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The History of the Study of Shock Wave's Mach Reflection from the Wedge

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ABSTRACT

This article discusses the history of study of shock-wave structures arising at a clash of the shock wave on a wedge. We introduce the concept of regular and Mach reflection. We consider von Neumann three-wave model containing a branching shock wave, main and reflected discontinuities, a tangential discontinuity (sliding surface) behind the shock waves line of intersection. Experiments on studying Mach reflection at low Mach numbers and small angle of the wedge are described. The main works devoted to J. Neumann paradox when at low Mach number Mach reflection, in accordance with the theory, cannot exist, but, nevertheless, is observed in the experiments. The work is useful for engineers and scientists to compile a basic overview of the problem of Mach reflection.

KEYWORDS
Shock waves; gas-dynamic discontinuity Mach reflection; Neumann model; Neumann paradox

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Introduction

The first shock-wave structures have been described by E. Mach (1878). In his work he described two types of shock wave reflection from an inclined surface:

– Regular reflection, which consists of two shock waves: an incident wave coming on a hard surface, and the reflections coming from the point of impact;

– Irregular reflection, which consists of three shock waves - incident, reflected, and the main one - all have a common triple point T (Figure 1).

This kind of reflection is called simple Mach, and the corresponding configuration, if it contains no other normal discontinuities - a triple configuration (TC) of shock waves.

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The experiment, performed by E. Mach was as follows: in two points located at some distance from each other, two sparks occurred simultaneously and created two spherical shock waves. Under these points a surface covered with soot was placed. The shock waves were leaving a distinct trace of their intersection points, starting in the middle between points of waves initialization. Next, the cut at its ends was divided into two symmetrical diverging lines.

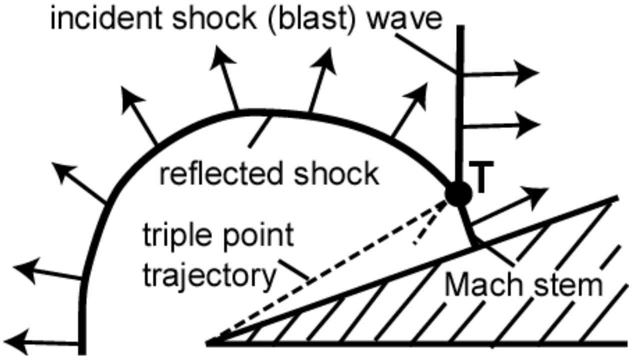


Figure 1. The formation of a triple configuration during a clash of the shock wave on a wedge. T-triple point

The resulting pattern shows that at the early stages of the interaction shock waves reflect from each other as if the reflection occurs in regular mode from an imaginary plane located midways between the waves initialization points. Then a Mach shock is formed.

Since only the trajectories of waves' intersection points were traced on a soot-covered surface, it took E. Mach a good spatial imagination to reconstruct the general picture of shock-wave interaction by these traces. Experiments of E. Mach are also interesting in that they created a foundation for the study of shock-wave structures. For a long time, the study of shock-wave structures (SWS) were reduced solely to the consideration of shock waves' Mach reflection from the wedge. Later other SWS were studied as well, for example those arising in the supersonic flow between the wedges.

Theoretical study of the regular reflection and triple configurations of shock waves was first performed by von J. Neumann (1943). In the description of TC von J. Neumann (1943) suggested that a tangential discontinuity (sliding surface τ) separating the flow behind the reflected σ_2 and the main σ_c shock waves comes from a triple point (Figure 2).

In reality a mixing layer is always formed on the sliding surface. Properties of the tangential and normal contact discontinuities are discussed in the book by L.D. Landau, and E.M. Lifshitz (2003). Such discontinuities are unstable. This type of instability is known as the Kelvin-Helmholtz instability. According to experimental studies of G.L. Brown and A. Roshko (1974) made in 1974, and of S.A. Ragab and J.L. Wu (1989), published in 1989, characteristic thickness of the Kelvin-Helmholtz area of instability in the stationary two-dimensional flows grows linearly on the longitudinal coordinate down the stream. Instability region quickly becomes a turbulent mixing zone.

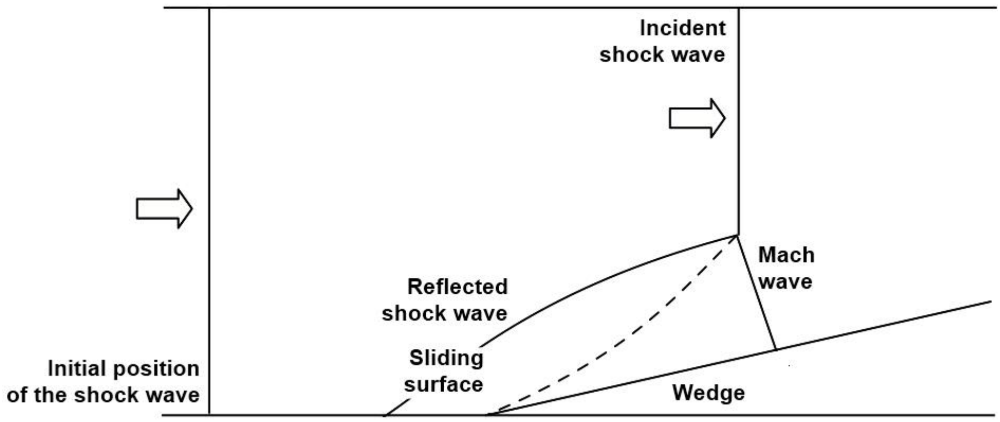


Figure 2. A model of von Neumann triple configuration

Literature review

J. Neumann (1943) formulated the theory of two- and three-wave reflection of a shock wave from a wedge. He also expressed the idea that reflection may be right (regular) and wrong (irregular, as an alternative to the right), and Mach reflection is one of the possible types of irregular reflection. J. Neumann also qualitatively described the kind of irregular reflection, which he called non-Mach and which later became known as "Neumann reflection" (Kawamura, 1956).

In this paper by W. Bleakney, C.H. Fletcher and D.K. Weimer (1949) and others the assumption of the existence of von Neumann on existence of tangential discontinuity behind the triple point was experimentally verified. In many experiments the contact discontinuity was observed, and in those cases when it is not observed, the theoretical value of the densities differential on its sides did not exceed the measurement error, and therefore it could not be detected. Thus, the model of von Neumann was proved.

Following the experimental confirmation Neumann's TC model was proved theoretically. R. Courant, and K.O. Friedrichs (1948) have shown that under ideal gas model three shock waves may exist in the same point only in presence of some another surface discontinuity. In his work Courant and Friedrichs also theorized that depending on the movement direction of the triple point (T on Figure 3) the Mach reflection can be divided into three types:

– Simple Mach reflection (as the shock wave propagates the triple point moves away from the wedge surface, the Mach stem increases, DiMR at Figure 3)

– Stationary Mach reflection (triple point moves parallel to the surface, StMR at Figure 3) and

– Reversed Mach reflection (triple point moves towards the surface, InMR at Figure 3).

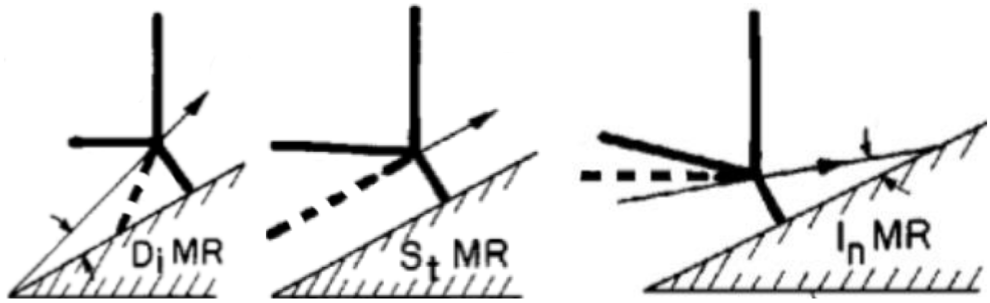


Figure 3. Simple D_iMR, stationary S_tMR and reversed I_nMR (inverse) Mach reflections of shock waves from the wedge

In 1967 Brides developed the result of Courant and Friedrichsau for the case of arbitrary equations of gas state (Breed, 1967). In 2008 V.N. Uskov together with M.V. Chernyshev (2009) presented the full research of stationary TC corresponding to von Neumann model, which was later generalized for non-stationary case (Uskov & Mostovykh, 2008). Von Neumann model for a real gas was considered by Lo for the case of oxygen (Law, 1970) for cases of nitrogen and argon. In these studies, the caloric properties of gases were described by a set of vibrational energy levels of molecules, to each of which the energy of its activation and its multiplicity was given. The work of J.-H. Lee, and I.I. Glass (1984) argues that in calculating of TC of oxygen O2 and nitrogen N2 it is possible to use the model of an ideal gas, and in the calculation of carbon dioxide CO2 and sulfur hexafluoride SF6 it is required to use a model of imperfect gas. Comparison of different models of diatomic non-perfect gas in the annex to TC calculation was performed by V.N. Uskov and P.S. Mostovykh (2011).

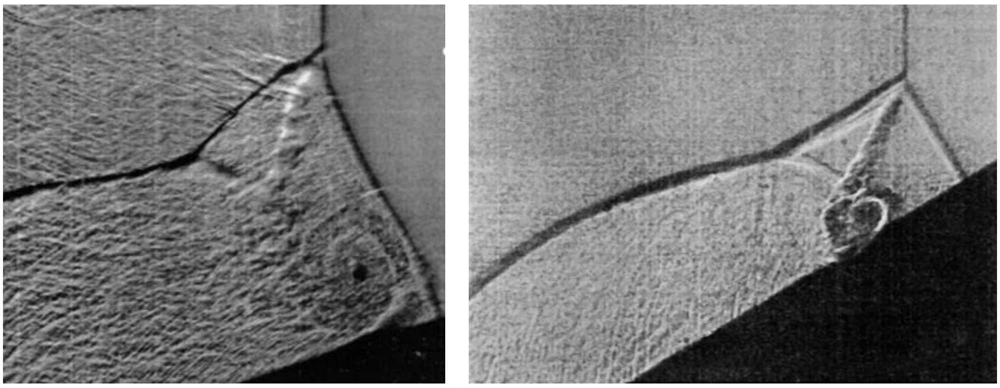
Purpose of the work is to show how the phenomena of Mach reflection of shock waves from the wedge was being studied on an example of the most significant scientific works. Irregular (Mach) reflection of gas-dynamic discontinuities (GDD) from obstacle is discussed. The researches of Mach and von Neumann, which created the foundation for the study of different types of shock waves triple configurations are presented. The history of studying triple configurations arising in supersonic jets is presented as well. Article also discusses the main types of Mach reflection and their classification.

Method

The analysis of leading local and foreign scientists, who studied this range of problems, is used in the work. In this research, comparative analysis is applied on a wide scale.

Data, Analysis, and Results

Experimental studies of the interaction of a traveling shock wave with a fixed wedge, performed by L.G. Smith (1945) have shown the dependence of wave reflection's nature on the angle at the wedge's apex. Smith also found that at high velocities of the incident shock wave a break is formed on the reflected shock wave in Mach reflection. Smith called such TC a complex Mach reflection (Figure 4a). At even higher velocities of incident shock wave the reflected shock wave itself acts as an incident one and forms a second triple configuration. This SWS is called double Mach reflection or the Mach-White reflection (Figure 4b).



a) b)

Figure 4. Complex (a) and double (b) Mach reflection

A detailed overview of the state of this question is given in W. Bleakney, and A.H. Taub (1949). As the authors note, there is poor compliance of the experimental results with the predictions of von Neumann theory for small wedge angles and in area of parameter values close to the transition from regular reflection to Mach reflection. The disagreement between the experimental data and the theory predictions, according to the review, cannot be explained by the wrongness of von Neumann's assumptions about the structure of TC.

Experimental studies and theoretical method of analyzing the shock polars, proposed in 1956, by R. Kawamura, and H. Saito (1956) also gave no satisfactory agreement between theory and experiment. W. Bleakney and A.H. Taub (1949) in the aforementioned review article, on the basis of experimental results obtained by them and on Smith's experimental data, and have plotted the wedge angle at which the transition of an incident shock wave to a Mach reflection occurs, by its intensity. According to them, the transition occurs in an area where regular reflection is theoretically impossible (required rotation angle of the flow is greater than the critical rotation angle on the shock wave). This criterion is called the criterion of disconnection (of the shock polar from ordinate axis) or von Neumann criterion. Smith's results were experimentally confirmed by Kawamura and Saito. It was also found that with increase of shock wave intensity the region of small angles at wedge's apex, for which the von Neumann theory is distant from the experiment, is reduced. In contrast, the range of angles at which the transition from regular reflection to Mach reflection occurs and both types of reflection are theoretically possible increases.

Paying attention to the fact that in a certain range of wedge angles the shock polar axis intersects both the ordinate and the upper branch of isomach, i.e. both regular and Mach reflection are theoretically possible, Neumann suggested another criterion of transition from regular reflection to Mach reflection, somewhat poorly calling it "the criterion of mechanical equilibrium." According to this criterion, the transition should occur at the moment when a shock polar crosses with an isomach at its peak, i.e., the intensity of the Mach stem in this case is equal to maximum for a given Mach number, which determines an isomach (Figure 5). SWS, which occurs at this is moment called a stationary Mach configuration (StMC). That is why this criterion was named “StMC criterion” by V. N. Uskov.

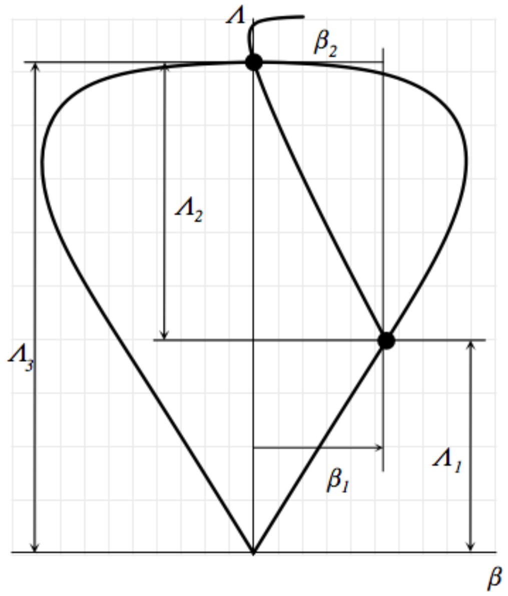


Figure 5. The solution on the plane of polars, corresponding to the transition from regular reflection to irregular one according to StMC criterion

On the figure 5: β_1 - flow rotation angle on the incoming shock, β_2 - flow rotation angle on the reflected shock, Λ_1 logarithm of incoming shock’s intensity, Λ_2 logarithm of reflected shock’s intensity, Λ_3 logarithm of Mach stem intensity.

Subsequent studies have shown that a hysteresis often occurs, i.e. with increase of the angle the transition from regular to irregular reflection occurs at intensity close to the criterion of disconnection, and with a decrease of the shock’s incline angle the reversed transition is closer to the StMC criterion. A lot of papers are dedicated to this issue.

In 70s – 80s of 20-th century the search for other criteria and/or the confirmation of von Neumann criterion was the subject of numerous experimental researches conducted in shock tubes. During the experiments, the disconnection criterion was confirmed by Henderson and Lozzi for steady flows of diatomic gases at oncoming wave’s Mach numbers from 1 to 4, by Hornung and Kichakoff for argon at incident shock wave’s Mach number up to 16, for pseudo-stationary cases – by Hornung, Oertel and Sandeman, for pseudo-stationary flows, simple, complex and double Mach reflection – in the works of G. Ben-Dor, and I.I. Glass (1980).

Guderley’s 3 – waves model for Mach reflection of a weak shock waves and other models.

In addition to von Neumann model, several other possible local patterns of the flow at the triple point of Mach reflection were suggested. In 1959, Sternberg suggested that in the immediate vicinity of the triple point the tangential discontinuity surface is not fully formed. Sternberg calculated the flow in the vicinity of TC triple point taking into account the gas viscosity.

Description of the triple point without using the SWS on GDD, i.e. without using the Rankin-Hugoniot relations was first undertaken in 1964 in the work of A. Sakurai (1964). He received an approximate analytical solution of Navier-Stokes equations in vicinity of triple point. A. Sakurai (1964) found gas-dynamic parameters in the vicinity of triple point depending on the polar angle and showed that at very low shock intensity his theory corresponds better to the experiment than von Neumann theory. At high shock intensities, on the contrary, von Neumann theory is more accurate. This result appears to be natural. In a close vicinity of interference point the influence of real gas properties is strong, and the shock waves cannot be considered infinitely thin. With the increase of distance from the point of interference the accuracy of equations' factorization in a row of by small parameter decreases.

It is worth to mention the very exotic hypotheses as. In the paper by V.G. Dulov (1973), it was suggested that it is not one, but two tangential discontinuity coming out from the triple point, however, this assumption didn't found experimental confirmation and didn't get the mathematical development. At present, the von Neumann model is considered generally recognized. Nevertheless, an unexplained Neumann paradox remains (Bleakney, Fletcher & Weimer, 1949). Recall that for Mach numbers less than a special number of

$$M_T = \sqrt{\frac{2-\varepsilon}{1-\varepsilon}}\ , \ \varepsilon = \frac{\gamma - 1}{\gamma + 1}\ , \tag{1}$$

the solution for irregular reflection of the shock wave from the wall with forming of a triple point is absent. However, it is observed experimentally, γ-adiabatic parameter equal to the ratio between the heat capacity at constant pressure and the heat capacity at constant volume. For nearly forty years the experiments were conducted, sometimes being very subtle, which clearly demonstrated that the three-wave theory does not work for the reflection of weak shock waves with Mach number of incident flow less than Mt.

For weak shock waves (small Mach numbers) Guderley suggested a four-wave model (Figure 6) with an additional rarefaction wave behind the reflected discontinuity. A similar pattern was researched in the work of E.I. Vasilev, and A.N. Kraiko (1999).

For a long time, it was not possible to obtain a numerical solution for this kind of flows as well, until E. I. Vasilev (1999) has shown that the problem lies in the lack of accuracy of numerical methods, the influence of "circuit" computing viscosity and parasitic oscillations of the solution, and the flow corresponds to Guderley's "four-wave" model. For this end, a numerical method with the highlight of discontinuities was used. Finally, V.N. Uskov constructed a harmonious classification of the interference of stationary gasdynamic discontinuities which showed that triple configurations are of three types, and the Guderley model is just a special case of interference of overtaking shocks. In addition, generally speaking, it is not quite correct, because shock wave called Guderley reflected (RS in Figure 6), is actually a second coming shock wave.

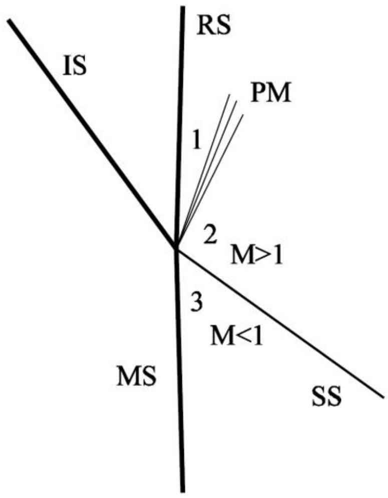


Figure 6. Configuration of SWS in Guderley's four-wave model.
IS - the incident shock, RS - reflected shock, MS - Mach stem, PM - Prandtl-Mayer, rarefaction wave, SS - tangential discontinuity.

Discussion

The development of jet technology in the 50s of 20-th century caused a series of studies on the problem of Mach disk's appearance in supersonic gas jet from Laval nozzle. These discs, which are, in fact, the normal shock waves in the jet axis, are clearly visible during the start of a rocket and jet engines.

The impossibility of regular shock reflection from the axis of symmetry without the forming of a Mach disc was first described in the work by D.A. Melnikov (1962). Indeed, on the axis of symmetry condition of inclination angle of the velocity vector and of the streamline curvature behind the reflected shock being equal to zero, but at y = 0 it is impossible. As the incident shock approaches the axis of symmetry its curvature Kσ tends to infinity, because Kσ ~ y-1, so the conditions for formation of Mach disk in nonisobaric jet are always created. For the calculation of flows with shock wave reflection from the symmetry axis Melnikov suggested an approximate method of calculation, in which the size of Mach "disc" was an empirically defined value. Works and consist the corrections of Melnikov's evidence that the existence of a strong configuration of GDD's regular reflection from the axis of symmetry is impossible.

A detailed analysis of this issue is performed in where various hypotheses of transition from the regular shock wave reflection from the axis to an irregular (Mach) are discussed. The most famous of these is so-called Abbet model, modernized in the works S.M. Dash, and D.T. Roger (1981). Abbet suggested that in the critical section behind the Mach disc the condition of "flow throat" is fulfilled, i.e. the flow velocity becomes equal to the local sonic speed (Figure 7). If a triple point location is placed on incident shock, thus the initial conditions for the calculation of the flow, limited by disk Mach and tangential discontinuity are determined.

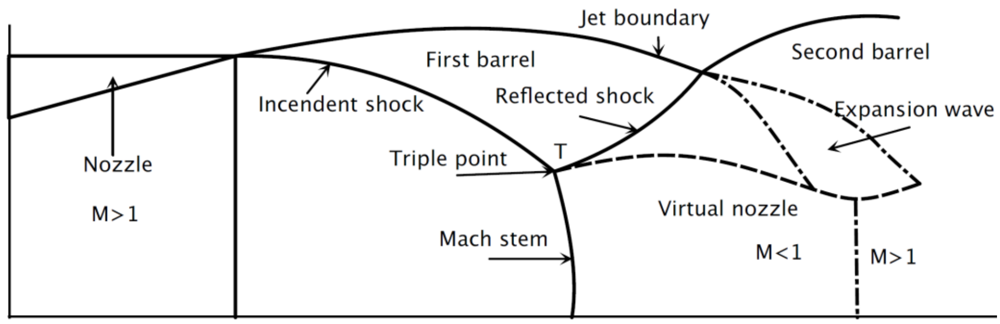


Figure 7. Explanation to Abbet procedure of finding Mach disk position in supersonic axisymmetric jet

Considering this flow one-dimensional, it is possible to perform the analysis in the same way as in the Laval nozzle. If the result in minimum section of flow behind the Mach disk speed is equal to the local sonic speed, then in Abbet-Dash procedure it is considered that the location of Mach disc in the jet is selected correctly. Continuing development of this model S.M. Dash has shown that it is close to the experimental results only in certain flow regimes and it is easier to calculate the Mach disk using implicit methods for solution of parabolic Navier-Stokes equations (Dash & Roger, 1981). Among other models well-confirmed is the criterion according to which the formation of Mach disc occurs when the intensity of the incident shock reaches $J = J_0$, corresponding to stationary Mach configuration. In StMC (Figure 5) the main shock wave is straightforward.

The characteristic intensity J_0 is obtained by solving the cubic equation, corresponding to the polars' intersection at the top of main polar.

$$\sum_{k=0}^{3} A_k J_0^k = 0, \quad \begin{aligned} &A_3 = 1-\varepsilon^2, \\ &A_2 = -\left(\left(1+\varepsilon-\varepsilon^2+\varepsilon^3\right)J_m + 1 + \varepsilon^2\right), \\ &A_1 = \varepsilon\left(1+J_m\right)\left[\left(1-\varepsilon\right)J_m - 2\right], \\ &A_0 = \left(1-\varepsilon\right)J_m\left(J_m-1\right), \\ &J_m = \left(1+\varepsilon\right)M^2 - \varepsilon. \end{aligned} \tag{2}$$

Indirect justification for J_0 criterion is the solution of the first order problem on shock waves (shocks) triple configurations, obtained by V.N. Uskov (2012) still in his doctoral study and published in 2012 (Uskov, Bulat & Prodan, 2012). Its essence lies in the fact that if at each point of suspended (falling on the axis of symmetry) shock a formal calculation of triple shock wave configuration (Figure 8) would be done, then with intensities $J < J_0$ triple configuration belongs to TC-1 type and the outgoing tangential discontinuity τ has a positive curvature (1 and 2 in Figure 8). At the point of shock, where $J = J_0$ (StMR), the curvature τ becomes negative, which corresponds to the prevailing of empirical understanding of tangential discontinuity form.

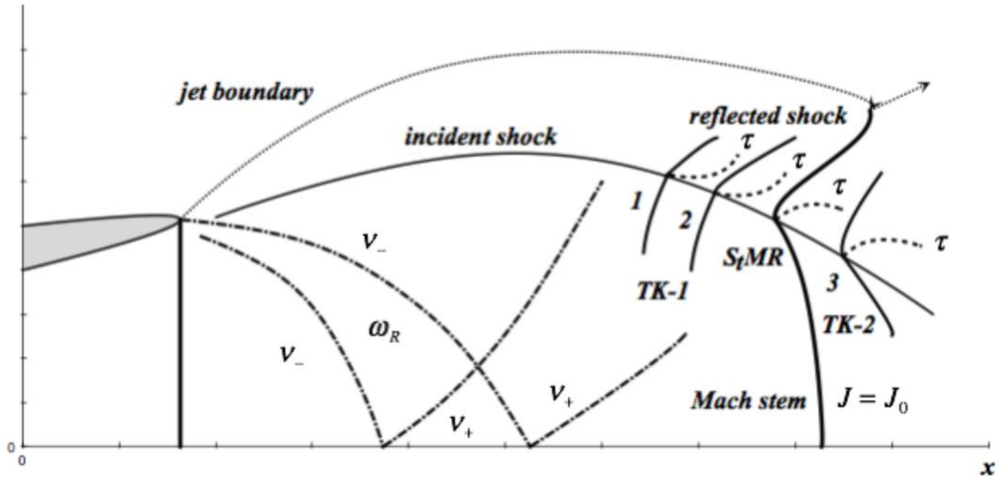


Figure 8. On the justification of stationary Mach configuration model for Mach disc in supersonic underexpanded jet

Triple configuration corresponds to the transition type TC-1/2 (StMC-StMR). When the triple point is located down the stream (point 3 in Figure 8) triple configuration would belong to the type TC-2.

Stationarity criterion of Mach configuration was strictly proved by P.V. Bulat in 2012 (Uskov, Bulat & Prodan, 2012), using the theory of features of smooth reflections, developed for shock waves under the leadership of V.I. Arnold. Comparison of calculation results with experimental results showed a good agreement. In the work a method for estimating the size of Mach shock in overexpanded jet is developed, with the dependence of its size on the pressure ratio being nonisobaric and becoming zero at the parameters of the jet corresponding the von Neumann criterion.

Thus, the question of the criteria for transition to Mach reflection in the axially symmetric case can be considered closed. Transition occurs when the incident shock wave reaches the intensity of J_0, appropriate to StMC.

Conclusion

We considered an irregular reflection of discontinuity from the wedge, the plane and the axis of symmetry, the wall plane. Information about the Mach disc in supersonic jets is provided. It is shown that at the Mach disc's point of origin on hovering shock the intensity of the shock must be equal to the special intensity J_0, corresponding to the intersection of reflected shock's polar with the top of the shock polar, built by the Mach number of incident flow. In other cases, the question of the criteria for transition to Mach reflection remained open. There are areas of ambiguous solutions. At low shock wave intensities the theory predicts the impossibility of shock wave's Mach reflection from an obstacle, but it is observed in the experiments. This phenomenon is known as von Neumann paradox. Experiments also show the hysteresis of reflection's characteristics depending on the direction of shock's angle change (increase or decrease). The study of irregular shock waves reflection remains a major problem.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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