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Interference of Same Direction Shocks

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ABSTRACT

The article considers the interference of shock of the same direction or, as they are called, catching-up shock waves. Purpose is to give a classification to the shock-wave structures that arise in this type of shocks interaction, and to determine the area of their existence. As a result of same direction shocks' intersection a shock-wave structure forms ate the intersection point, containing the main shock, tangential discontinuity and one more reflected gas-dynamic discontinuity, the type of which is not known beforehand. The problem of determining the type of reflected discontinuity is the main problem, which must be solved in the study of catching-up shocks' interference. The paper presents qualitative picture of shock-wave structures arising from the interaction of catching shock. The areas in which there is a regular and irregular interaction catching shocks. There are also areas in which the stationary solution is not available. The latest factor has determined the revival of interest in the theoretical study of given problem, because the facts of shock-wave structure's sudden destruction inside the air intake of supersonic aircrafts at high Mach numbers were discovered. Is also relevant to investigate the possibility of using catching-up oblique shock waves to create an over-compressed detonation in promising detonation air-jet and rocket engines.

KEYWORDS
Shock one-direction shocks, catching- up shocks, shock-wave structures, shocks interference, gas-dynamic discontinuities

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Introduction

Purpose is to give a classification to the shock-wave structures (SWS) that arise at the interaction of oblique same-direction shocks, to identify the existence areas of different types of SWS, to formulate a criterion for determining the type of reflected discontinuity, to describe the qualitative flow pattern at regular and irregular intersection of shocks (Solovchuk & Sheu, 2010). It is most simple to imagine the interaction of gas-dynamic discontinuities (GDD) of same direction in one-dimensional case, where one shock wave catches up with another (Figure 1a) (Taniguchi et al., 2014),. By analogy with the one-dimensional case the intersecting oblique shocks of same direction, i.e. such that turns the flow in the same direction, are called "catching-up" (Figure 1b) (Cai, Fan & Li, 2015). There can be several of such shocks in SWS and the more of them exist, the closer to

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such a configuration is to a centered isentropic compression wave (Figure 1c), which is an ideal structure for compression of supersonic flow and is widely used in the design of hypersonic air inlets (Bulat & Uskov, 2012).

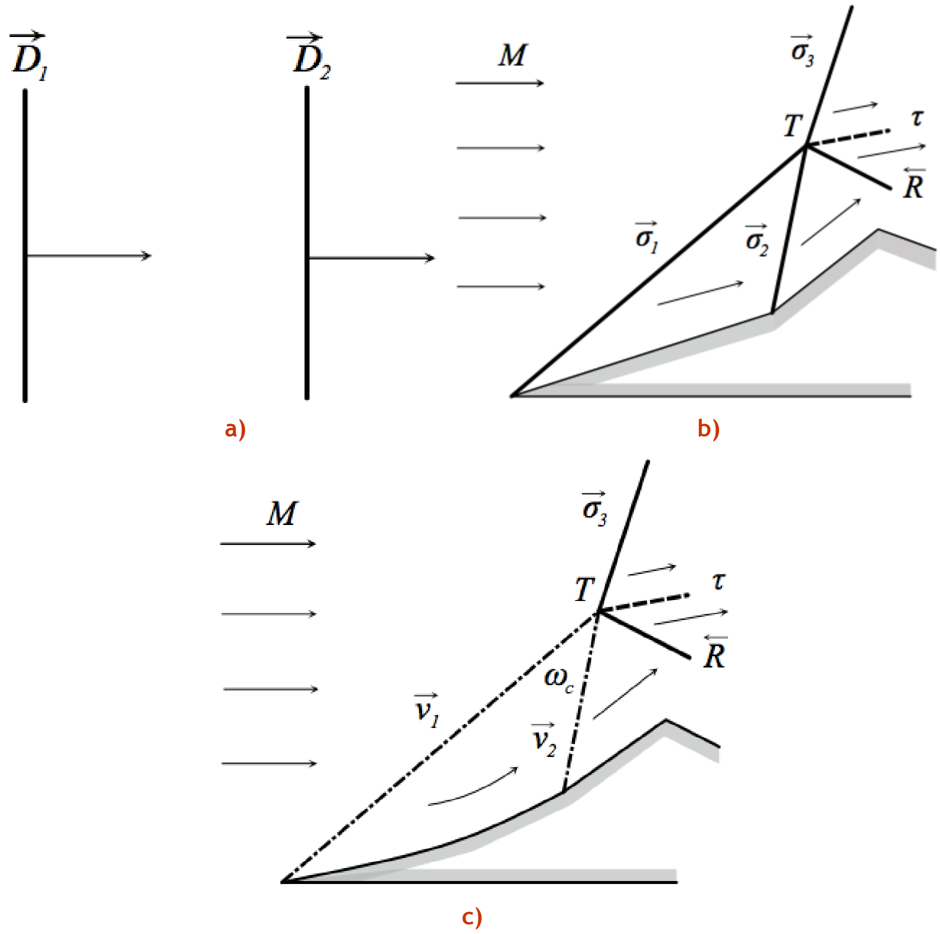
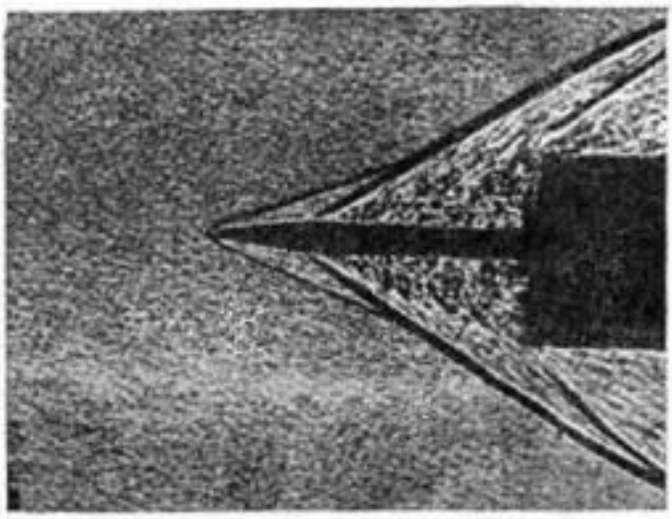


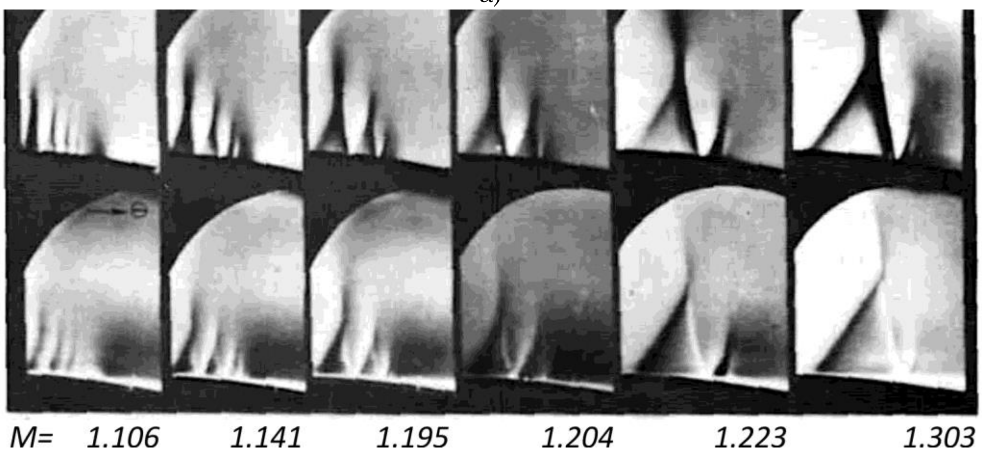
Figure 1. Interference of one-direction discontinuity. a) catching-up one-dimensional shock waves D_1 and D_2; b) catching-up oblique shocks; c) centered isentropic compression wave, M - Mach number, T - point of shocks' intersection, R - reflected discontinuity, σ_1, σ_2 - catching-up shocks , σ_3 - the main shock, τ - tangential discontinuity

The first studies of catching-up shock (CUS) appeared in the 50s and 60s of XX-th century and have been associated with the emergence of two-and three-shock air inlets of supersonic aircraft (Vekken, 1950; Roslyakov, 1965). V.N. Uskov (Adrianov, Starykh & Uskov, 1995) and A.L. Staryh (Uskov, Roslyakov & Starykh, 1987) made a great contribution to the development of the theory of shock catching-up shocks and to the identification of existence areas of different shock-wave structures (Bulat, 2013; Bulat et al., 2014a, 2014b) with their participation. A.V. Omelchenko and V.N. Uskov (1999) investigated the behavior of gas-dynamic variables behind SWS with oblique catching-up shock waves for extremum. V.N. Uskov (2000a, 2000b) fully solved the problem of the interaction of one-dimensional shock waves, moving in same direction (Uskov, 2000a), including those corresponding to some optimality criterion (Uskov, 2000b).

By the early 2000s it seemed that this phenomenon is studied almost completely. However, many details remain fragmented, for example, the existence area of different cases of CUS interference were studied only for certain Mach numbers (Uribe, 2016). The theory of transition from SWS of one type to another was also absent. Discovery of non-stationary processes during the flow around the head parts with a needle, extended into the flow of the (Figure 2a), and the study of the so-called Neumann paradox forced to return to CUS phenomenon (Vasiliev, Elperin & Ben-Dor, 2008). The essence of Neumann paradox is that at Mach numbers close to one the shock waves triple configuration cannot theoretically exist (Uskov & Chernyshov, 2006), but, nevertheless, they are visually observed (see Figure 2).



a)



b)

Figure 2. von Neumann paradox
a) catching-up shocks in the flow around the head of the aircraft with a needle extended into in the current, b) transonic flow around the profile.

Materials and Methods

The intensity of the reflected discontinuity J4 is determined from the equations

at $\bar{\sigma}_3$: $\beta_\sigma(M,J_4)+\beta_\sigma(\hat{M}_2,GJ_4)=\beta_\sigma(M,J_1)+\beta_\sigma(\hat{M}_1,J_2)$; (1)

at $\bar{\omega}_3$: $\beta_\sigma(M,J_4)-\beta_\omega(\hat{M}_2,GJ_4)=\beta_\sigma(M,J_1)+\beta_\sigma(\hat{M}_1,J_2)$; (2)

where

$$G=(J_1J_2)^{-1}\ ;\ J_1\in[1;J_s]\ ;\ J_2\in[1;J_{2s}], \quad (3)$$

$\hat{M}_i$ - Mach number of the i-th shock wave.

The conditions under which the secondary polar is located inside the main polar or outside it. Secondary polar $P_\sigma(\hat{M}_1)$ can be inside or outside the polar $P_\sigma(M)$, depending on the values of the derivatives $\Lambda'=(\partial\Lambda/\partial\beta)$ at the current point 1 of main polar $\Lambda_1'=(\partial\Lambda/\partial\beta)_1$ at the origin of secondary polar emanating from this point. Because

$$\Lambda_1'(\hat{M}_1)=\frac{(J+\varepsilon)\left[\gamma M^2-(J-1)\right]-(1+\varepsilon J)(J-1)}{\left[J(1+\varepsilon J)\left[(J+\varepsilon)(M^2-1)-(J^2-1)\right]\right]^{1/2}} \quad (4)$$

where $\varepsilon=(\gamma-1)/(\gamma+1)$, γ - adiabatic index and Λ' is defined by the formula

$$\frac{\partial\Lambda}{\partial\beta}=\left(J\frac{\partial\beta}{\partial J}\right)^{-1} \quad (5)$$

then, equating derivatives, we obtain a cubic equation for technical variable $x = J_f - \varepsilon$, where J_f – shock intensity at such point on the polar, at which the reflected discontinuity 4 degenerates into a characteristic (Figure 4b), with coefficients

$$\sum_{n=0}^{3}A_nx^n=0$$

$$A_3=1\ ;\ A_2=\left[(\gamma-1)^2-2\right]M^2+2\gamma-3\varepsilon;$$

$$A_1=(2-\gamma)\gamma M^4+2\left[(\gamma-1)\varepsilon-1\right]M^2+1+4\varepsilon+3\varepsilon^2;\ A_0=-\varepsilon(1+\varepsilon+\gamma M^2)^2. \quad (6)$$

Such points are called characteristic ones. The polar can have multiple characteristic points. Equation (6) written with regard to M^2 is square

$$aM^4+bM^2+c=0\ ;$$

$$a=\gamma\left[(2-\gamma)(J_f+\varepsilon)-\gamma\varepsilon\right];$$

$$b=\left[(\gamma-1)^2-2\right]\left(J_f+\varepsilon\right)^2+2\left[(\gamma-1)\varepsilon-1\right](J_f+\varepsilon)-2\gamma\varepsilon(1+\varepsilon);$$

$$c=(J_f+\varepsilon)^3+(2\gamma-3\varepsilon)(J_f+\varepsilon)+(2\varepsilon^2+4\varepsilon+1)(1+\varepsilon). \quad (7)$$

Discriminant $\Delta(\gamma;M)$ of the equation (7) turns to zero at $M=M_\Delta(\gamma)$. If $M<M_\Delta$, then there may be only one characteristic point f at $P_\sigma(M)$. If $M>M_\Delta$ >M then of the three real equation roots we must discard those that do not satisfy the inequalities $1<J_f<J_m$. Assuming Jf = 1in (7), we obtain the boundary Mach numbers that identifies the existence of characteristic points on the polar

$$(5-3\gamma)M^4-4(3-\gamma)M^2+8=0\,. \tag{8}$$

Whence

$$M_{f_1}=(a_1-b_1)^{1/2};\ M_{f_2}=(a_1+b_1)^{1/2}, \tag{9}$$

Where

$$a_1=\frac{2(3-\gamma)}{5-3\gamma};b_1=\frac{2\sqrt{\gamma^2-1}}{5-3\gamma}\,. \tag{10}$$

The above mathematical model can be used to find existence areas of SWS with different types of reflected discontinuities. It is possible to plot curves $M_f(\gamma)$ and $J_f(M,\gamma)$, corresponding to the characteristic of SWS, they will separate the areas with various types of reflected discontinuity.

The analysis of derivatives Λ' and Λ_1' in various existence areas of characteristic points f_i allows to conclude that at

$$\left[\begin{array}{l}\Lambda'>\Lambda_1';\\ J_1\notin\left[J_{f_3},J_{f_2}\right]\cup\left[J_{f_1},J_s\right]\end{array}\right. \tag{11}$$

the beginning of first polar $P_\sigma(\hat{M}_1)$ passes inside $P_\sigma(M)$ and at

$$\left[\begin{array}{l}\Lambda'<\Lambda_1';\\ J_1\notin\left[1,J_{f_3}\right]\cup\left[J_{f_2},J_{f_1}\right]\end{array}\right. \tag{12}$$

- outside of the main polar. Thus, the conditions (11-12) determine the type of reflected discontinuity in WSW with CUS at intensities of incoming shock close to the characteristic.

For γ = 5/4 within the range $M\in\left[M_{f_1},M_{f_2}\right]$ only one characteristic point f_1, and when $M>M_{f_2}$ - two points: f_1 and f_2. When γ = 5/4, there is an area $M_\Delta<M<M_{f_2}$, in which the polar compression are three characteristic points (Figure 7). In the range $M>M_{f_2}$ lower points f_3 are absent for any value of γ. Specific Mach number are summarized in Table 1.

Table 1. Specific Mach number at different adiabatic index

γ	1.1	1.25	1.4	1.67
M_Δ	1.62	2	2.46	4.125
M_{f1}	1.302	1.265	1.245	1.224
M_{f2}	1.666	2	2.54	∞

Results

One of the three possible SWS, shown in Figure 3 is formed. The main objective during the study of CUS regular interaction is to define the type of reflected discontinuity: a shock (see. Figure 3a), discontinuous parameters (see. Figure 3b) or centered rarefaction wave (see. Figure 3 c). The problem can be solved geometrically on the plane of polars. The original polar corresponds to the Mach number of incoming stream. If, on this the polar such point 1 is chosen, that the flow rotation angle β and the shock intensity J will be, respectively, equal to the flow reversal angle and the first shock intensity, then the shock polar released from point 1, will describe the flow behind this shock.

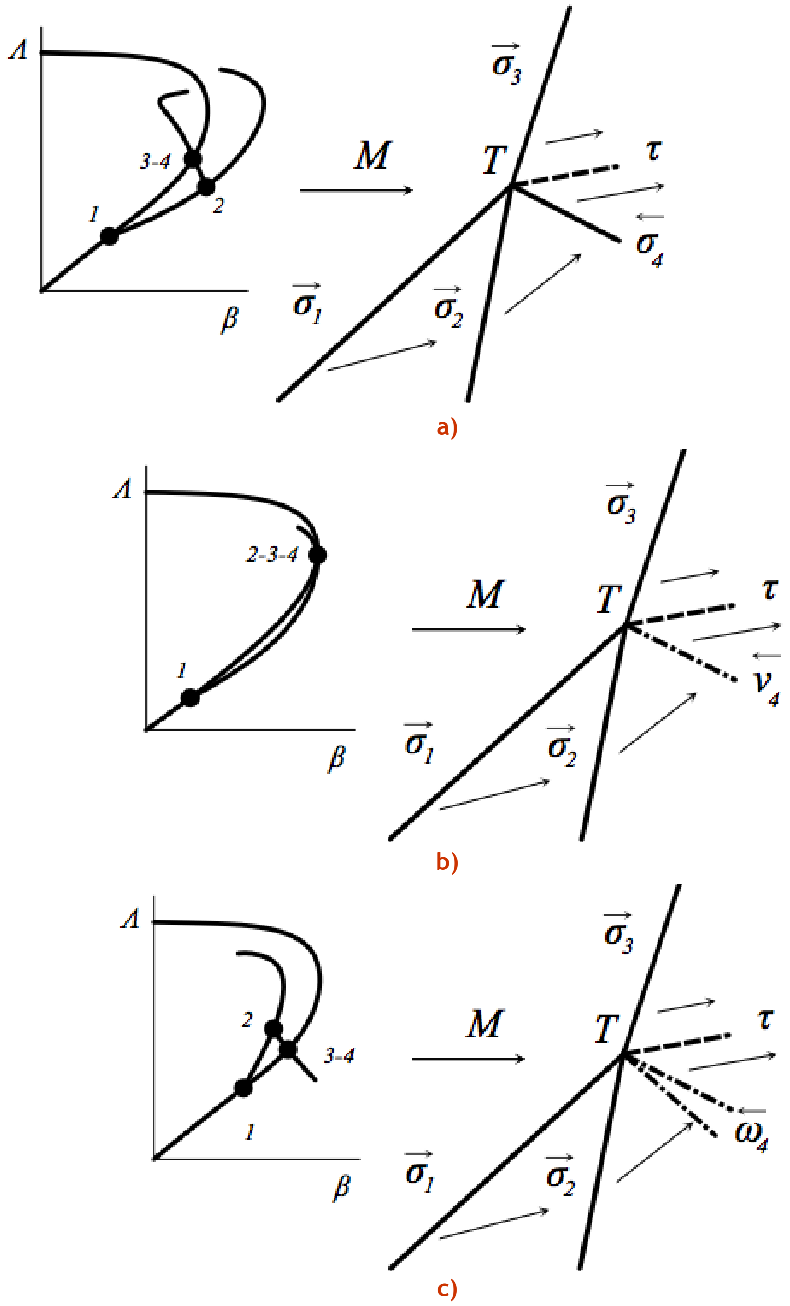


Figure 3. SWS and the solutions on the plane of polars with regular interaction of CUS. M – the Mach number in front of SWS with CUS, T - intersection point of gas-dynamic discontinuities, B - flow deflection angle at the discontinuity, $\Lambda = lnJ$ - logarithm of the discontinuity's intensity, σ - the shock, τ - tangential discontinuity

At this secondary polar the point 2 determines the intensity and flow rotation angle at the second catching-up shock. If the second polar is located outside the main polar, then the reflected discontinuity will be a shock, with parameters at point 3-4 in Figure 3a. If the second polar passes in the main, the reflected break - rarefaction wave, the intensity of which is defined as the point on the plane polar 3-4 in Figure 3c. These two cases are separated by characteristic SWS (Figure 3b), when the reflected discontinuity degenerates into a discontinuous characteristics.

This SWS corresponds to shock waves triple configuration of the TK-3 type (Uskov & Chernyshov, 2006). Algorithm for solving the problem of CUS interference for the case of regular interaction is quite obvious. It is only necessary to analyze the location of a point corresponding to the secondary polar on the main polar, relatively to characteristic points (Figure 4).

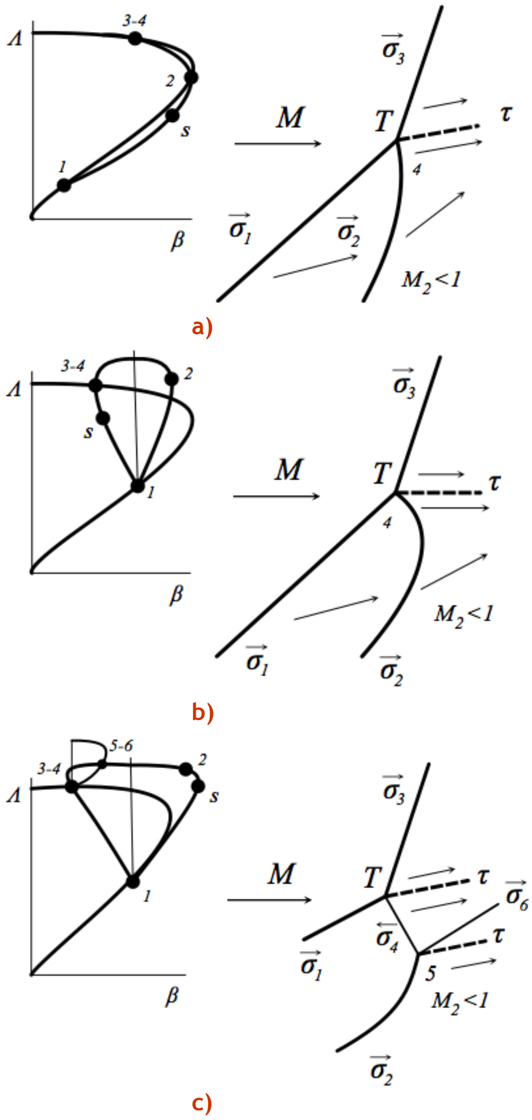


Figure 4. SWS and solutions on polar plane with irregular interaction of CUS at subsonic flow behind the second incoming shock

For this purpose, equations (6) - (9) are used. Situation is much more difficult, if the solution of equations (1) - (2) is absent. Triple points are formed on the incoming shocks and the interference becomes irregular. The cases of the flow behind the shock 2 being subsonic are considered on Figure 4.

SWS, shown in Figure 4a, belongs to triple configurations TC-3. Point 2, corresponding to the second shock, is located on the secondary polar above the sonic point s, in which the Mach number behind the shock is equal to 1, and the intensity of the shock $J = J_s$. Second catching-up shock curves. Its intensity changes from the parameters corresponding to point 2 to the parameters at point 3-4 in which the pressure and direction of velocity vectors on both sides of tangential discontinuity must be identical.

Depending on the M number and intensity of shocks 1 and 2, the second shock can curve so much that it changes its direction (Figure 4b). In this case, SWS belongs to the TC-2 class of triple configurations, and the point 3-4, corresponding to parameters at the triple point T is lying on the left branch of secondary polar. If the flow behind the shock 4 at the triple point T is supersonic, and behind the second incoming shock is subsonic, then SWS, shown in Figure 5c is formed. The mechanism of transition from the SWS shown in Figure 5b, to SWS in Figure 4c, and vice versa, has not been studied.

If the flow behind second catching-up shock is supersonic, but the solution of equations (1) and (2) is absent, then SWSs shown in figure 6 can form. If the flow behind shock is 4 subsonic (Figure 5a), then a triple point 5 is formed, in which incoming shock 2 and reflected shock 6 intersect.

With an increase in the M number the flow behind shock 4 may become supersonic. Then shock 2 shock 4 becomes counter to each other and SWS, typical for interference of counter shocks is formed (Figure 5b). The shock T4, emanating from the point T is counter relatively to shock 2 and incoming one for SWS composed of two triple points 5 and 8. With further increase of M the interaction of shocks becomes regular and points 5 and 8 are merged into one. Graph showing the existence area of characteristic SWS is shown in Figure 6.

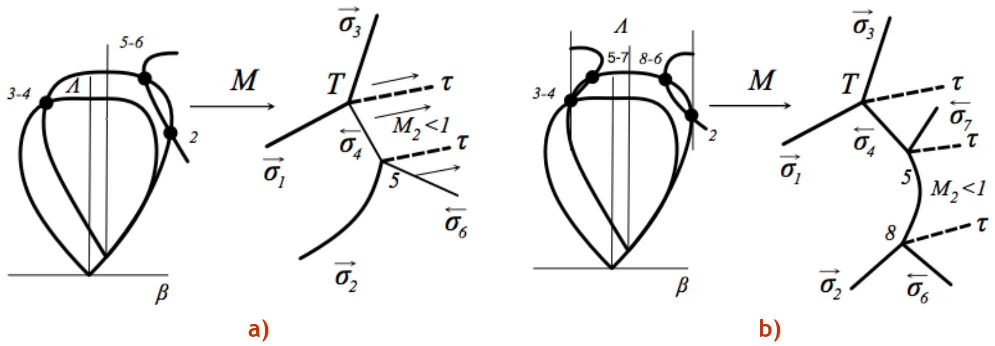


Figure 5. SWS and solutions on polar plane with irregular interaction of CUS at supersonic flow behind the second incoming shock

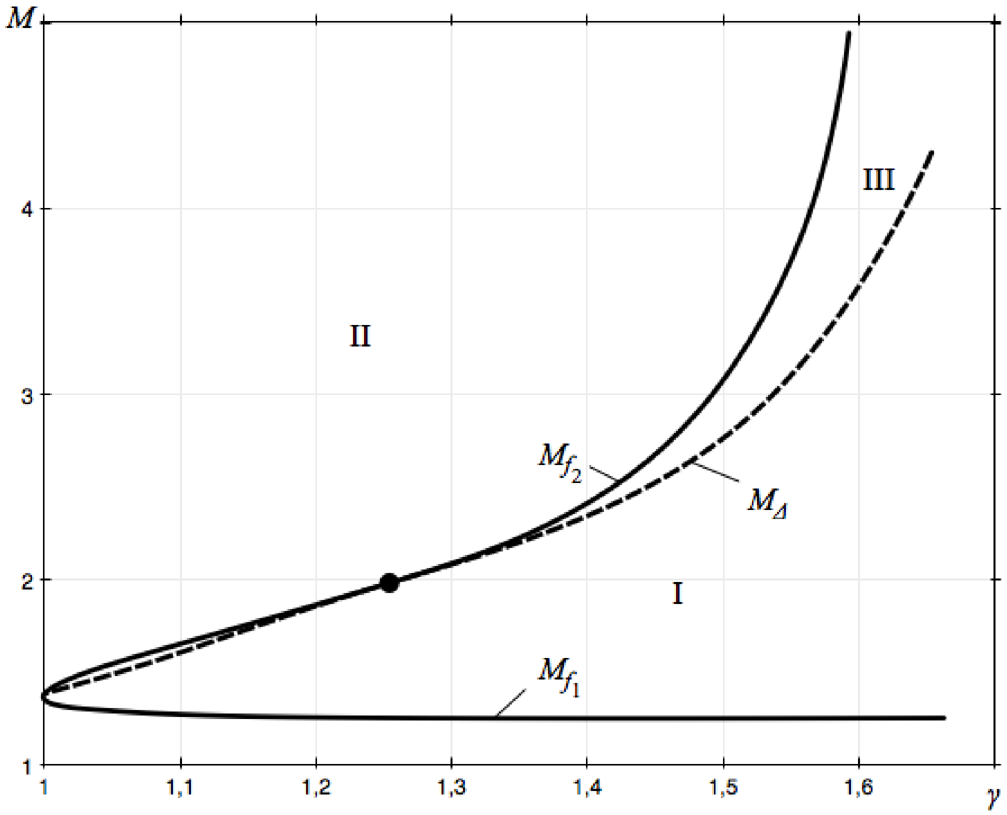


Figure 6. Dependence of characteristic Mach number on the adiabatic index. I - one characteristic point f_1, II - two characteristic points f_1 and f_2, III - three characteristic point f_1, f_2, f_3

Discussions

If the intensity of shocks 1, 2 (Figure 1) and the Mach number M are such that the flow behind the second shock is supersonic, the interference of CUS is regular.

It is clearly seen that at various Mach numbers there can be one to three characteristic points on the shock polar. Figure 7 shows the dependence of shock intensity on the M number in system of catching-up shocks, at which the characteristic SWS is formed. It is these SWS that are optimal by the criterion of degree of total pressure reduction. The box "A" shows the curves corresponding to characteristic points f_1 in enlarged scale. Dashed discriminant curve, corresponding to the vanishing of the discriminant in equation (7), separates the characteristic curves into two branches: the upper f_2 and lower f_3. Mach numbers M_Δ limit the existence area of the characteristic SWS f_2 and f_3 at the left. When $\gamma < 1.2$ discriminant curve goes into $J < 1$ area, hence only the upper branch of the characteristic curve f_2 exists. Thus, depending on the Mach number and the adiabatic index the characteristic SWS can arise at one, two or three points on the shock polar.

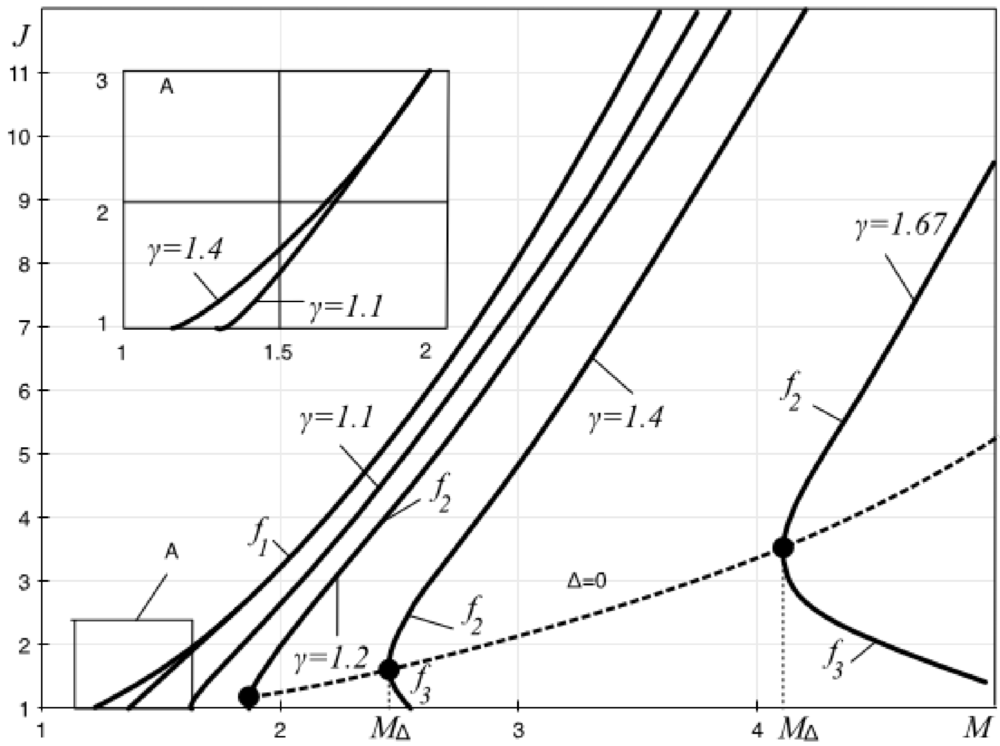


Figure 7. Characteristic points on the shock polar

This question arose particularly acute in the study of supersonic jets outflow from nozzles with Mach numbers less than 1.5 (Figure 8). Such nozzles are used today on supersonic military aircraft. The problem is that in this case the reflected shock wave is, strictly speaking, the one incoming, not outgoing. Consequently, the reflection of the symmetry axis cannot be the cause of its occurrence. Incomprehensible are as well the mechanisms of SWS transformation when changing Mach number within this range. At the same time, it is known that at small Mach numbers the supersonic jets have an increased level of acoustic emission, which indicates non-stationary flow pattern. All these issues require detailed study.

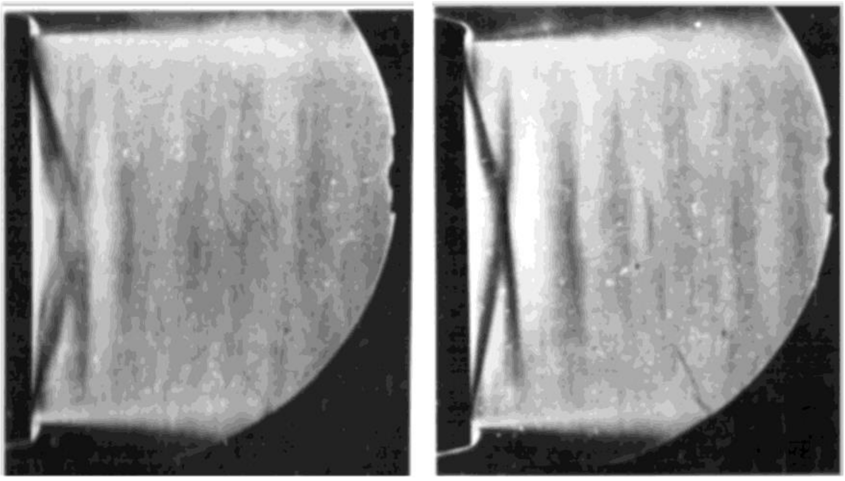


Figure 8. Reflection of shocks from the symmetry axis of supersonic jet flowing out of a nozzle with a Mach number close to one

The existence conditions of characteristic SWS separating the cases with reflected discontinuity - shock and the reflected discontinuity - rarefaction wave. This task of resembles the analysis of SWS arising in the center of the compression wave (Bulat & Bulat, 2013; 2014).

Obviously, at such point on the main polar that corresponds to characteristic SWS the primary and the secondary polars must have the same inclination angle, i.e. the order of polars contact is not less than first.

Conclusion

The paper provides studies results of qualitative flow pattern, which arises during catching-up shocks interaction. This is a problem of considerable importance in studying the flow at a nozzle section with Mach number not much greater than one, as well as in supersonic air intake, and around the heads of aircraft with the needle extended into the stream. The interaction of these shocks can be regular or irregular with the formation of triple points. There are areas in which a stationary solution is absent at all. This seriously complicates the work of supersonic air intakes on certain regimes. Transformation laws of SWS, which arise due to the interference of CUS have not been studied thoroughly. Separate studies to determine the mechanism of SWS transformation from one type to another during the change of flow parameters are required.

Characteristic SWS, in which reflected discontinuity degenerates into discontinuous characteristic, are of the greatest interest. These SWS have a number of extreme properties. In this work the existence areas of such SWS were found.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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