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Analyzing AI-based educational platforms for supporting personalized mathematics learning

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ABSTRACT

This study aimed to explore how AI-based educational platforms can support personalized mathematics learning. The three prominent AI-based educational platforms for mathematics were analyzed using a framework based on four dimensions: source, target, time, and adaptation method. Specifically, this study focused on providing illustrative examples for each dimension to gain insights into the potential of such platforms to support personalized mathematics learning in classroom settings. The findings revealed that all three platforms employed a variety of elements as sources of adaptation to facilitate personalized mathematics learning. They also adopted a dual-pathway approach to determine when to adapt, as well as a shared-controlled approach to how adaptation occurs. In terms of what to adapt, the platforms varied in their approaches to content, presentation format and degree of instructional support. However, KnowRe Math and ALEKS did not offer flexibility in terms of presentation format. Based on these findings, the implications for educators of integrating AI-based platforms for personalized mathematics learning in the classroom are discussed.

Keywords: AI-based educational platform, AI-based mathematics education, AI in Education (AIED), personalized learning, personalized mathematics learning

INTRODUCTION

In 1953, Skinner observed his daughter's mathematics class working on an arithmetic problem and noticed that the students who solved the problem quickly became bored, and those who had not yet finished became anxious (Watters, 2023). The students left the lesson with a disparity in their problem-solving speed, and the teacher took their work home, graded it, and returned it the following day. This observation made him realize the need for automated teaching to overcome the limitations of one-size-fits-all classroom instruction. He based his teaching on the principles of programmed instruction, in which students learn at their own pace and immediate reinforcement, or feedback occurs (Skinner, 1968). This anecdote illustrates how a high number of students in a classroom hinders the provision of immediate or consistent feedback, ultimately leading to learning gaps. Meanwhile, these challenges have been addressed through personalized instruction, using formative assessments to monitor learners' understanding and teacher-prepared activity sheets for practice. Similarly, Bloom's 2 sigma problem reported an educational phenomenon in which students who received one-to-one tutoring using mastery learning techniques outperformed classroom students by two standard deviations (Bloom, 1984). Research and practices have revealed the ongoing methodological challenges of personalized learning (PL) in education. That is to develop group instruction methods that can be as effective as one-to-one tutoring, but the education community has yet to address this need due to its lack of cost-effectiveness.

With the advancement of artificial intelligence (AI) in education, the ongoing challenge of implementing PL in the classroom is being revisited through a technological perspective. AI-based platforms are now being used to provide PL experiences that had long been envisioned by early theorists. In particular, Skinner's anecdote about the need for personalized immediate feedback finds a modern counterpart in AI-supported learning environments that respond to learner needs in real time, track progress, and adjust content dynamically. Along with these changes, many countries have attempted to implement PL using AI technology in public education. The U.S. Department of Education's Office of Educational Technology (2023) established a vision for how technology can be used to transform teaching and learning and has been carrying out its mission by promoting equal access to transformational learning experiences, which are tailored to individual learners' interests or levels enabled by technology. Since Singapore proposed personalized education through adaptive learning and assessment as one of its National Artificial Intelligence

Strategy policies (Smart Nation Singapore, 2019), the Ministry of Education has prioritized PL experiences, tailoring the pace and pathway to meet the needs of each learner. Additionally, South Korea has suggested a PL environment as a major direction in AI-based education policy and announced plans to develop AI Digital Textbooks (AIDT) to provide customized content based on learners' data collected during the teaching and learning process (Ministry of Education, 2023). While the government has emphasized the importance of using AI technology in education, the private sector has made significant progress in developing AI-based educational platforms. Research has shown that technology-enhanced adaptive PL, based on the advancement and popularization of AI, improves student learning performance (Xie et al., 2019) and encourages the expansion of related educational policies.

The central idea of PL has historically evolved along two major theoretical perspectives: the objectivistic and the relativistic (Martindale & Dowdy, 2010; Montebello, 2018; Şahin & Uluyol, 2016). The objectivistic perspective emphasizes knowledge mastery through personalized content, methods, and pace, often relying on reinforcement and structured instruction. In contrast, the relativistic perspective focuses on learner autonomy, encouraging students to regulate their own learning, reflect on their progress, and shape learning environments based on personal preferences. In the context of AI-based personalized mathematics learning (PML), it is crucial to integrate both perspectives—ensuring content mastery while also supporting learner-driven educational decisions. Recent AI-based mathematics platforms have been developed to embody this dual focus, reflecting the complementary strengths of both theoretical traditions.

Mathematics is one of the most extensively studied subjects in AI-based education due to its hierarchical and sequential nature (Holmes et al., 2019). This has led to substantial research on PL in mathematics using AI technologies. Among these, intelligent tutoring systems (ITS) are widely used for providing adaptive and personalized feedback (Shin, 2020) and have been shown to be as cost-effective as human tutoring (del Olmo-Muñoz et al., 2023; Kulik & Fletcher, 2016; Steenbergen-Hu & Cooper, 2013; Walkington, 2013; Wu et al., 2017; Zhang & Jia, 2017). Recent studies have examined the pedagogical features of AI-based platforms that support PL in mathematics (Azevedo et al., 2022; Park, 2020; Park et al., 2022; Yim et al., 2021), and demonstrated their effectiveness (Dani, 2016; Phillips et al., 2020). While these studies offer valuable insights into platform features, few have provided a comprehensive analysis of how these platforms support the adaptive dimension of PL. Thus, an integrated understanding of how AI technologies facilitate PML remains limited.

In technology-enriched environments, it is important to comprehensively consider how these environments can support PML rather than solely understanding their functional features (Vandewaetere & Clarebout, 2014). Given the growing emphasis on PL in mathematics, this study investigates how AI-based platforms support PML using the four-dimensional framework proposed by Vandewaetere and Clarebout (2014), offering a holistic view of the platforms' adaptive features. The research questions are as follows:

RQ1 To what extent do AI-based educational platforms incorporate elements that support personalized mathematics learning?

RQ2 In what ways do AI-based educational platforms provide adaptive support for personalized mathematics learning?

BACKGROUND

A Framework for AI-Based PL

Because AI-based PL is concerned with design, it is important to specifically explore its components in technology-based environments (Kim, 2023). For instructors to gain insights from learning data during PL with AI in the classroom and design teaching strategies based on them, it is necessary to examine what adaptive factors the AI-based system specifically reflects (Baker et al., 2019). Vandewaetere and Clarebout (2014) highlighted that adaptive technologies, such as AI and Educational Data Mining, have augmented traditional learner modeling in PL, but they argued that these technologies alone do not provide a sufficient description of the systems in PL. For a holistic understanding of PL, they proposed a theoretical framework that views PL in four dimensions: what data to adapt based on ("adapt to what"), what to adapt to ("what to adapt"), when to implement adaptive elements ("when to adapt"), and under whose control to adapt ("how to adapt"). This section examines Vandewaetere and Clarebout's (2014) four-dimensional perspective. These perspectives can provide a useful framework for analyzing whether AI-based platforms consider the multidimensional elements of PL.

Source of adaptation

First, the source of adaptation is divided into two parameters: learner parameters, where decisions about PL are based on learner characteristics (e.g., learning style) or learning outcomes (e.g., task completion time and learning outcomes), and learner-system parameters, allowing personalization based on the learner's interaction with the system. An example of this dimension is personalization based on learning style, which refers to the way an individual learner prefers to take in, retain, process, and recall information (Whittington & Raven, 1995). This is a learner characteristic that should be considered when optimizing the learning process. Previous studies have shown that adaptive learning environments can improve the effectiveness of PL by considering the learning style of each learner when making decisions on how to teach (Choi, 2017; Karadimce & Davcev, 2013; Papadimitriou & Gyftodimos, 2007; Shariffudin et al., 2012).

Target of adaptation

The target of adapation refers to what can be adapted to a PL system. This dimension can be adapted in three ways: the learning content, presentation format of learning content, and degree of guidance and support. First, to adapt to the learning content, each learner is provided with assignments or tasks with varying levels of difficulty. In the field of mathematics education, good math tasks should provide learners with diverse mathematical experiences and encourage the development of mathematical thinking (National Council of Teachers of Mathematics [NCTM], 2000). Stein et al. (1996), and Stein and Smith (1998) are well known for their categorization of tasks based on the levels of cognitive demand. Several studies have shown that the potential for math learning opportunities varies depending on the cognitive demands of the tasks (Basyal et al., 2023; Boesen et al., 2010; Remillard et al., 2014; Thompson et al., 2012). Jung and Lee (2020), Lee and Cho (2023), and Stein and Kim (2009) have also reported that math textbook tasks can affect the depth of mathematical thinking experienced by learners in school mathematics. Taken together, it is evident that the nature of the tasks affects mathematics learning. Therefore, personalizing tasks on an AI-based platform can enhance mathematics learning by customizing the learning content.

Second, adapting the presentation format of learning content is related to mathematical representations. Bruner (1964) posited that learners can form mathematical structures by providing experiences that align with the developmental sequence of enactive, iconic, and symbolic representations. Accordingly, the value of multiple representations has consistently been emphasized in mathematical learning and teaching (Arcavi, 2003; NCTM, 2000; Pape & Tchoshanov, 2001). Goldin and Nina (2001) also found that a relational understanding of different representations of the same concept enables effective mathematical thinking. More recently, Moreno-Armella et al. (2008) and Usiskin (2018) showed that, with the increasing use of technology as an educational tool, dynamic representations support mathematical understanding. Webb et al. (2008) demonstrated that teachers' capacity to use a range of visual representations enables them to develop individually customized lesson plans for their learners. The findings of these studies collectively indicate that the diverse use of visual representations in mathematics teaching and learning may be extended to AI-based platforms, with the objective of comprehending mathematical content and facilitating PML.

Third, to adapt to the degree of instruction, the available support is based on the scaffolding approach. Research on scaffolding in mathematics education is rapidly growing in conjunction with interest in sociocultural perspectives (Bakker et al., 2015). Scaffolding can be traced back to Wood et al. (1976), who used the term to describe the adaptive support of children's learning by adults or professionals based on Vygotsky's zone of proximal development. Scaffolding is an interactive process that occurs between teachers and students when both are actively involved in the learning process, and is characterized by contingency, fading, and transfer of responsibility (Van de Pol et al., 2010). In other words, providing adaptive scaffolding enables learners to accomplish tasks beyond their current abilities, and learners gradually assume more responsibility for learning with less scaffolding. Van de Pol and Elbers (2013) emphasized the importance of contingency in scaffolding, stating that providing contingent support allows learners to feel the right degree of challenge, leading to successful learning. Earlier research reported that scaffolding has been categorized based on the nature, purpose, or source of the interactions that occur during scaffolding (Azevedo et al., 2005; Cagiltay, 2006; Ge & Land, 2003, 2004; Greene & Land, 2000; Jackson et al., 1998; Kim & Hannafin, 2011; Lee et al., 2014; Saye & Brush, 2002). Experimental studies have shown that the scaffolding strategies utilized in each study help improve mathematical problem-solving skills (Cho & Kim, 2020; Schukajlow et al., 2015). These studies had different specific purposes, but they were designed to provide instructional assistance in problem-solving. From this, it is evident that scaffolding is characterized by the fact that it does not leave learners in their current state of learning but supports them in progress further. Furthermore, these efforts are ongoing in technology-enhanced environments and should be continued in AI-based environments.

Time of adaptation

Third, the time of adaptation refers to when the adaptation takes place. One is a static approach that determines the learner model before learning begins, and the other is a dynamic approach that continuously tracks learner information to update the learner model during the learning process. Another approach that combines the first two is a dual-pathway approach that initially determines the learner model based on learner parameters and then updates the learner model during the learning process based on learner-system parameters. In mathematics education, research on noticing has highlighted the significance of adaptive time in response. For example, Jacobs et al. (2010) identified professional observation of children's mathematical thinking as teaching expertise. This skill involves three components: attending to children’s mathematical strategies, interpreting their understanding as reflected in these strategies, and deciding how to respond based on that understanding and interpretation. In particular, the value of in-the-moment decision-making was highlighted as it relates to the time of adaptation to support PL. Meanwhile, ITS are known to support mathematics teaching and learning by providing customized feedback. According to Steenbergen-Hu and Cooper (2013), computerized mathematics learning based on ITS has a positive effect on general students and is valuable as an educational resource that supports math teaching and learning by providing immediate feedback. Previous research on noticing and ITS suggests that responding in the moment is beneficial for learning math.

Method of adaptation

Finally, the method of adaptation is a component related to who controls the learning process, which is divided into a learner-controlled method, where the learner has full control over the learning environment and content; a program-controlled method, where the developer or instructor has control; and a shared-controlled method, where the system first selects appropriate content considering the learner's characteristics and then allows the learner to make free choices within the range. This dimension relates to self-determination, the view that learners feel satisfied and intrinsically motivated by autonomy over their own learning processes (Deci & Ryan, 1985; Vallerand et al., 1997). Montebello (2018) emphasized the significance of self-determination theory, stating that in an AI-based PL environment, learners are more motivated to learn when they can design their own PL experience. Another important factor to consider when placing learners in control of their own learning is self-regulated learning. This involves the ability to monitor and manage one's learning processes, which can lead to improved academic performance. Self-regulated learning is an active process in which learners examine, regulate, and evaluate their cognition, behavior, and motivation to achieve their learning goals (Pintrich, 2004). In online learning environments that require a high level of proactivity, learners who lack self-regulated learning strategies may struggle to complete their learning successfully. Along with the importance of self-regulated learning in online learning, research on dashboards and feedback to support learners' self-regulated learning on AI-based platforms has attracted attention (Chen & Su, 2019; Duffy & Azevedo, 2015; Molenaar et al., 2019).

METHOD

Platforms to Analyze

The goal of this study was to investigate the potential of AI-based educational platforms as tools to facilitate PML in response to the challenge of meeting the needs of individual students in traditional classroom settings with many students. To this end, three platforms, KnowRe Math, Khan Academy, and ALEKS, were selected and analyzed due to their close alignment with school curriculum standards, which makes them particularly relevant for integration into formal educational settings.

KnowRe Math

KnowRe Math was designed for use in classrooms to help teachers with ever-increasing responsibilities and standards, aligned with the flexibility to integrate into a variety of curricula. KnowRe Math has been granted patents in South Korea and the U.S. for step-by-step learning, referred to as Walk Me Through, based on AI technology. It exclusively leverages this technology to provide prompts and questions to help students solve problems and collect data on each student's learning competencies to inform individualized math practices. It was designed to analyze the learning data revealed during the problem-solving process to identify weaknesses and help students understand higher-level concepts by improving their weaknesses. This technology aims to improve learners' math performance in a short period by calculating the probability of solving problems through an AI algorithm and recommending suitable problems to avoid repeating mistakes.

Khan Academy

Khan Academy is one of the world's most popular open educational resources with free web-based tutorial programs. It is operated by a non-profit organization with the mission of providing free education worldwide. Its instructional mathematics videos are aligned to practice problem sets in a variety of interactive formats and a real-time discussion board. In March 2023, the Khan Academy launched Khanmigo, a chatbot based on GPT-4, to support learners' PL. Unlike GPT-4, which provides the correct answer to a question, Khanmigo assists students in finding their own answers by identifying how they arrived at the answer or where they made a mistake (Khan Academy, n.d.-a).

ALEKS

ALEKS, provided by McGraw-Hill Education, is an AI-based platform that uses adaptive learning technology to identify the knowledge levels of individual learners and deliver PL content. ALEKS uses a mathematical algorithm to measure a learner's current level and recommends an appropriate learning path. To apply the algorithm, the learning content must be divided into independent conceptual units, and the relationships between the divided topics must be mathematically recorded.

Data Collection and Analysis

This study analyzed how three selected AI-based educational mathematics platforms facilitated PML using the multiple dimensions proposed by Vandewaetere and Clarebout (2014). An initial examination was conducted to determine whether the platforms incorporated elements supporting PML across each dimension. To this end, we actively explored and used all functional aspects of each platform in actual mathematics learning contexts and analyzed how each feature contributes to supporting PML. Specifically, the English version of Khan Academy, the public education version of KnowRe Math, and the free trial version of ALEKS were used for this analysis. Then, we selected representative cases to demonstrate how the platforms support PML, presenting best practices for each element. In this study, PML is defined as an approach that provides an environment in which learners take control of their learning process by making educational decisions tailored to their individual needs and learning pace. Based on this definition, each dimension and the meanings of sub-dimensions for analysis are delineated in **Table 1**. To ensure the reliability and validity of the analysis, two researchers independently coded the data and conducted cross-checks to verify consistency. The data and interpretations were repeatedly reviewed, and both member checking and expert review were employed to enhance the trustworthiness of the findings.

RESULTS

Whether AI-Based Platforms Have the Elements to Support PML

The overall results of the analysis of whether each platform supports PML in terms of the four-dimensional perspective on adaptive learning are presented in **Table 2**. All three platforms employed a variety of elements as sources of adaptation to facilitate PML. They also adopted a dual-pathway approach for when to adapt and a shared-controlled approach for adapting. On the other hand, regarding what to adapt, they all took various approaches to content, presentation format, or degree of instruction, but a presentation format was not found in KnowRe Math and ALEKS.

Table 1. Adaptive dimensions and elements for personalized learning by Vandewaetere and Clarebout (2014)

Dimension	**Elements**	**Definition**
Source (adapt to what)	Learner parameters	Adapting to learner parameters such as learner characteristics
	Learner-system parameters	Adapting to the behavior of the learner when interacting with the system
Target (adapt what)	Content	Adapting the content, for instance by differentiating the difficulty level of the tasks, or items
	Presentation	Adapting the presentation format of the learning content, for instance by hiding or highlighting links
	Support/instruction	Adapting the instruction and available support
Time (adapt when)	Static approach	Determining the learner model before starting teaching and learning activities
	Dynamic approach	Updating the learner model by continuously tracking learner information during the learning process
	Dual pathway approach	A first adaptation occurs after a single measurement of learner characteristics, and further modeling and adaptation occurring based on learner-system parameters
Method (adapt how)	Learner-controlled	The learner fully controls the environment and learning content
	System-controlled	Adaptation that is defined by the system or the instructor
	Shared-controlled	The system first selects an appropriate set of learning materials or tasks, taking into account learner characteristics to adapt for, and, after that, the learner being able to freely choose within this set of materials or tasks.

Table 2. The overall results on whether each platform has the elements to support personalized mathematics learning

Dimension (Elements)		KnowRe Math	Khan Academy	ALEKS
Source	Learner parameters	Learner characteristics and diagnostic assessment results		
	Learner-system parameters	Learner's session details (start time, time spent, level, repetitions, scores), item-level performance (number of attempts, incorrect answers, retakes), targeted assignments	Self-regulation and decision-making during learning (choosing supplementary videos and whether to retry)	Learner's session details (cumulative usage time, daily learning duration, learning start times)
Target	Content	Targeted assignments based on students' prior performance	Problems customized to learner level	Problems customized to learner level
	Presentation	N/A	Videos, practices, and quizzes selectable by learner	N/A
	Support/instruction	Step-by-step hints	Curriculum map, video transcripts, and on-demand access to hints and related content during practice	Built-in tools such as 'Dictionary'
Time (Dual-pathway)		Learner model built from diagnostic test; unit levels and follow-up problems adapt to performance in real time	Learner model updated through five mastery levels; time-based Mastery Challenges triggered by progress	Initial knowledge check builds learner model; system continuously adapts topic sequence based on performance
Method (Shared-controlled)		The system suggests tasks, but learners choose what to engage with.	Learners choose courses and lessons, decide whether to retry, skip, or use hints during practice.	Learners follow system recommendations but can select content order and repeat past topics.

How AI-Based Platforms Support PML by Each Dimension

This section presents examples of adaptive elements for each dimension. Rather than presenting all the examples of the platforms, this section focuses only on examples that could provide insight into using those platforms to support PML in classroom lessons.

Source of adaptation

Regarding the source of adaptation, AI-based educational platforms support PML by making decisions based on information from both learner parameters and learner-system parameters. These data are visualized on student and teacher dashboards to guide instructional decisions.

All three platforms utilize learner parameters—such as school, grade, class, gender, birthday, and diagnostic assessment results—to determine an appropriate starting point for AI-driven learning. For example, KnowRe Math evaluates prior knowledge through diagnostic tests and combines the results with learner information to set the initial difficulty level. These platforms also rely on learner-system parameters, which capture behavioral data generated through interactions with the system. These include metrics such as cumulative usage time, daily learning duration, and learning start times. Such data are recorded and displayed on dashboards. In KnowRe Math, the teacher dashboard shows each learner's session details, including start time, time spent, level, repetitions, and scores, while the student dashboard displays problem-solving scores, duration, and attempt counts. It also offers a detailed lesson summary with data such as number of attempts, incorrect answers, retakes, and targeted assignments. In addition, learner-system parameters encompass behavior related to self-regulation and decision-making during learning. For instance, Khan Academy allows students to select supplementary videos aligned with their current learning content (**Figure 1**), and during problem-solving, learners can choose whether to retry or proceed to the next item (**Figure 2**). This illustrates how AI-based platforms not only adapt based on behavioral data but also empower learners to direct their own learning paths, enhancing autonomy in the PML process.

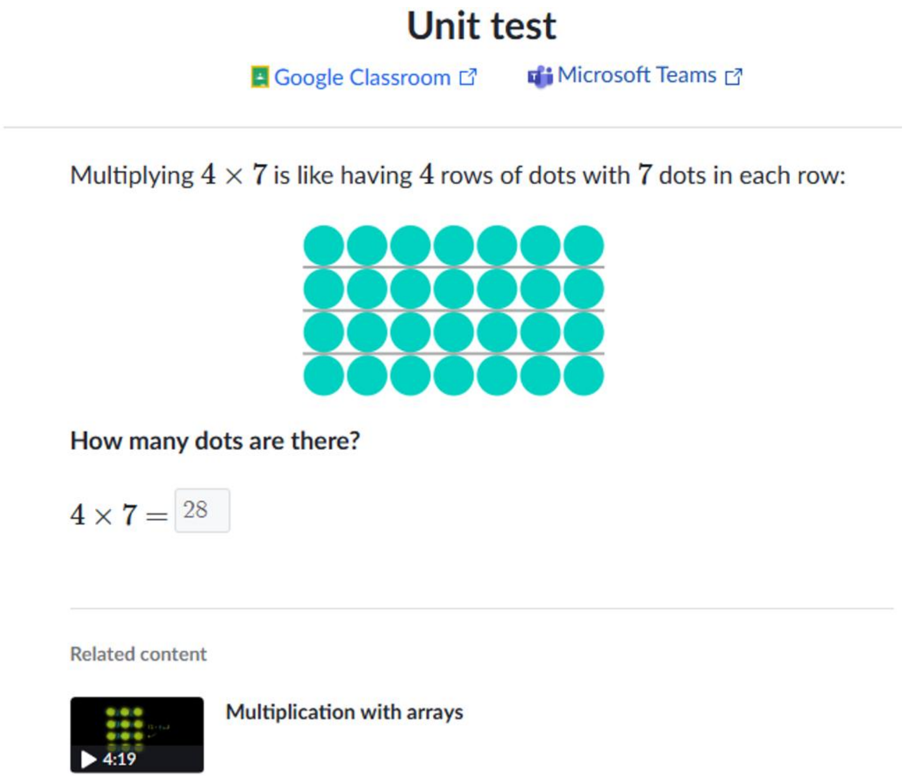

Figure 1. Examples of the learner-system parameters in Khan Academy (1) (Khan Academy, n.d.-b)

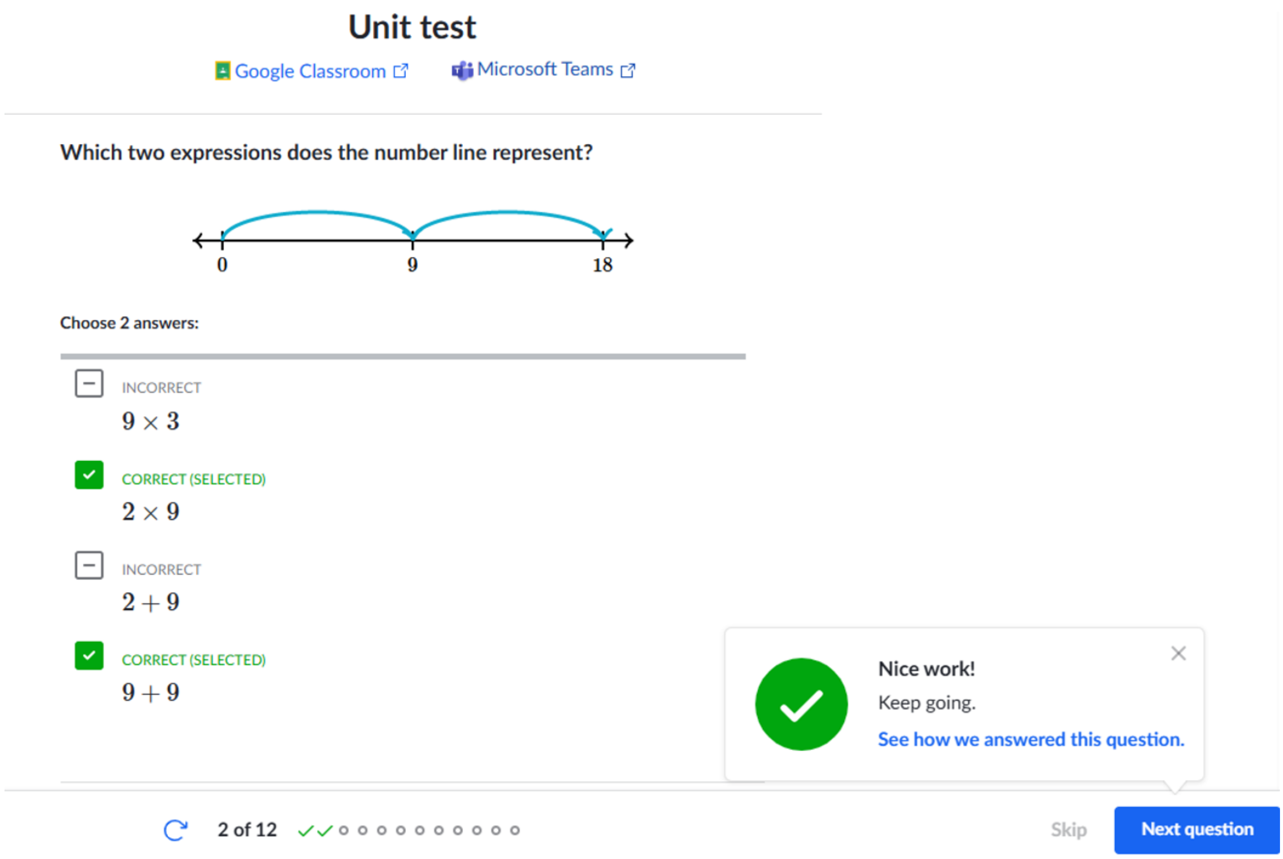

Figure 2. Examples of the learner-system parameters in Khan Academy (2) (Khan Academy, n.d.-b)

Target of adaptation

Regarding the target of adaptation, three aspects were analyzed: content, presentation, and support/instruction. All platforms aim to provide adaptive support that fosters mathematical understanding and problem-solving skills.

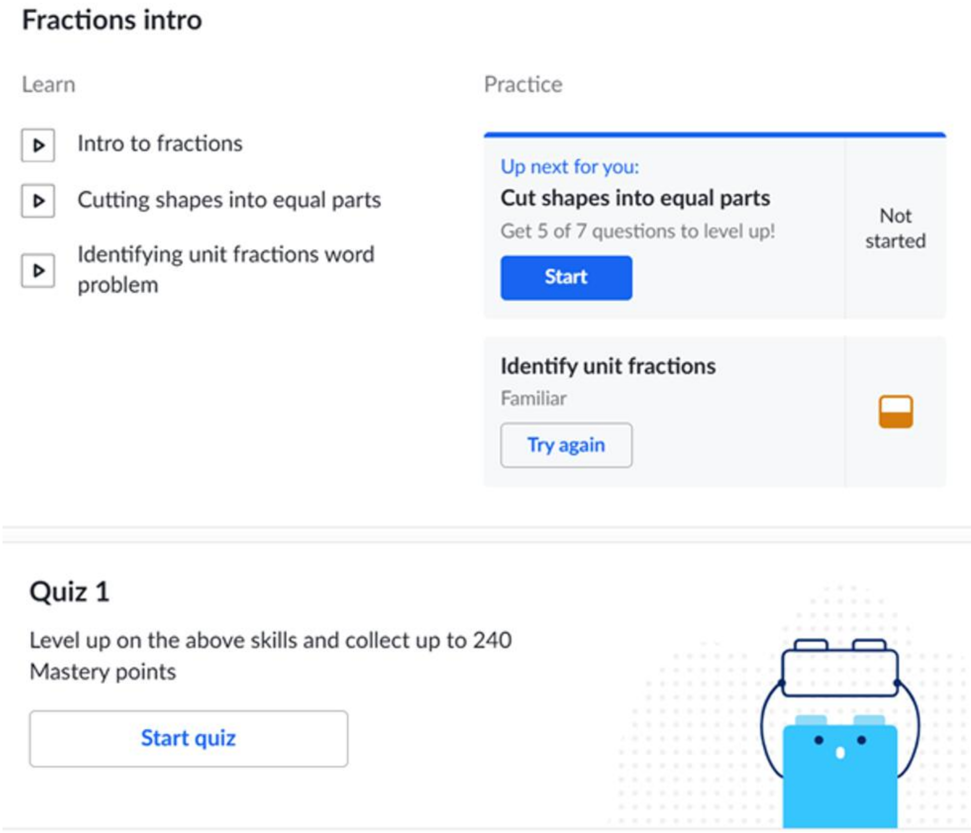

Figure 3. Example of the adaptation on presentation in Khan Academy (Khan Academy, n.d.-b)

The most prevalent method for personalizing learning content involves customizing the difficulty level of problems according to the learner's level of understanding. First, content personalization is mainly achieved by adjusting problem difficulty based on learners' understanding. In KnowRe Math, teachers assign lesson assignments, while the system generates targeted assignments based on students' prior performance. These targeted assignments include twin problems—items similar to those answered incorrectly — allowing students to focus on areas needing improvement.

Second, content presentation is primarily delivered through videos and practice exercises. In Khan Academy, each lesson includes 'Learn' (video instruction), 'Practice' (problem-solving), and 'Quiz' components (**Figure 3**). Students can choose which components to engage with, enabling flexible and self-paced learning.

Third, the platforms offer various forms of instructional support. ALEKS provides several built-in tools: the 'Dictionary' explains key terms (e.g., "expanded form"); 'See Also' links related concepts such as digit and place value; the 'Explanation' feature outlines problem-solving steps; and 'Show me pictures' offers visual aids. In contrast, KnowRe Math supports learners through interactive step-by-step hints that guide them in understanding the problem, planning a strategy, and executing the solution.

Time of adaptation

Regarding the time of adaptation, all three platforms adopt a dual-pathway approach. Initially, a learner model is established based on diagnostic evaluation results, and it is continuously updated through further interactions using learner or learner-system parameters.

In Khan Academy, the learner model is updated based on performance, categorized into five levels: mastered, proficient, familiar, attempted, and not started. Performance for each lesson is recorded on the dashboard (**Figure 4**), and mastery levels for individual problems are shown in Course Challenges or unit tests, determined by aggregated performance data (**Figure 5**). Additionally, the Mastery Challenge—a time-limited task—becomes available when a learner achieves a certain proficiency or mastery level, offering a personalized opportunity to advance further.

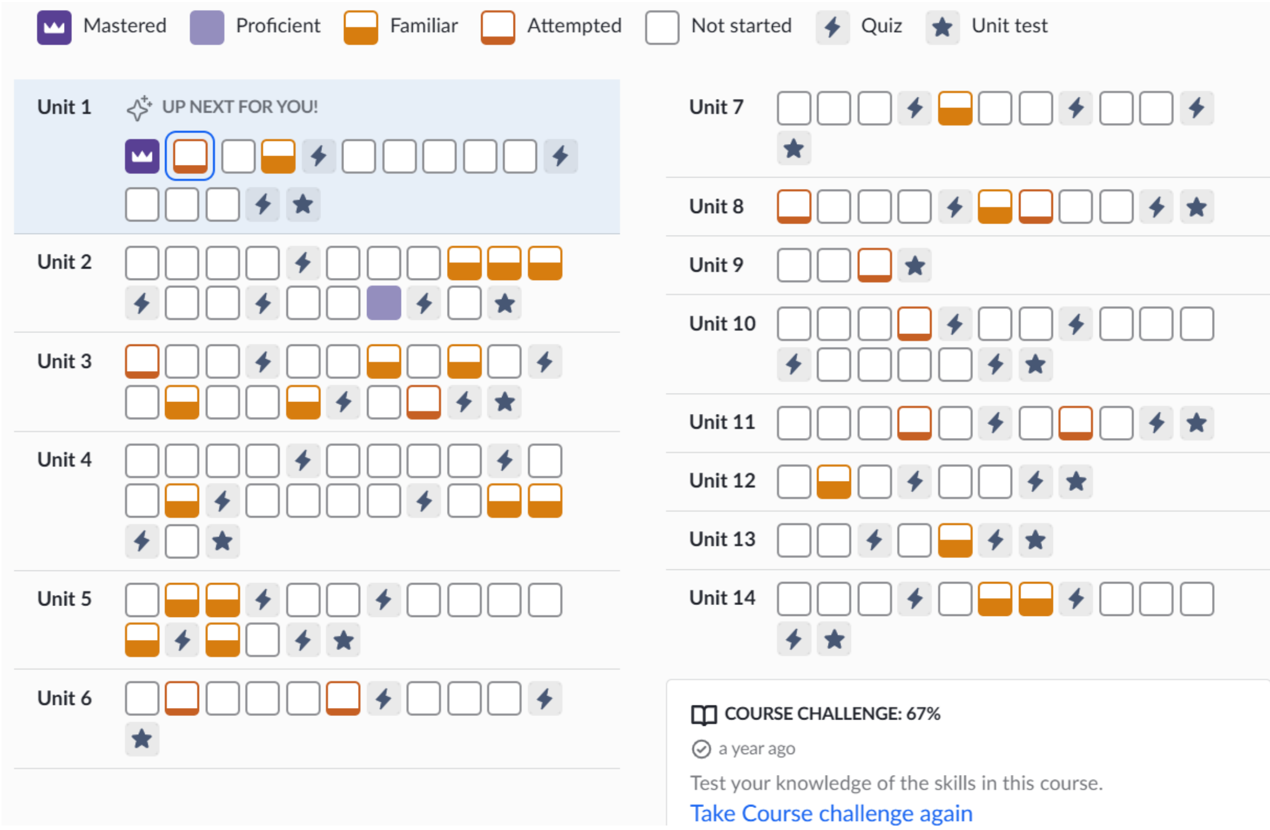

Figure 4. Examples of the dual pathway approach in Khan Academy (1) (Khan Academy, n.d.-b)

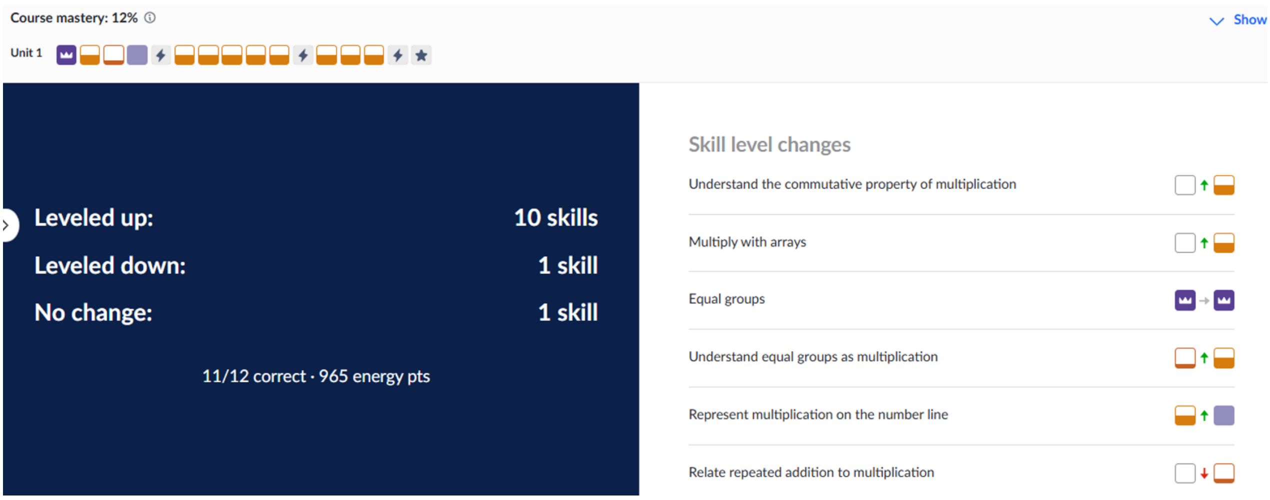

Figure 5. Examples of the dual pathway approach in Khan Academy (2) (Khan Academy, n.d.-b)

Method of adaptation

All three platforms adopt shared-controlled adaptation as the method of adaptation. While the AI-based tools recommend PML experiences, learners retain autonomy to accept or decline these suggestions. In KnowRe Math, which is based on mastery learning, targeted assignments are generated using results from previous lessons. Incorrectly answered problems are presented again, while similar problems are offered for correctly answered ones. Learners can review their performance and decide whether to retry specific questions or tackle related twin problems. In ALEKS, learners follow a system-generated learning path but can choose the sequence of content areas to study within that path. They can also create new or repeat past practice worksheets by selecting options from the menu. These features support PML by combining system-driven recommendations with learner choice, promoting active engagement tailored to individual needs.

DISCUSSION

Given that it can be challenging for a single teacher to support the individual learning needs of all students within the class period with many students, this study aimed to identify the features of AI-based educational platforms that can support PML. For this purpose, we analyzed how three AI-based platforms—Khan Academy, KnowRe Math, and ALEKS—can support PML according to Vandewaetere and Clarebout's (2014) four-dimensional perspective on adaptive learning. The four-dimensional perspective of adaptive learning focuses on the following components: source, target, time, and adaptation method. The results for each component are as follows.

First, the three platforms provide various types of information on learner parameters or learner-system parameters to teachers or learners on dashboards, and this information is used to make decisions about supporting PML. Sung (2023) used an AI-based platform in the classroom to support PML for fifth graders in South Korea and found that teachers could guide learners to increase classroom engagement using learner parameter information. Moreover, she found that learners could monitor their own learning progress using learner parameter information. Previous research has demonstrated that information on learner or learner-system parameters presented on a dashboard assists learners in monitoring their own learning activities, becoming cognizant of their learning status, and encouraging self-reflection (Verbert et al., 2014). Consequently, we can ascertain that information on learner parameters or learner-system parameters not only facilitates PML but also encourages learners to reflect on their own mathematics learning.

Second, the findings indicate that the elements of the targets to adapt were integrated with diverse system features to support PML. Identifying whether each element-supported PML was based on how each system feature was intended to develop mathematical understanding and problem-solving. In contrast to the traditional approach to e-learning, which prioritizes content delivery, recent developments in online learning have placed greater emphasis on the design of information presentations. Especially on AI-based educational platforms, the selection of targets to adapt to facilitate PML should be based on whether the system feature will result in a growth of abilities for mathematical understanding and problem-solving.

Third, an analysis of the time taken to adapt revealed that a dual-pathway approach was employed by the platforms to support PML. This approach involves establishing a starting point for PML and then updating the learner model based on learner information gathered from a variety of sources. This information is used to adaptively personalize the learning experience according to the learner's needs during the ongoing learning process. When these findings are considered in conjunction with previous research on noticing and ITS (Jacobs et al., 2010; Steenbergen-Hu & Cooper, 2013), which indicate how the instructor or system responds to learners' reactions during the learning process affects mathematics learning, it is obvious that PML is facilitated by immediate feedback from the teacher or system. Consequently, the continuous update of an individual learner model on AI-based platforms can be regarded as an adaptation of the learning paths followed by the learner model, whereby immediate feedback on the learning process and results are reflected. This instantaneous update of the learner model demonstrates that it provides an opportunity for PML.

Fourth, our analysis of how PL is adapted to AI-based platforms revealed that learners are not merely following the learning paths recommended by the instructor or the system; they are taking the initiative to choose their own learning path from the recommended ones. This represents shared-controlled adaptation of learning between the instructor, system, and learner. There are two perspectives on PL: one that prioritizes the mastery of knowledge, such as Skinner's programmed instruction, and the other that allows learners to design their own learning based on their interests and autonomy. Given the two distinct perspectives on PL, one emphasizing knowledge mastery and the other enabling autonomy- and learner-driven curriculum design based on personal interests, the learner-controlled method holds significant value in ensuring autonomy and choice, allowing for individual needs to be considered in PL. This approach facilitates self-directed learning, which may enhance motivation to learn (Deci & Ryan, 1985; Montebello, 2018; Vallerand et al., 1997).

This study demonstrated the potential of AI-based platforms to support PML in four dimensions: source, target, time, and method. In summary, the analysis concluded that the elements of source, time, and method should be considered within the formal dimension to support PML. Similarly, the elements of target may be considered from the content perspective to support PML. This study adds to the growing body of research on AI-based mathematics education by demonstrating the necessity of considering how to support PML not only in terms of content but also in terms of the form of learning by leveraging the features of the system.

CONCLUSIONS

With the growing emphasis on AI-based educational platforms in mathematics education, PL has evolved across multiple dimensions—such as data sources, adaptation targets, timing, and methods. This study analyzed how three AI-based platforms support PML across the four dimensions proposed by Vandewaetere and Clarebout (2014), providing practical implications for educators aiming to integrate such tools into their classrooms.

First, this study underscores the need for a multidimensional approach to designing PML, emphasizing that educators should move beyond merely identifying individual learning deficiencies and instead consider the diverse adaptive functionalities embedded in AI-based platforms. By understanding how each dimension contributes to PML, teachers can co-design learning experiences in partnership with platforms, moving beyond passive reliance on algorithmic recommendations. Although these platforms are often used for homework or supplementary activities (Pepin et al., 2016), this study suggests that effective use of dashboard data and adaptive elements can make them integral to classroom instruction.

Second, the study highlights the advancement of AI-based platforms and their evolving role in classrooms. Unlike early PL based programs that reacted only to learner behavior, current AI-based platforms use diverse data to adapt content, presentation, and support to learners' needs. This reflects a shift toward Human-Centered Learning Analytics (HCLA), which emphasizes collaboration among stakeholders—teachers, students, parents, and edtech developers—to use learning data meaningfully (Chatti et al., 2020; Shum et al., 2019). These platforms support educational decision-making by not only analyzing usage data but also incorporating decisions made by teachers and learners. Future research should explore how AI-based platforms can be embedded into HCLA frameworks to enhance PML.

Third, when implementing PML in classrooms with many students, a balance between mastery learning and learner agency is critical. The platforms examined offer structured curricula aligned with grade-level standards while allowing both teachers and students to select learning paths. This shared-controlled approach enables dynamic, individualized learning. As noted by the Florida Center for Instructional Technology (2019), meaningful technology integration requires more than functionality—it must actively engage students in their learning. Future research should examine how much agency learners truly exercise within platform constraints.

Lastly, while the development of AI-based tools continues to advance, the potential of these technologies to support PML ultimately depends not on the sophistication of their design alone, but on how teachers and students utilize them in practice. A well-designed platform may offer robust adaptive features, but its educational impact is mediated by the instructional choices made in the classroom. In particular, the teacher's agency in integrating and orchestrating these tools plays a pivotal role. The extent to which teachers can assert pedagogical control—by interpreting platform data, customizing learning paths, and aligning technological use with instructional goals—will significantly shape the potential for realizing PML in meaningful and sustainable ways.

Although this study is limited to three platforms, it contributes to understanding the potential of AI-based platforms for supporting PML in public education. These platforms can help address challenges in large classrooms, such as providing timely feedback and tailored support. Ultimately, this study extends prior research by examining how AI-driven features can support not only differentiated instruction but also learner-centered experiences, pointing to new directions for developing and applying PML in public school settings.

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