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Oblique Shock Reflection from the Wall

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ABSTRACT

Regular and Mach (irregular) reflection of oblique shock from the wall is discussed. The criteria for the transition from regular to irregular reflection: von Neumann criterion and the criterion of fixed Mach configuration are described. The dependences of specific incident shocks' intensities corresponding to the two criteria for the transition from regular to irregular reflection are plotted. The article demonstrates ambiguity region in which both regular and Mach reflection are not prohibited by conditions of dynamic compatibility. The areas in which the transition from one reflection type to another is only possible via shock, as well as areas of possible a smooth transition are described. The dependences of magnitude of this shock change in reflected discontinuity's intensity on the intensity of the incident shock are plotted. The article also provides dependences of reflected discontinuity's intensity on the intensity of incident shock falling on the wall for all types of reflections.

KEYWORDS
Shock, shock wave, Mach reflection, Neumann criterion, Neumann paradox

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Introduction

In XXI century the works on supersonic speeds aircraft were widely deployed (Gvozdeva, Borsch & Gavrenkov, 2012). The range of Mach numbers M = 1.3-1.6 was investigated for manned military aircraft, and of M = 3-4.5 for missiles with air jet engine (Gavrenkov & Gvozdeva, 2011). It was revealed that the shock-wave structure in the inlets at these modes behave themselves irregulary (Uskov, Bulat & Prodan, 2012). At high Mach numbers, there is a hysteresis, i.e. at the same flow Mach number two different shock-wave structures are observed (Bulat & Uskov, 2014). At low Mach numbers so-called irregular reflection of shocks from walls occur, which, in accordance with the theory, cannot exist on these modes. This caused a new wave of research in the field of shock wave interaction with each other and different forms of their reflection from the wall.

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The largest contribution was made by the Institute of Theoretical and Applied Mechanics (Ivanov et al., 2001, 2002; Ben-Dor et al., 2002). The current state of the issue is detailed in the monograph by G. Ben-Dor (2007).

For the first time the problem of shock wave reflection (for the non-stationary case) was considered in E. Mach (1978) and von Neumann (1943). They described two types of shock wave reflection from the inclined surface (Figure 1):

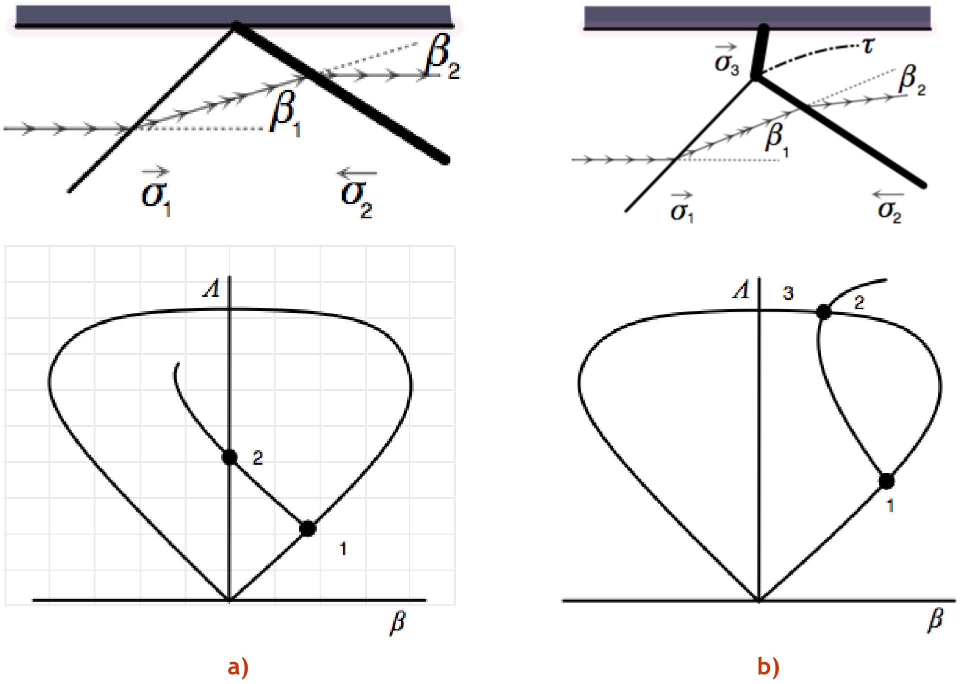


Figure 1. Regular (a) and irregular (b) reflection of oblique shock σ1 from wall

– *Regular reflection (RR)*, which consists of two shock waves: incoming wave incident on a solid surface, and the reflection wave, coming from the point of impact (Figure 1a);

– *Irregular reflection*, which consists of three shock waves - incident, reflected, and main one - having a common triple point (Figure 1b). Such kind of reflection is called Mach reflection (MR), and the corresponding configuration, if it contains no other normal discontinuities - a triple configuration (TC) of shock waves.

On figure 1: β_1 - flow rotation angle on incoming shock, β_2 - flow rotation angle on reflected shock, Λ - a logarithm of incoming shock's intensity (the ratio between static pressure behind the shock and the pressure before it), σ_1 - incoming shock, σ_2 -reflected shock and Mach stem σ_3 (main shock), respectively. The arrows in Figure 1 show the direction of shock (right or left). In the description of TC von Neumann suggested that tangential discontinuity (sliding surface τ) comes from triple point and separates the flow behind the reflected and the main σ_3 shock waves (Figure 1b).

The goal is to provide basic information about the problem of oblique shock reflection compaction from the wall, plane and the axis of symmetry (Uskov, Bulat & Prodan, 2012), as well as to perform parametric studies of intensity of incoming and outgoing discontinuities for various possible types of reflections. Regular and irregular (Mach) reflection are being discussed.

Method

The analysis of leading local and foreign scientists, who studied this range of problems, is used in the work. In this research, comparative analysis is applied on a wide scale.

Data, Analysis, and Results

Theoretical method of shock polars analysis, proposed in 1956 by R. Kawamura, and H. Saito (1956), has been applied to the analysis of irregular reflection. In the case of shock reflection from the wall on a polar plane the incoming oblique shock is corresponded by point 1 on the main polar (Figure 1 bottom), built by the Mach number M_1 of undisturbed flow.

At regular reflection (Figure 1a) the polar launched from point 1, corresponding to Mach number M_2 behind the shock 1 intersects the vertical coordinate axis at point 2, which corresponds to the reflected shock 2. In this case, the total rotation angle of the flow $\beta = \beta_1 - \beta_2 = 0$. The total stream compression degree by two shocks on a polars plane $\ln J_{1,2} = \Lambda_2 = \Lambda_1 + (\Lambda_2 - \Lambda_1)$, where Λ_1- the natural logarithm of intensity of the incident shock σ_1, and $(\Lambda_2 - \Lambda_1)$ - natural logarithm of intensity of reflected shock σ_2.

Mach number M_2 behind the first shock with the intensity of J is defined by the equation (Kawamura & Saito, 1956)

$$M_2 = \sqrt{\frac{M_1^2 - (1-E)(J+1)}{EJ}} \tag{1}$$

where E – shock adiabat of Rankine-Hugoniot

$$E = \frac{1+\varepsilon J}{J+\varepsilon} \, . \tag{2}$$

Von Neumann formulated the "criterion detaching", according to which the transition from regular to Mach reflection occurs when the secondary polar touches the vertical axis (Figure 2a and d). In his honor, this criterion is called von Neumann criterion.

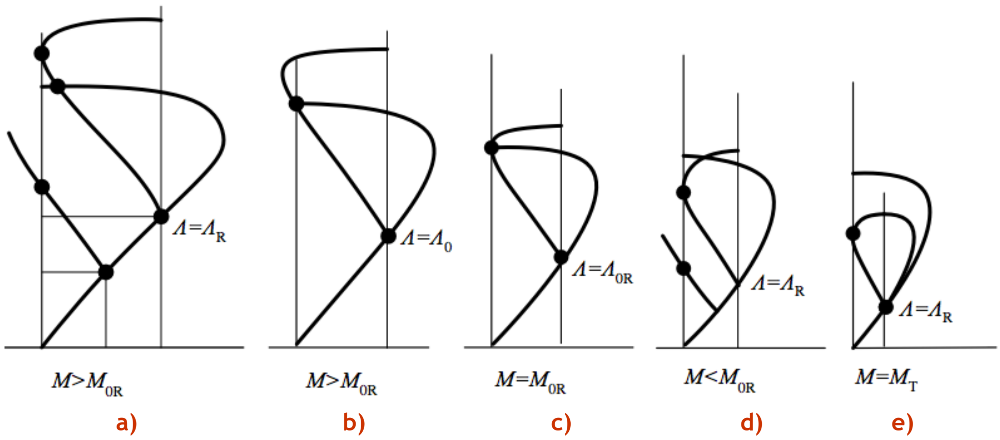


Figure 2. The transition from regular to Mach reflection

a - the transition in accordance with von Neumann criterion at high Mach numbers; b - the transition in accordance with the criterion of stationary Mach configuration, c) a case of special Mach number $M = M_{0R}$ in which both criteria are the same, d - transition in accordance with von Neumann criterion at low Mach numbers, e - limit Mach number $M = M_T$, limiting the area of triple configurations of shock waves' existence. Λ_R - special intensity of incoming shock at which the transition to Mach reflection in accordance with the criterion of von Neumann occurs, Λ_0 - special intensity of incoming shock at which the transition to Mach reflection in accordance with the criterion of stationary Mach configuration occurs, $\Lambda_{0R} = \Lambda_R = \Lambda_0$ - special intensity at $M = M_{0R}$.

For a given Mach number, the touch of ordinates axis by the polar is corresponded by shock intensity 1 equal to J_R (Bulat & Uskov, 2014), which is determined by solving the following equation:

$$\sum_{n=0}^{3} A_n x_R^n = 0, x_R = (1+\varepsilon)M^2 / \left(J_R + \varepsilon\right) \tag{3}$$

where

$$\begin{aligned}
&A_0 = -(1-\varepsilon)^2 L^4, L = (J_R - 1) / (J_R + \varepsilon),\\
&A_1 = 2(1-\varepsilon)(3-\varepsilon)L^2 - 4(1-\varepsilon)(1-3\varepsilon)L^3 + (1-\varepsilon)^4 L^4,\\
&A_2 = 2L^2(1-2\varepsilon-\varepsilon^2) - 4L - 1,\\
&A_3 = 1.
\end{aligned} \tag{4}$$

In equations (3) - (4) $\varepsilon = (\gamma-1) / (\gamma + 1)$, where γ is an adiabatic exponent equal to the ratio between specific heat capacity at constant pressure and the specific heat capacity at constant volume. Symbol A is introduced for convenience.

If the secondary polar does not intersect the y-axis, then the regular reflection is impossible, since limiting rotation angle of flow on the second shock is smaller than the angle on the first shock. In this case, triple configuration of shock waves forms, comprising a tangential discontinuity τ, which gradually turns the flow parallel to the wall. Pressure on the sides of the tangential discontinuity is equal and the velocity vectors are parallel, which is expressed in equalities $\Lambda 2 = \Lambda 3$ and $\beta 2 = \beta 3$. Mach stem bent and the logarithm of its intensity varies from $\Lambda 3$ to Λm. Λm point corresponds to the top of the main polar.

Neumann paid attention to the fact that in a certain range of wedge angles the shock polar intersects both the axis of ordinates and the upper branch of the main polar (Figure 2a and b), i.e. both regular and Mach reflection theoretically possible. Thus he suggested another criterion of transition from regular to Mach reflection, somewhat poorly calling it "the criterion of mechanical equilibrium". According to this criterion, the transition should occur at the moment when the shock polar crosses the isomah at its top, i.e., the intensity of the Mach stem in this case is equal to maximum for a given Mach number, which determines the Mach number isoline (Figure 2b). The shock-wave structure (SWS), arising during this is called stationary Mach configuration (SMS). That is why this criterion was named the SMC criterion by V.N. Uskov (Adrianov, Starykh & Uskov, 1995). SMC is corresponded by the Mach disk, which forms during reflection of a hovering shock wave from the axis of symmetry in supersonic jet (Uskov et al., 2012).

For a given Mach number, the SMC is corresponded by the characteristic intensity J0, which is defined by the equation

$$\sum_{k=0}^{3} A_k J_0^k = 0 \tag{5}$$

where

$$\begin{aligned} A_3 &= 1-\varepsilon^2, \\ A_2 &= -\left(\left(1+\varepsilon-\varepsilon^2+\varepsilon^3\right)J_m+1+\varepsilon^2\right), \\ A_1 &= \varepsilon\left(1+J_m\right)\left[\left(1-\varepsilon\right)J_m-2\right], \\ A_0 &= \left(1-\varepsilon\right)J_m\left(J_m-1\right), \\ J_m &= \left(1+\varepsilon\right)M^2-\varepsilon. \end{aligned} \tag{6}$$

Clearly, there is a special Mach number M_{0R}, such that the secondary polar, launched from point 1 of the main polar touches the ordinates axis exactly at the top of the main polar (Figure 2c). This case is corresponded by the equation

$$\frac{4A_1\left(\dfrac{A_1}{A_0}-3\dfrac{A_2}{A_1}\right)^2}{9-\dfrac{A_1A_2}{A_0}} = 9-\frac{A_1A_2}{A_0}-4\left(3-\frac{A_2^2}{A_1}\right) \tag{7}$$

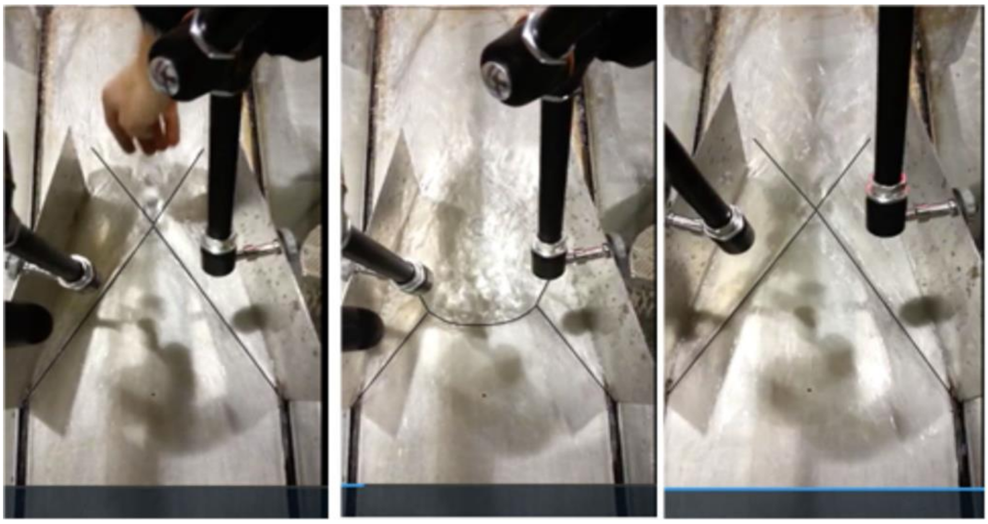
coefficients A_k are the same as in (4). At Mach numbers $<M_{0R}$, secondary polar touches the y-axis within the main polar (Figure 2d), so the transition from regular to irregular reflection occurs abruptly to the point of intersection of secondary polar's left branch with the subsonic region of main polar's right branch (Figure 2d).

At Mach numbers $M>M_{0R}$ secondary shock polar first touches the y-axis at its top (Figure 2b), and only then detaches from it (the highest point in Figure 2a). Accordingly, there are two theoretically possible transitions to irregular reflection:

– In accordance with the SMC criterion, with intensity of the incident shock equal to J_0, a smooth transition;

– In accordance with the detachment criterion, at intensity of incident shock equal to J_R, transition occurs abruptly at the intersection point of the main and secondary polars.

Subsequent studies have shown that a hysteresis often occurs, i.e. by increasing the intensity of the incident shock the transition from regular to irregular reflection occurs at intensity close to the detachment criterion, and with a decrease of shock's incline angle the transition occurs closer to SMC criterion. This issue is discussed in a large number of works, however, the question about the causes of hysteresis remained open. The authors conducted an experiment by hydro-analogy method and it clearly demonstrated that there is an area in which at the same flow parameters both and RR and MR can occur (Figure 3).



a) b) c)

Figure 3. Simulation of MR $\Leftrightarrow$ RR hysteresis by hydro-analogy method

Instead of reflection from the walls the interaction of two counter-shocks, generated by symmetrical wedges was studied. Such setup of experiment corresponds to the reflection of oblique shock from a symmetry plane. At the initial moment the reflection was regular. After creating a disturbance in the stream behind the shocks' intersection point with a hand (Figure 3a) the flow was transiting to Mach reflection (Figure 3b). Then a disturbance was brought into the stream before the Mach stem by a small increase in flow velocity. The flow was returning to regular shocks reflection (Figure 3c). Comparison of experiments with calculation showed that the transition from regular to irregular reflection takes place in accordance with Von Neumann criterion, i.e. when $J = J_R$, calculated for k=2 by the formulas (3) - (4).

For Mach numbers less than special number

$$M_T = \sqrt{\frac{2-\varepsilon}{1-\varepsilon}} \tag{8}$$

the solution for irregular shock wave reflection from the wall with formation of a triple point is absent, since secondary polar lies entirely within the main one at any intensity of the shock 1 (Figure 2e). However, it is observed experimentally (White, 1952).

This phenomenon is called Neumann paradox, since the first one to draw attention to it was, again, Neumann. For a long time, it was not possible to receive and numerical solution for this kind of flows until E.I. Vasilev (1999) has shown that the problem is the lack of numerical methods' accuracy, the influence of "circuit" computing viscosity and parasitic oscillations of solution, and the flow corresponds to a "four-wave" model of K.G. Guderley (1960). For this, a numerical method was used with the isolation of discontinuities (Vasilev & Olkhovsky, 2009). Nevertheless, the question remained open, because there are other explanations, such as non-stationary or three-dimensional nature of the flow on these modes, the arising of triple point of the curved mixing layer across which the pressure can be varied, etc.

Discussion

The following is an analysis of the reflected shock's intensity on the intensity of incident shock bath at RR, and at MR. Intensity of shock 1 varies from 1 Js, at which the stream behind this shock becomes subsonic.

$$J_S = \frac{M^2-1}{2} + \sqrt{\left(\frac{M^2-1}{2}\right)^2 + \varepsilon\left(M^2-1\right)+1} \quad . \tag{9}$$

For each adiabatic index a specific Mach number M_{0R} is defined, at which the intensity J_R, corresponding to von Neumann criterion, is equal to the intensity J_0, corresponding to SMC criterion. The results of the calculation are given in Table 1.

Table 1. Special Mach numbers

γ	1,67	1,4	1,25	1,1
M_{0R}	2,447	2,203	2,078	1,952

Gas was considered thermodynamically and calorically perfect. The results of these calculations are presented in Figure 4.

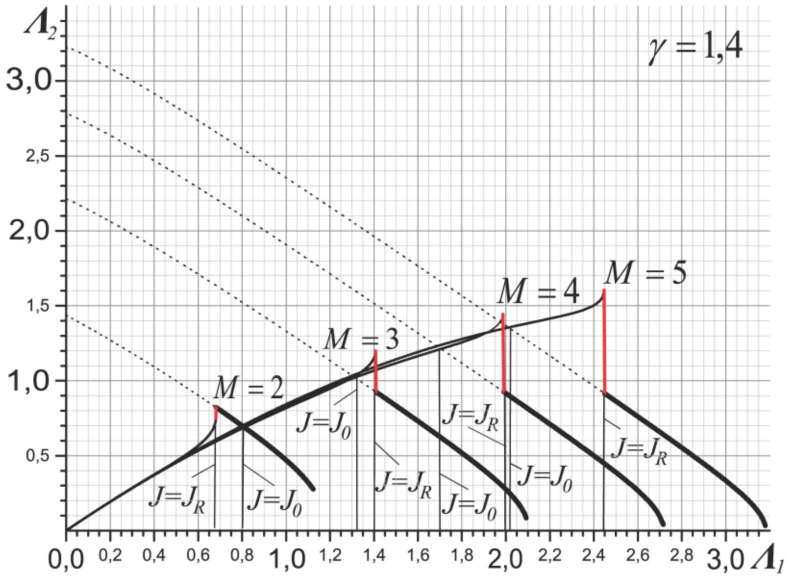


Figure 4. Dependence of reflected shock's intensity on the intensity of incoming shock

On figure 4: solid line - the intensity of the reflected shock, red line - the shock intensity of the reflected discontinuity during transition from regular to irregular reflection, dotted line - during regular reflection of incoming shock "fictitious" intensity of reflected shock, corresponding to intersection point of main polar and the polar of incoming shock. We see that in the region, in which

RR happens the secondary polar intersects with the y-axis (solid line) and a subsonic part of the main polar (dashed line). If the base stream Mach number $M<M_{OR}$,, then $J_R<J_0$ (M = 2 in Figure 4). When $J_1=J_R$ the shock wave system abruptly transfers to M_R, but the intensity of reflected shock J_2 increases (red line in Figure 4). If the base stream Mach number $M> M_{OR}$, then $J_R> J_0$ (M = 3,4,5 in Figure 4). When $J_1 = J_R$ shock wave system also abruptly transfers to M_R, but the intensity of reflected shock J_2 decreases (red line in Figure 4 for M = 3,4,5). Thus, depending on the Mach number of the main stream the intensity of reflected shock can abruptly increase or decrease (Figure 5). In theory, the transition from RR to RR is possible according to the SMC criterion at $J = J_0$. These intensities are marked on Figure 4, the thin vertical lines. It is clearly seen that there is no abrupt change in the intensity of the reflected shock, i.e. the transition from RR to MR is smooth. Thus, between the vertical lines J_R, J_0 an ambiguous solutions region is enclosed when both RR and MR are theoretically possible. For this important area, in which non-stationary and hysteresis phenomena are possible the boundaries for major γ are plotted. The diagram contains the special values of Mach numbers. Recall that when $M <M_{OR}$ SMC cannot exist, respectively, J_0 is not defined here. With increasing Mach number the ambiguous solutions region expands and the value of abrupt change in intensity of reflected shock growth as well, which creates the preconditions for emergence of oscillations. At Mach numbers $M <M_T$, triple configurations of shock waves theoretically should not occur, but are observed experimentally.

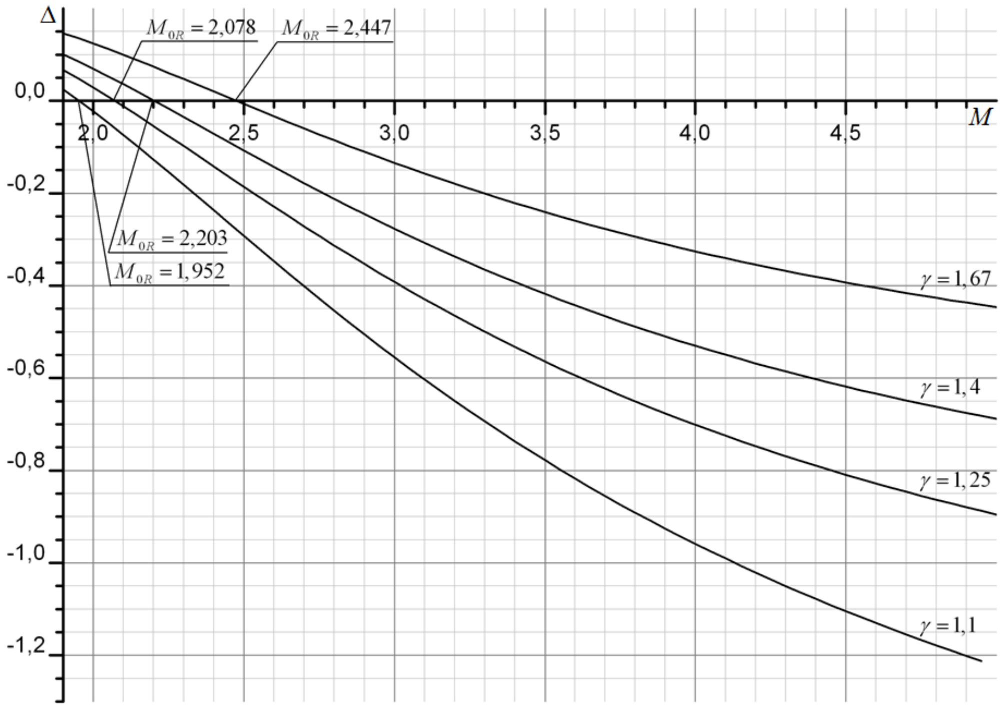


Figure 5. Dependence of the intensity difference of the reflected shock wave on the Mach number during abrupt transition from RR to MR in accordance with the detachment criterion (Von Neumann criterion)

Conclusion

The article discussed the regular and irregular reflection of a shock from a flat wall. At low shock waves' intensities (Mach number M <M_T) theory predicts an impossibility of shock wave's Mach reflection from an obstacle, but it is observed in the experiments. This phenomenon is known as von Neumann paradox. There are areas of ambiguous solutions. In these areas, the theory does not forbid the existence of both regular and Mach reflection. With the increase of Mach number the width ambiguous solutions areas increases. The transition from regular to Mach reflection can take place smoothly when the intensity of the incident shock reaches a specific intensity of J_0, when the secondary shock polar crosses the main polar at its top.

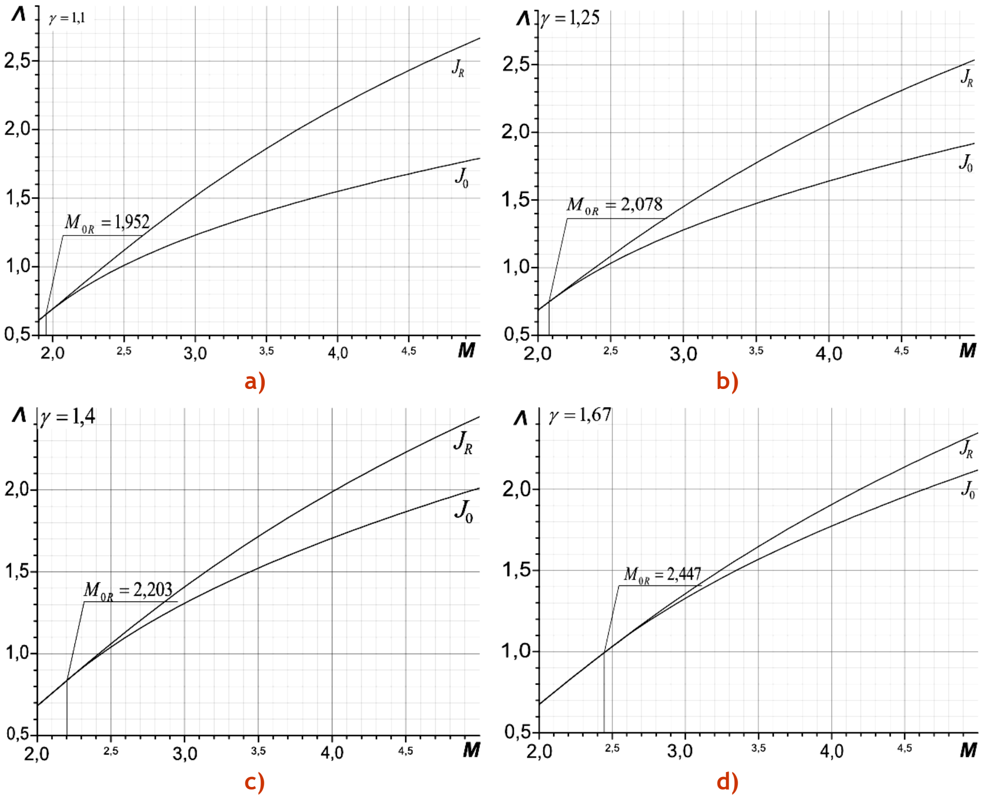


Figure 6. The area of ambiguity, in which both RR and MR of shock from the wall are possible

Such a transition is possible only when the Mach number is greater than specific number M0R, when secondary polar's point of contact with the y-axis coincides with the top of the main polar. During transition to Mach reflection in accordance with the detachment criterion (secondary polar touches y-axis) the restructuring of shock-wave structure is abrupt, i.e. the intensity of the reflected shock changes abruptly, when M <M_{0R} it abruptly increases, and when M <M_{0R} - abruptly decreases. Experiments show the hysteresis of reflection characteristics depending on the direction of shock's incident angle change (increase or decrease). All these questions require further theoretical and experimental study.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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