At this point, some of the significant opinions of the teachers about the training they received are, as follows:

1. Contributing to professional development.
2. Providing a new perspective for mathematics courses and problems.
3. Feeling differences in their teaching after the training received.
4. Increased student participation as a reflection of the training in classes that are difficult to control.
5. More willingness in students who have heard that their teachers have received such training.

**Orienteering** (Complexity due to unclear conditionality)

Orienteering is a nature sport where participants try to visit checkpoints with the help of a map and compass as soon as possible. Five athletes doing orienteering will race on a 5 km medium difficulty course. They have to reach 16 checkpoints and finish the course. Athletes arriving at the end of 1.5 hours will be disqualified. The first table shows the start times of the athletes. In the second table, while the finishing times of the athletes were posted every half hour, a list of names posted at 13:00 is given.

| Name | Start time |
|---|---|
| Meryem | 11:30 |
| Sude | 12:00 |
| Zerina | 12:11 |
| Tuğba | 13:14 |

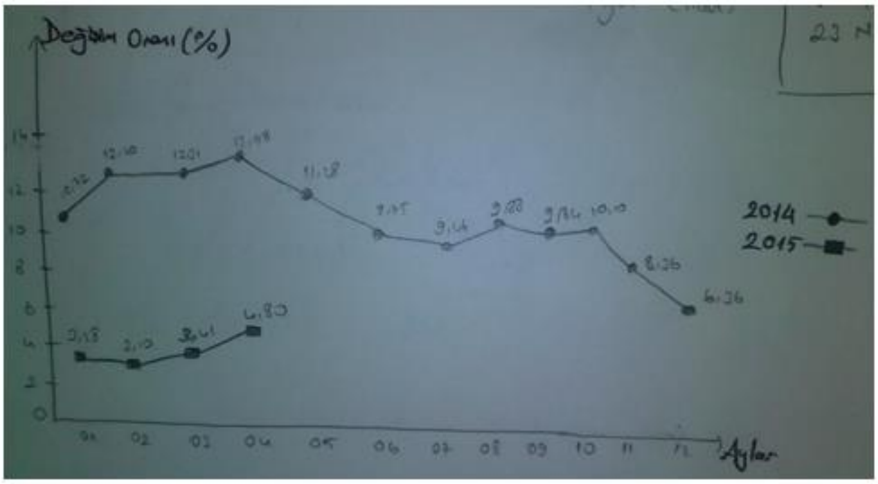
| Name | Start time |
|---|---|
| Meryem | 55:06 |
| Tuğba | 57:01 |
| Sude | 57:04 |
| Zerina | 59:06 |

If the course was for athletes who are new to orienteering rather than medium difficulty, there would be 11 checkpoints at 2 km. In this case, using the tables above, find out in how many minutes Tuğba can finish at the earliest.

**Economical Shopping** (Lack of information at the root of the question)

Ayşe goes shopping with her mother. Her mother says they will buy toilet paper, but they need to do market research first. They see that in Kaya Market, 12 pieces of toilet paper of brand X cost 11.90 TL, and 16 pieces of toilet paper cost 14.90 TL, while in in Beyaz Market, 24 pieces of toilet paper of the same brand cost 18.90 TL, and 32 pieces of toilet paper cost 27.90 TL. From which market should Ayşe and her mother buy toilet paper?

**Domestic Producer Price Index** (Complexity due to unclear relationship)




When the given table is examined, how has the monthly index changed compared to last year?

**Figure 6.** Examples of posed problems in the unsolvable category

6. A resource for teachers who want to teach life-based courses.
7. Willingness to actively continue to pose ML problems.
8. Dissemination and sustainability of training.

Consequently, it is understood that the teachers have benefited from and contributed to the in-service training they have received and that it has been an essential source of motivation for their professional teaching lives.

## CONCLUSION AND DISCUSSION

In the present study, which examined the process of posing ML problems, some clues about the nature of the process were obtained. Most of the teachers were able to pose open-ended problems. In another study on posing an ML problem, Sahin and Basgul (2018) determined that most of the problems posed by the pre-service teachers were open-ended and appropriate to the nature of PISA. On the other hand, some posed non-mathematical or unsolvable problems. Silver's (1994, p. 26) statement that "when someone poses a problem, s/he may not know whether the problem will have a solution or it will be unresolved" supports the fact that unsolvable problems were posed in this study, too.

We concluded the problems posed by teachers were generally "identification" type of problems with low complexity under the linguistic-syntactic perspective. These problems posed with the kind of identification also corresponded to the lowest level as defined by PISA (solvable by following a clear and single stimulus) (OECD, 2013). Considering that the number of relations involved in the semantic aspects of complexity, another dimension of complexity, is an indicator of the level of complexity (Marshall, 1995), it was found that teachers could only pose very few (4/111) problems with four and five relations. Similar to the results of Silver and Cai (1996), it was found that the teachers could only write problems with mostly two or three relations.

Despite the in-service training given to the teachers about ML problem writing, they included sentences ending with the verbs "write and calculate" in the ML problems created by the teachers. They did not prefer or could not pose problems with an estimated/approximate answer. As a result of these, it is thought that teachers are under the influence of textbooks and multiple-choice exams, and this limitation should be overcome first. Consistent with this result, Ozmen et al. (2012) concluded that teachers used the same or similar problems in the classroom under the influence of textbooks and existing national tests. Also, Guven et al. (2016) emphasized teachers preferred curriculum-dependent and routine problems in mathematics lessons. First and foremost, these constraints should be overcome. However, this is not a limitation that can quickly get over, and no matter how much in-service training is given, teachers need time to resolve such situations. Multiple-choice exams have now become a controversial issue in our country. For the continuation of such exams, recommendations such as introducing open-ended questions as part of the content have been included in the agenda (Altun, 2020).

Developing the skill of posing mathematical problems is essential, particularly for mathematics teachers (Lavy & Bershadsky, 2003; Silver & Cai, 1996; Southwell, 1998). As can be understood from their statements, the problem-posing process for many teachers participating in the study was a situation they had never experienced as a student in their academic life. In studies, it is stated that they have difficulty in posing problems suitable for mathematical literacy due to reasons such as not being familiar with these problem types (Demir & Altun, 2018) and lack of experience (Canbazoglu & Tarim, 2021). Teachers have an essential role in the problem-posing activities in mathematics courses, which reveals that they need to develop their problem-posing skills (e.g., Gonzales, 1996; Leung & Silver, 1997). At this point, the teachers stated that they benefited from the training provided, that they developed their awareness of ML problems and of the details of the process of writing these types of problems, and that they were aware of their deficiencies in this area and needed additional training to improve them.

If we assume that teachers often teach as they were taught. How can we hope that they will pose good problems for their students if they have not had the opportunity to realize the importance of problem posing during their pre-service education period (Stickles, 2011)? In this respect, the teachers personally experienced the problem-posing process modeled by mathematical content and methods. One question that remains to be answered by researchers is: How can teachers and students be persuaded about the potential and benefits of "problem-posing" for developing mathematics education and mathematical literacy (Hošpesová & Tichá, 2015)? In this study, which was conducted to answer these questions, we provided teachers with a training opportunity to be an ML problem poser and with an environment that they could carry into their teaching lives. In Ozgen's (2019) study, the mathematical literacy course had positive effects on problems posed for both teachers and teacher candidates.

Nicol and Bragg (2009), who focused on posing real-life problems, proposed encouraging teachers to "see" mathematics around them as a way of linking mathematics with real life. This idea supports the accuracy of the fact admitted by the teachers that "I cannot seem to come up with it when I think about it, but I realize that a situation in everyday life can be the subject for an ML problem." With this point of view, teacher trainers should design professional development activities so that teachers can take inspiration from their environment, art, or science. These activities will naturally lead to finding open-ended situations, which are difficult for teachers (Chapman, 2012; Nicol & Bragg, 2009). In this study, on the other hand, the contexts in which teachers could develop open-ended real-life problems were determined by the structural examination of existing ML problems. In this way, we presented a series of guiding and encouraging proposals for problem posing contexts both to the participant teachers of this study and to all teachers who had already taken up/will take up problem posing. When the problems posed by the teachers were examined, even though they were not limited to these sources, they considered them valuable since they were regarded as good examples. According to Gravemeijer and Doorman (1999), in contextual problems, which were defined as problem situations from real experiences, it was not surprising that context was recognized during real experiences. Within the scope of this study, the teachers stated that they could not easily find a context when they stopped and thought about it, but that the idea that a situation they encountered in real life might be the context of life occurred during a moment in real life. Thus, it was concluded that writing an ML problem was more difficult than writing a traditional exercise or word problem.

This study included a detailed process analysis of ML problem-posing training given to a group of teachers. When we examined the research results and the teachers' opinions, it was considered that this training was valuable and practical. Accordingly, it is foreseen that both in-service and pre-service teachers can be supported by problem posing and adaptation strategies, examples of which have been presented in this study (e.g., Nicol & Bragg, 2009; Prestage & Perks, 2007). Nicol and Crespo (2006) stated that they obtained a promising finding that mathematics teachers were willing to change textbook questions and creating their problems. Therefore, based on the word problems found in the middle school mathematics textbooks during problem-posing training in the present study, converting them into ML problems was considered a quality problem-posing strategy. Brown and Walter's (1983) "what if not?" teaching technique, commonly used in problem posing and similarly, and the problem-posing technique by Polya (1957) that can be expressed as "why should I make a calculation?" in parallel with the process of "looking back" in solving a mathematical problem are effective techniques. How these techniques have been applied can be examined in detail in the examples found in **Appendix A**. Posing a real-life mathematical problem is typically associated with more complex activities, which are more closely related to real-life problem situations (Chen et al., 2011). The need to investigate how mathematics taught at schools can be converted into "reality" (Aydin, 2014; Monaghan, 2007) is emphasized in the previous studies. One of these possible ways, "solving and posing ML problems based on real-life," was articulated in this study. Teachers should combine mathematics with everyday life and let their students confront the contextual ML problems they have posed. Thus, teachers' increasing awareness about problem posing, along with their pedagogical activities in their courses, will be able to facilitate the problem-posing process of their students as well (Kerekes et al., 2009; Lowrie, 2002). Similarly, it is also necessary to include examples of posing mathematical problems from real-life situations in the textbooks, which are part of the teaching process. Integrating ML problems into the units is necessary to deepen the mathematical concepts and establish a relationship with daily life. At this point, they indicate that teachers can continue to develop their problem-posing skills in their course preparation process in the future after completing their training.

Based on the study results, it is thought that teachers possess the potential to pose ML problems. On the other hand, it is foreseen that this potential can be developed by devoting more time to such problems and that in this way, teachers can also become ML problem posers in the future. It is clear that despite providing some significant results, this study has limitations that require further examination. To examine the direct connection between teachers' skills and ideas in terms of their experiences and behavior in real-life problem-posing activities in their classrooms, as also stated by Chen et al. (2011), are still needed experimental studies. The other issues that need to be investigated, as stated in this study, are as follows: (i) Even though it was identified that both in-service and pre-service training on the subject of ML could be helpful, the variety in the form of such training in the relevant literature has not yet achieved a clear structure. In the continuation of this study, the question of "how should quality ML training be given?" may be a research topic. (ii) The paths or strategies teachers follow in posing problems cannot be understood in this research process. Therefore, data triangulation with other resources such as observations, reports, and field notes will benefit a more in-depth analysis in this context. (iii) Posing an ML problem is not just a desk job. Teachers or researchers should keep this issue under "continuous observation" and make it a habit to take notes when they come up with the appropriate context.

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# APPENDIX A

### Based on Textbook Problems, the Technique of Writing Mathematical Problems–Why Should I Make a Calculation?

1. *How many meters is the perimeter of an area which is 420 square meters? Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we calculate the perimeter of a known area?*

**Examples of the proposed mathematical literacy problem for this problem:**

- Mehtap's father donates to her a plot of land with an area of 420 square meters. Even though Mehtap will not take any action on the land, she wants to fence off the land to protect it. How should she choose the land to keep the fencing costs to a minimum?
- The hall of our house is 36 m². I and my wife invited three families for the house-warming party. The maximum number of people expected from each family is five people. Is the size of the hall big enough for the invitees (There should be minimum 1.5 m² for one person)?

2. *"One of two taps fills an empty pool in 8 hours and the second one fills it in 24 hours. If both taps are on at the same time, how many hours does it take for the two taps to fill the pool?" Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we calculate how long it takes for the pool to be filled?*

**Examples of the proposed mathematical literacy problem for this problem:**

- Bahçeköy Municipality announces that the city water will be cut after 5 hours. As a precaution against the water cut, Tuğçe decided to fill the tank in the house from two different water sources. Based on her past experiences, one of the two water sources can fill the tank in eight hours and the other one can fill it in 24 hours. In order to be able to fill the tank, does Tuğçe have enough time to fill the tank by using both water sources?"

Instead of the two taps question, the situation of two people who do the same job by asking for help (one has a surgery appointment) from the other can create a context.

3. *There are three people in a hall. How many people will there be when there are two more than five times the number of people? Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we calculate the number of people?*

**Examples of the proposed mathematical literacy problem for this problem:**

- There must be at least 20 participants for a seminar to take place. Three people coming from a faraway place say that they know that to conduct the seminar, two people are on their way. Can the seminar take place if the people in the hall bring five more of their friends?
- In a summer school, three students who wish the Field writing course to be opened go to the Registrar's Office to discuss the opening of the course. The official in the Registrar's Office tells them "the number of students is insufficient". For the course to be opened, the students ask, "Two more people will come; if each of us call for five more friends, can the course be opened then?". The official tells them, "yes, you gather up the minimum number for the opening of the course." At least how many students are required for the course to be opened?

4. *"The total number of Nükhet's and Banu's walnuts is 150. If Nükhet gives Banu 5 walnuts, the number of walnuts is equalized. So, how many walnuts does Nükhet have? Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we add the walnuts?*
   - ✓ *WHY should we equalize the number of walnuts?*

**Examples of the proposed mathematical literacy problem for this problem:**

- Two people who own a garden agree on the total number of walnut trees at the time of maintenance to benefit from maintenance costs and provide maintenance for the garden. When they are told that the amount each of them is required to pay is 1,200 TL, one of the owners of the garden objects to the amount, saying "He has five more trees than me, so we can nott pay an equal amount!" Assuming that a total of 150 trees are to be provided maintenance, what kind of change should there be in the amount to be paid by the owners of the garden?

5. *When a person who moves 5 steps forward and two steps back is 10 steps away from the start, which step will be his/her next one? Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we calculate which step s/he takes?*

**Examples of the proposed mathematical literacy problem for this problem:**

- A piece of cloth is stuck in a loom 10 cm from the start. In the machine which operates 5 cm forward and 2 cm back, at which step may the piece of cloth be stuck?

6. *Two vehicles simultaneously move from city A to city B. Since the speed of the vehicles is 70 km and 85 km, how many hours later will the difference between them be 45 km? Convert this question into an ML problem. Let's discuss.*
   - ✓ *WHY should we make a calculation?*

**Examples of the proposed mathematical literacy problem for this problem:**

- A police vehicle located at point A is informed about a moving vehicle, which is 45 km away. The speed of the vehicle reported by satellite is detected as 70 km/h. Due to road conditions, the police vehicle can travel an average of 85 km/h. To avoid danger, the vehicle that is reported must be captured within two hours. Is it possible for the police to catch the reported vehicle during this time?

- A vehicle carrying pesticide travels at a speed of 70 km/h to the service area. The company that produces the drug compound realizes that there was a mistake in the process of making the compound. The substance to be added is sent off with a vehicle travelling at 85 km/h. However, since the vehicle carrying the medication is known to be 45 km away, can the replacement substance be delivered within the two-hour period which is the deterioration period of the drug?