A PRACTICAL APPROACH TO PROBABILITY
IN THE CONTEXT OF A SCIENCE FAIR

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ABSTRACT. In a society that generates information rapidly, schools have to fulfil their programmes imaginatively. Thus, extra-curricular activities may be helpful for the students to acquire wider knowledge than that they may get within the classrooms. On the other hand, randomness is present in almost all everyday decisions, mainly based on prior information so it is important to have at least a rough idea on how specific events may affect the chances of other events. We explore both ideas here in the context of a science fair, in which two high-school senior students conducted an investigation about conditional probability using a game called "Shut the box". We also want to pose, as a research question, if, after their participation in the science fair, these students have reached higher levels in probabilistic reasoning compared to their classmates or have acquired knowledge about concepts far beyond the official curriculum.

KEYWORDS. Science Fair, Conditional Probability, Bayes' theorem, extra-curricular work, project-based work, personal development.

1. INTRODUCTION

Although the classroom is the primary place where students acquire new knowledge, learning is not confined to it. Actually, learning may be traced in many situations in everyday life. Moreover, there are good arguments to take these other occasions of learning seriously into account. Moura (1995) states that foundations for that movement comes from the objective of "educating the citizen", be it within schools or within other contexts, looking to increase cultural awareness and aiming to prepare people for a more conscious and more participative life in a world, which is more and more influenced by science and technology.

Even for younger citizens, it is not possible to restrain education solely to formal schooling. Certain strands of didactic discussion also focus on the idea that students should acquire their knowledge within their daily life context, which includes practical applications as well. Unfortunately, restrictions in educational programmes do not allow students to transfer the acquired knowledge to the desirable levels of contextualization and application: in this case, extra-curricular activities may play an important role in students' education.

One of these possible extra-curricular activities is a science fair, that according to Brasil (2006, 20), "is shown to be an opening for the curiosity and interest of the student, the creativity and mobilization of the teacher, and for the academic and social life of the school". Mills (2002) states that "researchers in education and psychology support the theory that students learn by actively building or constructing their own knowledge and making sense out of this knowledge", and that individuals may construct new knowledge internally by transforming, organizing, and reorganizing previous knowledge as well as externally, through environmental and social factors that are influenced by culture, language, and interactions with others (see Cobb 1994, Greeno, et al 1996, or, Bruning, et al 1999).

On the other hand, it is well known that randomness is found in almost every daily activity and consistently has a great impact on our life. Hence, the teaching of probability and the development of scientific research in this area are of major importance. Furthermore, both teaching of and research in probability (and empirical research in a wide variety of disciplines) will eventually require contextualization and application.

In this paper, these ideas are illustrated by analyzing an extra-curricular activity developed by two high school senior students, which was presented at a science fair. The activity involved research about some probabilistic concepts by adapting a game named "Shut the box", which aimed to show the importance of measuring randomness, on the basis of prior information,

in order to make a decision in a game, which could well be part of our everyday life's experience. A simulation of the original game (with just two dice) may be retrieved at Barendregt (n. d.).

2. SCIENCE FAIR: STUDENTS' EDUCATIONAL PROCESS

As a part of a school's innovative development, the inclusion of extra-curricular activities are intended to support an integral development of the students at social, affective, cognitive, and psychological levels. According to Freire (2004), by interacting with our students and stimulating them to use their knowledge outside the classroom, we may expect them to realize that the one who teaches learns by teaching, and that one who learns teaches by learning.

Among these extra-curricular activities are science fairs. A science fair is, according to Ormastroni (1990, p. 7), "a public exposition of scientific and cultural work developed by students. The students give demonstrations, provide oral explanations, and answer questions about the methods and their conditions. There is an exchange of knowledge and information between students and the visiting public".

The social aspects within scientific education that a science fair may offer to participating students is noted in the definition given in the document written by the Science Teachers in the Training Centre of Rio Grande do Sul, CECIRS (1970, p. 2): "It is a cultural activity carried out by students, shown through the planned demonstrations and presentation of their work, their knowledge and understanding in a technical-scientific area. This creates cultural development as well as a better cohesion between the school and the local community." It is important not only that teachers encourage their students to participate in such a science fair, but also evaluate their performance at the fair afterwards.

The importance of students' own reconstruction of the concepts in the learning process seems indisputable. However, this does not guarantee that following such an active role for learner's results in complete and immediate learning; in this project, the students faced a lot of difficulties, especially in the calculations for the conditional probability tables, or the idea of the use of prior information, and the development of Bayes' theorem.

Such situations of learning difficulties establish a most delicate issue for the teacher; the students have the possibility of asking the teacher, but the development of the projects and their solutions must be done by them, so the teacher just may help by asking questions like "Why not trying this?" or "Have you checked those values in your table? Look carefully for the chances on

the dice". The teacher's role diverts to being a counsellor, he no longer is the prime and sole source for knowledge.

In this paper we are dealing particularly with the content of a project developed for the Science Fair of the Science Museum Universum. For the educational process of the participating students, the reader may find more details in the Annex where also the guidelines of this science fair are summarised up in English.

3. BAYES' THEOREM AT THE SCIENCE FAIR: THEORETICAL ASPECTS

During the science fair, the students presented the formulas and concepts to the public in three stages: *prior* knowledge, conditional probability, and Bayes' theorem.

- First Stage: Prior Knowledge – The students discussed with the audience how subjective knowledge about everyday situations might influence the private judgement of the probability of specific events (accidents, being late, etc) and the decisions as a consequence of the changed situation (make a detour, starting earlier, etc).

- Second Stage: Conditional Probability – The students used two-way tables to explain the concepts of dependent and independent events, and the concept of conditional probability to the audience.

- Third Stage: Bayes' formula – The students tried to explain the formal procedure to revise probabilities in the light of new events or information. They imbedded this discussion in playing the board game "Shut the box" and used the results of the previous stages to discuss the relative merits of various strategies of the game. The audience could learn from playing the game that strategies based on probabilistic concepts are better than their intuitive approach.

In the Annex, some of the ideas expressed by the students during their presentation for first and second stages are shown. Here, we will focus on the game and Bayes' formula.

In every stage the students used the following method: initially they presented some situations and/or questions, with the idea of allowing the public to develop some probabilistic reasoning, and then they explained their formal ideas underlying the situation. Since the

regulations of the science fair asked for a theoretical frame, part of the students' work at the fair was expected to show the formal, theoretical part of their research.

Figure 1. Student explaining the project to an audience member – Students at their stand

It was discussed with the public, in an informal manner, that Bayes' theorem is valid in many applications of the theory of probabilities. Nevertheless, there is a controversy about the nature of probabilities that the theorem uses. Essentially, adherents of the frequentist probability, admit probabilities only if they are based on experiments, which can be repeated and may thus be estimated by empirical relative frequencies, while those named as Bayesian statisticians also allow the use of qualitative information about the tendency of occurrence and non-occurrence of events to derive subjective probabilities.

The theorem is particularly useful to indicate how we should modify our subjective probabilities when we receive additional information for an experiment, although it also can be used when no "prior" information is available. Bayesian statistics proves useful for estimating unknown probabilities based on subjective prior knowledge; this approach enables ways to construct knowledge where the traditional approach fails – however, at the cost of using subjective information as the frequentists would object.

By explaining all this to the audience, the students' purpose was to establish the link between the ideas brought out in the first stage about prior knowledge and a new formula, Bayes' theorem.

The next step for the students was to present the whole development of the formula based on the previous discussion of the concepts. After that conceptual presentation, the students began to explore the game with the public, making comparisons between theory and practice, thereby completing the methodological cycle of the project.

4. THE GAME

The device used by the students was the game "Shut the box", with the rules about the use of dice changed to allow a wider probabilistic analysis. The device is a box containing a series of small boards (or slats), labelled from one to nine, which lie on an axis inside a box, in such a way that each individual board may work just as the box's cover does; this means, each board can be moved up and down individually, so they may lay inside the box in a vertical or horizontal position. A video showing the game with its original rules may be found at eHow.com (n. d.).

Figure 2. The game Shut the Box – used as "Bayesian Box".

The game begins with the boards up, or "opened", and the goal is "closing" them all, or putting them horizontally. In order to do this, two six-sided dice must be thrown. Which boards can be closed is decided either by the numbers shown by the single die, or by the sum of the two numbers on each die; it is up to the player to decide that. This is somehow different from the rules

traditionally used mainly in Europe, since those rules state that the player has to sum the points on the dice, and then "close" any combination of boards leading to that very sum. These traditional rules may be found at Masters Traditional Games (n. d.). Also, a little history about Shut the box may be found at The Online Guide to Traditional Games (n. d.).

Figure 3. Various dice used in the game.

In the students' project, extra dice are also allowed: the player can choose from a certain combination of dice, before starting playing; he could opt for two six-faced dice, a four-faced die and an eight-faced die, a two-faced die and a ten-faced die, or a single twelve-sided die and then throw the dice or die repeatedly. The rules of the original game were extended for this additional option of the choice of dice to allow a wider probabilistic analysis of potential strategies.

The aim is to "close" all the numbered boards of the box, and there are two ways for "closing" a board:

- first is adding the points shown on the dice and "closing" the board with the number of that sum;

- second is just taking notice of the single numbers on the die's faces and "closing" both boards with these single numbers, if possible.

At each step of the game, the dice have to be thrown and one (or two) boards have to be closed. The game ends successfully when all the boards are closed. As long as at least one of the boards can be "closed", the game continues with tossing the dice, but if at some point it is not possible to "close" neither that board fitting the sum nor boards with the one of the two single numbers, the player fails.

Some specific examples might enhance understanding the consequences of the rules of the game:

- If player gets 6 and 3 he has two options; firstly he could close the board labelled as 9 if there still is the board labelled by 9 "open", as the sum of the dice yields 9; or secondly, he could close both boards labelled 6 and 3 (the single numbers of the dice shown) if both boards are still "open". If 9 and 6 are already "closed", the player can "close" board 3. However, if 9 and 6 and 3 are already "closed", while still some boards remain "open", the player loses.

- If a combination of equal numbers occurs, like three and three, allows the player to "close" either the sum of the two dice, or the board with that repeated number as well; in this example, the player would close either board six or board three.

These rules give the player a problem of decision-making right from the start, of having to decide the combination of dice to play with. The player should formulate a strategy based on the convenience of "closing" the boards by sums or by the two single values observed, as long as this alternative "may be applied". An example of outcome in which the player does not have the chance to decide would be a six and four result, since there is no board labelled 10 and the player must "close" the four or the six board (or both). The same happens on a toss in which the sum presents a number on a board already "closed", or both numbers on dice are "closed" but the sum "open".

An analysis of the probabilities of getting a result between two and nine inclusive, by sums, and the probabilities of getting a certain combination of values on the dice, will allow the player to plan, or at least attempt a strategy.

As stated, a further variant in the game was included in this research. By playing with two six-faced dice, a total of twelve faces may be counted. Under that idea, before starting the game, the player is given the option of playing either with two six-faced dice, or with a die with eight faces and another one with four faces, or with one ten-faced and one two-faced die, or, the last option, with a single twelve-faced die.

All these options confront the player with the additional decision to choose a combination of dice first, with the target to increase the chances of winning. Also, depending on the selection of the dice, the actual strategy for the game could be different: the player has to decide on the combination of dice and then on how to play with them, by points or by sums in each throw, even

when the player may switch this last strategy related to sums or points from toss to toss depending on the boards closed already.

In order to help the public with the reasoning, some tables were presented: these tables included probabilities for the outcomes of the different combinations of the dice, as well as some probability distributions. Tables 1–4 are presented here as an example for the two six-faced dice.

So, the probability of having a particular number $x \in [1, 6]$, like 4 on one or both dice is

$P(X = 4) = 11/36 = 0.306.$

This is there are 11 results in which number four appears at least once.

And the probabilities of getting any number from two up to 12 by sums are:

$P(X = 2) = 1/36 = 0.028, \quad P(X=3) = 2/36 = 0.056$ etc.; see Table 3 for the results.

All these calculations were made with the idea that number one can be obtained just as a point on one die, and numbers seven, eight and nine only can be obtained by sums.

With the distributions constructed, the students presented the calculations for the probabilities of each result in tossing the dice, directly or by sums, as well as the calculations for the conditional probabilities of the sums given that the value on one of the dice is known, with the aim of observing the viability of the application of Bayes' theorem. Table 3 shows the probabilities for the sums, while Table 4 shows the conditional probabilities of each sum.

Using the formula for conditional probability, the probability of having a certain sum given a certain number on one of the dice is 2/11 or 1/11.

As an example:

$$P(\text{Sum} = 3 \text{ GIVEN one die shows } 2) = \frac{P(\text{one die shows 2 AND the other die shows 1})}{P(\text{one die shows 2})}$$

$$= \frac{2/36}{11/36} = 2/11 = 0.182$$

$$P(\text{Sum} = 4 \text{ GIVEN one die shows } 2) = \frac{P(\text{one die shows 2 AND the other die shows 2})}{P(\text{one die shows 2})}$$

$$= \frac{1/36}{11/36} = 1/11 = 0.091$$

Face	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 1. Probability distributions for each face in a six-faced die.

	1	2	3	4	5	6
1	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
2	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
3	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
5	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
6	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

Table 2. Probabilities for the outcomes with two six-faced dice.

Sum X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Table 3. Probability of different sums of two six-faced dice.

Sum	Given value of "second" die					
	1	2	3	4	5	6
2	$\frac{1}{11}$	—	—	—	—	—
3	$\frac{2}{11}$	$\frac{2}{11}$	—	—	—	—
4	$\frac{2}{11}$	$\frac{1}{11}$	$\frac{2}{11}$	—	—	—
5	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	—	—
6	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	—
7	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$
8	—	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{2}{11}$
9	—	—	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{2}{11}$
10	—	—	—	$\frac{2}{11}$	$\frac{1}{11}$	$\frac{2}{11}$
11	—	—	—	—	$\frac{2}{11}$	$\frac{2}{11}$
12	—	—	—	—	—	$\frac{1}{11}$
Total	1	1	1	1	1	1

Table 4. Conditional probabilities for the sums (with given value on second die).

This analysis using tables and formulas were developed by the students for the combination of ten and two-sided dice, for the eight- and four-sided dice, and partly for the twelve-sided die also. Comparing the tables and by experimenting with the game, they concluded that the combination of four and eight sided dice would be the best choice.

It was pointed out that ideally they should play a series of games with different combinations of dice, with the aim of checking whether the best strategy gained from experience was the same as the best derived from the calculation of probabilities. At the beginning it was confusing for the public to understand the idea of the activity, since the initial explanation given to them is quite complicated. As they started with the game, the most common errors were:

- If the player lost two or three games at the very beginning playing with the box, he or she concluded that the game was impossible to win.

- The player assumed that any combinations of dice, excluding the single twelve-faced die, offers the same chances of winning.

- At the beginning the player did not find a difference between closing the boards by points or by sums in terms of probabilities.

- Only after a few throws did they notice that this element could make the difference between winning and losing the game.

- At that point, the players started to appreciate the idea of closing just one board (that of the sum) or two as a part of a strategy.

After direct experience with the game, and once again with an explanation from the students, it was clearer to the public how to make use of conditional probabilities, and that probabilities play a decisive role for making decisions, and that prior information is important to improve one's decisions; in this case, how the different combinations of dice, the sums and points, may behave.

5. RESULTS, ANALYSIS AND INTERPRETATION

With the analysis of the tables, it was possible for the students and the public to draw certain conclusions through discussion. Some examples of this are:

1. In the "Total" line of each of the tables of conditional probabilities for the sums, it may be noticed that

    $$\sum_{i=1}^{k} P(A_i|B) = 1.$$

    This is something that students actually told the audience directly.

2. If we observe the tables for the combinations of each possibility of the pair of dice, and we calculate the probability of the possible sums, particularly those in which we are interested for the game, we find that there is no coincidence since they were calculated as unconditional results. But, as we have seen, the sum can be linked to the result observed in one of the dice; the total of the tables and calculations may be found in the student's project, which may be retrieved from the internet at Vargas & Muciño (2007, p. 8–14).

    This may be taken into consideration as a part of the game strategy by comparing the values. It can be observed that the values are different for the different pairs of the dice, with a maximum probability for certain combinations. Anyway, that does not mean that calculating the chances for each pair of dices gives the best strategy, since the decision about discarding the boards by sums or by points is also part of a strategy that could increase the chance of winning. This was also pointed out by the students.

3. For the combination of a ten-faced die with a two-faced one, values from two to nine may be reached both by single points and by sums. In the case of combining an eight-faced die with a four-faced one, just the values from two to eight have that property. And in the combination of two six-faced dice, only the numbers from two to six allow for these dual possibilities. The students informally tried to guide the audience to find this out while playing.

4. We can also observe in the combination of a ten-faced die with a two-faced one that certain sums have high probabilities, due to the options on the dice, but there are fewer alternatives.

    In the combination of two six-faced dice, matters are reversed: there are very many ways of getting different sums from a conditional event, but that reduces the probabilities for each one of these sums. And in the combination of an eight-faced die with a four-faced one it seems to be a certain balance between both former cases.

This leads us to assume that the best strategy is playing with this pair of dice, and by closing first the boards by values obtained by sums, particularly for the values of five and higher. The students just presented the tables to the audience, but did not point this out explicitly.

5. The probabilistic analysis was carried out just for the first toss of the dice. The students realized that developing an analysis for the second toss would be a very complicated task. They would have to take into consideration that the combination of dice would be no longer a decision to make, since it was taken already before the first toss, but that anyway it would affect the strategy as well since the player would be playing with that combination.

The decision of closing the boards by sums or by points would be made according to the results and/or the decisions taken after the first toss. In this way, conditional probability is not sufficient for developing an analysis similar to that they made, so Bayes' theorem would have to be applied, but only after a new and deeper analysis. The students realised that further analysis would be quite complicated.

Sadly, more accurate information about audience reactions is not possible, since as a part of the Fair's rules, the teacher was not allowed to stay on the first day of the event and his presence had to be very limited along the second day.

6. FINAL CONSIDERATIONS

It has been stated out that instruction completely restricted to the classroom may limit the acquisition of wider knowledge. It is possible that both participants in the science fair could learn and understand more advanced concepts in statistics and probability compared to their classmates by being at the event, which indicates a certain benefit of encouraging students to participate in extra-curricular activities.

A key feature of students' work in such a project is that they have not just to study and research about the subject, but to explain it to an audience. This could be seen not just because of the way they tackled the problems in the project or by the way they presented it to the researchers and to the general public at the fair, but also because they were able to develop the basic ideas of Bayes' theorem, which is beyond the scope of the syllabus included in the institutional program of the school these students attend. This means that they were not just able to learn about Bayes'

theorem in the course of their research, but they were also able to explain it to an audience that had no previous knowledge about it.

It is important to point out here that they were directed by their teacher to research about Bayes as part of the investigation, but they had to study the ideas underlying the theorem on their own. It is then a distinct possibility that their learning was supported not just by participating at the science fair, but by reading, researching, and working with the teacher.

We have stated as a research question if, after their participation in the science fair, these students have reached higher levels in probabilistic reasoning compared to their classmates, and have acquired knowledge about concepts far beyond the official curriculum. We believe that participating at the science fair at least gave the chance to the students of working with probability in a different way and at different level than their classmates, mainly because they had not just to research about a subject, but to explain it to an audience. And, by participating in the project, they got the opportunity of learning about a subject out of the programme; clearly, this could be done just by studying it and being at the science fair is not necessarily the only way of achieving this, but it was an extra consequence.

Moreover, the activity gave the opportunity to the public to think about how randomness affects their lives and how this concept could improve their decision processes. The positive feedback on the activity from the audience and the public may encourage future groups to participate in fairs to come with projects in probability. Such science fairs allow students to take a complete new role as active agents of their own learning processes, and to develop a capacity of critical analysis and investigative skills which qualifies them better not only for their later profession but also as emancipated citizens.

Finally, the teacher must be conscious of the fact that encouraging students in these activities demands good skills in the subject matter, here from probability and extra work with a different approach to the students, since he will be working closer with them, helping them in a project in a more direct way, but at the same time just guiding them without solving difficulties or participating along the research beyond giving orientations.

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ANNEX

For the annex, follow this link.

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Figure 4. Poster used by the students at the science fair.