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An Algorithm to Renegotiate Debt through Equivalent Equations and Transaction Costs: A Proposal for the Field of Financial Education

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ABSTRACT

Frequently, the shadow of financial insolvency is present in economies, whether between countries, companies or the general population (people) who contract the obligation of a loan, credit or other debt. Debt restructuring is one of the most effective financial tools to face this. Through mathematical modeling with algorithms designed for the inclusion of the variable transaction cost coefficient (TCC) and transaction cost amount (TCA), it was possible to demonstrate the feasibility to modify: V_{OD} to $V_{OD_{adjusted}}$ and $Y_{1...j} = \frac{V_{OD}}{V_{NSP}}$ to $Y"_{1...j} = \frac{V_{OD_{adjusted}}}{V_{NSP}}$.

Keywords: equivalent equations, debt restructuring, transaction cost, debtor, creditor

INTRODUCTION

Once again, this document addresses the issue of economic and financial insolvency, which businesses and individuals often face. This situation is aggravated when it comes to meeting commitments with financial institutions or suppliers of products and services.

Therefore, the debt contracted by debtors (customers) with creditors becomes a headache debtors often face. In most cases, cash flows do not give the economic margin to be able to face these commitments; hence, the renegotiation or restructuring of debt is a possible solution to this situation.

The owner of the debt portfolio -the creditor- must carry out actions to recover the capital owed to them. These legal actions could be through a commercial suit or through a possible renegotiation with the debtor.

In a study carried out by Moreno-García, García-Santillán, Bermúdez and Almeida (2015) they analyze and discuss that debt indicators go hand in hand with the degree in which the creditors of goods and services participate in the external financing of the company or people, depending who requests the loan or credit. However, we must also consider the inherent risk to which the parties are exposed (debtor and creditor), so the need to integrate a financial model of debt restructuring, the transaction cost, is questioned.

The Coase Theorem states that in the absence of transaction costs, the allocation of resources would be more effective regardless of how the property right is distributed (Peris-Ortis, 2018). It is also important to point out that companies face problems when they request a loan or credit that is greater than what they can actually afford to pay. That is, in some cases the profitability of the company is not enough to cover both the capital and the accrued interest.

Definitely over-indebtedness of debtors, in a strict sense, constitutes the level that exceeds a rational plan of income, even if the debtor is not insolvent; the commitments contracted as well interest at charge exceed

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their payment plan (Krausz, 2005). This coincides with Marin-Azabe and Araya-Vega (2008). They define it "as the inability to meet the payments for the financial commitments acquired, that is, the financial insolvency due to the excessive accumulation of debt".

It is important to note that, if the company or people (both debtors) observe a shortfall in their cash flows that affects the payment of debts owed to creditors, it is essential that companies consider approaching their creditors, before that the creditor attempts collection by judicial means. It is important to reach an agreement where debt restructuring can be carried out with new payment schemes suitable to the debtors' cash flow. Otherwise, the creditor will take legal action to collect the loan and / or credit granted as well as the interest stipulated in the contract.

In Mexico, there is the National Commission for the Protection and Defense of Financial Service Users (CONDUSEF), a surveillance body that carries out intermediation actions between the parties (debtor and creditor).

This commission has postulated the need for debt restructuring in all insolvency cases. This action has a purpose of modifying the established conditions of a bank loan for the benefit of the debtor when the debtor shows the institution his inability to comply with the agreed conditions, or when the debtor wants to take advantage of new financial market conditions that are more favorable (CONDUSEF, 2015).

With all the above, the question arises: What is the algorithm that allows valuing a debt restructuring that includes transaction costs? To answer this question, it is necessary to demonstrate through a mathematical modeling that includes the variable Transaction costs, the modification of:

$$V_{OD} \text{ to } V_{OD_{adjusted}} \text{ and } Y_{1...j} = \frac{V_{OD}}{V_{NSP}} \text{ to } Y''_{1...j} = \frac{V_{OD_{adjusted}}}{V_{NSP}}$$

LITERATURE REVIEW

Regarding debt restructuring or renegotiation, several studies explain how it could be done. The seminal paper by García-Santillán and Vega-Lebrúm (2008) demonstrates one of the first proposals. They propose an algorithm to evaluate the original debt; after that, identify a coefficient for the new scheme of payment and finally calculate the value of Y.

In the same idea Moreno-García, García-Santillán, Bermúdez and Almeida (2015) carry out a study where through a model of equivalent equations, they present a proposal with three scenarios to carry out a debt restructuring. This proposal is based on the assumption that the debtor has no financial solvency to fulfill its commitments. In the study they start from assumptions where some of the promissory notes have already expired and others have not yet expired. And the new proposals are presented in: equal payments, different payments, all in amounts and dates other than the original ones

Other works develop by García-Santillán, Howe and Venegas-Martínez (2016) shows how a company carried out a debt restructuring by financial insolvency to be presented at the trimesters to come. In their study, they propose a plan to renegotiate 17 payments because they will not be able to face since the cash flows will not be enough for it. With the proposal, the tension in the liquidity flow of the debtor was relaxed and the economic solvency improved satisfactorily.

Others proposals to debt restructuring have been suggested, for example Dedu, Lãzãrescu and Nitescu (2009) has suggested that the technique to debt restructure could be grouped in three categories: a).- Restructure a payment plan according to the debtor's economic possibility. B).- Cancel part of the debt, considering the cost of carrying out an embargo by the creditor, and c).- In the case of debtors, it would be convenient to exchange part of the debt for shares

In their study Dedu, Lãzãrescu and Nitescu (2009) refer the impact that economic crises have caused in the world, affecting the global and local economy, and in a very particular way, to the companies that have been affected in their economic solvency to face their commitments. In 2005, Manaligod (2005) analyzed the possible alternatives and implications of debt restructuring. Highlights the importance of reaching a debt restructuring agreement, for this, it suggests modifying the terms of the contract, the recovery of the credit becomes more secure and relaxes the tension of the debtors.

On the other hand, the restructuring of sovereign debt among countries has also been studied, highlighting some studies such as the works of Manaligod, (2005); Das, Papaioannou & Trebesch, (2012); Das, Papaioannou & Trebesch, (2014). Although these studies are related to debt renegotiation models, other macroeconomic

Table 1. Value of the original debt

Promissory notes	Expired and not expired	Year	Days to focal date	Interest rate	Amount (Thousand dls.)
1	*bfd*	2019	297	E_{ir}	$17.50
2	*bfd*	2019	190	E_{ir}	$20.00
3	*bfd*	2019	140	E_{ir}	$17.00
4	*bfd*	2019	111	E_{ir}	$18.00
5	*bfd*	2019	60	E_{ir}	$19.00
6	*fd*	2019	0	-	$15.00
7	*afd*	2020	21	R_{ir}	$11.50
8	*afd*	2020	87	R_{ir}	$12.00
9	*afd*	2020	121	R_{ir}	$14.00
10	*afd*	2020	161	R_{ir}	$22.00
11	*afd*	2020	221	R_{ir}	$19.50
12	*afd*	2020	290	R_{ir}	$17.00
13	*afd*	2020	321	R_{ir}	$15.00

variables in this type of renegotiation are used, which are not exactly those used in the quantitative technique proposed in this document.

About this topic, it is recommended to the companies take into account that the debt restructuring can affect some specific areas of business, such as: the production area, inventories, accountancy and finance, among others. The debtor must be aware of these effects of the debt restructuring on cash flows and those key indicators of liquidity and economic solvency that reflect the financial position of the business.

With the different proposals for debt restructuring through the equivalent equations model (García-Santillán & Vega-Lebrún 2008; García-Santillán, Escalera-Chávez & Venegas-Martínez, 2013; García-Santillán, Venegas-Martínez & Escalera -Chávez, 2014; Moreno-García, García-Santillán, Bermúdez & Almeida, 2015; García-Santillán, Rojas-Kramer, Venegas-Martínez & López-Morales, 2016; García-Santillán, Howe and Venegas –Martinez, 2016), the relevance of including an additional variable that protects the owner of the capital given on loan is questioned. The variable that we refer, is the transactions cost.

In theory, the debtor's lack of payment damages the financial capacity of the creditor, who´s the owner of the borrowed capital, considering that the creditor does not recover his capital in the terms previously agreed upon; therefore, it is necessary to include the transaction costs.

About transaction costs, in 1937 in his work "The nature of the firm" Ronald Coase talked about this type of costs called The Coase Theorem, which states that in the absence of these transaction costs, the allocation of resources would be more effective, regardless of the way in which property rights are distributed (Peris-Ortis, 2018). Namely, the theory of transaction costs has been a key element in economic theory. In essence, can be defined as the costs of transferring property rights, that is, the cost to establish and maintain ownership of a property. As in this case it would be the property of the borrowed capital.

Based on the foregoing, to develop the algorithm that integrates the transaction costs variable into the equivalent equations model, a hypothetical assumption is made in the following terms: A debtor seeks to debt restructure due to the absence of cash flows to meet its commitments. From the review of the projected cash flows for the years 2019 and 2020, one year before the proposed focal date for the renegotiation, the agreement is reached to restructure the promissory notes that have expired and also those that have not yet expired.

The agreement between debtor and creditor, stipulates that the promissory notes must be revalued at the focal date, for which the effective nominal rate of 13.5% capitalizable every 28 days is used for documents that have already expired. The same nominal interest rate in its real rate format will be used to discount the documents that have not yet expired. In addition, an inflation rate of 3.8% per year is considered.

In addition, 2.5% for transaction costs in favor to the creditor is stipulated. This penalty is for the effect on the creditor's assets, not recovering their capital as originally established. The resulting amount is prorated proportionally based on the revalued promissory notes in V_{OD}.

The promissory notes list that makes up debt restructure agreement is shown in **Table 1**.

Hypothetical scenarios for the new payment scheme in debt restructuring:

a) 12 equal payments (**Table 2**)

Table 2. Value of the new scheme of payment (scenario a)

Promissory notes	Expired and not expired	Year	Days to focal date	Interest rate	Amount (Thousand dls.)
1	*bfd*	*2019*	*15*	*Eir*	*?*
2	*fd*	*2019*	*0*	-	*?*
3	*afd*	*2019*	*35*	*Rir*	*?*
4	*afd*	*2019*	*60*	*Rir*	*?*
5	*afd*	*2019*	*120*	*Rir*	*?*
6	*afd*	*2020*	*180*	*Rir*	*?*
7	*afd*	*2020*	*240*	*Rir*	*?*
8	*afd*	*2020*	*270*	*Rir*	*?*
9	*afd*	*2020*	*300*	*Rir*	*?*
10	*afd*	*2020*	*330*	*Rir*	*?*
11	*afd*	*2021*	*390*	*Rir*	*?*
12	*afd*	*2021*	*450*	*Rir*	*?*

Table 3. Value of the new scheme of payment (scenario b)

Promissory notes	Expired and not expired	Year	Days to focal date	Interest rate	Amount (Thousand dls.)
1	*bfd*	2019	28	E_{ir}	$12.50
2	*bfd*	2019	15	E_{ir}	$10.00
3	*fd*	2019	0	E_{ir}	$9.00
4	*afd*	2019	35	E_{ir}	$8.00
5	*afd*	2019	60	E_{ir}	$7.00
6	*afd*	2019	120	-	$8.50
7	*afd*	2020	180	R_{ir}	$7.50
8	*afd*	2020	210	R_{ir}	$6.00
9	*afd*	2020	240	R_{ir}	$6.50
10	*afd*	2020	270	R_{ir}	$10.50
11	*afd*	2021	480	R_{ir}	¿?

Table 4. Value of the new scheme of payment (scenario c)

Promissory notes	Expired and not expired	Year	Days to focal date	Interest rate	Amount (Thousand dls.)
1	*afd*	*2019*	*50*	*Eir*	*$27.50*
2	*afd*	*2019*	*110*	*Eir*	*$27.50*
3	*afd*	*2019*	*160*	*Eir*	*$27.50*
4	*afd*	*2020*	*222*	*Rir*	*$27.50*
5	*afd*	*2020*	*270*	*Rir*	*$27.50*
6	*afd*	*2020*	*321*	*Rir*	*¿?*
7	*afd*	*2020*	*470*	*Rir*	*¿?*
8	*afd*	*2021*	*511*	*Rir*	*¿?*

b) 10 different payments and an amount at the end for the difference (**Table 3**)

c) 8 payments, 5 with similar amounts of $ 27.5 and the three remaining unknown with similar amounts (**Table 4**)

METHOD

To develop the financial modeling of debt restructuring, we carry out the following procedure:

a.- Become nominal interest rate in its Effective rate and in its Real interest rate, as follow:

$$E_{ir} = \left[1 + \left({}^{N_{ir}}/_{m}\right)^{t/m} - 1\right] * 100 \text{ and } \mathrm{R_{ir}} = \left[\frac{(\mathrm{Eir} - \mathrm{I_{nf}})}{(1 + \mathrm{I_{nf}})}\right] * 100$$

where: E_{ir} = effective interest rate, N_{ir} = nominal interest rate, R_{ir} = real interest rate, t = time, m = type of capitalization and I_{ir} = inflation rate.

b.- Restructuring debt with equivalent equations – Original formulation

Table 5. Notations formulas

fd	Focal date	*n*	Time ($\sum t/m$)
bfd	Before focal date	V_{OD}	Value of the original debt
afd	After focal date	V_{NSP}	Value of the new scheme of payment
$i_{d/m}$	Accurate interest rate (for discount) ($\sum i_d/365*m$)	$Y_{1\ldots j}$	Equal payment
$i_{indx/m}$	Accurate interest rate (indexed) ($\sum i_{indx}/365*m$)	E_{ir}	Effective interest rate
PN_1	Promissory note 1	R_{ir}	Real interest rate
m	Capitalization	I_{nf}	Inflation

According to García-Santillán et al. (2014), the algorithms that allow us revalue the original debt with compound interest is as follow:

1.- Revaluating the original debt, using the effective interest rate E_{ir} and the real interest rate R_{ir}:

$$V_{OD} = \sum_{1\ldots j}^{bfd} PN_{1\ldots j.bfd}\left[1+\left(\frac{E_{ir}}{t}*m\right)\right]^{t/m} + PN_{fd} + \sum_{1\ldots j}^{afd}\frac{PN_{1\ldots j.afd}}{\left[1+\left(\frac{R_{ir}}{t}*m\right)\right]^{t/m}} \quad (1)$$

Otherwise, if we use the nominal accurate interest rate $i_{indx/m}$ and the discount nominal accurate interest rate $i_{d/m}$, them the formula is modified in:

$$V_{OD} = \sum_{1\ldots j}^{bfd} PN_{1\ldots j.bfd}\left[1+\left(\frac{i_{indx}}{m}\right)\right]^{t/m} + PN_{fd} + \sum_{1\ldots j}^{afd}\frac{PN_{1\ldots j.afd}}{\left[1+\left(\frac{i_d}{m}\right)\right]^{t/m}} \quad (2)$$

2.1.- The next step: evaluate the new scheme of payment, according to:

$$V_{NSP} = \sum_{1\ldots j}^{bfd} X_{1\ldots j.bfd}\left[1+\left(\frac{E_{ir}}{t}*m\right)\right]^{t/m} + X_{fd} + \sum_{1\ldots j}^{afd}\frac{X_{1\ldots j}}{\left[1+\left(\frac{R_{ir}}{t}*m\right)\right]^{t/m}} \quad (3)$$

where X= 1, So we have:

$$V_{NSP} = \sum_{1\ldots j}^{bfd} 1_{1\ldots j.bfd}\left[1+\left(\frac{E_{ir}}{t}*m\right)\right]^{t/m} + 1_{fd} + \sum_{1\ldots j}^{afd}\frac{1_{1\ldots j}}{\left[1+\left(\frac{R_{ir}}{t}*m\right)\right]^{t/m}} \quad (4)$$

Once again, using the exact nominal interest rate (for indexing) and the exact interest rate (for the discount), in addition to considering all X of *1 ... j* with a value of 1 in all cases, then the formula becomes:

$$V_{NSP} = \sum_{1\ldots j}^{bfd} 1_{1\ldots j.bfd}\left[1_{1\ldots j}+\left(\frac{i_{indx}}{m}\right)\right]^{t/m} + 1_{fd} + \sum_{1\ldots j}^{afd}\frac{1_{1\ldots j}}{\left[1+\left(\frac{i_d}{m}\right)\right]^{t/m}} \quad (5)$$

3.- Finally, we must calculate the amount of each equivalent payment, from

$$Y_{1\ldots j} = \frac{V_{OD}}{V_{NSP}} \quad (6)$$

Notation used in the formulas previously described, is shown in **Table 5**.

Inclusion Cost Transactions

In this new proposal, we include in the equivalent equations model the variable "Transaction Costs", which is proportionally distributed in each promissory note restructured in V_{OD} (1), in accordance with the following procedure:

From the original formula (1):

$$V_{OD} = \sum_{1...j}^{bfd} PN_{1...j.bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + PN_{fd} + \sum_{1...j}^{afd} \frac{PN_{1...j.afd}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}} \quad (7)$$

The value of each of the revalued promissory notes is obtained depending on the position in the timeline, that is, the value of the revalued debt resulting from the valuation of each promissory note (before the focal date, on the focal date and after the focal date).

For the modality of restructuring, where the inclusion of the Transaction Cost is agreed as part of the requirements proposed by the creditor to his debtor, the mathematical model must be restructured based on the prorated function according to the amount of each restructured promissory note in relation to the total debt revalued.

First, we calculate the transaction cost coefficient for each of the promissory notes that make up the revalued debt from the following function:

$$TCC = \frac{V_{OD}}{PN_{1...j}}...$$

Likewise, the amount of the transaction cost that has been agreed is calculated according to:

$$TCA = V_{OD} * i_{tce}$$

where:

TCC= Transaction cost coefficient

TCA= Transaction cost amount

PN = Promissory notes (1 to *j*)

V_{OD}= Value of the original debt

i_{tce}= Estimated transaction cost rate (agreed in the renegotiation)

Now we obtain each promissory note revalued according to its position within the timeline, starting from the focal date that has been established for restructuring:

$$V_{OD} = \sum PN_{1....j\ .bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + PN_{x...fd} + \frac{\sum PN_{1...j.afd}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}} \quad (8)$$

With the formula (8) we obtain V_{OD} which is the sum of all revalued promissory notes. Now the formula is modified with the new variable of the model, the agreed rate of the weighted transaction cost, the coefficient of the transaction cost, as well as the transaction cost amount, all from the formula (9):

$$V_{OD_{adjusted}} = \sum_{1...j}^{bfd}\left[\left({PN"_{bfd}}_{1...j}\Big/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right] + \left[\left({PN"_{fd}}\Big/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right] + \ldots$$
$$\ldots + \sum_{1...j}^{afd}\left[\left({PN"_{afd1...j}}\Big/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right] \quad (9)$$

where:

$V_{OD_{adjusted}}$ = Value of the adjusted original debt

${PN"_{bfd}}_{1...j}$ = Promissory notes revaluated before focal date

${PN"_{fd}}$ = Promissory notes revaluated in the focal date

${PN"_{afd1...j}}$ = Promissory notes revaluated after focal date

i_{tce} = Estimated transaction cost rate (agreed in the renegotiation)

V_{OD} = Value of the original debt revaluated in (1)

Evaluate the new payment scheme in (3 and 3.1.) or (4) depending the agreed interest rate:

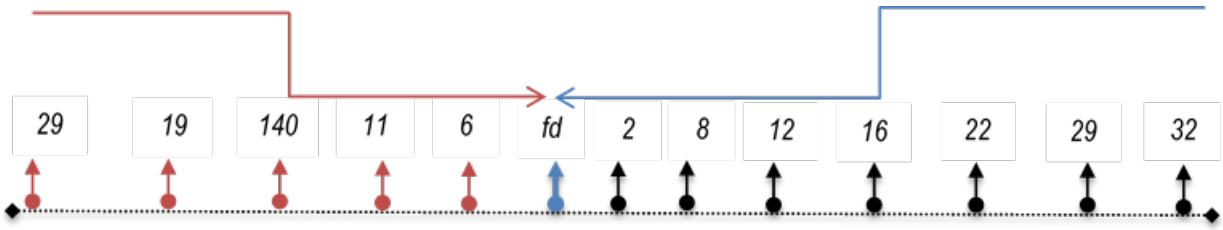
Finally calculate the amount of each equivalent payment, from:

$$Y^{''}_{1\ldots j} = \frac{V_{OD\_adjusted}}{V_{NSP}}$$

DEVELOP OF THE FINANCIAL MODELING

Scenario a).- 12 equal payments (**Table 2**)

An overview of all promissory notes in the timeline



First step: We obtain nominal interest rate according to:

$E_{ir} = \left[1 + (^{N_{ir}}/_{m})^{t/m} - 1\right] * 100 : E_{ir} = \left[1 + (\cdot^{135}/_{365} + 28)^{365/28} - 1\right] * 100$

$E_{ir} = [1 + (.01035616)^{13.0357143} - 1] * 100 : E_{ir} = [1.14374246 - 1] * 100$

$E_{ir} = 14.374246$

$R_{ir} = \left[\frac{(E_{ir} - I_{nf})}{(1 + I_{nf})}\right] * 100 : R_{ir} = \left[\frac{(.14374246 - 0.038)}{(1.038)}\right] * 100$

$R_{ir} = \left[\frac{(.10574246}{(1.038)}\right] * 100 : R_{ir} = [0.10187135] * 100 : R_{ir} = 10.187135$

Second step: We evaluate the original debt

$$V_{OD} = \sum_{1\ldots j}^{bfd} PN_{1\ldots j.bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + PN_{fd} + \sum_{1\ldots j}^{afd} \frac{PN_{1\ldots j.afd}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}} \tag{10}$$

$$V_{OD} = \sum_{1\ldots j}^{bfd} \$17.50_{bfd}\left[1 + .\left(\frac{.14374246}{365} * 28\right)\right]^{297/28} + \$20.00_{bfd}\left[1 + .\left(\frac{.14374246}{365} * 28\right)\right]^{190/28} + \ldots$$

$$\ldots + \$17.00_{bfd}\left[1 + .\left(\frac{.14374246}{365} * 28\right)\right]^{140/28} + \$18.00_{bfd}\left[1 + .\left(\frac{.14374246}{365} * 28\right)\right]^{111/28} + \ldots$$

$$\ldots + \$19.00_{bfd}\left[1 + .\left(\frac{.14374246}{365} * 28\right)\right]^{60/28} + \$15.00_{fd} + \frac{\$11.50_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{21/28}\right]} + \ldots$$

$$\ldots + \frac{\$12.00_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{87/28}\right]} + \frac{\$14.00_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{121/28}\right]} + \frac{\$22.00_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{161/28}\right]} + \ldots$$

$$\ldots + \frac{\$19.50_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{221/28}\right]} + \frac{\$17.00_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{290/28}\right]} + \frac{\$15.00_{afd}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)^{321/28}\right]}$$

$$V_{OD} = \sum_{1\ldots j}^{bfd} \$17.50\,[1.1233585] + \$20.00[1.0772541] + \$17.00[1.0563635] + \$18.00[1.0444331] + \ldots$$

$$\ldots + \$19.00[1.0237779] + \$15.00 + \frac{\$11.50}{[1.0058554]} + \frac{\$12.00}{[1.0244822]} + \frac{\$14.00}{[1.0342120]} + \frac{\$22.00}{[1.0457772]} + \ldots$$

$$\ldots + \frac{\$19.50}{[1.0633680]} + \frac{\$17.00}{[1.0839635]} + \frac{\$15.00}{[1.0933460]}$$

$$V_{OD} = \$19.66 + \$21.55 + \$17.96 + \$18.80 + \$19.45 + \$15.00 + \$11.43 + \$11.71 + \$13.54 + \$21.04 + \cdots$$

$$\ldots + \$18.34 + \$15.68 + \$13.72$$

$$V_{OD} = \$217.87$$

Now, we calculate V_{OD} with (9) integrating the variable "Transaction Cost weighted"; therefore, we obtain:

$$V_{OD_{adjusted}} = \sum_{1\ldots j}^{bfd}\left[\left({}^{PN''_{bfd_{1\ldots j}}}/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right] + \left[\left({}^{PN''_{fd}}/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right] + \ldots$$
$$\ldots + \sum_{1\ldots j}^{afd}\left[\left({}^{PN''_{afd1\ldots j}}/_{V_{OD}}\right) * (V_{OD} * i_{tce})\right]$$

$$V_{OD_{adjusted}} = \$19.66\left[\left({}^{\$19.66}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$21.55\left[\left({}^{\$21.55}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$17.96\left[\left({}^{\$17.96}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$18.80\left[\left({}^{\$18.80}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$19.45\left[\left({}^{\$19.45}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$15.00\left[\left({}^{\$15.00}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$11.43\left[\left({}^{\$11.43}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$11.71\left[\left({}^{\$11.71}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$13.54\left[\left({}^{\$13.54}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$21.04\left[\left({}^{\$21.04}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$18.34\left[\left({}^{\$18.34}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \$15.68\left[\left({}^{\$15.68}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right] + \ldots$$
$$\ldots + \$13.72\left[\left({}^{\$13.72}/_{\$217.87}\right) * (\$217.87V_{OD} * 0.025)\right].$$

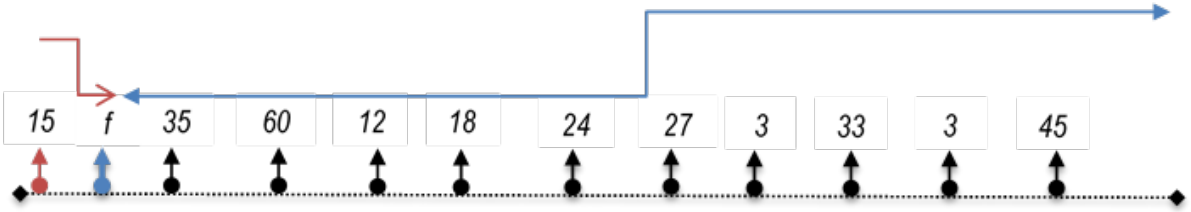
$$V_{OD_{adjusted}} = \$19.66[(0.090) * (\$5.45)] + \$21.55[(0.099) * (\$5.45)] + \$17.96[(0.082) * (\$5.45)] + \ldots$$
$$\ldots + \$18.80[(0.086) * (\$5.45)] + \$19.45[(0.089) * (\$5.45)] + \$15.00[(0.069) * (\$5.45)] + \ldots$$
$$\ldots + \$11.43[(0.052) * (\$5.45)] + \$11.71[(0.054) * (\$5.45)] + \$13.54[(0.062) * (\$5.45)] + \ldots$$
$$\ldots + \$21.04[(0.097) * (\$5.45)] + \$18.34[(0.084) * (\$5.45)] + \$15.68[(0.072) * (\$5.45)] + \ldots$$
$$\ldots + \$13.72[(0.063) * (\$5.45)].$$

$$V_{OD_{adjusted}} = \$19.66[(\$0.49)] + \$21.55[(\$0.54)] + \$17.96[(\$0.45)] + \$18.80[(\$0.47)] + \ldots$$
$$\ldots + \$19.45[(\$0.49)] + \$15.00[(\$0.38)] + \$11.43[(\$0.29)] + \$11.71[(\$0.29)] + \ldots$$
$$\ldots + \$13.54[(\$0.34)] + \$21.04[(\$0.53)] + \$18.34[(\$0.46)] + \$15.68[(\$0.39)] + \ldots$$
$$\ldots + \$13.72[(\$0.34)].$$

$$V_{OD_{adjusted}} = \$20.15 + \$22.08 + \$18.41 + \$19.27 + \$19.94 + \$15.38 + \$11.72 + \$12.01 + \ldots$$
$$\ldots + \$13.88 + \$21.56 + \$18.80 + \$16.08 + \$14.06$$
$$V_{OD_{adjusted}} = \$223.32$$

The next step is to calculate the V_{NSP} with (11)

The timeline for the new scheme of payment is:



$$V_{NSP} = \sum_{1\ldots j}^{bfd} 1_{1\ldots j.bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + 1_{fd} + \sum_{1\ldots j}^{afd}\frac{1_{1\ldots j}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}} \quad (11)$$

$$V_{NSP} = \sum_{1\ldots j}^{bfd}\left[1 + \left(\frac{.14374246}{365} * 28\right)\right]^{15/28} + 1_{fd} + \sum_{1\ldots j}^{afd}\frac{1_3}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{35/28}} + \ldots$$
$$\ldots + \frac{1_4}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{60/28}} + \frac{1_5}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{60/28}} + \frac{1_6}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{120/28}} + \ldots$$
$$\ldots + \frac{1_7}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{180/28}} + \frac{1_8}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{240/28}} + \frac{1_9}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{270/28}} + \ldots$$
$$\ldots + \frac{1_{10}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{300/28}} + \frac{1_{11}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{390/28}} + \frac{1_{12}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{450/28}}$$

Table 6. Payment for scenario a

PN number	Year	Days to focal date	Amount (Thousand dls.)	Total
1	2019	15_{bfd}	$19.64	$19.64
2	2019	0_{fd}	$19.64	$39.28
3	2019	35_{afd}	$19.64	$58.92
4	2019	60_{afd}	$19.64	$78.56
5	2019	120_{afd}	$19.64	$98.20
6	2020	180_{afd}	$19.64	$117.84
7	2020	240_{afd}	$19.64	$137.48
8	2020	270_{afd}	$19.64	$157.12
9	2020	300_{afd}	$19.64	$176.76
10	2020	330_{afd}	$19.64	$196.40
11	2021	390_{afd}	$19.64	$216.04
12	2021	450_{afd}	$19.64	$235.68

$$V_{NSP} = 1[1 + (0.01102682)]^{15/28} + 1 + \frac{1}{[1 + (0.00781479)]^{35/28}} + \frac{1_4}{[1 + (0.00781479)]^{60/28}} + \dots$$
$$\dots + \frac{1_5}{[1 + (0.00781479)]^{120/28}} + \frac{1_6}{[1 + (0.00781479)]^{180/28}} + \frac{1_7}{[1 + (0.00781479)]^{240/28}} + \dots$$
$$\dots + \frac{1_8}{[1 + (0.00781479)]^{270/28}} + \frac{1_9}{[1 + (0.00781479)]^{300/28}} + \frac{1_{10}}{[1 + (0.00781479)]^{330/28}} + \dots$$
$$\dots + \frac{1_{11}}{[1 + (0.00781479)]^{390/28}} + \frac{1_{12}}{[1 + (0.00781479)]^{450/28}}$$

$$V_{NSP} = 1_1[1.0058922] + 1_2 + \frac{1_3}{[1.00977801]} + \frac{1_4}{[1.01682079]} + \frac{1_5}{[1.03392451]} + \frac{1_6}{[1.05131594]} + \frac{1_7}{[1.06899990]} + \dots$$
$$\dots + \frac{1_8}{[1.07795312]} + \frac{1_9}{[1.08698132]} + \frac{1_{10}}{[1.09608514]} + \frac{1_{11}}{[1.11452215]} + \frac{1_{12}}{[1.13326929]}$$

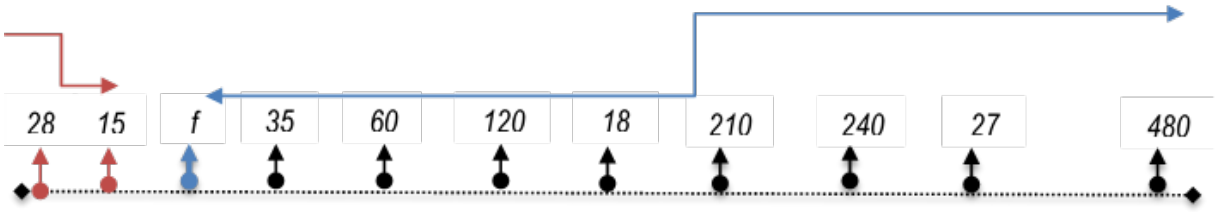
$$V_{NSP} = 1.0058922 + 1 + 0.9903167 + 0.9834575 + 0.9671886 + 0.9511889 + 0.9354538 + 0.9276841 + \dots$$
$$\dots + 0.9199790 + 0.9123379 + 0.8972455 + 0.8824028$$
$$V_{NSP} = 11.3731469$$

$$Y''_{1\dots j} = \frac{V_{OD\_adjusted}}{V_{NSP}} = \frac{\$223.32}{11.3731469} = \$19.64\_dls.\_per\_each\_payment$$

Scenario b).- 10 different payments and an amount at the end for the difference (**Table 6**)

Again, as a first step, we take the Value of the original debt adjusted $V_{OD_{adjusted}} = \$223.32$

The second step is to calculate the V_{NSP} according to the timeline:



From (11)

$$V_{NSP} = \sum_{1\dots j}^{bfd} 1_{1\dots j.bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + 1_{fd} + \sum_{1\dots j}^{afd} \frac{1_{1\dots j}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}}$$

Table 7. Payment for scenario b

PN number	Year	Days to focal date	Amount (Thousand dls.)	Total
1	2019	28_{bfd}	\$12.50	\$12.50
2	2019	15_{bfd}	\$10.00	\$22.50
3	2019	0_{fd}	\$9.00	\$31.50
4	2019	35_{afd}	\$8.00	\$39.50
5	2019	60	\$7.00	\$46.50
6	2019	120	\$8.50	\$55.00
7	2020	180	\$7.50	\$62.50
8	2020	210	\$6.00	\$68.50
9	2020	240	\$6.50	\$75.00
10	2020	270	\$10.50	\$85.50
11	2021	480	\$157.50	\$243.00

We have:

$$V_{NSP} = \$12.50_{bfd.1} + \$10.00_{bfd.2} + \$9.00_{fd.3} + \$8.00_{afd.4} + \$7.00_{afd.5} + \$8.50_{afd.6} + \ldots$$
$$\ldots + \$7.50_{afd.7} + \$6.00_{afd.8} + \$6.50_{afd.9} + \$10.50_{afd.10} + \frac{1_{11}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{480/28}}$$

Since we know the amount of each promissory note (1 to 10) in the new scheme of payments, then we calculate the coefficient of promissory notes number 11

$$V_{NSP_{(PN_{11})}} = \frac{1_{11}}{[1.00781489]^{17.1428571}} = \frac{1_{11}}{[1.14276075]} = 0.8750738$$

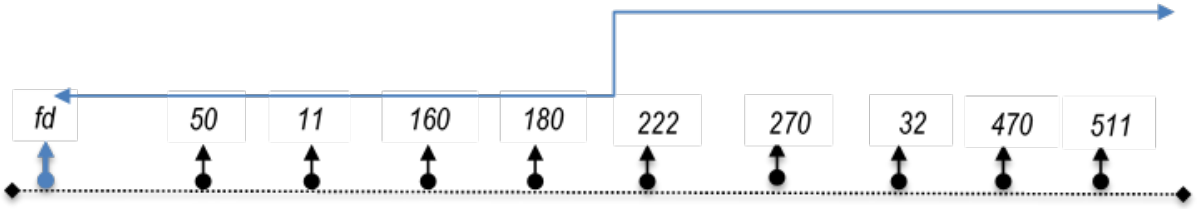
$$Y''_{11} = \frac{V_{OD\_adjusted} - [PN''_{1\ldots10}]}{V_{NSP}} = \frac{\$223.32 - \begin{bmatrix} \$12.50_{bfd.1} + \$10.00_{bfd.2} + \$9.00_{fd.3} + \ldots \\ \ldots + \$8.00_{afd.4} + \$7.00_{afd.5} + \ldots + \$8.50_{afd.6} + \ldots \\ \ldots + \$7.50_{afd.7} + \ldots + \$6.00_{afd.8} + \$6.50_{afd.9} + \ldots \\ \ldots + \$10.50_{afd.10} \end{bmatrix}}{0.8750738} =$$

$$Y''_{11} = \frac{\$223.32 - \$85.50}{0.8750738} = \frac{\$137.82}{0.8750738} = \$157.50\_dls.\_payment\_11$$

Scenario c).- Eight payments, five with similar amounts of \$27.50 and the three remaining unknown, with similar amounts (**Table 7**)

As a first step, also we take $V_{OD_{adjusted}} = \$223.32$

The second step is to calculate the V_{NSP} according to the timeline:



From (11) $V_{NSP} = \sum_{1\ldots j}^{bfd} 1_{1\ldots j.bfd}\left[1 + \left(\frac{E_{ir}}{t} * m\right)\right]^{t/m} + 1_{fd} + \sum_{1\ldots j}^{afd} \frac{1_{1\ldots j}}{\left[1+\left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}}$

We have:

$$V_{NSP} = \$27.50_{afd.1} + \$27.50_{afd.2} + \$27.50_{afd.3} + \$27.50_{afd.4} + \$27.50_{afd.5} + \ldots$$
$$\ldots + \sum_{6\ldots8}^{afd} \frac{1_{6\ldots8}}{\left[1 + \left(\frac{R_{ir}}{t} * m\right)\right]^{t/m}}$$

Table 8. Payment for scenario c

Promissory notes	Year	Days to focal date	Amount (Thousand dls.)	Total
1	2019	50_{afd}	$27.50	$27.50
2	2019	110_{afd}	$27.50	$55.00
3	2019	160_{afd}	$27.50	$82.50
4	2020	222_{afd}	$27.50	$110.00
5	2020	270_{afd}	$27.50	$137.50
6	2020	321_{afd}	$32.27	$169.77
7	2020	470_{afd}	$32.27	$202.04
8	2021	511_{afd}	$32.27	$234.31

Since we know the amount of each promissory note (1 to 5) in the new scheme of payments, we can calculate the coefficient of promissory notes number 6, 7 and 8.

$$V_{NSP} = \$27.50_{afd.1} + \$27.50_{afd.2} + \$27.50_{afd.3} + \$27.50_{afd.4} + \$27.50_{afd.5} + \ldots$$

$$\ldots + \sum_{6\ldots8}^{afd} \frac{1_{6.}}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{321/28}} + \frac{1_7}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{470/28}} + \ldots$$

$$\ldots + \frac{1_8}{\left[1 + \left(\frac{.10187135}{365} * 28\right)\right]^{511/28}}$$

$$V_{NSP} = \$27.50_{afd.1} + \$27.50_{afd.2} + \$27.50_{afd.3} + \$27.50_{afd.4} + \$27.50_{afd.5} + \ldots$$

$$\ldots + \sum_{6\ldots8}^{afd} \frac{1_{6.}}{[1.00781479]^{11.4642857}} + \frac{1_7}{[1.00781479]^{16.7857143}} + \frac{1_8}{[1.00781479]^{18.25}}$$

$$V_{NSP} = \$137.50 + \sum_{6\ldots8}^{afd} \frac{1_{6.}}{[1.00781479]^{11.4642857}} + \frac{1_7}{[1.00781479]^{16.7857143}} + \ldots$$

$$\ldots + \frac{1_8}{[1.00781479]^{18.25}}$$

$$V_{NSP} = \$137.50 + \frac{1_{6.}}{1.09334601} + \frac{1_7}{1.13958815} + \frac{1_8}{1.15265218}$$

We separate the unknown payment: 1_6, 1_7 and 1_8

$$V_{NSP\ (PN_{6,7,8})} = 0.9146235 + 0.8775100 + 0.8675644$$

$$V_{NSP(PN_{6,7,8})} = 2.6596979$$

$$Y''_{(PN_{6,7,8})} = \frac{V_{OD\_adjusted} - [PN''_{6\ldots8}]}{V_{NSP(PN6,7,8,)}} = \frac{\$223.32 - [\$137.50]}{2.6596979} =$$

$$Y''_{(PN_{6,7,8})} = \frac{\$85.82}{2.6596979} = \$32.27\_dls.\_payment\_6,\_7,\_and\_8$$

After calculating every hypothetical scenario, now in **Table 9**, we show the summary.

Table 9. Summary of each hypothetical scenario

Scenario a		**Scenario b**		**Scenario c**	
Original debt \$217.50; E_{ir} 14.3742460.% (m, c/28 days) and R_{ir} 10.187135.% (m, c/28 days)					
TCC	$V_{OD}/PN_{1\ldots j}$	TCC	$V_{OD}/PN_{1\ldots j}$	TCC	$V_{OD}/PN_{1\ldots j}$
TCA	$V_{OD} * i_{tce}$	TCA	$V_{OD} * i_{tce}$	TCA	$V_{OD} * i_{tce}$
V_{OD}	\$217.87	V_{OD}	\$217.87	V_{OD}	\$217.87
$V_{ODadjusted}$	\$223.32	$V_{ODadjusted}$	\$223.32	$V_{ODadjusted}$	\$223.32
$\sum Y"$	\$235.68	$\sum Y"$	\$243.00	$\sum Y"$	\$234.31
a).- *12 equal payments (table 2)*		*b).- 10 different payments and an amount at the end for the difference (table 3)*		*c).- 8 payments, 5 with similar amounts of \$ 27.5 and the three remaining unknown with similar amounts (table 4)*	

DISCUSSION

In relationship to the results of the hypothetical sceneries shown in **Table 9**, we discuss the following:

To develop the adjusted model of equivalent equations proposed in this document, we start from the total of the debt that we wish to restructure. In this case, the amount of debt is \$ 217.50 (**Table 1**). This amount to be revaluated in VOD for the three scenarios, an updated value of \$ 217.87 which is taken as reference for the three scenarios developed in this work is obtained. With these previous considerations and the results obtained, each scenario is now analyzed according to the following:

In scenario a) the result of VOD before including the transaction costs variable is \$217.87 (thousands of dollars) and adjusted VOD \$ 223.32 (thousands of dollars), with a spread of \$ 5.45 representing 2.5% of the costs of transaction in favor of the creditor as an incentive, considering that he does not recover his capital granted in loan or credit to the debtor, as previously agreed.

In the same line, the amount of \$235.68 (thousands of dollars) corresponding to 12 equal payments, presents an increase of 8.36% which represents approximately \$18.18 (thousands of dollars) in real terms with respect to the original debt that will be restructured (\$ 217.50). This difference includes, in addition to the transaction cost, those amounts derived from all the adjustments in the updated values of the promissory notes that have already expired and the discounted values of those promissory notes that have not yet expired, to later determine the new scheme through VNSP of payments (ΣY ").

In scenario b), the amount of \$243.00 (thousands of dollars) corresponding to the 10 payments proposed by the debtor. Here there is an increase of 11.72% which represents approximately \$25.50 (thousands of dollars) in real terms with respect to the original debt (\$ 217.50). Likewise, this difference includes transaction costs and the amounts derived from all the adjustments of the promissory notes overdue, as well as the promissory notes not yet overdue. The above to determine through the VNSP equation, the new payment scheme (ΣY").

The last scenario c), an amount of \$234.31 is obtained, corresponding to the eight payments agreed upon in the restructuring. This represents an increase of 7.73% equivalent to \$16.81 (thousands of dollars) in real terms with respect to the original value of the debt to be restructured (\$ 217.50). As in the previous cases, this differential integrates the transaction cost and the amounts derived from the adjustments made in VOD of all the promissory notes that are restructured in the new payment scheme (ΣY").

FINAL REMARKS

From the results of the hypothetical scenarios proposed for financial modeling, we have the following reflection:

As described at the beginning of this document, debtors face serious problems with their creditors when they cannot meet their commitments to pay the debt. Companies (be they small or medium), often go through stressful times due to the loss of liquidity in their cash flows. Even large companies are not exempt from this phenomenon. To address these events, it has been suggested in several studies that one of the best financial options to be taken into account is renegotiation of the debt in a new payment scheme, modifying the payment dates and the amounts of each of the promissory notes.

In previous studies development by García-Santillán and Vega-Lebrúm (2008), García-Santillán, Venegas-Martínez and Escalera-Chávez (2014), had suggested the importance of identifying a common factor between debt valued at a given focal date and the new proposal of payments. This algebraic operation allows us to know the value of Y and Y".

We should emphasize that the proposal of the equivalent equation model has the purpose of seeking balance between the parties. The debtor seeks a deferral over time, allowing a better management of his cash flow, improving the management of his working capital and generating better solvency and liquidity indicators, in order to pay his commitments. On the other hand, considering that the creditor will not be able to recover his capital, then, the creditor would request an additional benefit, being in this case the moratorium interest accrued and transaction cost in his favor.

Therefore, both parties will benefit: the debtor postpones the payment date according to the dates on which he estimates he will have the cash available to pay and, on the other hand, the creditor will receive more money in compensation for waiting.

In the words of García-Santillán, Howe and Venegas-Martínez (2016), "this proposal is intended to offer a fast solution for those debtors who, facing insufficient financial resources, might consider making a debt restructuring with its creditors to pay their debts". [sic]

Finally, we can say that this model is not the only option to renegotiate debt; it is at least a scheme which seeks to set up a mutual balance between the needs of both debtors and creditors.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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REFERENCES

Das, U. S., Papaioannou, M. G., & Trebesch, C. (2012). Sovereign Debt Restructurings 1950-2010: Concepts, Literature Survey, and Stylized Facts. IMF Working Paper 12/203. International Monetary Fund.

Das, U. S., Papaioannou, M. G., & Trebesch, C. (2014). Restructuring Sovereign Debt: Lessons from Recent History. En S. Claessens, A. Kose, L. Leaven & F. Valencia (2014). Financial Crises, Consequences and Policy Responses (pp. 593-620). Washington, D.C.: IMF International Monetary Fund.

Dedu, V., Lăzărescu, S. A., & Nitescu, D. C. (2009). Banking Restructuring Techniques in the Economicañ Crisis. Theoretical and Applied Economics, Asociatia Generala a Economistilor din Romania. *AGER, 11*(540), 27-32.

García-Santillán, A., & Vega-Lebrún, C.A. (2008). Reestructuración de la deuda mediante un factor común y la modelización con ecuaciones equivalentes. Contribuciones a la Economía. Retrieved from https://goo.gl/ZWdR9x

García-Santillán, A, Howe Andrew J., & Venegas-Martínez F. (2016). Short- and Long-term Debt Restructuring through Equivalent Equations: The Case of a Company in the Services Sector. *Istanbul University Journal of the School of Business, 46*(1), 133-142.

García-Santillán, A. Venegas-Martínez, F., & Escalera-Chávez, M. (2014). Modeling Restructuring Debt with Equivalent Equations: Theoretical and practical implications. *American Review of Mathematics and Statistics, 2*(2), 91-106. https://doi.org/10.15640/arms.v2n2a5

Moreno-Garcia, E., García-Santillán, A., Bermúdez, A., & Almeida, P.C. (2015). Restructuring debt proposal in three hypothetical scenarios: Equal payments, different amounts and one unknown payment, different amounts and three unknown payment. *Journal of Progressive Research in Mathematics, 4*(1), 233-246.

Manaligod, M. G. (2005). Debt restructuring: alternatives and implications. *Notes of Business Education, 8*(1), 1-6.

Peris-Ortiz, M (2018). Teoría de costes de transacción. In *Diccionario empresarial Wolter Skluwer*. Retrieved from https://goo.gl/C3VFdC