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A guide to the analysis of didactic suitability of probability textbook lessons

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ABSTRACT

The aim of this article is to buid a Guide to the Analysis of Probability Textbook Lessons for secondary education (students aged 12-14), using the framework of Didactic Suitability. The facets, components and indicators of the didactic suitability construct are applied to categorize and organize didactic-mathematical knowledge on teaching and learning of probability in secondary education derived from research on the subject. The guide constitutes a systemic structure of descriptors (gradable and recognizable features in the textbook lesson) associated with the different components of partial suitability, which can facilitate the teacher's decision-making on how to use a textbook lesson in the classroom to optimise the instructional process.

Keywords: didactic analysis, textbook, teacher training, didactic suitability, probability

INTRODUCTION

The importance of the textbook as a support for teaching and learning processes in the school context warrants that it be treated as an object of study, and for specific research methodologies to be developed (Bel & Colomer, 2018; Rezat et al., 2021; Schubring & Fan, 2018). According to Schubring and Fan (2018), the research on mathematics textbooks has received increasing attention as it has opened to new formats such as electronic resources, encompassing historical reflections and international comparisons. However, there are few studies that develop or propose some kind of instrument, grounded in the discipline itself, to guide the global and integral assessment of textbook lessons (Castillo et al., 2022a). Thus, the criteria for analysing textbooks depend on the aspects that concern researchers in relation to their quality: Their alignment with the curriculum (Bhatti et al., 2017; Hashmi et al., 2018), the types of tasks or their cognitive demand (Alayont et al., 2023; Gea et al., 2022), reasoning (Bingölbali & Bingölbali, 2020; Jäder et al., 2015), the representations or languages used (Alkhateeb, 2019; Díaz-Levicoy et al., 2016), and the detection of conflicts or errors (Arteaga & Díaz-Levicoy, 2016).

In the specific case of probability, which is the focus of this article, the research on textbooks shows a greater presence of contents linked to the classical meaning to the detriment of the frequentist or subjective approaches (Alsina & Vásquez, 2016; Gómez-Torres & Batanero, 2014; Han et al., 2011; Sánchez, 2009; Vásquez & Alsina, 2015b). Consequently, the privileged context is that of games of chance, and the proposed situations are not sufficiently representative and balanced (Fukuda & Kamimoto, 2018; Nakawa, 2020; Nakawa & Kimura, 2021) to develop adequate probabilistic literacy (Gal, 2012). Even when textbooks consider both theoretical and experimental probability, they differ in the presentation of the relationship between them. While some are written to teach students to distinguish between experimental and theoretical probabilities, others emphasize that experimental probability converges to theoretical probability, including inadequate explanations of the law of large numbers (Ishibashi, 2022). Moreover, the treatment of probability in textbooks is not always in line with what different curricular regulations propose (Alsina & Vásquez, 2016; Rodríguez-Muñiz et al., 2019), which is why Alsina and Vásquez (2016) recommend reviewing the probability content in textbooks to ensure that students achieve the learning objectives set in the curriculum. Studies such as those carried out by Díaz-Levicoy and Roa (2014), Gómez-Torres et al. (2013, 2014) or Vásquez and Alsina (2015a, 2015b, 2017a) focus on the analysis of problem situations, linguistic elements, concepts, properties, procedures, and arguments showing their relationship with the four meanings of probability. Deficiencies are observed in the processes of connections, reasoning, proof, and communication (Vásquez et al., 2021) and problems of a routine nature (Nakawa & Kimura, 2021) or little presence of mathematical modelling activities (Díaz-Levicoy & Roa, 2014; Trelles-Zambrano et al., 2022).

These works focus on partial mathematical aspects of the treatment of probability in textbooks, without considering other factors (such as attention to the curriculum, ensuring progression in learning, motivation, or content sequencing, among others) that are also crucial for assessing resource suitability. This leads us to recognize the need to design a specific instrument for the comprehensive analysis of a probability textbook lesson, which includes the multitude of factors that affect instructional processes, integrating the results of research and didactic-mathematical knowledge about this content.

Thus, the objective of this work is to develop a guide for the analysis of probability textbook lessons for the first cycle of secondary education (students aged 12-14). A lesson is a segment of the textbook dedicated to a specific mathematical topic, in our case, probability, which corresponds to the didactic trajectory that the teacher will deliver in the classroom to address the teaching of that topic.To elaborate this instrument, we rely on the didactic suitability construct (Godino et al., 2023; Breda et al., 2017; Godino, 2024; Godino et al., 2016) proposed within the Ontosemiotic Approach (Godino, 2024; Godino et al., 2007). The theory of didactic suitability has been used to organize reflection on the adequacy of these and other educational resources (Balcaza et al., 2017; Castillo & Burgos, 2022; Godino et al., 2006; Monje et al., 2018). This research builds on the work previously carried out by Castillo et al. (2022a) in which the authors develop a guide for the analysis of a generic lesson from a mathematics textbook, by Castillo et al. (2022b), where the focus is in proportionality; and by Beltrán-Pellicer et al. (2018a), where the authors review the criteria and indicators of didactic suitability with the purpose of creating a guide to assess curricular guidelines on probability.

The guide developed in this article constitutes a tool that facilitates the assessment of the suitability of probability textbook lessons, to support the study processes of probability in the school context. Its structure through indicators associated with the different dimensions that condition instructional processes offers a comprehensive and articulated view that ensures systematic and thorough analysis, while also considering the specificity of the resource being analyzed (a textbook lesson). This instrument can be useful for both researchers (hoping to facilitate research on the analysis of probability lessons in different educational contexts) and curriculum developers, as well as for practicing or trainee teachers. In particular, assuming that a secondary school mathematics teacher has decided to use a lesson from a textbook as a resource to support the teaching and learning process on probability, the analysis guide is presented as a tool that allows them to assess its adequacy, identify aspects that need to be improved or reinforced in the lesson, and substantiate decision-making to optimize its management in class.

THEORETICAL FRAMEWORK

Meanings of Probability

Different interpretations of the concept of probability have been proposed throughout history and are currently used in teaching (Borovcnik & Kapadia, 2014).

The intuitive meaning of probability emerged in ancient times in association with games of chance and religious rituals, in situations requiring the expression of personal degrees of belief about the occurrence of particular events and the qualitative assignment of probabilities.

The classical (also referred as a priori, theoretical, or Laplacian) meaning corresponds to the early mathematical approaches, stemming from the correspondence between Pascal and Fermat and synthesized in Laplace's rule. The probability of an event is calculated deductively without the need to conduct an experiment (Chernoff, 2008) as the ratio of the number of favorable cases to the number of possible cases, provided that all outcomes were equally probable (Borovcnik & Kapadia, 2014). This definition is circular and not applicable to situations where it cannot be assumed that elementary events are equiprobable (for example, tossing of pins or the cork of a bottle).

Bernoulli's demonstration of the first law of large numbers was the origin of the frequentist interpretation of probability, in which experimental (empirical or objective) probability is defined as the limit towards which the relative frequency of an event tends when repeating the experiment, a large number of times, and hence, rendered a posteriori probability (Chernoff, 2008). Although this definition extends the probability calculation to experiments with non-equiprobable events, it does not allow us to obtain the true value of the probability, but only an estimate of the same; moreover, it requires the independence of successive trials that should be performed under identical conditions (Borovcnik & Kapadia, 2014). By means of Bayes' theorem, the value of the (a priori) probability for an event could be revised based on new data to transform it into an a posteriori probability. At the beginning of the 20th century, mathematicians such as Finetti or Ramsey relied on this theorem to define subjective (also referred as Bayesian or personal) probability as a degree of belief based on personal experience (Chernoff, 2008). With this, the field of application is broadened, but probability loses its objective character as it is conditioned by a given system of knowledge (Borovcnik & Kapadia, 2014).

The works on Borel measure made it possible for Kolmogorov to subsequently develop the formalization of probability theory. Kolmogorov defined probability as a measurable function that fulfilled some axioms and that allowed the development of all the results known at the time on probability calculation.

These different approaches to probability have been interpreted within the Ontosemiotic approach as pragmatic meanings (Batanero & Díaz, 2007). The meaning of a mathematical object refers to the systems of operative and discursive practices carried out by a person (personal meaning) or that are shared within an institution (institutional meaning), to solve a type of problem-situations (Godino et al., 2007). To analyse mathematical activity, six types of primary objects are considered according to their nature and function: Languages (terms, expressions, notations, graphics) in their various registers (written, oral, gestural, etc.), problem-situations (extra-mathematical applications, exercises), concepts (introduced through definitions or descriptions), propositions (statements about concepts), procedures (algorithms, operations, calculation techniques) and arguments (statements used to validate or explain propositions and procedures). These entities relate to form configurations (epistemic or cognitive) that are understood as networks of intervening and emerging objects from the systems of practices (institutional or personal) and the relationships that occur between them.

Each of these interpretations of probability (intuitive, classical, experimental, subjective, axiomatic) differs "not only in the definition of probability, but also in the related problems, tools, representations, properties, and concepts that have emerged to solve various problems" (Batanero & Díaz, 2007, p. 116).

Didactic Suitability

The theory of didactic suitability is motivated by the need to develop specific instructional proposals that serve as a guide for the teacher in the phases of design, implementation, and evaluation of teaching practice, moving from a descriptive-explanatory didactic to a didactic oriented towards effective intervention in the classroom (Breda et al., 2018; Godino, 2013, 2024; Godino et al., 2007).

In the Ontosemiotic approach, the didactic suitability of a teaching-learning process is understood as the degree to which it (or a part of it) meets certain characteristics that allow it to be qualified as optimal or suitable to achieve the adaptation between the personal meanings achieved by the students (learning) and the institutional meanings intended or implemented (teaching), taking into account the circumstances and available resources (environment) (Breda et al., 2018). These institutional meanings are also representative of the global reference meaning.

Didactic suitability is a gradable trait of teaching and learning processes that involves the coherent articulation of the following six facets (Godino, 2024; Godino et al., 2007):

- Epistemic suitability: Degree of representativity of the implemented institutional meanings with respect to reference meaning.
- Cognitive suitability: Degree to which the implemented meanings are in the potential development zone of the students; proximity of the achieved personal meanings to the implemented meanings.
- Interactional suitability: Degree to which the didactic configurations and trajectories allow identifying potential semiotic conflicts and resolving the conflicts that occur in the instruction process. Any mismatch between institutional-type meanings; for example, between the reference meaning and the one implemented in a textbook lesson, is called epistemic conflict, while the disparity between the meaning manifested by a subject (personal) and the reference one (institutional) is considered a cognitive conflict.
- Mediational suitability: Degree of availability and adequacy of material and temporal resources for the development of the teaching-learning process.
- Affective suitability: Degree of student involvement (interest, motivation, etc.) in the study process.
- Ecological suitability: Degree to which the study process conforms to the educational project of the centre, the school, and society, and to the environment in which it is developed.

For each of the facets or dimensions involved in instructional processes, a system of components, subcomponents, and general empirical indicators is identified, which constitute a guide for systematic analysis and reflection and provide criteria for the progressive improvement of teaching and learning processes (Breda et al., 2018; Godino, 2013):

> Suitability criteria should be understood as a correctness standard that establishes how a teaching and learning process should be carried out. These criteria should be understood as correct standards emanating from the argumentative discourse of the scientific community, when it is oriented towards achieving consensus on what can be considered as best practice (Breda et al., 2018, p. 264, own translation).

These principles, a priori, guide how things should be done, and a posteriori, allow the evaluation of the implemented teaching and learning process. The siuitability criteria constitute a powerful tool for organizing the reflection and evaluation of both one's own and others' instructional processes (Breda et al., 2017, p. 1895), facilitating the analysis of didactic sequences designed and implemented with the aim of improving mathematics teaching (Garcés et al., 2021). These suitability criteria should be enriched and adapted according to the specific mathematical content intended to be taught (Breda et al., 2017), but also to the type of instructional process that needs to be analyzed, classroom instructional processes, curricular materials, textbooks, and educational videos (Balcaza et al., 2017; Beltrán-Pellicer et al., 2018a, 2018b; Castillo et al., 2022a, 2022b; Godino et al., 2006; Monje et al., 2018).

Assessing the suitability of a specific instructional process (planned, designed, or implemented) involves making a judgment about the degree of compliance with suitability criteria, which entails observing empirical indicators. Suitability indicators are understood as features that should be noted in a suitable instructional process were formulated for these components (Breda et al., 2018; Godino, 2013).

Given that a textbook lesson reflects the instructional process planned by the author to address the teaching and learning of specific content, it is possible to apply the didactic suitability construct and its breakdown into facets, components, and indicators for its systematic analysis. However, to carry out this analysis, it is necessary to consider the specificity of the instructional process, for which some of the general indicators may cease to be applicable. Indeed, didactic suitability is relative to the circumstances of the study process (Breda et al., 2018; Godino, 2013), so the application or relegation of certain indicators depends both on the educational context and the nature of the instructional process itself. Once reviewed and selected those indicators in the different facets that can be considered to evaluate the suitability degree of the instructional process (such as a textbook lesson), they must be enriched and particularised according to the specific topic to be taught (Breda et al., 2018), in our case, probability. Therefore, the development of an instrument that guides the assessment of the didactic suitability of the instruction process planned in a textbook lesson on probability requires compiling, analysing, and systematising didactic-mathematical knowledge resulting from research on teaching and learning of probability, as well as research related to the analysis of textbooks in mathematical education (Beltrán-Pellicer et al., 2018a; Castillo et al., 2022a, 2022b).

Probablistic Literacy and Reasoning

The previous definition of didactic suitability describes the conditions required in an instructional process to attribute it to the value of optimal. Essentially, this value is achieved when there is coupling between teaching and learning and rich mathematics are implemented, that is, representative of its general meaning and consistency with the educational project and society, considering personal and contextual circumstances.

Probability is a part of mathematics that plays a fundamental role in many disciplines. Citizens require basic probability literacy (Gal, 2005, 2012) and reasoning (Batanero & Borovcnik, 2016) to make informed decisions under uncertainty, in their social and professional performance. This has motivated the inclusion of probability in secondary school curricula, starting even in the primary stage in many countries such as Australia, the United States, or Spain (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020; Common Core State Standards Initiative [CCSSI], 2015; Ministerio de Educación y Formación Profesional [MEFP], 2022).

Therefore, a general criterion of didactic suitability for probability instructional processes would be to provide students with a probabilistic literacy (Gal, 2005) necessary to face the random situations that surround them in their personal, social, and professional lives, developing their probabilistic reasoning. This should guide the development of educational materials, particularly textbooks, that support the design and planning of instructional processes with high level of didactical suitability.

Gal's (2012) model of probabilistic literacy includes the following competencies:

a) understanding the fundamental probabilistic notions of variability, randomness, independence, and predictability/uncertainty,
b) calculating or estimating the probabilities of events, in everyday random situations, including simple, compound, or conditional probabilities,
c) properly using the language of chance, both the mathematical language and the one used to refer to randomness in everyday life,
d) recognizing the role of probability in different contexts and in personal and social discourse,
e) knowing the critical questions that can be raised about information related to chance, the type of conclusions that can be drawn with the available information, the relationship of the reliability of a prediction with the size of a sample, and the possible bias in the data.

In addition to these skills, Gal (2012) considers that a person with probabilistic literacy should have a critical attitude towards probabilistic information, control of biased beliefs about probability, and an appreciation of probability as a tool to work in random situations in which the person is involved.

Sánchez and Valdez (2017) add to the probabilistic literacy the development of sufficient probabilistic reasoning to solve probability problems and use arguments to prove the truth of a probabilistic statement, the validity of the solution to the problems, or the possibility of generalizing the information to another population or context. Probabilistic reasoning involves the ability to:

a) Balance personal beliefs or conceptions about randomness with formal mathematical elements involved in the situation,
b) Understand that there are no algorithms to achieve a certain result in random situations,
c) Discriminate between randomness and causality,
d) Become aware of the influence of prior probabilities in making a probability judgment,
e) Understand the asymmetry of conditional probabilities,
f) Recognize the theoretical nature of independence,
g) Properly interpret very small or very large probabilities, distinguishing them from impossibility and certainty,
h) Correctly interpret correlation and association (Batanero & Borovnick, 2016).

The competencies that constitute Gal's (2012) model of probabilistic literacy and the indicators of probabilistic reasoning by Sánchez and Valdez (2017) may be considered in an instrument to assess the didactic suitability of instructional processes, particularly those outlined in a textbook lesson on probability.

METHOD

The objective of this work is to develop a guide for analyzing the didactic suitability of probability textbook lessons. To elaborate on this guide, it is necessary to explicitly define indicators grounded "in research findings or in expert judgments assumed by the academic community, often embodied in national and international curriculum guidelines" (Godino et al., 2012, p. 333).

Godino et al. (2012) develop a methodology of content analysis for the progressive improvement of instruments for assessing the didactic suitability of instructional processes through the content analysis of curriculum proposals, to identify agreements and complementarities with existing instruments. This methodology "involves the realization of meta-analyses to identify principles, values, and means assumed by basic and applied research on which there is a certain degree of consensus to be applied in educational practice" (Godino et al., 2021).

Content analysis is applied to process and review qualitative dimensions, describe trends and characteristics of content, as well as formulate valid inferences from these data (Godino et al., 2012). It requires delimiting units of analysis in selected texts, in our case, research on textbook analysis, curriculum guidelines, or other documents from the academic and scientific community that gather expert judgments or research findings on the teaching and learning of probability. These are classified according to the facets and components of the Theory of Didactic Suitability. Thus, for example, within the epistemic facet, the categories considered are problem situations, linguistic elements/representations, regulatory elements (concepts/definitions, procedures, propositions), arguments, and relations.

In the second phase, these units are compared with the criteria and indicators proposed in the existent instruments for the analysis of didactic suitability (Beltrán-Pellicer et al., 2018a; Castillo et al., 2022a; Godino, 2013) to assess whether the information contained in these units reiterates, complements, or adds new elements regarding the already established indicators. Specifically, we build upon previous didactic suitability assessment instruments:

a) The guide for the analysis of mathematics textbook lessons developed in Castillo et al. (2022a) from the content analysis of research referring to the analysis of textbook lessons,

b) The guide to evaluate the didactic suitability of official Peruvian curriculum materials on probability, developed in Cotrado et al. (2022) based on a review of research on the meanings of probability in curriculum materials (Batanero et al., 2005; Gómez-Torres et al., 2013, 2014).

Due to the specificity of the resource, some components, and indicators in certain facets of didactic suitability (Beltrán-Pellicer et al., 2018a; Godino, 2013) ceased to be pertinent in the analysis of curricular guidelines (Cotrado et al., 2022). Therefore, it was necessary to analyze whether those didactic suitability indicators from Godino (2013) that had been omitted in the guidelines for probability curriculum materials in Cotrado et al. (2022) were still irrelevant for textbook lessons or if, considering the guide for the analysis of mathematics textbook lessons by Castillo et al. (2022a), it was necessary to readjust them for the new didactic suitability analysis guide on probability. For example, teacher-student interaction (interactional facet) in a curriculum material, or specifically in a textbook, is not bidirectional, which entails a reformulation of the indicators by Godino (2013) associated with this component. It no longer makes sense to "[The teacher] Recognizes and resolves students' conflicts (asks appropriate questions and gives answers, etc.)" (p. 123). Similarly, it is not possible to assess whether "[the students] try to convince themselves and others of the validity of their statements, conjectures, and answers, using mathematical arguments" (p. 123). Thus, suitability in relation to interaction among students is evaluated in terms of the ways in which the lesson author seeks to promote dialogue or communication among students. Similarly, in relation to mediational suitability, the component "Number of students, schedule, and classroom conditions" (Godino, 2013, p. 125) becomes irrelevant. However, it was necessary to include new indicators that did not appear in the guidelines for official curriculum materials but were applicable in the case of textbook lessons, such as the processes component in the epistemic facet, or those indicators related to clarity, complexity of proposed tasks, or the use of error as a source of learning in the cognitive facet (Beltrán-Pellicer et al., 2018a; Godino, 2013).

Finally, the content analysis of key research on different aspects of teaching and learning probability in secondary education, which can be applied in the analysis of textbook lessons, allows for the comparison and enrichment of the criteria and indicators proposed in both instruments, to construct a textbook lessons analysis guide for probability in secondary education.

Guide for the Analysis of Probability Lessons in Textbooks

As a result of the content analysis described in the previous section, the main ideas addressed in the found and reviewed works have been considered, which allow us to complement some indicator or establish new criteria that needed to be incorporated to the previous instruments (Beltrán-Pellicer et al., 2018a; Burgos et al., 2020) to get the guide for the analysis of probability lesson in textbooks. This information is categorized, summarized, and organized (according to components and subcomponents of each dimension) into tables (**Table 1** to **Table 6**) that collectively form the guide for the analysis of the didactic suitability of a probability textbook lesson for the first cycle of secondary education (students aged 12-14)[1].

The guide constitutes a systemic structure of descriptors (gradable and recognizable features in the textbook lesson) associated with the different components of partial suitability and grounded in didactic-mathematical knowledge about probability. Researchers or teachers can use the guide (**Table 1** to **Table 6**) as a rubric to assess didactic suitability and professionally identify those aspects of the text that require adaptations to optimize instruction.

[1] The focus of this work relies more specifically on the stage of secondary education between the ages of 12 and 14. While the age of the students to whom the lesson is directed significantly influences some indicators (for example, those related to concepts or properties considered fundamental for the educational level, or the prior knowledge required by students), in other cases, it does not depend on this (for example, education in values or inter and intra disciplinary connections).

DISCUSSION

Epistemic Suitability

The epistemic suitability indicators allow assessing the degree of representativity of the intended institutional meanings with respect to reference meaning. The reference meanings of probability contemplated in the current secondary curricula are the intuitive, subjective, frequentist, classical, and to a lesser extent the axiomatic (Batanero & Borovcnik, 2016; Beltrán-Pellicer et al., 2018a). Each of these meanings differs not only in the definition of probability itself but also in the concepts, properties, and procedures that arise to solve the characteristic problem-situations of each one. Thus, the situations proposed in the textbook lesson must be representative of the reference meanings, be heterogeneous with respect to their typology, context, and complexity level, but also allow relating the different meanings of probability (Nakawa, 2020; Nakawa & Kimura, 2021). For Sánchez and Valdez (2017), achieving appropriate reasoning about randomness requires a knowledge and pertinent use of probabilistic language, as well as making evident the existence of an explicit or underlying probability that can be approximated through relative frequencies. They propose using situations where the classical definition is applied, as this provides a priori probability to compare it with the relative frequencies that result from repeating the experiment many times. The lesson must distinguish the frequentist meaning from the classical one, as well as explain the difference between "calculating the probability", from the classical meaning, and "estimating the probability", from the frequentist one. The finiteness of the number of outcomes that allow the assignment of probabilities according to the classical meaning must be specified, as well as ensuring the equiprobability of the elementary events, so that Laplace's rule can be applied (Gómez-Torres et al., 2014).

The frequentist meaning allows for connecting statistics and probability, as it uses the concept of relative frequency from statistics and applies it in probability calculations. Additionally, its characteristic procedures include the posteriori collection of data and their statistical analysis (Batanero et al., 2016). The lesson must specify that relative frequencies and frequency distributions vary in each series of trials, providing spaces for reflection so that students can discriminate between a frequency estimate, a value that varies, and the probability, which is always a theoretical value (Batanero et al., 2016). It should also insist on the increase in the reliability of the estimate with the sample size, an essential property to understand the law of large numbers and the notions of variability and precision.

In this sense, Sánchez and Valdez (2017) also consider that the inferences and interpretations made by students in probabilistic situations involving the classical and frequentist meanings of probability are influenced by the level of reasoning and understanding achieved about the ideas of variability, randomness, and independence" (p. 139, free translation).

Conditional probability and the handling of the subjective meaning of probability are accessible from the early levels of secondary education (student aged 12-14) through their intuitive and experimental interpretations (Kazak & Leavy, 2018; Rodríguez-Muñiz et al., 2022). The subjective meaning is used when handling previous information, new information, and personal experiences and beliefs, as an attitudinal element, to estimate a probability (Kazak & Leavy, 2018). Following the recommendations of Rodríguez-Muñiz et al. (2019), it seems important to include experiments on subjective probability through everyday life situations instead of archetypal examples, such as urns, coins, dice, cards, or lottery.

The importance of articulating the frequentist and theoretical perspectives in probability teaching has been defended by several authors (Batanero, 2014; Martin & Theis, 2016). On the one hand, the use of the frequentist perspective allows for the notion of sample variability, which is impossible from theoretical perspective. Moreover, the theoretical probabilistic perspective is not predictive, meaning it offers no certainty associated with the occurrence of an isolated event (unless it is certain). However, the calculation of theoretical probability somehow maintains reasoning in a deterministic perspective, as it allows obtaining a precise and definitive answer, the ratio between the number of favourable and possible cases if all cases are equiprobable (Batanero, 2014; Jones & Thornton, 2005; Martin & Theis, 2016).

The lesson must make explicit the relationship of probability calculation with proportional reasoning, both in explanations and examples as well as in the proposal of activities. Proportional reasoning, understood as the ability to establish multiplicative relationships between two quantities and to extend that relationship to another pair of quantities (Lamon, 2007), involves a sense of covariation and multiple comparisons in relative terms. This reasoning is considered as a basic component of probabilistic reasoning as it is an integral part of the components of sample space analysis, the proportional quantification of probabilities, and the understanding and use of correlations (Bryant & Nunes, 2012).

It is advisable that the situations proposed in the lesson are first centered around physical experimentation, then computer simulation, and finally mathematical formalization. It seems to be desiderable that these situations should be open, require data searching, ask questions about the results, connect the collected results with the theoretical models that can explain them, change sample sizes, and discuss the effects of the changes produced. The lesson must pay attention to essential processes in the teaching and learning of mathematics, such as modelling, communication, argumentation, and generalization (Martínez & Penalva, 2006; National Council of Teachers of Mathematics [NCTM], 2000; Stylianides, 2009). Modelling activities are characterized by being sufficiently open and complex for students to bring into play their prior knowledge and creative capacity to suggest hypotheses, propose models that explain the behaviour of the phenomenon in question in mathematical terms, communicate their results, review and reflect on the extension of the solutions to similar situations (Trelles-Zambrano et al., 2022). Argumentation processes are related to learning, conceptual understanding, competence in solving mathematical problems, as well as the development of creativity and affections, something especially notable in probability (Nussbaum, 2011; Reuter, 2023). In addition, authors such as Sánchez and Valdez (2017) consider employing arguments to prove the truth of a probabilistic statement, the validity of the solution to the problem, or the possibility of generalizing the information to another population or context, important features of probabilistic culture.

Table 1. Components, subcomponents, and epistemic suitability indicators

Subcomponent	Indicators
	Component meanings
Problems	- A representative showcase of everyday situations for the student is included where they must distinguish between the random and the deterministic, including the simulation of random experiments. - Situations are proposed to introduce and develop the different meanings of probability (intuitive, subjective, frequentist, and classical). - Situations are presented that allow distinguishing when one meaning or another of probability is applicable. - The statements of the problems are presented without errors, contradictions, and ambiguities. - Students are prompted to formulate problems involving random experiences and simulation.
Languages	- Different types of representation (verbal, tree diagram, tables, symbolic, graphics) are used to model problems and mathematical ideas, analyzing the relevance and potential of one or another type of representation. - Language level appropriate to the target students. - Students are prompted to use, interpret, and translate between different expressions and representations of probability (verbal, tree diagram, tables, symbolic, graphic) through the proposed tasks.
Concepts	- The fundamental concepts of probability are defined for the corresponding educational level: random and deterministic experiment, sample space, favourable and possible cases, event, certain and impossible event, simple and compound event, dependent and independent events, frequency, relative frequency, convergence, simulation, experimentation, variability, equiprobability, probability. - The notions of probability as a degree of belief, classical probability, and frequentist probability are discussed, differentiating theoretical probability and the approximate value of probability. - In the proposed situations, students have to recognize, generate or negotiate definitions identifying the associated approach.
Propositions	- The fundamental propositions are stated correctly and adapted to the corresponding educational level. Sufficient and necessary propositions are established to recognize and characterize the different meanings (classical, frequentist, subjective). - The fundamental propositions about probability are stated and developed: range, probability of the impossible event, certain event and the complementary, elementary algebra of events. - The conditions that allow the application of Laplace's rule (finiteness of the sample space and equiprobability) are clearly established. - The properties related to the frequentist meaning are comprehensibly stated: variability of frequencies in each series of trials; increase in the reliability of the estimate with the sample size; tendency of relative frequencies to stabilize (law of large numbers). - The properties that characterize conditional probability as probability and when it is applicable are clearly developed. The properties related to the independence of events are introduced at an intuitive and experimental level. - Situations are proposed where students have to generate or negotiate propositions about the different meanings of probability.
Procedures	- The fundamental procedures linked to the classical meaning are correctly worked: construction of the sample space, distinction of favourable and possible cases, determination of the equiprobability of elementary events, application of Laplace's rule, proportional comparison of probabilities; calculation of joint probability in independent and dependent experiments. - The fundamental procedures linked to the frequentist meaning are properly implemented: estimation of probabilities from repetitions of the same random experiment, calculation and representation of frequencies, interpretation of tables and graphs. - Possible results of a random phenomenon are quantified based on previous experiences and their contrast with the conditions in which said phenomenon occurs. - In compound experiments, the associated uncertainty is analyzed, and conditional probabilities are determined. -Situations are proposed where students have to generate or negotiate procedures for different meanings.
Arguments	- The propositions are argued appropriately for the corresponding educational level, using simulation of experiments and supported by diagrams, tables, or graphs. - The procedures are justified relying on the concepts and properties used. - Situations are proposed in which students must adequately explain and argue the procedures or propositions used.
	Component relations
Communication, argumentation	- Various random situations are promoted where the student has to describe, explain, formulate, and verify conjectures that involve probabilistic reasoning.
Modeling	- Situations are proposed that allow the student to use simulation as a model of the situation, recognizing the difference between model and reality. - Open extra-mathematical situations are presented for the student to propose models that explain the behaviour of random phenomena in mathematical.
Generalization	- Students are given the opportunity to recognise regularities, analyse them and generalise relationships and properties involved in probabilistic situations.

Given these considerations, **Table 1** includes the indicators for assessing the epistemic suitability of a probability lesson in a secondary school textbook, according to the components and sub-components of this facet.

Ecological Suitability

The ecological dimension considers the degree to which the objectives, contents and situations are adapted to the curricular guidelines, whether the proposed activities are open to innovation, as well as the extent to which the contents contribute to the social and professional training of the students, connecting with other areas of mathematics, as well as with other disciplines.

To promote probabilistic literacy, it is essential to consider various contexts that require decision-making, such as the natural world, physical, technological, medicine, justice and crime, finance and business, research and statistics, and gambling (Gal,

Table 2. Indicators of ecological suitability

Subcomponent	Indicators
Adaptation to the curriculum	- The purposes, meanings of probability, its development, and the planned evaluation in the material correspond to the curricular guidelines.
Openness to innovation	- Activities are open to innovation and reflective practice.
Socio-professional adaptation	- The probability contents considered contribute to the socio-professional training of the student.
Education in values	- Democratic values training is contemplated, avoiding graphic or verbally discriminatory expressions.
Intra and interdisciplinary connections	- Tasks allow the contents of probability to be related to other intra and interdisciplinary contents. - Situations promote probabilistic literacy: various contexts requiring decision-making are considered, understanding the role and implication of probabilistic problems, and fostering reflective questioning of discourses (context, source, analysis process, and meaning).

2005). Proper probabilistic literacy involves understanding the role and implications of probabilistic problems in different areas and in personal or public discourse (Gal, 2005). The lesson should offer students the opportunity to ask critical questions about the context (to what extent does it imply randomness?), the source of the information, the analysis process followed, the meaning of the message, and how it is interpreted, allowing for reflection when dealing with probabilities (Gal, 2005).

In relation to values education, according to Braga and Belver (2016), it is recommended that the textbook lesson should avoid transmitting stereotypes and the presence of discriminatory elements, both in the text and images.

Table 2 provides a structured way to evaluate the ecological suitability of a probability lesson in secondary education, ensuring that it aligns with curricular guidelines, promotes innovation, and fosters a holistic understanding of probability in various contexts.

Cognitive Suitability

The cognitive suitability indicators consider factors that ensure a progressive adaptation of the intended institutional meanings to the achieved personal meanings of students (Godino, 2013).

The textbook lesson must consider the knowledge and skills that are prerequisites so that students can establish the necessary connections to successfully face new content and assess their achievements (Observatório Nacional de Textos Escolares [OBNATE], 2023). Specifically, students should be familiar with rational numbers and proportionality and be competent to quantify and estimate probability both subjectively and through the verification of the stabilization of relative frequencies in random experiments. They should also use linguistic registers such as bar diagrams and tables to represent information, as well as to calculate and compare probabilities in simple experiments where the Laplace rule is applicable, applying basic counting techniques (ACARA, 2020; MEFP, 2022).

Achieving progression in learning means, firstly, that the intended content has a manageable level of difficulty, and that concepts, representations, procedures, and properties are presented in an increasing degree of complexity (Monterrubio & Ortega, 2012). Secondly, it requires considering the limitations students may encounter in studying probability, through activities that allow diagnosing previous ideas, errors, and difficulties (Braga & Belver, 2016). For instance, success in analyzing and comparing probabilities requires an adequate level of proportional reasoning (Bryant & Nunes, 2012).

A literature review of the aspects that influence the complexity of proportionality tasks (Author, date) allows us to consider some factors that intervene in the difficulty degree of probability comparison tasks. Problems that involve the first or second terms of a ratio being related (from favourable to unfavourable, favourable to possible, or among favourable or unfavourable), or where there is a divisibility relationship between their terms, are easier for students. In this regard, Pérez Echeverría et al. (1986) defined four levels of difficulty in probability comparison problems based on the required strategy.

In Level 1 problems, the number of favourable or unfavourable cases is the same (no need for the use of fractions to solve them). In Level 2 problems, there is proportionality between the favourable and unfavourable cases within the same urn or between favourable and unfavourable cases in two urns (they can be solved by establishing a correspondence in one urn and observing that the relationship is the same in the other urn). Level 3 corresponds to problems that present a multiplicative relationship only between the favourable cases of both groups (or urns in our case) or only between unfavorable cases, or between favourable and unfavorable cases within a single group (once the ratio between favourable or unfavorable cases is established, you can compare if the existing one between the other terms is smaller or greater). In Level 4 problems, there is no multiplicative relationship between favourable and unfavorable cases within each group. It requires working with fractions, finding a common denominator.

The calculation or comparison of probabilities begins by describing or enumerating the set of elements in the sample space, the correct determination of which is an essential part of solving the problem. Understanding the sample space, according to Langrall and Mooney (2005), is fundamental for probabilistic reasoning. This understanding requires recognizing all possible outcomes of an experiment, describing them completely, and relating the sample space to the likelihood of each experiment's outcome (Horvath & Lehrer, 1998), the latter being the most difficult ability for students. The results of Hernández-Solis et al. (2021a, 2021b) suggest that pupils find it easier to construct a sample space compatible with a possible and equiprobable event, while few students correctly construct a sample space corresponding to the certain and impossible event. They also observe less difficulty in constructing sample spaces in the roulette context than in the ballot box context, with the biggest difference being in the equiprobable event.

Although research related to understanding the frequentist meaning is still scarce, results show that students find it challenging to understand the variability of relative frequency and the expected value in different samples. Students have

Table 3. Indicators of cognitive suitability

Subcomponent	Indicators
Prior knowledge	- Prior knowledge (fractions, fraction equivalence, ratio and proportion, percentages, calculation and comparison of simple probabilities, basic counting techniques, statistical graphs and tables) is considered according to the educational level. - The intended contents have a manageable difficulty across their various components.
Individual differences	- Tasks are designed with a low floor and high ceiling, meaning all students can engage at some level, and all have the opportunity to face challenges. - Extension and consolidation activities are included. - Access, achievement, and support for all students are promoted, supporting the use of various correct strategies that are then interconnected, including informal strategies and intuitive reasoning.
Progression in learning	- Situations with varying levels of difficulty are anticipated (multiplicative relationship between favourable and/or unfavourable cases, whole and non-whole ratios in probability comparison, determination of sample space in the context of urns and roulette wheels; simple and compound experiments; determining the proportion in the sample given the population's proportion or vice versa; relationship between the proportion value in the population and the expected relative frequency, effect of sample size on variability). - Potential conceptual and procedural biases and errors are highlighted, particularly representativeness and equiprobability biases, outcome approach, and the law of small numbers. - Errors are used as a learning resource.
Evaluation/ assessment	- Evaluation and self-assessment instruments are proposed. - The included evaluation methods are suitable for assessing whether students establish connections between mathematical objects and the meanings of probability. - The evaluation methods included allow for assessing whether students achieve the appropriation of the intended knowledge and skills (conceptual and propositional understanding; communicative and argumentative competence; procedural fluency, etc.). - Different levels of understanding and competence are considered in the evaluation.

difficulties in understanding that samples share characteristics of the population from which they are drawn, because they perceive them as independent sets. For small samples students' intuitions about randomness are adequate, managing to propose sequences without repeated patterns and with reasonable length runs (García et al., 2014).

Begué et al. (2018) analyse secondary school students' intuitive understanding of two sampling properties:

1) Expected sample proportion and relationship with the proportion in the population,
2) Variability of the proportion in the sample and the effect of sample size on this variability.

In relation to the first one, students have greater difficulties in estimating the expected value of the sampling proportion when the probability of the event of interest is provided from frequency information. In addition, they overestimate the variability of the sample proportion in large samples, and it depends on the context of the problem in small samples. On the second property, they show difficulties in understanding the variability intrinsic to the sampling process (Begué et al., 2018).

Other research points to students' difficulties in calculating probabilities in compound experiments and interpreting tree diagrams (Batanero & Sánchez, 2005). The context also influences the difficulty level of problems (Batanero & Álvarez-Arroyo, 2024; Cañizares, 1997).

It is important that the textbook considers the usual biases in the components of probabilistic reasoning at this educational stage (Beltrán-Pellicer & Giacomone, 2021; Beltrán-Pellicer & Godino, 2019). Errors and biases in probabilistic reasoning should be seen as learning opportunities in the lesson. The limited experience of students with gambling and the influence of subjective factors promote the formation of biases that interfere with other correct probabilistic intuitions. It is suggested that the lesson should contemplate a varied and rich probabilistic context to bring out and correct students' incorrect intuitions in the field of probability (Cañizares, 1997). For example, those of the representativeness heuristic (Tversky & Kahneman, 1982), the equiprobability bias (Lecoutre, 1992), the isolated outcome approach (Konold, 1991) or insensitivity to sample size (Tversky & Kahneman, 1974).

Finally, it seems to be important that the textbook provides spaces that allow the student to become aware of their level of competency achievement and their shortcomings (Monterrubio & Ortega, 2012). In particular, it should allow assessing whether students establish connections between different mathematical objects (concepts, procedures, propositions, etc)[2] and between the corresponding meanings of probability.

Table 3 provides a structured summary of the components and indicators of cognitive suitability, that should be understood from the previous synthesis of research on probability learning.

Affective Suitability

Instructional resources, including textbooks, exert a significant influence on the shaping of attitudes and beliefs (Santaolalla, 2014). To structure the indicators in the affective facet, we consider attitudes, emotions, beliefs, and values (Beltrán-Pellicer & Godino, 2019).

The presence of situations that encourage active collaboration is valued, where one feels comfortable formulating hypotheses, motivating argumentation under equal conditions and respecting strategies and explanations different from one's own. In this regard, the role of language, particularly non-verbal language, is crucial. The material should allow to encourage students to adopt

[2] In Batanero and Díaz (2007), the different objects are related to the respective meanings of probability.

Table 4. Indicators of affective suitability

Subcomponent	Indicators
Interest and emotions	- Inclusion of didactic-contract-interesting activities and tasks in the textbook. - Space is reserved for students to express and reflect on emotions (blockage, curiosity, despair, confidence, etc.) in response to proposed situations. - Contextual situations and motivating elements are proposed (paradoxes, humour, or games). - Logical reasoning, original ideas, or practical/realistic work are facilitated and enhanced. - Self-esteem is promoted, avoiding rejection, phobia, or fear of mathematics.
Attitudes	- Texts (explanations, examples, etc.) and proposed situations aim to generate a relatable attitude through language. - Proposed situations foster a mathematical attitude (perseverance, responsibility, etc.). - Flexibility in exploring mathematical ideas and alternative problem-solving methods is stimulated. - Argumentation is supported by tasks aimed to be developed in conditions of equity.
Beliefs	- Situations involve students' metacognition and beliefs about the social context in which they learn. - A wide range of probability applications is offered, e.g., medicine, risk analysis, education, management, climate, or voting. - The idea that mathematics is a cultural construct is promoted. - The aesthetic and precision qualities of mathematics are highlighted.
Values	- The value and utility of probability in students' daily life decision-making are considered and emphasised.

Table 5. Indicators of interactional suitability

Subcomponent	Indicators
Author→Student interaction	- The author provides a clear, well-organised presentation, using language tailored to students, considering the peculiarities of probabilistic vocabulary, specifying key concepts of the topic: random and deterministic experiment, sample space, favourable and possible cases, event (certain and impossible events, simple and compound, dependent and independent), absolute frequency, relative frequency, convergence, variability, equiprobability, probability, independence. - Situations are promoted where consensus is sought based on the best argument. - Various argumentative resources are used to involve and capture students' attention.
Student interactions	- Tasks are provided to foster dialogue, communication, and debate among students in which different viewpoints are explained, justified, and questioned using mathematical arguments.
Autonomy	- Tasks are proposed for students to take responsibility for their studies: explore examples and counterexamples to investigate and conjecture; use various strategies to establish connections, solve problems, and communicate their conclusions.

a flexible attitude in exploring ideas and solving problems, as well as a critical stance or questioning of messages that may be confusing, incomplete, or biased (intentionally or otherwise), guiding them in formulating questions about data or conclusions from surveys or another empirical research (Gal, 2005).

Regarding emotions, it is valued that the proposed contents and tasks are of interest and relate to the students' social life, fostering positive emotions through humour, play, or paradoxes, and establishing connections with the history of mathematics and other disciplines. It is also important to reserve spaces to make explicit emotional states (curiosity, satisfaction, frustration, anxiety, etc.) when solving tasks, promoting self-esteem, and avoiding rejection of mathematics (Beltrán-Pellicer & Godino, 2019; Castillo et al., 2022b; Cotrado et al., 2022).

Beliefs can be modified through consistent work on emotions and attitudes (Vila & Callejo, 2004). For this, situations must involve students' metacognition and offer a wide range of social applications that highlight the importance of probabilistic reasoning. The situations proposed in the lesson should aim for the student to develop a positive view of themselves as an individual capable of making probabilistic reasoning and evaluating the associated risk in relevant uncertain situations (Gal, 2005).

Table 4 provides a structured summary of the affective suitability indicators, serving as criteria to assess the quality and relevance of probability lesson textbook lessons in this dimension.

Interactional Suitability

Considering the specificity of the instructional process intended to be assessed, a textbook lesson, the author-student interaction (at least in the traditional book) is not bidirectional. This leads to considerations of whether there is a clear and well-organised presentation of the content, highlighting the key concepts of the topic, etc. Interaction among students is considered through the inclusion of tasks or situations that promote dialogue and communication between students, considering flexible groupings (Braga & Belver, 2016). Moreover, situations should be considered that allow students to be spontaneous with their probabilistic ideas and intuitions, developing autonomy to address real-world problems.

> To enable students to make intriguing inferences and solve problems based on the definitions of probability (classical and frequentist), it is advisable to design teaching strategies to concurrently develop, alongside the mastery of the relevant calculations, the students' reasoning levels about and with the fundamental ideas.
>
> (Sánchez & Valdez, 2017, p. 140)

Table 5 includes the indicators finally considered for each of these components of interactional suitability.

Mediational Suitability

In the mediational dimension, material and temporal resources are considered. In the teaching of probability, information and communication technologies are fundamental, both using specific programmes (spreadsheet) and applets that can be found on the internet. The extent to which the lesson promotes the incorporation of complementary materials to the book itself is valued (Braga & Belver, 2016; Castillo et al., 2022a). The use of these materials in the textbook should aim to experiment, simulate, and focus effort on understanding rather than repetitive probability calculations. For instance, designing irregular dice with flexible paste or paper and using them in games like horse racing or versions of Beano (predecessor of modern bingo) allows for a connection with the frequentist meaning through experimentation, avoiding overuse of Laplace's rule (Beltrán-Pellicer & Giacomone, 2021). The internet provides access to numerous statistical websites from which students can obtain data to develop probabilistic sense. In the GeoGebra repository, various simulations are available, allowing conjectures to be made and contrasted with experimentation.

The assessment of suitability in the time component leads to considering whether the sequencing of content is appropriate, whether sufficient space is reserved to cover more challenging content, and whether the time required to cover content and activities is feasible within the course programme. In this regard, there is some consensus in the mathematical education research community about the key probability concepts that should be included in teaching this content to early secondary school students and what a relevant didactic sequence should look like. Various curricular proposals and research results in mathematics education (Batanero & Álvarez-Arroyo, 2024; Beltrán-Pellicer et al., 2018a; NCTM, 2000) suggest starting with intuitive ideas of chance and probability centred on the students' context and experiences, allowing identification of how students perceive chance and randomness, and whether they can differentiate random from deterministic experiments. Simulations are essential for understanding the law of large numbers and the connections between the notions of relative frequency and probability. As Martin and Theis (2016) point out, the articulation of the frequentist and theoretical perspectives in probability teaching “does not imply a sequential treatment of the two perspectives without any link between them but is achieved through a complementarity of these two perspectives" (p. 346, free translation). However, both perspectives are rarely addressed in their multiplicity or complementarity in probability teaching, particularly in textbooks at different levels (Martin & Theis, 2016).

In Beltrán-Pellicer and Giacomone (2021), the design and rationale of a proposal for teaching probability in the early years of secondary education (12-13 years old) are presented, based on problem-solving to articulate the different meanings of probability in secondary education: Intuitive, frequentist, and classic, grading their abstraction and complexity of associated practices. Connecting with the subjective meaning can be achieved by proposing situations in which new information is incorporated and by discussing how our degree of belief about the occurrence of the event changes. The authors highlight the fundamental role of everyday language by asking the student to formulate sentences in two ways: on the one hand, given an event, assign it a probability expressed as a personal degree of belief; on the other hand, given an expression that indicates probability, assign it an event.

Assigning a number to the personal degree of belief allows introducing the probability scale, both with the classic and frequentist meanings. The articulation of these last two meanings requires careful planning, so that the frequentist meaning is not seen as a simple empirical verification of the classic. To do this, it is necessary:

a) To propose situations that involve events whose probability cannot be theoretically calculated exactly,
b) To show the need for equiprobability of events when it is possible to determine the probability (dice throwing, ball extraction, etc.) using Laplace's rule,
c) o include in the didactic sequence situations were assuming equiprobability makes no sense or leads to errors.

An intuitive approach to the idea of variability and determining what is an acceptable number of repetitions of an experiment is key to reasoning about uncertainty (Begué et al., 2018; Martin & Theis, 2016). As suggested by Begué et al. (2018), the most frequent activity in the frequentist approach is the calculation of the relative frequency of an event of interest, progressively increasing the sample size, to empirically show the convergence of relative frequency to theoretical probability. Therefore, "it would be important to complete these activities with others related to the extraction of a large number of samples of the same size from a population, to analyse the variability of this statistic and gradually arrive at the idea of the sample distribution of the statistic." (Begué et al., 2018, p. 75). Research in probability teaching at this stage also points to the need to initiate instruction on concepts associated with sampling to promote its understanding, from the first courses of secondary school (Begué et al., 2018).

With these considerations in mind, we include in **Table 6** the indicators of mediational suitability for assessing a probability textbook lesson.

Table 6. Indicators of mediational suitability

Subcomponent	Indicators
Material resources	- The use of manipulative materials (dice, coins, cards, balls) as well as audiovisual and computer resources (software) is promoted, providing valid experiences to progress in the different meanings of probability (intuitive, subjective, frequentist, and classic). - The models and visualisations used or proposed in tasks allow for contextualising and connecting the fundamental definitions and properties of probability. - The sources used are explicitly stated and are diverse.
Sequencing	- The sequencing of content and activities is appropriate: it begins with an intuitive approach, connecting with the subjective meaning and personal degree of belief; the classic and frequentist meanings are articulated, sample variability is considered, empirically analysing the convergence of relative frequency to theoretical probability; an informal approach to sampling is worked on. - Enough space is dedicated to content that presents greater comprehension difficulty.

CONCLUSIONS

In this article, we have described the development of an analysis guide for a probability textbook lesson for the first cycle of secondary education (students aged 12-14) based on the Didactic Suitability Theory (Breda et al., 2017, 2018; Godino et al., 2016; Godino et al., 2012). Taking into account previous works by Beltrán-Pellicer et al. (2018a), Castillo et al. (2022a), and Cotrado et al. (2022), the indicators are the result of reformulating these, considering the review of specific backgrounds of the type of instructional process intended to be assessed, as well as the involved mathematical content. The textbook lessons analysis guide for probability incorporates the most relevant aspects as specific criteria to assess the overall didactic suitability of a textbook lesson, which has been previously chosen to guide the implementation of the teaching and learning process of probability with students at the beginning of secondary education. The guide should be understood as a tool that facilitates the teacher's decision-making on how to use a textbook lesson in the classroom to optimise the instructional process.

The sequence of practices proposed in a textbook determines a planned instructional process, which the teacher deciding to use it must evaluate and adapt based on the specific needs of their students and the educational context in which it unfolds (Brown, 2009). For efficient use of textbooks, teachers must adopt a critical stance towards them, which cannot be achieved without the mathematical and didactic knowledge that allows them to identify their potential and shortcomings (Castillo & Burgos, 2022). For this reason, the analysis and evaluation of their relevance or suitability should be an essential aspect in mathematics teacher training programmes (Braga & Belver, 2016; Castillo & Burgos, 2022; Yang & Liu, 2019).

On the one hand, various studies highlight biases in probabilistic reasoning and the difficulties of both pre-service and in-service teachers in teaching probability (Alonso-Castaño et al., 2019; Batanero et al., 2012, 2015; Begolli et al., 2021; Chernoff & Russell, 2012; Gea et al., 2017; Gómez-Torres et al., 2013; Ortiz et al., 2012; Vásquez & Alsina, 2015a, 2017b, 2019). On the other hand, the analysis of textbook lessons, besides being a competence to be developed in teachers, allows for diagnosing, evaluating, and developing didactic-mathematical knowledge that a teacher must possess to optimally teach some content, particularly probability (Braga & Belver, 2016; Yang & Liu, 2019).

Leveraging the opportunities offered by the textbook lesson analysis in teacher training requires the development of guides that allow future or practising teachers to have guidelines for analysing the didactic suitability of a textbook lesson to address a specific mathematics topic (Castillo & Burgos, 2022). These instruments should not only act as a means for lesson analyses to be more analytical but should also facilitate informed decision-making on the use of a textbook lesson as a resource to support the teaching and learning process of a topic. Therefore, they must be specific to the mathematical content and the educational context in which it takes place.

The guide developed in this research provides a systematic and reflective analysis tool that pre-service and in-service teachers can use on a probability lesson to, firstly, recognise the characteristics that make it more or less relevant, and secondly to identify ways of managing or making decisions for change to optimise teaching and learning, becoming aware of and reinforcing their didactic-mathematical knowledge in the process.

Developing a guide to assess the didactic suitability of a textbook lesson dedicated to a specific content, such as probability, and to a specific school stage, such as the beginning of secondary education, has a clear limitation concerning the characteristics of the instructional process under analysis. Despite the effort to identify common features in curriculum guidelines and research on what is considered optimal or appropriate for teaching students of these ages (12-14 years) from mathematics education perspective, we are aware that textbook lessons vary from one country to another. Therefore, it would be necessary to assess its applicability not only in the Spanish or Peruvian context (where it has already been used) but also in other international contexts. The guide for analyzing textbook lessons on probability is not a finished product and needs to be strengthened by incorporating the considerations of users (researchers, teachers, and teacher educators) and the reflections that arise from its application. In this sense, it will be necessary to design and implement training actions with pre-service or in-service teachers, similar to others developed in the context of proportionality (Author, date), in which teachers become familiar with the guide and can apply it in the analysis and decision-making process of managing textbook lessons on probability.

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