

## Optimal Shock-Wave Structures

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### ABSTRACT

The article demonstrates the tasks that require designing of shock-wave structure with extreme values of the total pressure recovery coefficient, the relationships between flow velocity, static pressure, temperatures, and acoustic constant of the medium - the acoustic impedance. The problem of designing optimal shock-wave structure for supersonic air intakes of external, internal and mixed compression. The transition from the study of shock-wave processes to managing them, as well as to the construction of shock-wave structures with desired properties is accompanied by the increasing complexity of the mathematical apparatus and numerical techniques. Long-term efforts in this area have allowed performing a parametric study of SWS with properties, extreme by some parameter. In terms of optimal control theory problems of SWP management are formulated. Optimal combination of shocks composed of oblique shocks of same direction and a closing straight shock. Another optimal combination can include oblique incident shock and one reflected from the wall and closing direct shock. The problem of designing optimal triple of configurations shock waves arises in the study of supersonic air. It intakes work at nonisobaric mode, flows in three-dimensional nozzles with nozzle shocks, detonation waves is discussed.

### KEYWORDS

Shock-wave structure, shock wave, Mach reflection,  
optimal shock-wave structure, air intake, 3D-nozzle

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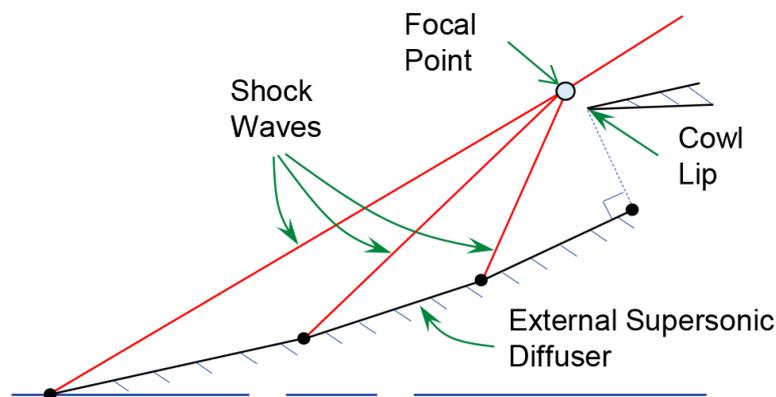
### Introduction

The development of technology created a task of managing the supersonic flows, which contain waves and gas-dynamic discontinuities. This raised the question of criterion for optimal control and, as a consequence for system of shock waves and discontinuities, extreme by some parameter. Thus, from the study of passive shock-wave processes (SWP), researchers have moved to the construction of SWS with specified properties.

First and foremost, there was the problem of minimizing the total pressure loss in SWS of multi-shock supersonic air intake (Figure 1).

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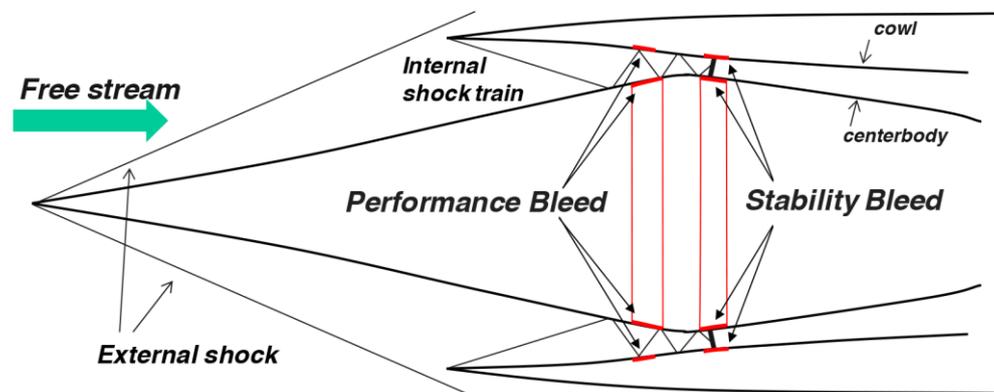
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**Figure 1.** The air intake with external compression

The optimal SWS should consist of oblique shocks of equal intensity (Petrov & Ukhov, 1947; Petrov, 1950). Closing direct shock should have a slightly lower intensity, but this difference can be ignored. The SWS consisting of shocks of equal intensity would be optimal. This significantly eased the problem of designing supersonic air intakes with external compression, because compression ratio is its predetermined parameter. Regardless of earlier work the behavior of total pressure loss function behind a series of oblique shocks and a direct wave.

The development of aircraft, the increase of their flight velocity up to  $M = 3$ - $3.5$  and more ( $M$  - Mach number equal to the ratio between velocity and the local sonic speed) has led to the use of air intakes with mixed compression, inside which a re-reflection of shock waves with closing direct shock occurs (Figure 2). The optimal deceleration of supersonic flow in such devices is, in particular, considered in (Uskov et al., 1998).



**Figure 2.** The air intake with a mixed (internal and external) compression

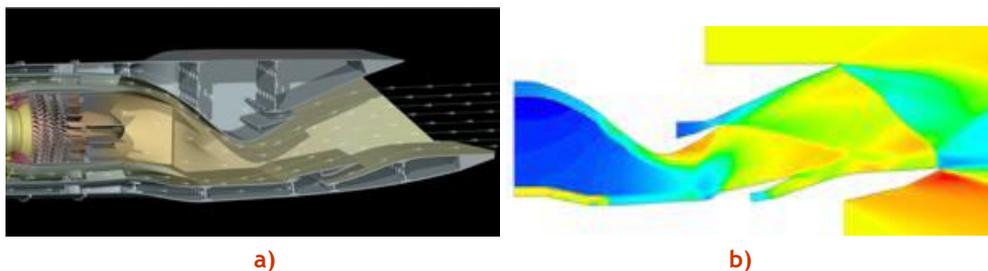
Works on the optimal (extreme by the set parameters) SWS (OSWS) became systematic only in 90th (Uskov & Omelchenko, 1995) and their subsequent works (Omelchenko & Uskov, 1997b), in which the studied shock-wave structures using optimal control theory. The main dependences of the gas-dynamic variables on the extremum behind an SWS in the following cases of practical importance:

- One-dimensional traveling waves (Uskov, 2000);
- Deceleration of supersonic flow in the diffuser (Malozemov, Omelchenko & Uskov, 1998);
- Optimal system composed of the shock and the rarefaction wave (Omelchenko & Uskov, 1997b) taking into account the fact that the angle of flow rotation should not exceed the limit value for rotation angle on the shock (Omelchenko & Uskov, 1996);
- SWS with the highest possible deflection angle (Omelchenko & Uskov, 1998);
- Optimal system of catch-up shock in multi-shock air intakes (Omelchenko & Uskov, 1999).

The study of extreme properties of shock waves triple configurations both stationary and non-stationary. In 2006 two staged works on optimal SWSs (Uskov & Chernyshov, 2006a) and extreme control problems of jet streams (Uskov & Chernyshov, 2006b) were published. A group of works work has been devoted to optimal triple configuration, first to stationary Mach ones (Uskov & Chernyshov, 2001), and then to all the rest: for stationary (Uskov & Chernyshov, 2002; 2006a) and the non-stationary case (Uskov et al., 2008).

During the study of detonation processes and progress in the development of jet engines with combustion in a system of shock waves the idea of organizing the detonation fuel combustion behind the direct shock wave (Mach stem) appeared. This process must be of high thermodynamic efficiency. It has been shown that during reflection within a nozzle the stationary Mach configuration are optimal by the criterion of difference of total pressure recovery coefficients on the system of oblique shocks and the Mach stem. They turned out to be optimal by other parameters as well. Then the study of extreme properties of the stationary Mach configurations themselves was conducted. Researchers' attention to triple configuration was attracted by the tasks of designing three-dimensional nozzles, similar to those planned to use in advanced engine ADVENT (Figure 3a). On particular modes of their work complicated a SWS is observed, containing triple points and the Mach stems (Figure 3b).

Triple configurations arise as well when external supersonic stream flows around the air intake, working at non-isobaric mode (Figure 4).



**Figure 3.** flows with shock waves in three-dimensional nozzle of ADVENT type (Bulat, Zasukhin & Prodan, 2012).



**Figure 4.** Flow around the air intake on non-isobaric mode

Successful numerical and analytical studies have raised the question of the possibility of transition from solving elementary problems of waves and discontinuities interference to the construction of SWS with specified properties and their optima regulation.

Supersonic stream hold priority in solving the problems of optimal jet flows control (Uskov et al., 2006) and of the external flow around an aircraft (Silnikov, Chernyshov & Uskov, 2014; Uskov & Chernyshov, 2013; 2014). This work laid the foundation for the design of aircraft's external contours having a maximum aerodynamic efficiency at supersonic speeds.

## Methods

### *Minimum pressure of nozzle launch*

When the total pressure before the nozzle is insufficient compared with the environment pressure a launch shock wave (LSW) forms inside the nozzle and on this wave the supersonic flow is decelerated to subsonic speed. With total pressure increase the LWS moves closer to the nozzle exit. The total pressure, at which the LSW reaches the nozzle section, is called launch nozzle pressure. With further increase in the total pressure an overexpansion supersonic jet flows out from the nozzle exit, and on the edge of the nozzle oblique shock is formed, which falls on the axis (plane) of symmetry. The lower the total nozzle launch pressure is, the smaller are the losses. The minimum launch pressure is reached at the nozzle geometric Mach number of  $M_a = M_T$ , where

$$M_T = \sqrt{(2 - \varepsilon) / (1 - \varepsilon)} \quad (1)$$

$\varepsilon = (\gamma - 1) / (\gamma + 1)$ , where  $\gamma$  – the adiabatic index, equal to the ratio of the specific heat at constant pressure to the specific heat at constant volume. For air,  $M_T = 1.483$ , and the launch pressure  $P_0 = 1.49p_\infty$ , wherein  $p_\infty$  - environment pressure.

### *Shock waves creating the maximum dynamic pressure*

The maximum pressure in the flow with a given pressure is observed at  $M = \sqrt{2}$ . Let us find the intensity of the oblique shock, which is to set-tion of the Mach number of the flow (given the velocity of propagation of the shock wave) provides maximum velocity pressure behind the shock

$$\frac{\rho_2 v_2^2}{\rho_1 v_1^2} = \frac{M_2^2}{M_1^2} J \quad (2)$$

where the subscript "1" indicates the parameters of the undisturbed flow in front of the wave, "2" – the parameters behind the wave,  $\rho$  – density,  $v$  – velocity,  $J$  – the intensity of the oblique shock (oblique shock wave). Differentiating (2), we obtain the desired intensity of optimal oblique shock

$$J_v = (\sqrt{1 + \varepsilon J_m} - 1) / \varepsilon \quad (3)$$

where  $J_m = (1 + \varepsilon)M^2 \cdot \varepsilon$  – the intensity of direct shock (direct shock wave). Maximum velocity pressure, corresponding to the optimal intensity of oblique shock (3)

$$\max \left( \frac{\rho_2 v_2^2}{\rho_1 v_1^2} \right) = \frac{1}{\varepsilon} \left( 1 + \frac{2(1 - \varepsilon)}{\varepsilon M^2} \right) - \frac{2(1 - \varepsilon)}{\varepsilon M^2} \sqrt{\frac{1 + \varepsilon}{\varepsilon} \left( \frac{1 - \varepsilon}{\varepsilon} + M^2 \right)} \quad (4)$$

tends to  $1 / \varepsilon$  as  $M \rightarrow \infty$ . At Mach numbers  $M < \sqrt{2}$ , the dynamic pressure behind the oblique shock wave is always smaller than before it. At high Mach numbers the maximum velocity pressure is created by an oblique shock wave with an intensity of (3).

### **The shock wave deflecting the flow with minimal losses**

To deflect the flow at maximum angle at the lowest cost of energy is required in the design of jet and aerodynamic controls. For  $M > \sqrt{2}$  there is the envelope around the collection of polars, which touches the polar, corresponding to a given Mach number, at the point  $J_e = M^2 - 1$ . At this point, the maximum rotation angle at a given shock intensity is achieved. If we fix the flow rotation angle  $\beta$ , then the point lying on the envelope defines the Mach number at which the intensity of the shock, turning the flow will be minimal

$$M_e = \left( \frac{2}{1 - \gamma \sin^2 \beta} \right)^{1/2}. \quad (5)$$

Conversely, if you the shock intensity is set, it is possible to find the Mach number at which the angle of deflection is maximal

$$M_e = \sqrt{J + 1}. \quad (6)$$

### **Parameters behind a series of shocks**

By replacing one shock with a series of weaker shocks, it is possible to optimize the SWS. In order to build a methodology for calculating such structures, let us consider how the parameters of the series of shocks are defined.

In a stream containing a series of shocks, gas-dynamic functions behind the  $n$ -th shock depend on the intensity of shocks and gas-dynamic functions of undisturbed flow.

$$p_n / p = \prod_{i=1}^n J_i \quad (7)$$

$$\theta_n = \theta + \sum_{i=1}^n \chi_i \beta_i \quad (8)$$

$$\rho / \rho_n = \prod_{i=1}^n E_i ; \quad (9)$$

$$T_n / T = \prod_{i=1}^n J_i E_i \quad (10)$$

$$P_{0n} / P_0 = \prod_{i=1}^n J_i (E_i J_i)^{-(1+\varepsilon)/2\varepsilon} \quad (11)$$

where  $E_i$  for the  $i$ -th shock is expressed by the Rankine - Hugoniot equation of shock adiabat

$$E_i = \frac{1 + \varepsilon J_i}{J_i + \varepsilon}. \quad (12)$$

Known relation for the Mach number behind  $i$ -th shock

$$M_i = \left[ \frac{M^2 - (1-E)(J+1)}{EJ} \right]^{1/2} \quad (13)$$

allows to calculate the Mach number behind the series of  $n$  shocks

$$M_n = M^2 \prod_{i=1}^n A_i - (1-\varepsilon) \sum_{i=1}^n \left( B_i \prod_{j=i+1}^n A_j \right), \quad A = (EJ)^{-1}, \quad B = A(J^2 - 1) / (J + \varepsilon). \quad (14)$$

The total ratio of pressure increase on  $n$  shocks

$$J_n = \prod_{i=1}^n J_i. \quad (15)$$

The angle of flow rotation behind a series of shocks

$$\beta_n = \sum_{i=1}^n \chi_i \beta(J_i, M_{i-1}). \quad (16)$$

Direction indicator of the shock  $\chi = \pm 1$  for the left and right shock. For any  $i$ -th shock is possible to construct a shock polar by the Mach number behind the  $i-1$  shock. The dependence of flow rotation angle at  $i$ -th shock on its intensity and the Mach number in front of it is determined by the equation

$$\operatorname{tg} \beta = \sqrt{\frac{J_m - J}{J + \varepsilon}} \frac{(1 - \varepsilon)(J - 1)}{(J_m + \varepsilon) - (1 - \varepsilon)(J - 1)}. \quad (17)$$

### **Air intake with external compression - a series of oblique shocks of same direction and the closing direct shock**

A system with a maximum value of the coefficient of total pressure conservation has the property that the normal component of Mach number in front of all oblique shocks is the same. Consequently, the ratio of the total pressures as well as static pressures, densities and temperatures for all of oblique shocks are also the same. A closing direct shock of the optimal system at  $1.5 < M_H < 5$  is slightly weaker than oblique shocks (Abramovich, 1991), so with a bit of error it can be assumed that all the shocks of OSWS in this case should have equal intensity.

### **Intake with mixed compression - two oblique shocks of different directions (shock's reflection from the wall) and closing direct shock**

The increase in flight velocity leads to necessity to use a mixed compression air intake. The simplest embodiment for such intake can be SWS, consisted of the incident shock, falling on solid surface, the shock, reflected from this surface and the closing direct shock. Let us show that the decelerations pressure behind a direct shock increases if the regular reflection of the shock from a wall (or from a

plane of symmetry) will occur in front of it. There is such intensity of incoming shock wave, whereby it will be the greatest increase in the corresponding structure is optimal. For a given SWS consisting of three shocks from the equation (11) we have

$$I_3 = P_{03} / P_0 = \prod_{i=1}^3 J_i (J_i E_i)^{-(1+\varepsilon)/2\varepsilon} \quad (18)$$

where  $J_1, J_2$  are intensities of incoming and reflected shock,

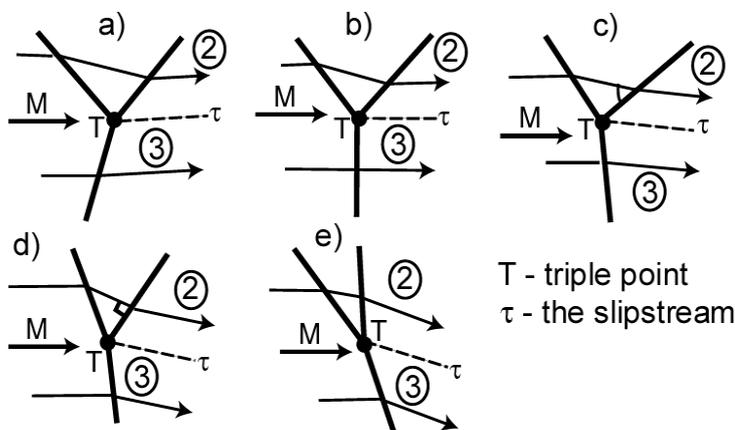
$$J_3 = J_m = (1 + \varepsilon) M_2^2 - \varepsilon \quad (19)$$

$M_2(M_1, J_1)$  - Mach number behind the reflected shock wave, calculated by the formula (14). Knowing the dependence  $J_2(J_1)$  we can, from (18), determine a function  $I_3(M; J_1)$ , in which the Mach number is a parameter. The graph of this function, referred to the ratio of the total pressure loss  $I_m$  at a single direct shock, shows how many times you can increase the deceleration pressure  $P_{03}$  at the cost of previous reflection of oblique shock. The inclination angle  $\sigma_1$  of incoming shock wave is an argument of the function  $I_3$  and is associated with the intensity  $J_1$  by the formula

$$\sigma = \chi \arcsin \left[ \frac{J + \varepsilon}{J_m + \varepsilon} \right]^{1/2}. \quad (20)$$

### Triple configuration of shocks

Triple configuration shock waves are SWSs consisting of one incoming shock, one main shock and the third shock, which can be both incoming and reflected (Figure 6). From the triple point at which all three shocks come together, the tangential discontinuity  $\tau$  definitely comes out.



**Figure 6.** Triple configurations of shock waves. a) 1-TC, b) TC-1/2, c) TC-2, d) TC-2/3, e) TC-3, 2 - second shock (reflected for cases a-d, incoming for case e), 3 - the main shock

Depending on the direction of flow deflection at the shocks 1-3 the triple configurations refers to first, second or third type. In the case when the first (branching) and the main shocks (3 in Figure 6) in the vicinity of the triple point deflect the flow in different directions (see Figure 6a), a triple configuration refers to the first type (TC-1). Triple configuration with direct main shock (Figure 6b) is called stationary Mach configuration (SMC or TC-1/2) and is the

border between the first and second types. In the triple configuration of the second type (TC-2), the first and the main shocks deflect the flow in the same direction and shock 2 - in the opposite direction (Figure 6c). Configuration with direct (normal to the velocity vector of flow before it) shock 2 is the transition from the second to the third type, and is denoted as TC-2/3 (Figure 6d). Figure 6e shows the configuration of the third type (TC-3), all shocks which deflect the flow in one direction.

Intensities (the ratios of static pressures) of  $J_1$ ,  $J_2$ ,  $J_3$  shock waves in triple configurations, Mach number in front of the triple point and the index  $\gamma$  of the adiabatic gas are connected by conditions of consistency on a tangential discontinuity. These conditions are written in the form:

$$J_1 J_2 = J_3 \quad (21)$$

$$\beta_1 + \beta_2 = \beta_3. \quad (22)$$

Each of stream rotation angle at  $i$ -th shock is connected to the intensity  $J_i$  of this shock and to Mach number in front of it formula (17). Mach number behind the shock  $i$  is also connected with the Mach number in front of it and with the intensity of this shock:

$$M_i = \sqrt{\frac{(J_i + \varepsilon)M^2 - (1 - \varepsilon)(J_i^2 - 1)}{J_i(1 + \varepsilon J_i)}}. \quad (23)$$

The triple configurations, in which the relations between certain flow parameters on a tangential discontinuity are extreme, are called optimal. As can be seen from the statement of problem, triple configuration is determined by its two parameters, such as Mach number of the undisturbed flow and intensity  $J_i$  of the branching shock. Controlling the last parameter, it is possible to create the optimal triple configurations without rebuilding the flow in front of them.

Let us consider different optimality criteria of triple configurations.

Ratio of the total pressures on the sides of the discontinuity  $\tau$  is defined through coefficients of total pressure loss on the shock:

$$I = (JE^\gamma)^{-(1-\varepsilon)/(2\varepsilon)}. \quad (24)$$

Applying this formula to each shock of the triple configuration, taking into account (21) for the ratio of the total pressures on a tangential discontinuity we will obtain

$$I_0 = \frac{p_{02}}{p_{03}} = \left( \frac{E_3}{E_1 E_2} \right)^{\frac{1+\varepsilon}{2\varepsilon}}. \quad (25)$$

This criterion reflects the minimum losses during the flow's passage through the system of two oblique shocks 1 and 2 in the triple configuration compared to the main shock 3 (e.g., Figure 4).

The relations of other thermodynamic parameters of perfect gas on a tangential discontinuity are in power dependence from  $I_0$ .

For example, the ratio of densities

$$\frac{\rho_2}{\rho_3} = \frac{E_3}{E_1 E_2} = I_0^{2\varepsilon/(1+\varepsilon)} \quad (26)$$

temperatures

$$\frac{T_2}{T_3} = \frac{E_1 E_2}{E_3} = I_0^{-2/(1+\varepsilon)} \quad (27)$$

sonic velocities

$$\frac{a_2}{a_3} = \sqrt{\frac{E_1 E_2}{E_3}} = I_0^{-\varepsilon/(1+\varepsilon)} \quad (28)$$

acoustic impedances (acoustic constant of medium)  $z = \rho a$

$$\frac{z_2}{z_3} = \sqrt{\frac{E_3}{E_1 E_2}} = I_0^{\varepsilon/(1+\varepsilon)}. \quad (29)$$

Consequently, the extremes of  $I_0$  value are extremes of other thermodynamic quantities as well, and the triple configuration, optimal for them, match. The criterion (26) determines the efficiency of gas compression in wave compression devices (wave compressors, wave pressure exchangers). Criterion (27) has the determining importance for detonation combustion, especially if gas-dynamic initiation of detonation is used.

If optimizable ratio depends on the flow rate velocities on the sides of the tangential discontinuity, the task of optimization is not simplified to finding the extremum of total pressures. For example, relations of velocity pressures

$$I_d = \frac{d_2}{d_3} = \frac{M_2^2}{M_3^2}, \quad d = \rho V^2; \quad (30)$$

of consumption functions

$$I_q = \frac{q_2}{q_3} = \frac{M_2}{M_3} \sqrt{\frac{E_3}{E_1 E_2}}, \quad q = \rho V; \quad (31)$$

of stream impulses

$$I_j = \frac{j_2}{j_3} = \frac{1+\gamma M_2^2}{1+\gamma M_3^2}, \quad j = p + \rho V^2 \quad (32)$$

are not expressed solely through density ratios at the shocks. Therefore, triple configurations, optimal by these criteria are different from optimal by  $I_0$ . Criterion (30) is used in problems related to the study of explosions' affecting factors (Gelfand, Silnikov & Chernyshov, 2010), and in technologies for metals hardening using powerful vibrations of shock-wave structures.

Lines corresponding to the criteria of optimality  $I_0$ ,  $I_d$ , cross the line SMC (fixed Mach configuration) at the point e. This means that the QMS at a Mach number equal to  $M_e$

$$M = M_e = \sqrt{4-3\varepsilon + \varepsilon^2} / (1-\varepsilon) = 2.254 \quad (33)$$

is optimal for all the above criteria. Indeed, at the point e, intensities coming in SMC (1) and reflected (2) shocks are:

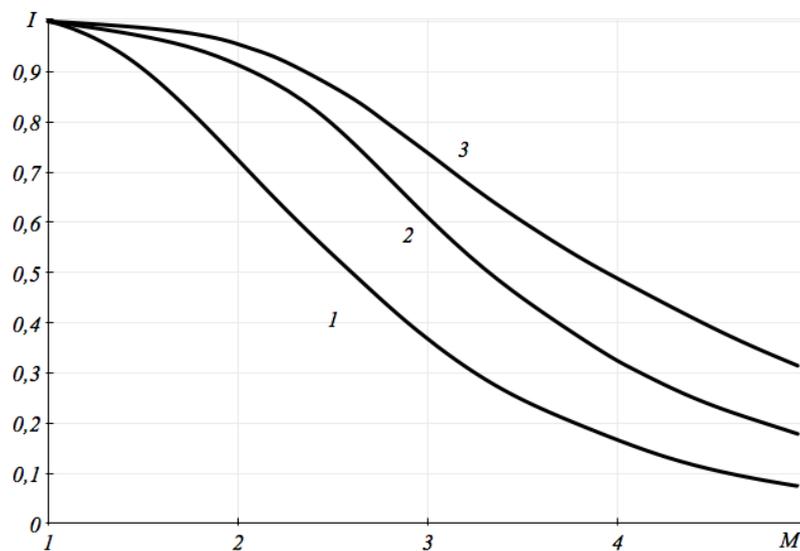
$$J_{1e} = J_{2e} = 2 / (1-\varepsilon). \quad (34)$$

## Results

### *Air intake with external compression*

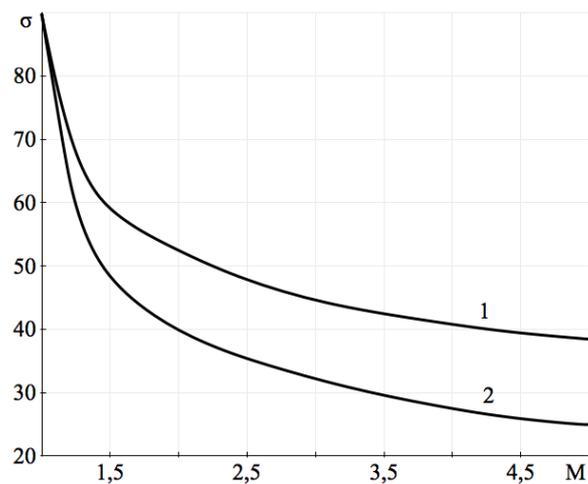
The rate of total pressure recovery  $I$  increases as the number of shocks in SWS grow (Figure 7). In the limit, when the system, composed of a series of oblique shocks of the same direction and a closing direct shock, is replaced by the isentropic compression wave (Bulat & Bulat, 2013; Bulat & Uskov, 2012; Bulat, 2014).

Figure 8 shows the dependence of the two angles of oblique shocks (in degrees) in the optimal triple shock wave structure.



**Figure 7.** The total pressure recovery ratio behind the shock system.

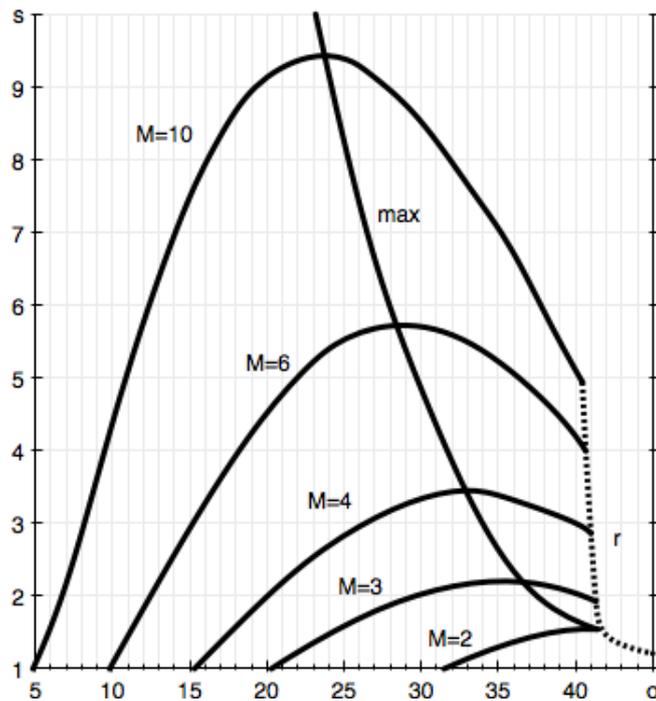
1 - Direct shock, 2 - one direct and one oblique shock, 3 - two oblique and one direct shock.



**Figure 8.** The angle  $\sigma$  of the first (1) and second (2) oblique shocks in optimum triple shock wave structure

### Air intake mixed compression

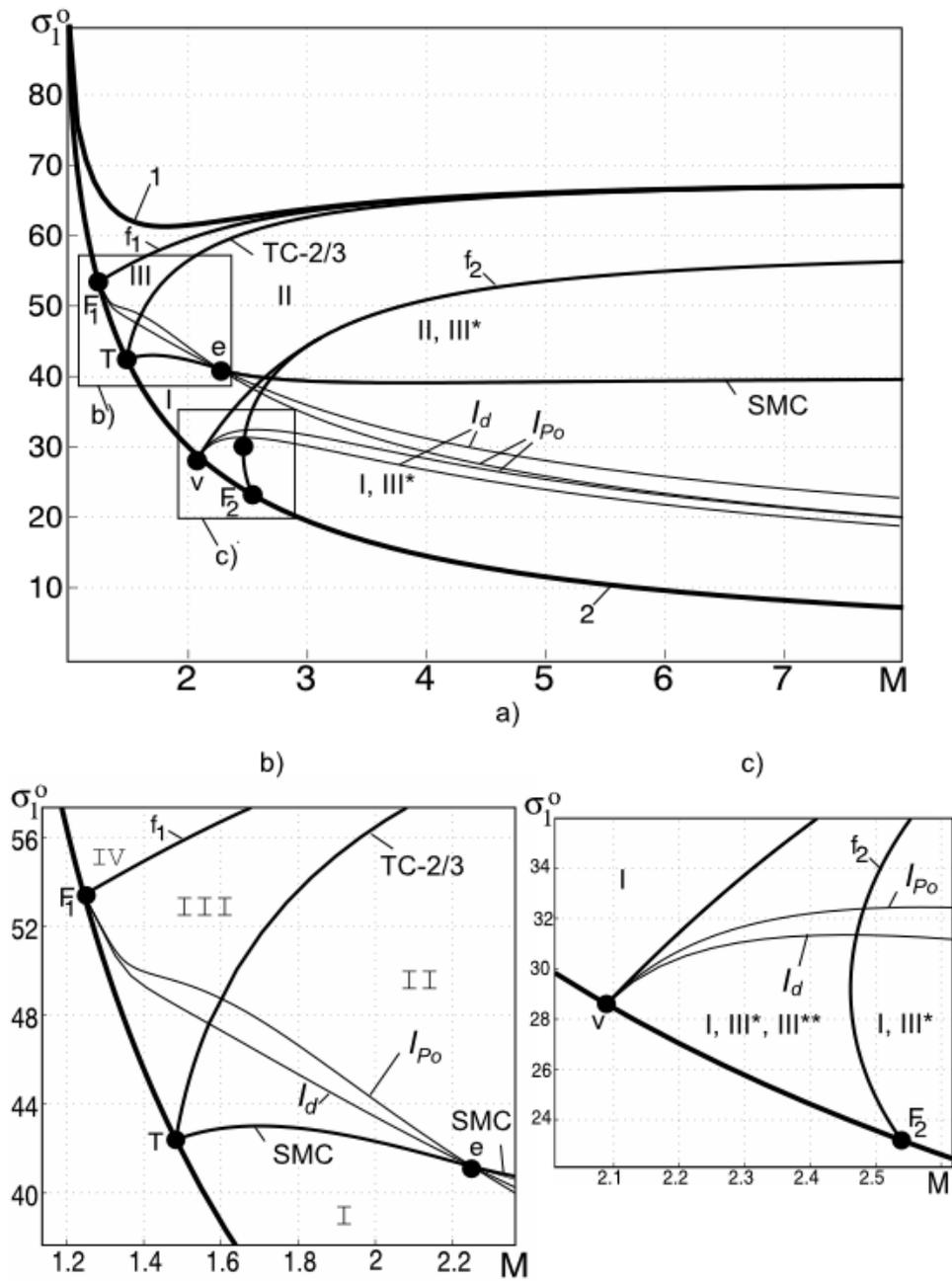
Figure 9 shows the reduction of the total pressure loss from the first shock wave angle (in degrees) compared with a normal shock wave in OSWS consisting of incident and reflected shocks (Figure 2). Dotted line  $r$  marked angles of the first shock, in which there is a transition from regular to Mach reflection of shock from the wall. Max line marked angles shock corresponding optimal OSWS providing the highest recovery rate of the total pressure. Note that if one oblique shock front gives a direct increase in  $P_{02}$ , for example, for  $M = 3$  is 1.8 times, the reflection from the wall to the right oblique shock  $P_{03}$  increases 2.25 times.



**Figure 9.** The dependence of the degree of increase in  $S$  stagnation pressure of the system fall-present, reflected and direct shock waves from the angle of the first shock and Mach number  $M$ .

### Optimal triple configuration of shocks

The flow properties of the triple configuration determined by the system (21-22) and the relations to the shock waves. In the problem of finding OSWS using optimality criteria (26-32) control parameter is the angle of the first shock  $\sigma_1$ . Triple configuration, the optimal relative total pressures, as well as criteria (26-29) are shown  $I_{P0}$  curve in Figure 10.



**Figure 10.** Optimal triple configuration

When  $M > M_e$  optimal configurations belong to the first type. As you know, in the right of the line  $v-f_2$ , the plane polars there are two additional points of intersection of the main and secondary polar corresponding configuration overtaking shock waves in one direction (configuration  $III^*$ ,  $III^{**}$ ). From the point  $v$  in Figure 10 line-out  $I_d$  and  $I_{P0}$ , corresponding to the optimal triple configuration  $TC-3^*$ . Right line  $f_2$  second (lower) the intersection of polars degenerates.  $III^{**}$  triple configuration may also be optimal in Figure 10 that corresponds to the line  $v-f_2$ .

Configuration optimal for relations velocity heads correspond to the plane of the curve  $I_d$ . Other criteria are not considered here. Both curves describing the optimal triple configuration, go from one point  $f_1$  (Mach number  $M = 1.245$ ).  $f_1$  line is the boundary of the region of existence of triple configurations of shock waves. In the region III are the optimal configuration of the third type, and in the region II - second.

### Discussions

The main destructive factor in the explosion is a high velocity pressure, which is created by a moving gas behind the shock wave, not only by high static pressure, as is commonly thought. This property is used in many industrial processes. For example, there is a technology of metal surface layers' hardening using the particulate by launching them with a shock wave. The shock wave, traveling along the barrel of shock tube arrives into chamber, in which the launched particles are supplied and carries them with itself. The higher the velocity pressure behind the shock wave is, the higher is the efficiency of such device.

Optimal shock wave system for a given Mach number at the entrance of air intake is such system that provides maximum value of total pressure conservation.

Studies have shown (Uskov & Chernyshov, 2014; Bulat & Bulat, 2013) that the total pressure losses in the supersonic diffuser are decreased by replacing the direct shock with a system of weaker oblique shocks, for which the speed remains supersonic, with the closing weak direct shock that translates flow into subsonic one.

By replacing one shock with a series of weaker shocks, it is possible to optimize the SWS. In order to build a methodology for calculating such structures, let us consider how the parameters of the series of shocks are defined.

In a stream containing a series of shocks, gas-dynamic functions behind the  $n$ -th shock depend on the intensity of shocks and gas-dynamic functions of undisturbed flow.

The comprehensive study of extreme properties of supersonic jet flows, triple configurations of shock waves and cases of waves and discontinuities interference, typical for external flow around the aircraft was performed. A number of obtained relations are directly applicable to the problem of designing OSWS.

### Conclusion

Demonstrated tasks that require design shock-wave structure with extreme values of the coefficient of restitution of the total pressure, the relationship dynamic pressure, static pressure, temperature, acoustic constant of the medium - the acoustic impedance. The method of calculating the flow parameters for a series of shocks. The results are used to construct an optimal DPS for supersonic air intakes external and mixed compression. On the example of the air intake, working at off-design operation, analyzed the problem of finding the optimal triple configurations of shock waves. It is shown that the optimal configuration

of the triple criterion of the total pressure is also optimal for the criteria of the temperature difference, the density and acoustic constant of the medium. Other criteria, such as the difference velocity heads correspond to their optimal configuration. Built the region of existence of triple configurations of shock waves, which is marked by the optimal full pressure and velocity head configuration.

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### Disclosure statement

No potential conflict of interest was reported by the authors.

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