IEJME

International Electronic Journal of

Mathematics Education

Volume 4, Number 3, October 2009 www.iejme.com

Special issue on "Research and Developments in Probability Education"
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RESEARCH AND DEVELOPMENTS IN PROBABILITY EDUCATION

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ABSTRACT. In the topic study group on probability at ICME 11 a variety of ideas on probability education were presented. Some of the papers have been developed further by the driving ideas of interactivity and use of the potential of electronic publishing. As often happens, the medium of research influences the results and thus – not surprisingly – the research change its character during this process. This paper provides a summary of the main threads of research in probability education across the world and the result of an experiment in electronic communication. For convenience of international readers, abstracts in Spanish and German have been supplied, as well as hints for navigation to linked electronic materials.

KEYWORDS. Research, probability education, ICME 11, electronic publishing, interactivity.

1. BACKGROUND

Does probability education need to be seen as discrete and separate from statistics education? This has been an ongoing debate for many years, particularly since statistics has sometimes been seen as dominant in school education where data handling has been a key theme as part of the movement of mathematics for all. Conversely probability has been seen as harder and less relevant within this approach. However, probability is an important discipline in its own right, and actually contains the key underpinning concepts to understand and use data sensibly. The argument for seeing probability as discrete has been promoted by various schools of thought in mathematical education and also within the International Statistical Institute (ISI) and its

ISSN: 1306-3030

educational wing, the International Association for Statistical Education (IASE). This finally led to two discrete topic groups at the International Congress on Mathematical Education (ICME 11) for probability and statistics respectively.

This paper focuses on the results of the topic study group 13 (TSG13) on probability issues in education and also introduces the ten papers which have been developed further for this publication. Ideas from the oral presentations (with links to the ICME web-site) are also included to give a flavour of the rich and varied set of ideas being studied within this field of research.

The reason that only ten papers have been included has as much to do with pragmatism as other scientific reasons. It was also partly due to the fact that these authors were willing and able to join us in the project to re-work their papers integrating the ideas of interactivity and electronic publishing. Moreover, IEJME made an exception to allow ten papers to be included, with a looser limit both on length of papers, as well as allowing linked extra materials where necessary. A strict refereeing process was used and the list of those involved is given below. As editors, we insisted that interactivity and electronic links had to be included to make good use of the facilities now available.

As readers will know these papers are published electronically in IEJME. Regular readers will also be aware that relatively few papers in electronic journals make use of the range of technology available in this medium of publication. This is partly because of underlying technical constraints which are discussed below. Nevertheless, we encouraged all authors to make some use of the interactive possibilities with the result that *all* papers in this issue of IEJME include some interactive or electronic components. Some helpful rules for navigating through the electronic documents might be seen from this link. For the convenience of readers of other languages an overview on the contributions is offered by abstracts in Spanish and German.

2. OVERARCHING THEMES IN PROBABILITY EDUCATION RESEARCH

Probability and statistics are relatively new disciplines in school mathematics to complement the traditional topics of arithmetic, algebra and geometry. Both have only recently been introduced into the mainstream school curricula and they now feature in virtually all countries. While the application-oriented statistics is undisputed in its relevance, discussion about the place of probability is more ambivalent. Reduction of probability to the classical conception, mainly based on combinatorics, or its perception as a solely mathematical discipline with its close connection to higher mathematics, are sometimes used as arguments to abandon it in favour of the statistics part. However, there are key reasons – which are developed in the papers published within this special issue of the journal – for a strong role for probability within mathematics and stochastics curricula:

1. Misconceptions on probability affect people's decisions in important situations, such as medical tests, jury verdicts, investment, assessment, etc.
2. Probability is essential to understand any inferential procedure of statistics.
3. Probability offers a tool for modelling and "creating" reality. For example, modern physics cannot be formulated without reference to probability concepts.
4. The concepts of risk (not only in financial markets) and reliability are closely related to and dependent upon probability.
5. Probability is an interesting subject in its own right and worthy of study.

Thus the challenge is to teach probability in order to enable students to understand and apply it. The focus has to be on creating approaches to probability that are more accessible and motivating. Additionally, the frequentist and subjectivist views of probability, and connections of probability to practical applications should be taken into account. Simulation is one such strategy, as is visualization of abstract concepts; there are more. The use of technology helps to reduce the technical calculations and focus the learner on the concepts instead. The world of personal attitudes and intuitions is another source for success or failure of teaching probability.

These challenges could not be fully met by a single working group at a conference. However, many valuable contributions were made, which will be clear from this journal. This follows the main themes emerging in the separate sessions, which nevertheless do overlap: Conditional Probability and Bayes' Theorem; The School Perspective: Pre- and Misconceptions; The Teachers' Perspective: Pre- and in-service Courses; Impact of Technology; Fundamental Ideas.

Conditional Probability and Bayes' Theorem

Conditional probability and Bayesian inference are important ingredients of university teaching, including courses for non-mathematical students. Many different types of errors have been investigated in isolation. According to C. Batanero & C. Diaz (Spain), a synthesis is missing. There is neither a study investigating connections between various types of misconceptions, nor an analysis whether misconceptions are related to mathematical knowledge, i.e. whether they decrease with better achievement in mathematics. Consequently, they have developed a test with (mainly familiar) items, and administer it to university students. Data are analyzed by means of factor analysis. They describe some phenomena, which remain even with higher mathematics education, but in general a significant decrease in misconceptions is found with a higher level of mathematics. For interrelations between several misconceptions, the result is less optimistic as these misconceptions seem to be quite isolated and persistent. As a consequence of this investigation, endeavour in mathematics education in probability has to be fostered while the types of misconceptions still have to be singly put to the fore in teaching again and again in order to facilitate students' understanding.

P. Huerta (Spain) criticizes a serious flaw of some existing research which does not take the structure of the posed problems into account. Then one cannot generalize from the results obtained to other kinds of problems with conditional probabilities. He describes the mathematical structure of "ternary problems" and classifies 20 different types of problems with conditional probabilities of which only one subclass (and from it mainly one type of task) has been used in existing research. A graph with all problems is used to visualize the grade of difficulty of a special problem at hand. By this deep structural analysis, Huerta develops a plan for future empirical research to cover all types of conditional probability problems to enhance the insights, which might be gained. In later stages of research he plans to extend the research from mere analysis of a subject's behaviour to classroom analysis in order to evaluate teaching interventions for their relative success. Other papers, described below also refer to conditional probability and Bayes' theorem, including by those by Vancsó, Martignon & Krauss, and Trevethan et al.

The School Perspective: Pre- and Misconceptions

There has been a trend away from misconceptions, which may be changed by suitable teaching, starting with pre-conceptions, which should be taken up and refined in teaching. Such a change of focus in research may be traced throughout all papers, which mainly reported on empirical research.

With D. Abrahamson (USA), the experiment is based on a single child (Li, 11 years) in an in-depth interview after a teaching phase in a classroom environment where the urn experiment was replaced step by step by the computer environment. Abrahamson is not only interested in the personal understanding of the child but in the learning trajectory of the child and how the interaction of the representation of the notions by different media can influence learning positively and possibly speed it up. He is careful to let the interplay of the different embodiments of the same notion work as the main input for learning (and not the teacher or interviewer).

In the binomial experiment he uses "four blocks" as a unifying element; they change their appearance from "spoons" to scoop samples to the building elements of the combination tower of all possibilities, and finally build up the histogram of repeated samples. As a valuable side effect of the approach, the histogram with these building blocks gets a "greenish" impression, which resembles the proportion of green marbles in the urn.

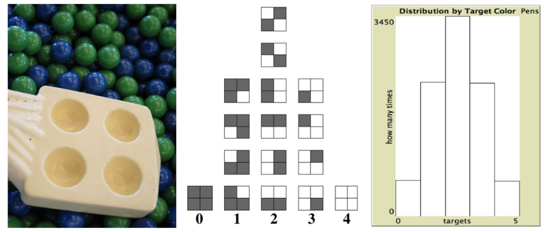

Figure 1. "Four blocks" as unifying elements.

The empirical studies by F. Chiesi & C. Primi (Italy) and L. Zapata (Colombia/USA) deal with heuristics in the tradition of Kahneman & Tversky. The Italian study deals with the development of "negative" and "positive recency" with age. They compare 9, 11, and 25 years olds in order to imitate a longitudinal survey.

From a bag with blue and green marbles a marble is drawn repeatedly with replacement. The result "all marbles of the same colour" is presented (the marbles are not actually drawn). The numbers of both colours are known – they are varied from equal numbers to a strong bias to either of the colours.

According to the "negative recency", people predict a change: with four blue they would predict a green one for the fifth draw. With the "positive recency", they predict that colour, which continues the series "observed". How frequently are these heuristics used, and are they independent of the proportion of the colours in the bag? Interestingly, the study shows an increase of the normative (correct) solution first (from age 9 to 11), but then this drops down (age 25). To a similar extent, "negative recency" decreases (from age 9 to 11) first and then increases again (age 25). With the "positive recency" there is a decrease (9 to 11) and it remains amongst adults (25) at this level. There have to be more in-depth investigations to clarify whether such a "development" can be confirmed (of course it is not truly longitudinal).

Conditional probabilities are considered to be difficult; Bayesian inference is no less difficult. However, their role in a successful curriculum in stochastics is undisputed. L. Martignon & S. Krauss (Germany) discuss a class experiment on this very topic with 10 year olds. With the help of Wason cards, they have managed to initiate learning steps in the children.

Which cards do you necessarily have to turn around in order to check, whether the following rule holds for the set of 4 cards? "If one side of a card exhibits a vowel, its other side must exhibit an odd number"

Children solve the task within its logical context badly, but improve with contexts closer to everyday life. Martignon & Krauss use 32 cards (equal numbers for all types) and let the children turn the "right" cards. In this statistical variant the children act quite successfully. The researchers move on to represent the Wason cards by tinker cubes, which can be put together to build towers. With two colours they represent the cards now with the advantage that both sides may be seen simultaneously. By using such media they describe learning steps in proportional thinking, right from the beginning in connection to probabilities. They report encouraging results from their pilot projects.

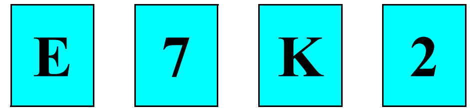

Figure 2. The original Wason cards.

K. Rolka (Germany, working jointly with S. Prediger) presented a study of 12 year olds playing a game of fortune with tokens moved forward on a playing board by the result of a die. The icosahedron used had more red sides than sides of any other colour whence it favoured the red token. The discussion amongst the children reveals how they interact and argue fiercely in favour of their strategy. Finally they jointly agree on a strategy. Views such as "there are more red sides on the die" and "the red token wins more often" were advocated likewise. The *common* struggle for a strategy seems to generate a better understanding of the value of their strategy – they seem to be much more aware of the chance that a token of a different colour could win as a result of their discussion; much more than they were aware by simply playing or *observing* the game. The social situation of the class with the children interacting in their discussion is a feature of Rolka's investigation – the situation is thought also to be exemplary for later teaching.

The Teachers' Perspective: Pre- and in-service Courses

This topic covers pre-service and in-service education as well teachers' conceptions of teaching and of probabilistic notions. Some of the following contributions would also fit the heading of fundamental ideas as well.

We all know about the pitfalls of the interpretation of results from statistical tests or from confidence intervals. These originate from the reduction of the interpretation of probability to situations, which may be repeated independently in the same manner. On this issue there has been a vigorous debate not only in the foundations of statistics but also in the didactical community. Taking this as a starter, Ö. Vancsó (Hungary) decided to develop a parallel course in classical *and* Bayesian statistics. He compares the debate on which approach is better, with the geometry debate of the 19th century. This led to a key component of modern mathematics, which is no longer devoted to the analysis of *absolutely* true statements. Similarly, it is a false dichotomy, to teach *either* classical statistics *or* Bayesian statistics. Both theories offer a consistent theory of probability. His didactical motto is "You will understand a theory much better if you contrast it to another". Accordingly he works on a conception for teaching both schools of statistics in *parallel*, without favouring either of the approaches. He tried his ideas and refined them in several cycles in teacher pre-service, and reports about his positive experiences: "Now I have really understood what is meant by confidence intervals" one of his students exclaimed.

An interesting extra-curricular activity is presented by H. Trevethan et al (Mexico) who describes a project in the context of a science fair. The concept of such fairs envisages that a group of students works on a project together with the aim of presenting this project to the general public at the fair. Aside of the interaction with the public, a jury of experts evaluates the projects including the performance of the groups at the fair. There are awards in various disciplines to win. The activity is aimed at students with a special interest in the subject. . The advantages to this approach include the autonomous activity of the students, their own responsibility, presenting in public etc. The students must be well prepared to face the imponderables of a live presentation. In the case at hand it deals with the game "Shut the box", which is certainly open to complex stochastic strategies. To elaborate on these strategies, to play against people from the audience with success, and to finally explain these strategies to the curious audience, changes the role from a (too often passive) student to one who is responsible for what goes on – that *is* of lasting effect on students. This authentic (and not artificial) transfer of responsibility could well be taken up more often in teaching in class. Mathematically, conditional probabilities and Bayes' theorem are the key concepts to develop winning strategies.

K. Lyso (Denmark) presented an innovative starter for an elementary probability course for teacher students. He started with an empirical investigation with students. He inspired their motivation and built a bridge between their intuitions and the mathematical concepts. He referred to these items and their discussion later in the course with success. Lyso uses a battery of quite normal tasks covering the main primitive concepts including two-stage experiments. The distinctive feature is how the items are dealt with after the written test. It can have a lasting effect, depending on how one opens the discussion about which solutions are feasible, or which reconstruction of the task would make sense and therefore lead to a sensible solution even if it does not coincide with the "normative" solution. One result is the documentation of an inclination by students to reformulate two-stage experiments into one-stage tasks and getting a wrong answer

– but they remain unwilling to see why their reconstruction is misleading:

> *Anna has three red, two green and one blue pencil in her pen case. She asks Maria to pick out two pencils without looking. Anna thinks that the probability that both of the pencils are red is 1/5 but Maria thinks that the probability is 1/3. Does either of them have the correct answer?*
>
> *"I agree with Maria. There are a total of 6 pencils and Maria picks out 2 pencils; this leads to the probability 2/6=1/3 for red pencils." (More than 20%)*
>
> *"No, one has to divide the red pencils, 3 pieces, by the total number of pencils, 6 pieces, = 3/6 = 50% chance for a red pencil." (About 15%)*

L. Zapata (Columbia/USA) investigated well-known tasks from Kahneman & Tversky. The conjunction of two statements is frequently assumed to be more probable than each of the single statements – this may be due to the fact that the conjunction of both statements resembles much more authenticity whence it is more "plausible" and thus judged to be more likely. Remember the technical term plausibility for the likelihood function, which uses the similarity in relation between the two terms. Other tasks in the study relate to the law of large and small numbers, which are linked to the fact that people are usually quite unaware of the huge influence of sample size on variability of results observed. New in Zapata's study are single interviews with *teachers*. Her target was to clarify what may be learned from new as well as more experienced teachers. Are the latter teachers better in anticipating such difficulties and can they meet the situation with suitable media or representations?

She tried to derive meta-knowledge for teachers from her in-depth interviews with teachers. Surprisingly, or possibly unsurprisingly, novice teachers repeat the same misleading intuitive conceptions as their students and thus are not really able to help them. This result is yet another argument to include (at least) one didactical course on the subject in teacher education at university and not restrict education to simply mathematical courses. Probability is much more prone to such difficulties than other topics in mathematics.

V. Kataoka (Brazil) reported a series of workshops of in-service education. Teachers' difficulties are surprisingly similar to those observed elsewhere. The interactive approach of involving teachers, however, could well lead to more innovative approaches worldwide. One special experiment used in the workshop illustrates the importance of suitable models and data sampled by randomness (when do you really have data from *random* samples?)

> *We break a stick randomly into three pieces. Afterwards, the subjects are asked to form a triangle of the three pieces. Finally the success rate is determined with which triangles actually could be formed. Try it with spaghetti – without explaining in advance what you plan.*

Success rates of 75% are not rare. In contrast to it, there are (at least) two models for randomly breaking the stick (with 25% and 19% success rates). The obvious discrepancy between the theory and the model lets us gradually start to doubt whether we can break the stick truly randomly into 3 parts. As a conclusion, relative frequencies may sometimes be of no value to estimate an unknown probability. This enriches the usual discussion about the convergence of relative frequencies by focussing on the underlying assumption of randomness of the data. Analogous examples are abundant but are less emotionally laden than spaghetti.

S. Anastasiadou (Greece, working with T. Chadjipantelis) developed a battery of simple items to research relations between algebraic and graphical skills in student teachers. Using similarity diagrams, she corroborates a widespread lack of skill to change between different representations of a task or a notion. This missing conceptual flexibility hinders a deeper comprehension of the notions. With different representations, students seem to learn *different* concepts – they do not necessarily notice that the representations deal with the same notion only in a different form.

Impact of Technology

Technology can be viewed in at least two very distinct ways. In one aspect the media such as Powerpoint is used to demonstrate ideas to students. The other aspect relates to the software tools for students to use interactively, such as Excel, Fathom, Tinkerplots, etc. Some software is generic (e.g. Excel) and some software is designed specifically for probability such as Fathom and ChanceMaker. In practice there is more software relating to statistics, though probability software is growing.

There has been no systematic evaluation of the possibilities and limitations of new media in the study group. However, as far as the presenters focused on teaching or in-service, they freely used various kinds of software in a substantial way. Spreadsheets (Excel), Fathom, or Tinkerplots were used for efficient calculations but also for illustrating key ideas such as the concept of distribution and the law of large numbers, with presentations by S. Inzunsa (Mexico) and R. Peard (Australia).

New media indirectly form the backbone of the research of D. Pratt, writing with R. Kapadia (England), on shaping the experience of naïve probabilists. By means of intentional sequences of the program ChanceMaker, he supplies new and challenging experiences to learners in order to shape their intuitions and strategies. Pratt talks of new challenges for designers of software and teachers using this software. In a fusion of control over the initial parameters (via randomness) and representations of results (histograms for the distribution of data or statistics like the mean), he seeks to prepare new insights into randomness, which should widen and refine intuitive notions, which might have been too narrow (and thus from time to time misleading) previously. Software offers more efficient, graphically orientated possibilities to supply (in fast motion) and order experience with randomness. How is it possible to form such basic ideas – with the support of new media? According to Pratt, a new world of up-to-date unknown intuitions might emerge, which would affect concepts and their understanding.

S. Ireland & J. Watson (Australia) report the results of in-depth interviews covering issues on the relations between empirical and theoretical aspects of probability. The interviews build on two course units with 12 year olds, which covered coin tossing and its tabulation of the results by the joint effort of the class and then the same experiments done by Tinkerplots, which is becoming more popular. Using the software has widened the students' experience, which is reflected by their relative success. Some questions remain open for further scrutiny. Can the computer really generate randomness? How can one read diagrams from the software correctly, e. g. their scale? How can we ensure that the children have sufficient experience in proportional thinking and fractions?

Studies of the use of software are being undertaken. Pratt has made a series of conjectures on design and teaching activities. Abrahamson also uses Logo as a software tool to explore binomial distributions. Peard based his teaching units entirely on Excel; Inzunsa used Fathom simply as a tool to support teaching; Ireland & Watson use Tinkerplots to enrich experiential evidence of children in an experiment to analyze the behaviour of relative frequencies.

Fundamental Ideas

A panel discussion directly addressed fundamental ideas in probability teaching. There were also presentations devoted to single probabilistic ideas like random variables, distribution, and expectation, or to the convergence of relative frequencies; the only one related to the central limit theorem was 'lost' prior to the congress. A further topic on "Revising probabilities – Bayes' theorem – independence" was the target of so much research that it formed a separate group.

In his approach to probability, Peard goes back to the roots of motivating concepts by problems and questions from games of fortune. Interestingly, however, he does not argue with the potential to explain the notions much better by the context, in which they emerged. Games of fortune have meanwhile been discredited by their closeness to combinatorics (which is not always easy to understand) and by their artificiality (we want and have to teach real applications to our students). Instead he promotes the application argument as games of fortune have been spread so much nowadays (at least in Australia) that they have become an important business sector, which is still growing very fast. It is therefore necessary to enable students to be familiar with this business and clarify wherein their chances truly lie.

R. Kapadia (England) discusses tasks from the national tests and concludes from the poor achievement of students therein that teaching compared to 20 years behind has not really improved substantially. This may be rooted back to recent trends as statistics, mainly that simple data handling is favoured at the cost of probability in the curricula. And, as far it concerns probability, teaching still focuses too much and narrowly on equal likelihood and experimental probability, there is not enough work on misconceptions, and risk as a concept related to probability is hardly discussed. Clearly, if people judge probabilities, they will have a strong bias towards "equal probabilities", especially when they are (or feel) confronted with two possibilities. The fundamental idea of judging probabilities and risks subjectively – coloured and supported by qualitative and objective information – has still not found a sustainable form of teaching.

Y. Wu (China) presented a panoramic view of the Chinese situation. School reality is centrally organized; only two textbooks are in use in the whole country. Despite the clash in cultures, the approach towards probability is surprisingly similar to that promoted in Western countries. M. Borovcnik (Austria) outlined some peculiarities of stochastic thinking, which make it so different from other approaches:

- There is no direct control of success with probabilities – the rarest event may occur and "destroy" the best strategy.
- Interference with causal re-interpretations may lead a person completely astray.
- Our criteria in uncertain situations may stem from "elsewhere" and may be laden with emotions – probability and divination have a common source in ancient Greece.

With these features of stochastic thinking in mind, paradoxes like the stabilizing of relative frequencies even though new events have full-fledged variability may not seem special. We are by far not so open to rational views when we are faced with uncertainty. The aspects referred to will influence our perception of the situation and what we would be prepared to accept as a mathematical solution. One difficulty may also lie in a primitive attribution of an ontological character of probability to situations. Probability does not exist – it is only *one of many views* to reflect on phenomena of the real world.

3. THE ICME GROUP AND PERSPECTIVES FOR THE FUTURE

ICME and IASE

The participants came from Europe, USA, Australia and Latin America, with equal representation from the English, the Spanish world, and the "rest". The graphs below illustrate the variety of approaches in the accepted papers. More details of the presentations are available from the conference web-site. The hope is that ICME will continue to organize topic study groups on probability and statistics separately. This did in fact split the potential audience as all the study groups were held at the same time. However, the great interest in the group on probability as well as the number of persons who attended the parallel statistics group confirm that we can attract more people to these areas by two separate groups.

The split into the two groups allowed a more convenient focus for the pertinent presentations and discussions. It showed that – against the international trend towards statistics in international curricula – there is substantial interest from the research perspective in probability issues as being highly relevant for any teaching and learning of statistics. This also applies to the joint study group of ICME and IASE, held prior to the ICME congress where a panel discussion about a vital role for probability within curricula met a strong echo and led to a lively discussion on the role of probability within educational research and in curricula.

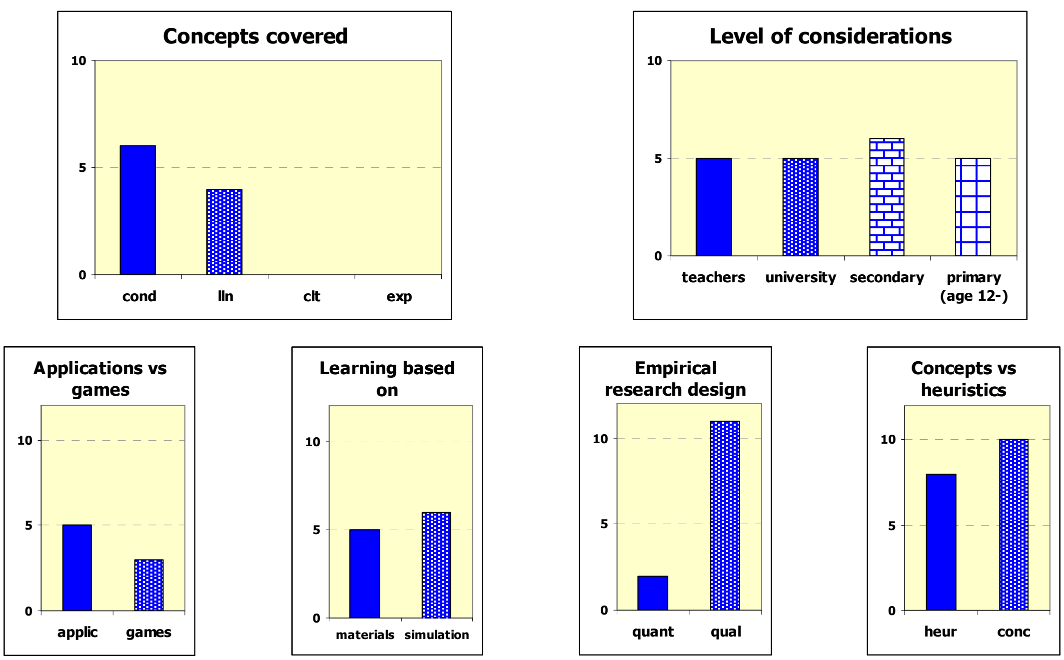

Figure 3. Frequencies of various categories into which the presentations may be attributed – Cond: conditional probability and Bayes, lln: law of large numbers, clt: central limit theorem, exp: expected value

Figure 4. Geographical origin and language families.

As can be seen from the first bar chart on *concepts covered*, there is limited research on expected values and the normal distribution. These two ideas are certainly quite sophisticated but both are important for many professionals in their future careers, especially those who are not mathematicians or statisticians. The notion of the so-called bell curve is often mentioned in the literature, but it would be valuable to know the extent to which it is understood, as well as the underlying conditions and assumptions to ensure its validity in an application. The notion of expected value and expectation is less common but is also not well researched.

Perspectives for the future

In a plenary session of TSG13, R. Kapadia deepened his elaborations from his initial presentation and enriched the discussion with a list of topics requiring instructional endeavour arising from the sessions:

- People use their experience in order to judge probabilities incompletely and – even worse – in a haphazard manner.
- People have difficulties in judging very small and very high probabilities especially if these are connected to adverse consequences.
- People are inclined to attribute equal chances to the possibilities, especially if there are two.
- People attribute probabilities and process them neglecting even the most basic rules (e. g. all probabilities sum to 1).
- Sharing and testing ideas across different countries will help promote deeper understanding.
- Further empirical testing using shared instruments will yield more insights.

Risk is seen as an increasingly important concept and requires further research. In the panel discussion, Watson referred to the growing interest of the community to such issues. Low probabilities are hard to understand, relating to the use of relative risk. These ideas are mentioned by Martignon & Krauss, following the pioneering work of Gigerenzer (2002). Utility is another key idea. These show the importance of probability in understanding and applying data, beyond frequencies.

Finally, the articles in this special issue speak for themselves about probability education. They show markedly that the community regained interest in probability education as a topic of increasing relevance. Future generations of students might thank the authors as they would profit in understanding all of statistics better.

ACKNOWLEDGEMENT

Both the Alpen-Adria University Klagenfurt and The London Mathematical Society awarded a small educational grant to support this interactive publication.

We are grateful to ICMI who invited the first-named editor to chair a topic study group at ICME 11. The organizing team consisted of: M. Borovcnik, C. Batanero, D. Pratt, Y. Wu.

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LINKS

Topic Study Group 13 on "Probability" at ICME 11: http://tsg.icme11.org/tsg/show/14; see also the website of IASE at: http://www.stat.auckland.ac.nz/~iase/publications.php

Topic Study Group 14 on "Statistics" at ICME 11: http://tsg.icme11.org/tsg/show/15

Joint ICME/IASE study: http://www.ugr.es/~icmi/iase_study/

SOFTWARE

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APPENDIX A.

SOME FEATURES PECULIAR TO THIS SPECIAL ISSUE OF IEJME

World-wide scientific community with the commonly accepted common language English

The scientific community nowadays is international and covers the whole world. As in older times, there is an urgent need for a common language to use. Latin was the common language for parts of the world after the Renaissance; however, it is now dead. For reasons of convenience, the community uses English. This means that many researchers have to learn a foreign language and even become expert in it in order to be fully accepted.

There has been much discussion about how the community can collate the creative potential amongst activities worldwide. After a decision in favour of one language, the world is divided into native speakers and non-native speakers. The whole community profits from a worldwide enterprise and the native speakers are somewhat privileged. Thus, they could repay this privilege and cooperate with non-native speakers to help them to get their ideas well formulated in the accepted common language, as a sort of quid pro quo.

The debate has been fierce and one of the editors still has the statement of a native speaker in his ears that "it cannot be my responsibility – you cannot expect that work from me". However, professional translators are of no real help as long as they are not involved in research in the field. One can only encourage researchers to form international partnerships and then help each other.

To translate a research paper is a creative act. It is not possible to translate a paper without delving into the scientific background. That is why professional translations are sometimes poor. Transferring to another language instead of translating means a change of role to become a co-researcher; this may also interfere with the research already done. Independent researchers are not used to that. They might cooperate with a person right from the beginning whom they choose deliberately. But when their paper is ready, they then have to re-work their ideas again. At several steps of the review process, we formed pairs of English speaking reviewers and authors to improve the English. We also planned that one of the editors, a native English speaker was willing to accept the burden of improving the English.

To connect to other language families and to co-operate between languages, to intensify the network between different cultures, abstracts of all the articles are offered also in Spanish and German. These may also be accessed via the contents paper of this issue to get an overview in Spanish or German. There are also links to these abstracts in each paper.

Interactivity

Another aim of this special issue was to use the specific potential of an electronic source of publishing. The style of communicating research results will and should change in the times of Web 2.0. From the beginning, there were ideas to try to develop the papers towards the direction of interactivity. Of course multi-media can also blur ideas. We had to deliberate carefully about the possibilities, and were unsure of how to proceed. In research, there is always the element of surprise. Our ideas included

- Internet addresses e.g. for more material of the author explaining special questions,
- movies related to interviews used in research experiments,
- Excel sheets with paradigmatic calculations to illustrate a difficulty or a possibility to reveal important aspects,
- images or sequences of images to show intermediate results derived in other software
- photographs to increase the authenticity

Authors went in different directions, some delivered authentic material like questionnaires, student interviews; others presented their videos of interviews; others wrote applets to demonstrate key concepts and ideas. Such material is of interest if a reader wants to go back to the roots of the data and develop his/her own conclusions. This may also serve as template for younger researchers on how to organize their own research project.

One further advantage of electronic publishing is also space. There is no obvious limit. Yet there are limitations such as the capability, capacity and time of authors to develop their papers further; also, the willingness of reviewers to read, evaluate and suggest improvements of the materials. The ideas also have to be realized by technological skills, which also vary. Technicians cannot always help as they are not involved in research issues. Thus we had to prepare ourselves for this challenge. The readers will hopefully appreciate the result as a first step in this direction and can look forward to more interactivity in the future in electronic publishing.

Style of referencing is a further issue, which can be dealt with differently in electronic publishing. For scientific papers it is important to give a reason for statements and to show how the presented ideas fit into existing research. In the extreme, this leads to references cluttering the text and possibly hiding the ideas in the background. On the other hand, careful referencing the sources will not include why one refers to such a source, and what role this source plays within a wider context of research; evaluative statements about references used are rare and would nearly always intrude the flux of argument of the text and the presented ideas.

In one of the articles we have experimented with a new publicizing format by extracting the intellectual background from the main text and consolidating and expanding it in notes that should prove useful for readers who wish to delve deeper into the issues underlying and motivating the research. In this way, a companion document with research notes emerged; the single notes are linked from the right places within the text.

"Rules" for Navigating between the Documents

A key decision for electronic publishing is whether to encode documents in html or pdf. The portable document format has the advantage that a huge percentage of the layout of documents is really preserved independently of the browser used. However, for linking with documents, there are drawbacks. It is not possible to link from any document to any position of another document. Moreover, the effort to establish all the documents in html is enormously higher than for pdf; the appearance of a document may also differ in the various browsers used to read in the internet – thus the decision of IEJME was for pdf format.

For the PC world, Adobe Flashplayer 10 is advised as the plug-in to read pdf documents; as with other plug-ins, some of the links might not work. Within Mac world, the browsers have such plug-ins already integrated. However, in testing the links, some problems were reported especially when plug-ins were used to view the pdf documents instead of the built-in preview. The problem is that in Mac world pdf documents are often *downloaded* instead of *viewed in the browser*. Of course, a downloaded document will become isolated and lose all its links. We cannot solve such issues for all browsers but some help may be found here for Safari, or here for Firefox and Mac. We will update our hints for navigation between pdf documents here.

Cell Press and Elsevier have launched a project called Article of the Future dealing with multi-media. To facilitate such projects, the computer world needs to work constructively together to solve the problems emerging from the fact that there are PCs and Macs. It is similar to issues discussed above about native English speakers and others.

An annex of all appendices gives an overview of extra materials like glossaries or movies connected to a paper. To navigate between the pdf documents, links can be used, which are encoded in a simple blue colour with no underline (the underline is a strong highlighting and would interfere with the text). If the reader follows a link to another text document, a link back is supplied by an arrow with a descriptor like To the article (Or, To the glossary). However, as it is, such a link always can lead only to the *first* page of the linked document.

Thus, for convenient navigation back to the same place from where one was linked to the extra document, it is advisable to use the *previous button* of Adobe or the browser; these buttons look like ; it could also be used in the opposite direction to the next open document ; such an icon cannot be integrated in the text as it can never be programmed to be a "variable" link, i.e. a link to the previous document. Sometimes – for no obvious reason – such a previous button has to be used several times until one gets back to the starting point in the original document. The arrow is sometimes misleading. Yet there is a reason to add it. A document may be retrieved by a search function in the Internet; in such a case this sign leads to its parent document. Otherwise it would remain an orphan. To get back to the main document from Powerpoint animations, it is best to use the ESC button of the keyboard.

Feedback is welcome if the reader notices that some links do not work, or a movie will not open to play. Perhaps we can fix the issue, or at least we will be able to learn for the next time. On the Internet many issues that work on the level of PC or Mac, are differently organized; if we get sensitized to it, we can improve. Feedback is also requested on the effectiveness and desirability of the interactive approach we have taken – this should be sent to the first-named editor who has undertaken most of the technical work, with able support from technical colleagues at the University of Klagenfurt.

APPENDIX B. ACKNOWLEDGEMENT OF REFEREEING WORK

As editors of this special issue we want to express our gratitude to the many persons who helped us over the last two years including the reviews and revisions for ICME 11. It is on the one side the authors who were exceptionally devoted to work, re-work, and revise the papers. They were open to our ideas including our experimenting in communicating research. A journal cannot exist with the background of the community. We are proud that we got so much help from quite a few reviewers who increased the quality of research and its presentation. It is the place here to honour their contribution to this issue of IEJME. We list them alphabetically:

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